A Survey on Topology in Motion Planning (proposal)

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Motion Planning is a computational problem that, given a object and a set of obstacles, how can we move the object from a source point to a destination point without colliding with any obstacle. The problem has many robotic applications, such as autonomy, automation, artificial intelligence, etc. It is studied by both computational geometry community and robotics community. Many algorithms have been designed to solve this problem in various settings. A lot of motion planning techniques utilize topology to help generating concise maps and efficient planners [1]. We want to do a survey on how topology has been exploited in motion planning and see if we could be inspired by some of those ideas for future work.

In the motion planning problem, we use *configuration* to describe the object's position and possibly direction, and the *configuration space* is often referred to all possible configurations(states). Typically, every configuration A consists of a finite set of real numbers, and we can see the configuration space X as a subset of the Euclidean space of which the dimension is the number of values we used to encode one state of the object In this setting, every point in this subset represents a state of the object, and the configuration space is naturally combining with topology [4]. Many research have been done for exploiting the topological properties of the configuration space in the problem. Farber studied the topological complexity $\mathbf{TC}(X)$ in the motion planning problem which is used to measure the discontinuity of the planning process for some configuration space X, and emphasize it as the minimal number such that there are this many different motion planning rules. They also gave the algorithm to compute this topological complexity [3], and a randomized algorithm computing a collision free plan.

Choset and Rizzi tried to use topological maps of the work space to partition the configuration spaces into simple regions for motion planning [1]. The topological map is a map that each homotopy class in the free space corresponds to a homotopy class in the map. While many applications of motion planning start utilizing homotopically distinct paths rather than a single path between source and destination configurations, Denny *et al.* proposed an approximate metric of measuring the homotopic similarity between two classes, and apply this metric to perform faster sample-based motion planning [2].

Martinetz and Schulten introduced the Topology Representing Network(TRN) in 1994 [6]. It has been applied in many applications for learning the topology property of the data in Robotics including motion planning. For example, Zellar *et al.* utilized this idea in sensor-based robot motion planning. Fu *et al.* also designed a vision-based planning method using TRN [5].

The goal of our survey is exploring those applications of topology on motion planning and/or possibly some other results we did not get chance to mention above.

References

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