CS 7301.003 Fall 2020 Author A. One (aa0123456)

Homework 1 Problem 1 Jiashuai Lu (jxl173630)

(a) Truthfully write the phrase ***"I have read and understand the course policies."***

Solution: I have read and understand the course policies. ■

(b) Prove that the composition of two PL homeomorphisms of the plane is another PL homeomorphism.

Solution: Kyle likes to start inline math mode using "\(" and end it using "\)", but most online guides start and end math mode with "$". Whatever you prefer is fine! ■

(c) Suppose ϕ is a PL homeomorphism with complexity x and ψ is a PL homeomorphism with complexity y. What can you say about the complexity of the PL homeomorphism $\psi \circ \phi$?

Solution: Sometimes, you don't want to use inline math.

$$1 < 2 = 6/3$$

■

(d) Prove that for any simple n-gon P, there is a piecewise-linear homeomorphism $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ with complexity $O(n)$ that maps the polygon P to a triangle.

Solution: Sometimes, it's convenient to use pseudocode:

```
SolveHomework(problems[1..n]):
  Set up boilerplate for writing homework.

  for i ← 1 to n
      Read problem problems[i]
      for j ← i down to 1
          ThinkAboutProblem(i)
      Write solution to problem problems[i]

  Turn in homework
```

■

(e) Prove that for any two simple n-gons P and Q, there is a piecewise-linear homeomorphism $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ with complexity $O(n^2)$ such that $\phi(P) = Q$.

Solution: Does this problem have too many parts? ■

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Homework 1 Problem 2 Jiashuai Lu (jxl173630)

(a) Prove that every connected plane graph has either a vertex with degree at most 3 or a face with degree at most 3.

Solution: I guess it's hard to fit lot of big objects in the plane. ■

(b) Prove that every simple bipartite planar graph has at most $2n-4$ edges.

Solution: Like really hard. ■

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Homework 1 Problem 3

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Let G be an arbitrary plane graph, let T be an arbitrary spanning tree of G, and let e be an arbitrary edge of T. Color the vertices in one component of $T \setminus e$ red and the vertices in the other component blue. Prove that any face of G is incident to either zero or two edges that have one red endpoint and one blue endpoint.

Solution: Sometimes a figure is useful. This is a topology course, after all. ■

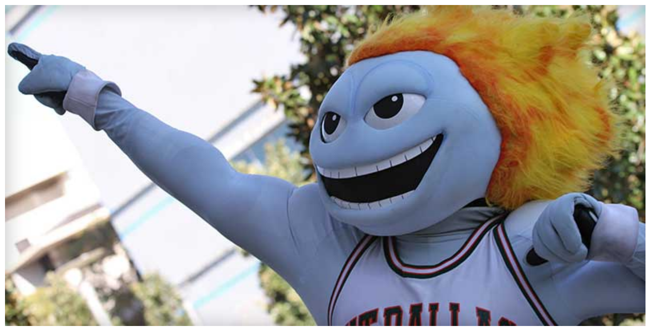

Figure 1. Temoc is blue, like certain subsets of vertices.