🎯Purpose of the Phase in Quantum State Preparation

1. Relative Phase = New Information

A balanced superposition alone isn't enough to represent a general qubit state. The *relative phase* between the $|0\rangle$ and $|1\rangle$ terms carries unique information.

Example:

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \quad |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

These two states differ **only** by a phase factor, yet they behave very differently under interference and measurement.

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2. Bloch Sphere Interpretation

- Without a phase: a Hadamard (`H`) puts the qubit on the equator at 0° longitude (the state $|+\rangle$).
- With a phase gate `P(θ)`: the state rotates around the equator to longitude θ.

➡️The phase lets us **steer the qubit around the Bloch sphere's equator**, changing how it interacts with future gates and measurements.

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3. Why It Matters in Teleportation

The teleportation protocol needs to prove it can transmit *any* state, not just simple ones like $|0\rangle$ or $|+\rangle$.

By preparing a state with a relative phase:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\theta}|1\rangle)$$

we demonstrate that **even fragile phase information** is preserved when the qubit is teleported.

If Bob recovers this exact state, it shows the protocol works for general states — not just basis states.

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4. Practical Purpose

Relative phases underpin: - **Quantum interference** (Deutsch, Grover, Shor's algorithms) - **Quantum communication** (encoding and transmitting info via phase) - **Quantum error correction** (must preserve phase as well as amplitude)

✅ The phase change in teleportation demos isn't arbitrary — it's proof that the protocol can handle **general qubit states with arbitrary phase**, not just the "easy" ones.