Confusion Matrix in Quantum Readout Mitigation

Overview

A **confusion matrix** is a 4×4 table (for two qubits) that captures how often a quantum computer *misreports* measurement results. It quantifies bias and readout errors, allowing us to mathematically correct raw experimental data.

Why We Need It

Real quantum hardware rarely returns perfect measurement results. Even if you prepare a clean computational basis state like `|00⟩`, the device might occasionally report `01`, `10`, or `11` because of:

- measurement crosstalk
- thermal excitations
- amplifier bias
- classical bit-flip errors
- imperfect discrimination of readout signals

The confusion matrix tells us *how often* each type of misclassification happens.

How It's Built

There are four 2-qubit basis states:

```
|00⟩, |01⟩, |10⟩, |11⟩
```

You run a calibration experiment:

1. Prepare each basis state exactly.
2. Measure it many times.
3. Record how often the hardware reports each possible outcome.

That gives you a probability table like this (example only):

Prepared → / Measured ↓	00	01	10	11
00	0.963	0.021	0.010	0.006
01	0.026	0.942	0.010	0.022
10	…	…	…	…

Prepared → / Measured ↓	00	01	10	11
11	…	…	…	…

Each row sums to 1.0 and represents:

$$M_{i,j} = P(\text{measured } j \mid \text{prepared } i)$$

This matrix tells you exactly how measurement errors distort ideal results.

How It's Used

Suppose your Bell experiment yields a measured probability vector:

```
p_measured = [P(00), P(01), P(10), P(11)]
```

Readout noise means:

```
p_measured = M · p_true
```

So we invert the confusion matrix:

```
p_true ≈ M⁻¹ · p_measured
```

This undoes the misclassification and recovers an estimate of the *actual* quantum-state populations.

Why It Works

Readout errors are mostly **classical** and **stable** over short time windows. That means if Torino misreads 01 as 00 in your calibration runs about 2% of the time, it will misread it the same way in your Bell-state run. So the calibration matrix acts like a fingerprint of the measurement noise.

Correcting with the inverse matrix recovers fidelity that the hardware lost *only during measurement*, not during the actual quantum evolution.

Practical Results

You observed this effect directly:

- Raw $\langle Z \otimes Z \rangle$: **0.951**
- Mitigated $\langle Z \otimes Z \rangle$: **0.970**

The improvement comes from removing readout bias, not changing the underlying quantum physics.

Summary

A confusion matrix is:

- A calibrated description of how the device misreports measurement outcomes.
- A 4×4 stochastic matrix for two-qubit experiments.
- A tool to reverse classical noise in measurement.
- A crucial part of near-term quantum error mitigation.

By preparing known states, measuring the device's mistakes, and mathematically undoing them, you reveal the “truer” behavior of the quantum system you prepared.