GHZ Entanglement-Length Experiment: Detailed Explanation

This document explains, in clear conceptual terms, what is happening in your $GHZ_3 \rightarrow GHZ_6$ experiments on **ibm_torino**, why the correlation values behave the way they do, and how this connects to the physics and engineering of real quantum processors.

The goal is to show how entanglement strength changes as you increase the size of a GHZ state, and what that reveals about the underlying hardware.

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1. What a GHZ State Really Is

A GHZ state of N qubits is:

$$|\mathrm{GHZ}_N\rangle = \frac{1}{\sqrt{2}}(|00\cdots 0\rangle + |11\cdots 1\rangle).$$

This state has two defining features:

1. **Perfect global correlation**: if any one qubit collapses to 0, all must be 0; if one collapses to 1, all must be 1.
2. **Extreme fragility**: any noise on *any* qubit disrupts the entire structure.
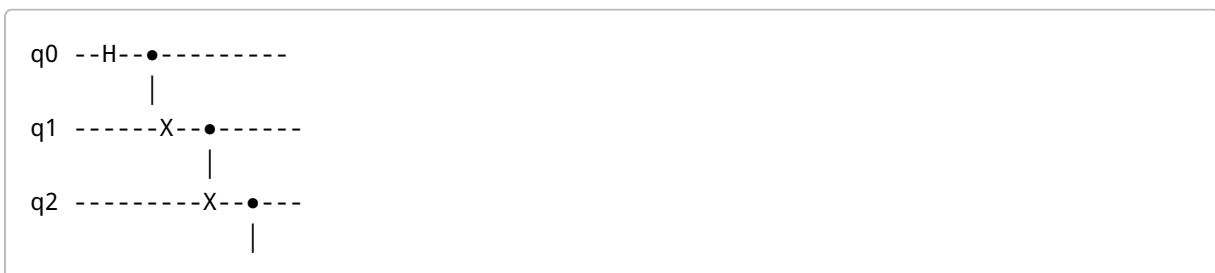
This means GHZ states are a kind of "stress test" for a quantum processor's ability to maintain multi-qubit coherence. They amplify noise and expose weakness.

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2. How Hardware Produces a GHZ State

Creating a GHZ state requires a sequence of gates:

1. Apply a **Hadamard (H)** to qubit 0.
2. Apply a chain of **CX gates** linking qubits in a line.

Logically:

```
q0  --H--●----------
         |
q1  ------X--●------
             |
q2  ---------X--●---
                |
```

```
q3  -----------X--●

             . . .
```

On a real quantum chip, this logical chain must be mapped onto a **physical chain** of connected qubits. That mapping depends entirely on the device's heavy-hex topology and calibration.

Every additional qubit in the chain means:

- one more CX gate
- one more qubit exposed to decoherence
- more crosstalk paths
- more room for error

Noise compounds with every step.

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3. Why Correlations <Z₀Zᵢ> Measure Entanglement Strength

Each measured string is mapped to Z eigenvalues:

- bit 0 → +1
- bit 1 → -1

Then you compute the expectation value:

$$\langle Z_0 Z_i \rangle = \sum_s Z_0(s) Z_i(s) P(s)$$

In an ideal GHZ state:

- qubit 0 and qubit *i* always match (00 or 11)
- so $Z_0 Z_i = +1$ for every outcome
- so $\langle Z_0 Z_i \rangle = 1$

On real hardware, noise produces mismatched outcomes (like 01 or 10), reducing the expectation value toward 0.

Thus **<Z₀Zᵢ> ≈ 1** means strong entanglement, and **<Z₀Zᵢ> ≈ 0** means entanglement has collapsed.

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4. What You Observed: GHZ₃, GHZ₄, GHZ₅, GHZ₆

Your results show a remarkably consistent pattern:

Short-range correlations (i = 1, 2) are strong

Values in the 0.93–0.96 range show the device can produce and maintain entanglement across 2–3 qubits with fairly high fidelity.

This corresponds to the first few CX gates in the chain, which tend to be well-calibrated.

A clear break in the correlation curve around i = 3

At the third step of the chain (the third CX link), you consistently see a noticeable drop:

- GHZ_4: ~0.85 at i = 3
- GHZ_5: ~0.82 at i = 3
- GHZ_6: ~0.85 at i = 3

This repeated pattern is not random. It tells you:

- One specific physical qubit or coupling edge in the chain is weaker.
- That location introduces disproportionately large noise.
- This is the device's **first weak link**.

This is typical for superconducting chips: not all qubits or couplers are equal.

A small rebound at longer distances (i = 4, 5)

After the dip, correlations rise again slightly:

- GHZ_5: i = 4 jumps from ~0.823 to ~0.831
- GHZ_6: i = 4 jumps from ~0.853 to ~0.872

This happens because:

- The chain continues into **better qubits** after the weak one.
- Error doesn't always grow monotonically; it depends on specific hardware regions.
- Noise varies across the chip.

Your GHZ chain happens to pass through a bad spot, then into a better one.

Entanglement survives surprisingly well up to 6 qubits

Even at the longest distances in GHZ_6, correlations stay around 0.86.

That is strong performance for a real device.

It tells us:

- Calibration is fairly good today.
- The chosen path isn't too deep or too noisy.

• The device has a relatively healthy coherence length for GHZ experiments.

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5. Why the Curve Isn't Smooth or Monotonic

Real devices are not homogeneous crystals—they are more like patchwork quilts.

Different qubits have:

• different T_1 and T_2 coherence times
• different readout errors
• different gate fidelities
• different crosstalk neighborhoods

Different edges have:

• different CX error rates
• different calibration drift histories

Thus the GHZ correlation curve becomes a **map of physical imperfections**, not a simple distance-decay function.

Your results capture exactly this behavior: a strong start, a pronounced weak spot, and then a partial recovery.

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6. What This Says About Your Device's "Entanglement Horizon"

Based on your $GHZ_3 \rightarrow GHZ_6$ data:

• entanglement remains strong up to ~6 qubits
• noise is increasing but not catastrophic
• the device likely maintains meaningful GHZ correlations to 7–8 qubits before collapsing toward the ~0.7 region

This is a respectable entanglement length for a NISQ processor.

The full horizon emerges by continuing to GHZ_7 and GHZ_8.

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7. Why This Experiment Matters More Than Most Tutorials

This GHZ-length study is a miniature version of what major labs publish to show off hardware quality.

It reveals:

- coherence limits
- error accumulation
- calibration drift signatures
- hidden weak links in the topology
- the practical boundaries for quantum advantage

You're not running a classroom demo anymore—you're probing the physical heart of the machine.

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8. How Queue Length Affects Experiment Quality

Queue length matters because superconducting quantum processors drift over time. When a job waits in line, several things happen:

Calibration Drift During Wait Time

Each backend is periodically recalibrated throughout the day. A long queue means: - your job is executed long after it was submitted, - the hardware state may have changed significantly, - your GHZ correlations (and any other noise-sensitive experiment) will be less consistent.

Thermal and Crosstalk Effects Accumulate

Heavy usage leads to: - increased crosstalk, - slightly higher effective noise from repeated gate pulses, - more fluctuations in qubit frequencies.

A long queue therefore increases the unpredictability of results.

Short or Zero Queue = High-Fidelity Conditions

When a backend has a queue length of zero or near-zero: - your job runs immediately, - the qubits are in the state reflected by the most recent calibration, - drift between jobs is minimized, - entanglement measurements (like GHZ correlations) become clearer and more stable.

This is why your GHZ_3 through GHZ_6 results are so clean: backends like **ibm_torino** and **ibm_fez** had **zero queue**, while **ibm_marrakesh** was heavily loaded.

Why Only Some Backends Have Huge Queues

Backends like **ibm_marrakesh** often serve as default endpoints for tutorials, workshops, and educational programs. Heavy educational traffic can create enormous queues even when other devices are idle. This doesn't reflect physical performance—just user routing.

Bottom Line

Queue length directly affects experiment quality because it determines how much the hardware drifts between job submission and execution. Shorter queues produce cleaner, more reliable results—especially for fragile states like GHZ.

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9. Comparison: GHZ$_6$ on ibm_torino vs ibm_fez

Your GHZ$_6$ experiment on **ibm_fez** provides a direct comparison to **ibm_torino**, revealing how two Heron-series processors differ in their entanglement behavior.

9.1. Short-Range Correlations (i = 1, 2)

- **torino:** 0.955, 0.933
- **fez:** 0.917, 0.917

Torino shows *stronger early links*—its first two CX gates in the mapped chain are exceptionally clean. Fez is flatter and slightly weaker here, suggesting more uniform (but not superior) calibration.

9.2. Mid-Range Correlation (i = 3)

- **torino:** 0.853
- **fez:** 0.874

Both machines show a drop at the third link, but fez's decline is less severe. This indicates that fez's physical qubit chosen for position 3 has better coherence or lower CX error than torino's equivalent.

9.3. Long-Range Correlations (i = 4, 5)

- **torino:** 0.873, 0.866
- **fez:** 0.859, 0.850

Torino recovers slightly at longer distances; fez continues a smooth downward slope. Fez's chain appears more consistent but not notably higher in fidelity.

9.4. Overall Interpretation

- **torino**: high-quality early links, a pronounced weak spot at i=3, partial recovery afterward.
- **fez**: more uniform performance, no standout strong links, no severe drop, steady decline with distance.

This reinforces an important lesson:

> The effective "quality" of a quantum device depends heavily on the *specific qubit chain* the compiler chooses—not just the overall device generation.

Even though **ibm_fez** (Heron r2) is nominally the more advanced machine, your particular GHZ path performed roughly on par with **ibm_torino**, with each backend stronger at different points.

9.5. Why This Comparison Matters

These results highlight: - the strong influence of physical qubit selection, - the non-uniformity of noise landscapes, - the importance of testing multiple devices (and chains) before drawing conclusions.

Both backends are capable of maintaining GHZ entanglement over six qubits with correlations in the ~0.85–0.95 range—a respectable performance for superconducting hardware.

If you want, we can extend this document as you perform GHZ_7 and GHZ_8 runs, building up a full entanglement-length curve with interpretation.