Tarea 1

Friday, July 17, 2020 10:12 PM

Problema 1

a) $X_t = \beta_0 + e_t$

i) $E(X_t) = E(\beta_0 + e_t)$

$= \beta_0 + \cancel{E(e_t)} \rightarrow 0$, recordar que $e_t \sim iid(0,1)$

ii) $Var(X_t) = Var(\beta_0 + e_t)$

$= \underbrace{\cancel{Var(\beta_0)}}^{0} + \cancel{Var(e_t)}^{1}$

$+ 2\, Cov(\cancel{\beta_0}, e_t)_{0}$

$= 1$

iii) $Cov(X_t, X_{t-1}) = Cov(\beta_0 + e_t, \beta_0 + e_{t-1})$

$= \cancel{Cov(\beta_0, \beta_0)}^{0} + \cancel{Cov(e_t, \beta_0)}_{0}$

$+ Cov(\beta_0, \cancel{e_{t-1}})_{0} + Cov(\cancel{e_t}, e_{t-1})$

$= 0$

b) $X_t = \beta_1 X_{t-1} + e_t$

i) $E(X_t) = E(\beta_1 X_{t-1} + e_t)$

$= \beta_1 E(X_{t-1}) + \cancel{E(e_t)}^{0}$

Si X_t es estacionaria $E(X_t) = E(X_{t-1})$

$E(X_t)=0$

ii) $Var(X_t) = Var(\beta_1 X_{t-1} + e_t)$

$= \beta_1^2 \, Var(X_{t-1}) + \cancel{Var(e_t)}$ 1

Si X_t es estacionaria, $Var(X_t) = Var(X_{t-1})$

$$Var(X_t) = \frac{1}{1-\beta_1^2}$$

iii) $Cov(X_t, X_{t-1}) = Cov(\beta_1 X_{t-1} + e_t, X_{t-1})$

$= \beta_1 \, \underbrace{\cancel{Cov(X_{t-1}, X_{t-1})}}_{} \; Var(X_t) = Var(X_{t-1})$ +

$\cancel{Cov(e_t, X_{t-1})}^{\,0}$

Definimos $\gamma_1 \equiv Cov(X_t, X_{t-1})$

Es decir γ_1 es la covarianza con 1 rezago de diferencia.

$Cov(X_t, X_{t-1}) \equiv \gamma_1 = \beta_1 Var(X_t)$

$$\gamma_1 = \frac{\beta_1}{1-\beta_1^2}$$

c)

$X_t = \beta_1 u_{t-1} + u_t$, $u_t \sim iid(\mu, \sigma^2)$

i) $E(X_t) = E(\beta_1 u_{t-1} + u_t)$

$$= \beta_1 \mu + \mu$$

$u_t \sim iid.$

ii) $$Var(X_t) = Var(\beta_1 u_{t-1} + u_t)$$

$$= \beta_1^2 \sigma^2 + \sigma^2 + 2\,\overset{0}{\cancel{Cov(\beta_1 u_{t-1}, u_t)}}$$

$$= \beta_1^2 \sigma^2 + \sigma^2$$

iii) $$Cov(X_t, X_{t-1}) = Cov(\beta_1 u_{t-1} + u_t, \beta_1 u_{t-2} + u_{t-1})$$

$$= \beta_1^2 \overset{0}{\cancel{Cov(u_{t-1}, u_{t-2})}} + \beta_1 \overset{0}{\cancel{Cov(u_t, u_{t-2})}} + \beta_1 Cov(u_{t-1}^2) + \overset{0}{\cancel{Cov(u_t, u_{t-1})}}$$

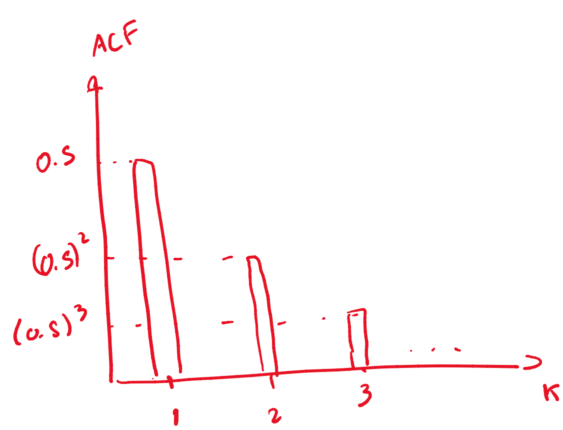
$$= \beta_1 \sigma^2$$

Problema 2

a) $$X_t = 0.5 X_{t-1} + e_t$$

ACF. Sea ρ_k la correlación entre X_t, X_{t-k}

$$\rho_1 = \frac{Cov(X_t, X_{t-1})}{Var(X_t)} = 0.5$$



$$\rho_2 = \frac{Cov(X_t, X_{t-2})}{Var(X_t)} = (0.5)^2$$

$\vdots$

$$\rho_K = \frac{Cov(X_t, X_{t-K})}{Var(X_t)} = (0.5)^K$$

$$\rho_k = \frac{Cov(X_t, X_{t-k})}{Var(X_t)} = (0.5)^k$$

1 2 3 k

PACF

$$\frac{\partial X_t}{\partial X_{t-1}} = 0.5$$

b) $X_t = 0.5 X_{t-1} - 0.2 X_{t-2} + e_t$

ACF

$$\rho_1 = \frac{Cov(X_t, X_{t-1})}{Var(X_t)} = \frac{\gamma_1}{Var(X_t)}$$

$$Cov(X_t, X_{t-1}) = Cov(0.5 X_{t-1} - 0.2 X_{t-2} + e_t, X_{t-1})$$

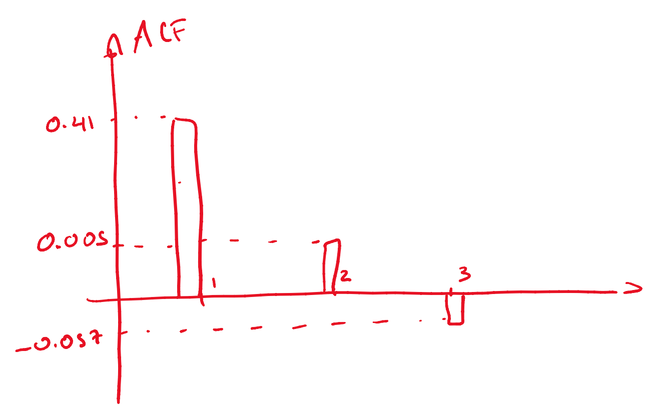
$$Cov(X_t, X_{t-1}) = 0.5\, Cov(X_{t-1}, X_{t-1}) - 0.2\, Cov(X_{t-1}, X_{t-2}) + \underbrace{\cancel{Cov(e_t, X_{t-1})}}_{0}$$

$$\gamma_1 = 0.5\, Var(X_t) - 0.2\, \gamma_1$$

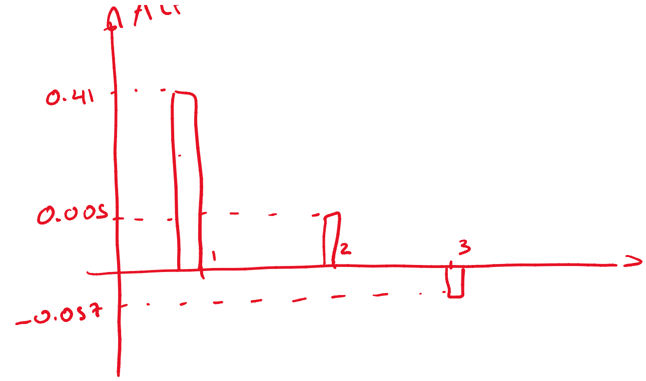
$$\gamma_1 = 0.41\, Var(X_t)$$

$$\rho_1 = \frac{0.41\, Var(X_t)}{Var(X_t)} = 0.41$$

$$\rho_2 = \frac{Cov(X_t, X_{t-2})}{Var(X_t)} = 0.005$$



$$\rho_1 = \frac{0.41\ Var(X_t)}{Var(X_t)} = 0.41$$

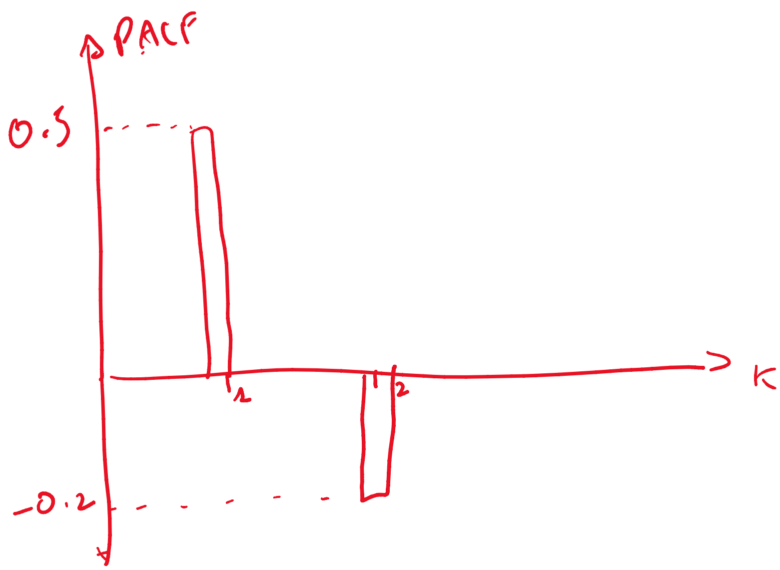


$$\rho_2 = \frac{Cov(X_t, X_{t-2})}{Var(X_t)} = 0.005$$

$$\rho_3 = \frac{Cov(X_t, X_{t-3})}{Var(X_t)} = -0.057$$

PACF

$$\frac{\partial X_t}{\partial X_{t-1}} = 0.5 \quad , \quad \frac{\partial X_t}{\partial X_{t-2}} = -0.2$$



c) $X_t = u_t + 0.5\, u_{t-1}$

ACF

$$\rho_1 = \frac{Cov(X_t, X_{t-1})}{Var(X_t)}$$

$$= \frac{Cov(u_t + 0.5u_{t-1},\ u_{t-1} + 0.5u_{t-2})}{Var(u_t + 0.5u_{t-1})}$$

$$= \frac{0.5\ Var(\cancel{u_t})}{1.25\ Var(\cancel{u_t})}$$

$$= 0.4$$

$$\rho_2 = \rho_3 = \dots = 0$$

PACF

$$X_t = (1 + 0.5L)\,u_t$$

$$\frac{X_t}{1 - (-0.5)L} = u_t$$

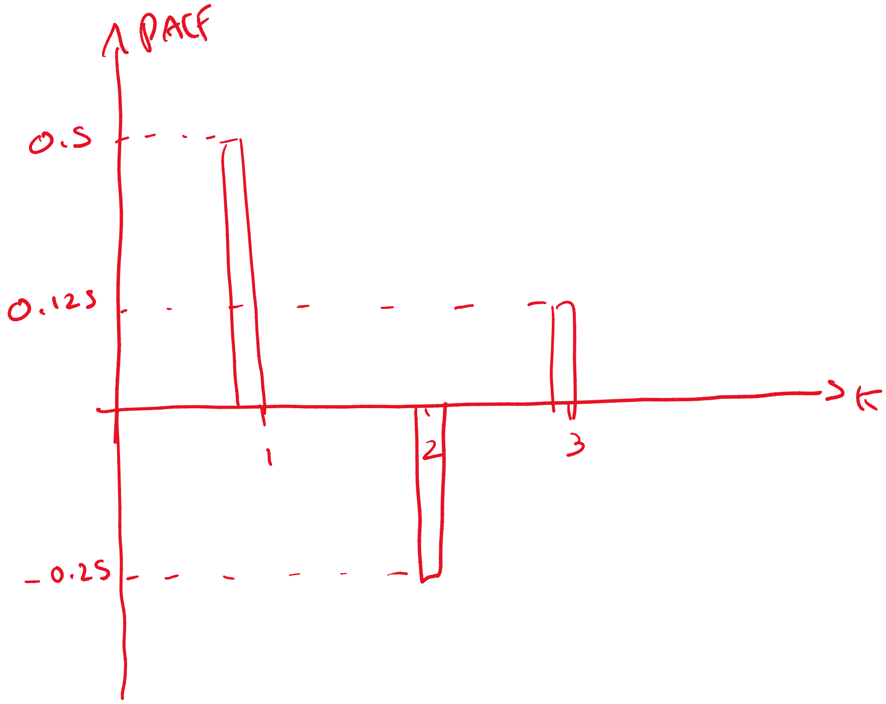
$$u_t = [1 + (-0.5)L + (-0.5)^2L^2 + (-0.5)^3L^3 + \dots]\,X_t$$

$$u_t = X_t - 0.5X_{t-1} + 0.25X_{t-2} - 0.125X_{t-3} + \dots$$

$$X_t = 0.5X_{t-1} - 0.25X_{t-2} + 0.125X_{t-3} + \dots u_t$$

$$\frac{\partial X_t}{\partial X_{t-1}} = 0.5 \qquad \frac{\partial X_t}{\partial X_{t-2}} = -0.25\ , \quad \frac{\partial X_t}{\partial X_{t-3}} = 0.125$$

↑ PACF



Problema 3

a) $X_t = 1.05\, X_{t-1} + e_t$

Dado que $\beta_1 = 1.05 > 1 \Rightarrow$

el proceso NO es estacionario

$$X_t - 1.05\, L X_t = e_t$$

$$(1 - 1.05L)\, X_t = e_t$$

$$\Phi(L) = 1 - 1.05L$$

¿Cuál es el valor de L, tal que $\Phi(L) = 0$?

$$0 = 1 - 1.05L$$

$0 = 1 - 1.05L$

- $L < 1$ Dado que la raíz es menor a uno el proceso es NO estacionario
- Si hubiera sido > 1, entonces el proceso es estacionario

b) $X_t = \overbrace{0.6}^{\beta_1} X_{t-1} + \overbrace{0.10}^{\beta_2} X_{t-2} + e_t$

Recordemos que

$$X_t - \beta_1 X_{t-1} - \beta_2 X_{t-2} = e_t$$

$$\Phi(L) = 1 - \beta_1 L - \beta_2 L^2$$

Si las raíces de esta ecuación cuadrática son > 1, entonces X_t ES ESTACIONARIO

Una forma complementaria es trabajar con la inversa ecuación característica, es decir

$$z^2 - \beta_1 z_1 - \beta_2 = 0$$

De esto obtenemos la ecuación

$$r_1, r_2 = \frac{\beta_1^2 \pm \sqrt{\beta_1^2 + 4\beta_2}}{2}$$

$|r_1 \text{ y } r_2| < 1$

De esto se generan 3 condiciones.

1) $\beta_1^2 + 4\beta_2 > 0 \rightarrow (0.6)^2 + 4(0.1) > 0$

2) $\beta_1 + \beta_2 < 1 \rightarrow 0.6 + 0.1 = 0.7 < 1$

3) $\beta_2 - \beta_1 < 1 \rightarrow 0.1 - 0.6 = -0.5 < 1$

Entonces el proceso es estacionario

c) $X_t = 0.8 X_{t-1} - 0.10 X_{t-2} + e_t$

1) $\beta_1^2 + 4\beta_2 > 0 \longrightarrow (0.8)^2 + 4(-0.10) > 0$

2) $\beta_1 + \beta_2 < 1 \rightarrow 0.8 - 0.10 = 0.70 < 1$

3) $\beta_2 - \beta_1 < 1 \rightarrow -0.10 - 0.8 = -0.90 < 1$

Entonces el proceso es estacionario

Problema 4

a) $X_t = 0.5 X_{t-1} + 0.10 X_{t-2} + e_t$

$$X_t - 0.5 L X_t - 0.10 L^2 X_t = e_t$$

$$(1 - 0.5L - 0.10L^2) X_t = e_t$$

$$\Phi(L) = 1 - 0.5L - 0.10L^2$$

La ecuación característica inversa es:

$$z^2 - 0.5z - 0.10 = 0$$

$$r_1, r_2 = \frac{0.5 \pm \sqrt{(0.5)^2 + 4(0.10)}}{2}$$

$$r_1 = 0.6531$$

$$r_2 = -0.1531$$

b) $X_t = 1.10 X_{t-1} - 0.2 X_{t-2} + e_t$

$$r_1, r_2 = \frac{-1.10 \pm \sqrt{(1.10)^2 + 4(0.2)}}{2}$$

$$r_1 = -1.2388$$

$$r_2 = 0.1589$$