APPM 4600 HW#2

1.a show $(1+x)^n = 1+nx+o(x)$ as $x \to 0$ | if $f(x) = o(g(x)) \to \lim_{x\to a} \frac{f}{g} = 0$

$o(g(x)) \to g(x) = x \qquad f(x) = (1+x)^n - 1 - nx$

$$\lim_{x\to 0} \frac{(1+x)^n - 1 - nx}{x} = \frac{0}{0} \otimes \xrightarrow{H} \lim_{x\to 0} \frac{n(1+x)^{n-1} - 1}{1} = \frac{0}{1} = \boxed{0} \checkmark\checkmark$$

1b show $x\sin(\sqrt{x}) = O(x^{3/2})$ as $x \to 0$ | $f(x) = O(g(x))$ if $f(x) \leq c \cdot g(x)$

$c \in \mathbb{R};\ x \geq x_0$

$$\lim_{x\to 0} \frac{x\sin(\sqrt{x})}{x^{3/2}} = \lim_{x\to 0} \frac{\sin(\sqrt{x})}{\sqrt{x}} = \frac{0}{0}$$

$$\stackrel{H}{=} \lim_{x\to 0} \frac{\frac{\cos(\sqrt{x})}{2\sqrt{x}}}{\frac{1}{2\sqrt{x}}} = \lim_{x\to 0} \cos\sqrt{x} = \boxed{1} \checkmark\checkmark$$

1c. $e^{-t} = o(\frac{1}{t^2})$ as $t \to \infty \qquad f(t) = e^{-t} \quad g(t) = t^{-2}$

$$\lim_{t\to\infty} \frac{e^{-t}}{t^{-2}} = \lim_{t\to\infty} \frac{t^2}{e^t} = \frac{\infty}{\infty} \otimes \xrightarrow{H} \lim_{t\to\infty} \frac{2t}{e^t} \stackrel{H}{=} \lim_{t\to\infty} \frac{2}{e^t} = \boxed{0} \checkmark\checkmark$$

1d $\int_0^\epsilon e^{-x^2}\,dx = O(\epsilon)$ as $\epsilon \to 0 \qquad f(\epsilon) = \int_0^\epsilon e^{-x^2}\,dx \quad g(\epsilon) = \epsilon$

$$\lim_{\epsilon\to 0} \frac{\int_0^\epsilon e^{-x^2}\,dx}{\epsilon} = \frac{\int_0^0 e^{-x^2}\,dx}{0} = \frac{0}{0} \otimes$$

$$\stackrel{H}{=} \lim_{\epsilon\to 0} \frac{e^{-\epsilon^2}}{1} = \frac{0}{1} = \boxed{0}$$

APPM 4660 HW#2

2. $\underline{A}\bar{x}=\bar{b}$ where $\underline{A}=\frac{1}{2}\begin{bmatrix}1 & 1\\ 1+10^{-1} & 1-10^{-1}\end{bmatrix}$ & $\bar{b}=\begin{bmatrix}1\\1\end{bmatrix}$

$\bar{x}=\begin{bmatrix}1\\1\end{bmatrix}$ $\underline{A}^{-1}=\begin{bmatrix}1-10^{10} & 10^{10}\\ 1+10^{10} & -10^{10}\end{bmatrix}$ find perturbation in $\underline{b}$ of $\begin{bmatrix}\Delta b_1\\ \Delta b_2\end{bmatrix}$

a. find an exact formula for Δ in solution b/w exact problem & preturbed problem Δx

$\frac{1}{2}\begin{bmatrix}1 & 1\\ 1+10^{-1} & 1-10^{-1}\end{bmatrix}\begin{bmatrix}x_1=1\\ x_2=1\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$ $\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}1-10^{10} & 10^{10}\\ 1+10^{10} & -10^{10}\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}$

$\rightarrow 1\cancel{-10^{10}+10^{10}}=x_1$ $1\cancel{+10^{10}-10^{10}}=x_2$

Let $\bar{b}\,\Delta$ $\begin{bmatrix}1\end{bmatrix}\begin{bmatrix}\Delta x_1\\ \Delta x_2\end{bmatrix}=\begin{bmatrix}1-10^{10} & 10^{10}\\ 1+10^{10} & -10^{10}\end{bmatrix}\begin{bmatrix}\Delta b_1\\ \Delta b_2\end{bmatrix}$

$\boxed{\Delta x_1=\Delta b_1 \quad \Delta x_2=\Delta b_2}$

b. find the condition # of $\underline{A}$

$\text{cond}(A)=\|A\|_\infty \|A^{-1}\|_\infty$

$\|A\|_\infty=\frac{1}{2}[1+1]=1$ or $\frac{1}{2}[1+\cancel{10^{-1}}+1-\cancel{10^{-1}}]=1$ so, 1

$\|A^{-1}\|_\infty=(1-10^{10}+10^{10})=1$ or $(1+10^{10}-10^{10})=1$, so, 1

condition number $=1$

APPM 4600 HW#2

3. relative condition # $K_f(x)$ | $f(x)$

for $\tilde{x} = x + \delta x$, $\delta x \to 0$, gives upper bound on relative error on output $\tilde{y} = f(\tilde{x})$

$$\frac{|f(x) - f(\tilde{x})|}{|f(x)|} \leq K_f(x) \frac{|x - \tilde{x}|}{|x|}$$

$$K_f(x) = \left| \frac{x f'(x)}{f(x)} \right|$$

Let $f = e^x - 1$

a. relative condition #?

$$K_f(x) = \left| \frac{x e^x}{e^x - 1} \right| \cdot \frac{\frac{1}{e^x}}{\frac{1}{e^x}} = \boxed{\frac{x}{1 - e^{-x}}}$$

$$\lim_{x \to \infty} K_f(x) = \frac{\infty}{1} = \infty$$

$$\lim_{x \to 0} K_f(x) = \frac{0}{1-1} = \frac{0}{0} \otimes \text{ LH} \rightarrow \lim_{x \to 0} K_f(x) = \frac{1}{1 + e^{-x}} = \frac{1}{2}$$

$$\lim_{x \to -\infty} \frac{-\infty}{1 - \infty} \otimes \overset{H}{=} \lim_{x \to -\infty} \frac{1}{1 + e^{-x}} = \frac{1}{1 + \infty} = 0$$

for x large $K_f(x)$ ill-conditioned

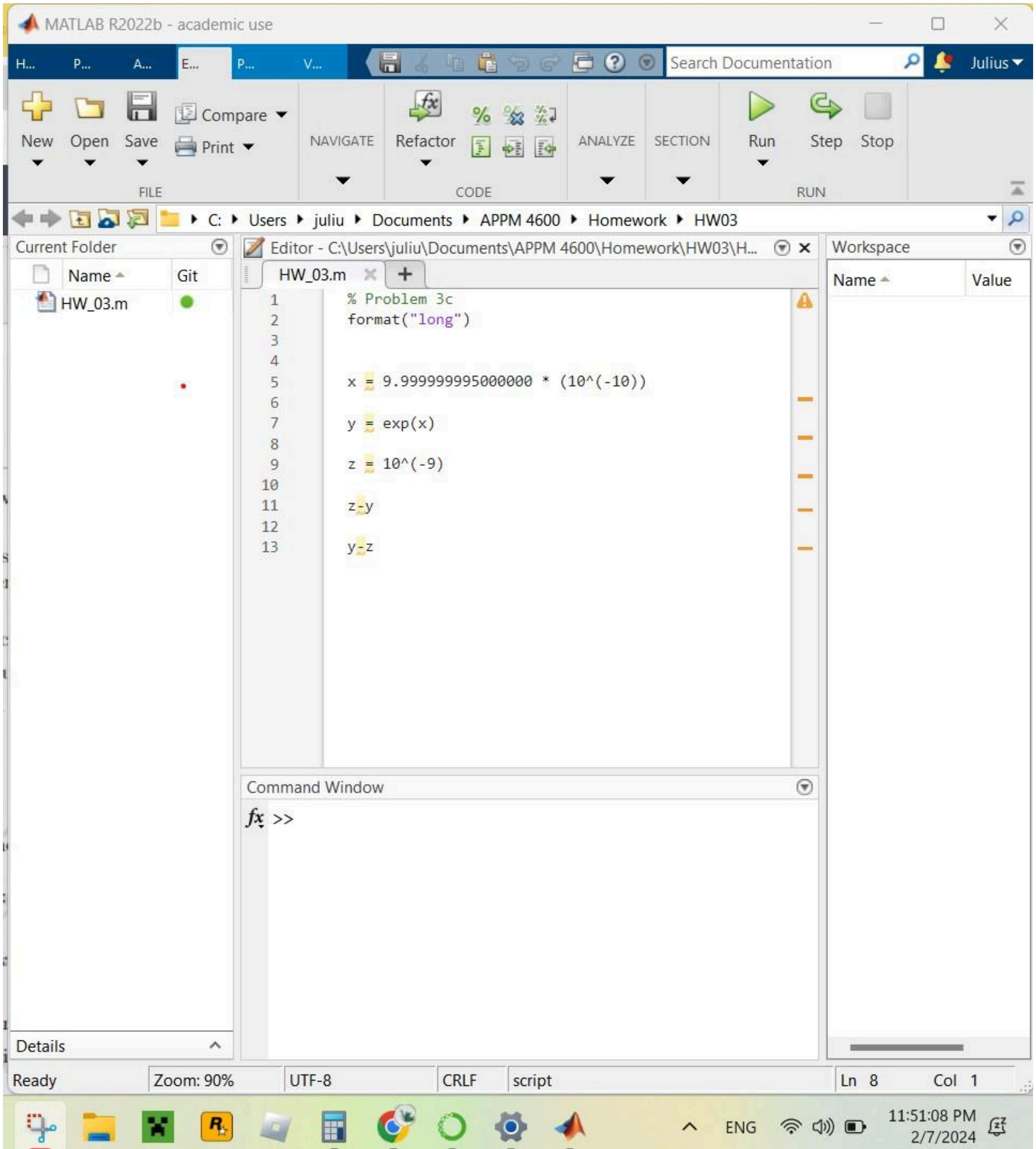
b. $K_f(x) = \left| \frac{x e^x}{e^x} \right| = |x|$ not ~~stable as $x \to \infty$~~

$K_f(x)$ ill-conditioned as $|x| \to \infty$, so it is not stable

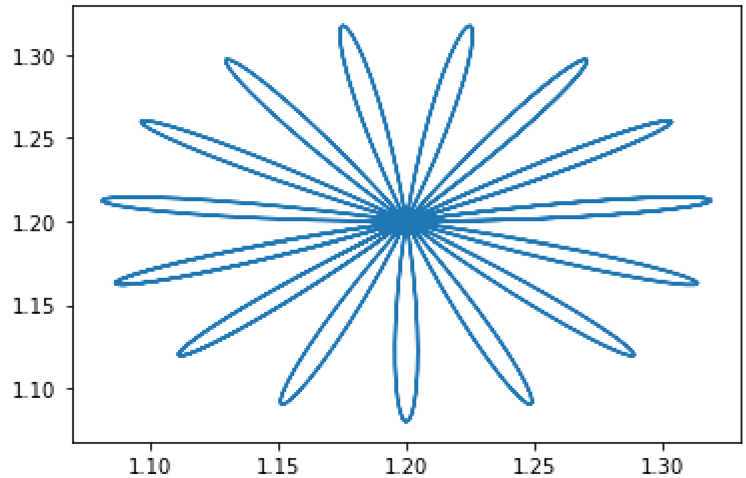
Scanned with CamScanner

APPM 4600 Homework 2

3c.



4b



lab.py hi.py HW03.py untitled1.py

```python
# -*- coding: utf-8 -*-
"""
Created on Wed Feb  7 23:54:45 2024

@author: juliu
"""

import numpy as np
import matplotlib.pyplot as plt
import random

t = np.arange(0,np.pi+np.pi/60,np.pi/30)

y = np.cos(t)

def sum(n):
    for k in range(1,n+1):

        sum = 0
        sum = sum + t[k]*y[k]
    print("The sum is: ", sum)


sum(4)

theta = np.arange(0,2*np.pi + 1, 0.01)

x = 1.2*(1 + 0.1*np.sin(15*theta)*np.cos(theta))
y = 1.2*(1 + 0.1*np.sin(15*theta)*np.sin(theta))

#plt.plot(x,y)
#plt.show()

for i in range(1,11):
    p = random.uniform(0,2)
    xtheta = i * (1 + 0.05*np.sin((2+i)*theta+p))*np.cos(theta)
    ytheta = i * (1 + 0.05*np.sin((2+i)*theta+p))*np.sin(theta)

    plt.figure()
    plt.plot(xtheta,ytheta)
    plt.show()
```