1. Weighted Giuded Filter

The original guided filter [1] uses a box window in the filter design. However, this causes a rotationally asymmetric response and does not reflect the spatial relationship. So, this can be remedied by adopting a Gaussian weighted window instead. In this case, the problem becomes to find a_k , and b_k that minimizes the following cost function in the window ω_k :

$$E(a_k,b_k) = \sum_{i \in \omega_k} \exp\left(-\left\|\mathbf{x}_i - \mathbf{x}_k\right\|^2 / \sigma_g^2\right) \left(\left(a_k I_i + b_k - p_i\right)^2 + \varepsilon a_k^2\right),$$

where p,q, and ε are the input image, output image, and regularization parameter, respectively, and \mathbf{x} denotes a pixel position. The output can be obtained by

$$q_i = \overline{a}_i I_i + \overline{b}_i$$
, where $\overline{a}_i = \frac{1}{|\omega|} \sum_{k \in \omega_k} a_k$ and $\overline{b}_i = \frac{1}{|\omega|} \sum_{k \in \omega_k} b_k$.

Following is the Algorithm for the Guided filter.

Algorithm Guided Filter.

Input: filtering input image p, guidance image l, radius r, regularization ε

Output: filtering output *q*.

1: $mean_I = f_mean(I)$

 $mean_p = f_mean(P)$

 $corr_l = f_mean(I.* I)$ $corr_l p = f_mean(I.* p)$

2: var_I = corr_I - mean_I.* mean_I

cov_lp = corr_lp - mean_l.* mean_p

3: $a = cov_lp./(var_l + \varepsilon)$

 $b = \text{mean}_p - \text{a.* mean} I$

4: $mean_a = f_mean(a)$

 $mean_b = f_mean(b)$

5: $q = \text{mean}_a$.* $I + \text{mean}_b$

For applying a Gaussian window, you need to use a 2D Gaussian filter for f_mean () in the algorithm.

$$G(\mathbf{x}) = \frac{1}{2\pi\sigma^2} \exp\left(-\left\|\mathbf{x}\right\|^2 / \sigma^2\right)$$

- Write your program for the Gaussian weighted guided filtering algorithm.
- Given test noisy color images (afghan_noise#.png), convert them into grey images using I = (R + G + B)/3.
- Clean the grey images by your algorithm by setting I = p. How about setting I as one of R, G and B?
- Modify your algorithm for multichannel input and test it on the color images.
- Show the noisy images and filtered images with different parameter settings.
- Provide the best result, its PSNR value and the running time.

$$PSNR = 20 \cdot \log_{10} \left(\frac{MAX_I}{\sqrt{MSE}} \right)$$
, where MAX_I is the maximum possible pixel value of the image,

and
$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [q(i,j) - \overline{q}(i,j)]^2$$
, \overline{q} : original clean image, q : denoised image

• [Extra Credit] Do you have any idea to improve the performance or enhance the running time?

2. Weighted Median Filter

The original median filter is to replace a pixel value by the median value of its neighboring pixels. The median can be computed from a histogram

$$h(\mathbf{x},i) = \sum_{\mathbf{x}' \in N(\mathbf{x})} \delta(p(\mathbf{x}') - i),$$

where $N(\mathbf{x})$ is a local window near \mathbf{x} , p is the pixel value, i is the discrete bin index, and $\delta(\cdot)$ is the Kronecker delta function.

The weighted median filter is a modification of it in which the pixels are weighted in the local histograms such that

$$h(\mathbf{x},i) = \sum_{\mathbf{x}' \in N(\mathbf{x})} W(\mathbf{x},\mathbf{x}') \delta(p(\mathbf{x}') - i).$$

The weigh function $W(\mathbf{x}, \mathbf{x}')$ can be the box filter, bilateral filter, or guided filter.

- Now, implement the weighted median filters with different weight functions of
 - i) Box filter
 - ii) Gaussian filter
 - iii) Bilateral filter [2]
- Test your algorithm on the noisy test (grey) images (monkey_noise#.png) with varying sizes of filters $((r+1) \times (r+1), r=2,4,8)$, with appropriate parameters for each weight filter.
- Show and compare your results qualitatively and quantitatively.
- Report your best results with the PSNR, and the running time.
- [Extra Credit] Do you have any idea to improve the performance or enhance the running time?

Implementation & Submission instructions:

- Implementation instruction: You can use MATLAB, C++, or Python for your implementation.
- Submission instructions:
- Upload the electronic file that includes the report, source code, and data in a single zip format with the name "ICV_assignment#2_yourname.zip" on the ETL class homepage.
- The report should include a brief description of the problems, code, results, and discussions
- Reference
- [1] Guided Image Filtering, by Kaiming He, Jian Sun, and Xiaoou Tang, in TPAMI 2013
- [2] Bilateral Filtering for Gray and Color Images, by C. Tomasi and R. Manduchi in ICCV1998

Note: All works should be individual-based. NO copy is allowed.