Histogram of Oriented Gradients (HOG) Conceptual Understanding and Problem Cases

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What is HOG?

Histogram of Oriented Gradients (HOG) is a feature descriptor that:

- Focuses on **edge directions**
- Encodes **local shape information**
- Is robust to **illumination changes**

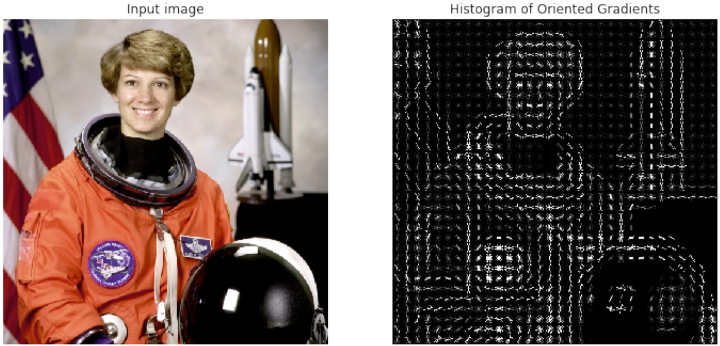
Key Idea: Object appearance is well characterized by the distribution of local intensity gradients.

Why Gradients Instead of Pixels?

- Pixel values change with lighting
- Gradients capture **changes**, not absolute values
- Edges define object shape

$$\text{Edges} \Rightarrow \text{Shape} \Rightarrow \text{Object Identity}$$

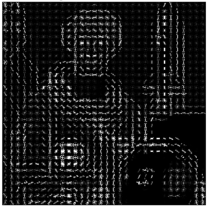
HOG Visualization Example



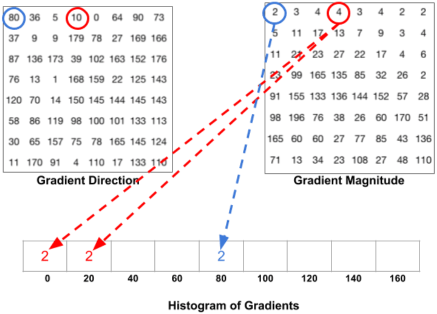
HOG: Image vs Gradients

Input image

Histogram of Oriented Gradients



Original Image



HOG Visualization

Step 1: Gradient Computation

For image intensity $I(x, y)$:

$$G_x = I(x+1, y) - I(x-1, y)$$

$$G_y = I(x, y+1) - I(x, y-1)$$

- G_x: horizontal edge strength
- G_y: vertical edge strength

Step 2: Magnitude and Orientation

$$\text{Magnitude } M = \sqrt{G_x^2 + G_y^2}$$

$$\text{Orientation } \theta = \tan^{-1}\left(\frac{G_y}{G_x}\right)$$

- Orientation range: 0°–180°
- Magnitude = vote weight

Step 3: Cell-wise Orientation Histograms

- Divide image into **cells** (e.g., 8×8 pixels)
- Each pixel votes to an orientation bin
- Typical bins: 9 (20° each)

$$\text{Cell Feature} = \text{Histogram of Orientations}$$

Step 4: Block Normalization

- Group cells into **blocks** (2×2 cells)
- Normalize histogram vector

$$\hat{v} = \frac{v}{\sqrt{\|v\|^2 + \epsilon}}$$

Purpose: Illumination and contrast invariance

Final HOG Descriptor

- Concatenate all normalized block histograms
- Produces a high-dimensional feature vector
- Used as input to SVM / ML classifier

$$\text{Image} \Rightarrow \text{HOG Feature Vector}$$

Real-Time Example: Pedestrian Detection

- Vertical gradients $\rightarrow$ legs and torso
- Horizontal gradients $\rightarrow$ shoulders
- Diagonal gradients $\rightarrow$ arms

HOG captures shape, not texture.

Problem Case 1: Rotation Sensitivity

- HOG is not rotation invariant
- Rotated object changes orientation bins

Example: Upright person vs tilted person

- Object size changes
- Fixed cell size causes mismatch

Solution (Partial): Image pyramids

Problem Case 3: Background Clutter

- Strong background edges dominate histograms
- Foreground object suppressed

Limitations of HOG

- No semantic understanding
- Manual feature design
- Outperformed by CNNs in complex tasks

Where HOG is Still Useful

- Classical computer vision pipelines
- Low-resource systems
- Explainable ML models
- Educational purposes

Summary

- HOG encodes local gradient distributions
- Robust to illumination
- Sensitive to rotation and scale
- Strong baseline before deep learning

Objective of Gradient Computation

- To detect intensity changes in the image
- Gradients capture edge information
- HOG uses gradients instead of raw pixels

$$\text{Edges} \Rightarrow \text{Shape} \Rightarrow \text{Object Representation}$$

Gradient Operators Used in HOG

For a pixel at location (x, y):

$$G_x = I(x+1, y) - I(x-1, y)$$

$$G_y = I(x, y+1) - I(x, y-1)$$

- G_x detects **vertical edges**
- G_y detects **horizontal edges**

Example Image Patch (Given)

Consider the following 3×3 grayscale image patch:

$$\begin{bmatrix} 40 & 45 & 50 \\ 60 & \color{red}{80} & 120 \\ 70 & 90 & 110 \end{bmatrix}$$

- Center pixel $= I(x, y) = 80$
- We compute gradients at the center pixel

Step 1: Computing G_x

Formula:

$$G_x = I(x+1, y) - I(x-1, y)$$

From the patch:

$$I(x+1, y) = 120, \quad I(x-1, y) = 60$$

$$\boxed{G_x = 120 - 60 = 60}$$

Interpretation:

- Positive value
- Intensity increases left $\rightarrow$ right
- Strong vertical edge

Step 2: Computing G_y

Formula:

$$G_y = I(x, y+1) - I(x, y-1)$$

From the patch:

$$I(x, y+1) = 90, \quad I(x, y-1) = 45$$

$$\boxed{G_y = 90 - 45 = 45}$$

Interpretation:

- Positive value
- Intensity increases top $\rightarrow$ bottom
- Horizontal edge present

Step 3: Gradient Magnitude

Formula:

$$M = \sqrt{G_x^2 + G_y^2}$$

Substitute values:

$$M = \sqrt{60^2 + 45^2}$$

$$\boxed{M = \sqrt{3600 + 2025} = \sqrt{5625} = 75}$$

Meaning: Strength of the edge

Step 4: Gradient Orientation

Formula:

$$\theta = \tan^{-1}\left(\frac{G_y}{G_x}\right)$$

$$\theta = \tan^{-1}\left(\frac{45}{60}\right)$$

$$\boxed{\theta \approx 36.87^\circ}$$

- Edge direction $= 37^\circ$
- Used for histogram binning in HOG

Summary of Computation

Quantity	Value
G_x	60
G_y	45
Magnitude (M)	75
Orientation (θ)	36.87°

This vector contributes to the HOG histogram.

Exam Answer (How to Write)

Sample Answer:

The horizontal gradient G_x is computed by subtracting the left neighboring pixel intensity from the right neighboring pixel intensity. The vertical gradient G_y is computed using top and bottom neighbors. Gradient magnitude represents edge strength, and orientation represents edge direction, which is used for histogram binning in HOG.

Key Takeaway

- $G_x \rightarrow$ vertical edges
- $G_y \rightarrow$ horizontal edges
- Magnitude $\rightarrow$ edge strength
- Orientation $\rightarrow$ edge direction

HOG builds robust shape descriptors using these gradients.