$\begin{array}{c} A \\ Laboratory \ Report \\ on \end{array}$

Ultrasonic Diffraction Experiment

by

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Experiment No. 6

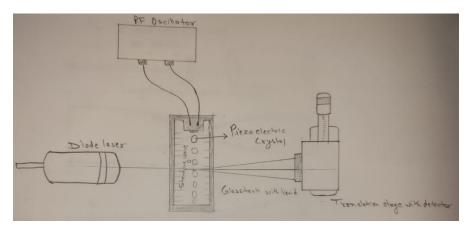
Ultrasonic Diffraction Experiment

I. Aim:

To determine the velocity of ultrasonic wave in liquid using ultrasonic diffraction apparatus, and hence measure the bulk modulus of the given liquid and estimate the compressibility of the liquid.

II. Apparatus:

Laser mount, diode laser with power supply, glass tank with liquid, glass tank holder, crystal with mount, RF oscillator, optical rail of length 1500 mm, cell mount with linear translation stage and pinhole detector, output measurement unit.



III. Theory:

Ultrasonic waves are generated by a transducer and propagate through a liquid medium. The reflections occur at the bottom of the cell, comprising a flat glass plate, leading to the formation of stationary or standing waves due to interference between incident and reflected waves. The velocity (v) of these ultrasonic waves in the liquid can be calculated using the equation,

$$v = f \times \Lambda$$

Where f is the frequency of the crystal oscillator, and Λ is the wavelength of the ultrasonic wave.

The wavelength Λ is further expressed as,

$$\Lambda = \frac{n \times \lambda}{\sin(\theta)}$$

Where n is the order of diffraction, λ is the wavelength of the laser used, and θ is the angle of diffraction. θ can be determined by the equation,

$$\theta = tan^{-1} \left(\frac{D}{L} \right)$$

Where D represents the order length and L is the distance from the crystal oscillator to the detector.

The bulk modulus β of the liquid is related to the density ρ and the velocity ν of the ultrasonic waves by $\beta = \rho \times \nu^2$.

The compressibility κ of the liquid can be calculated by the relation,

$$\kappa = \frac{1}{\beta} = \frac{1}{\rho \times v^2}$$

IV. Procedure:

- 1. First, clean all the glass equipment and fix the laser mount on the optical rail.
- 2. Fill the glass container with distilled water and place it on the tank holder and place the glass tank holder on the optical rail.
- 3. Place the laser on the laser mount and fix the crystal on the mount such that it is fully submerged in water and connect it to the RF oscillator.
- 4. Set up the cell mount with the moving part on the rail. Put in the pinhole detector and connect it to the measurement device.
- 5. Turn on the laser and the measurement tool. Make sure the laser beam runs parallel to the crystal's surface by adjusting their positions.
- 6. Use the adjustments on the laser mount to position the laser beam such that the standing wave forms.
- 7. Double-check that everything is lined up properly to get accurate measurements.
- 8. Ensure the laser spot stays on the detector stage and fine-tune the oscillator frequency for a distinct fringe pattern on both sides of the central bright spot.
- 9. Utilize the micrometer-driven stage to move the detector to the edges of the diffraction pattern.
- 10. Scan the pattern at close intervals of $0.05 \, mm$, and record the reading of the micrometer and corresponding value of the output detector.

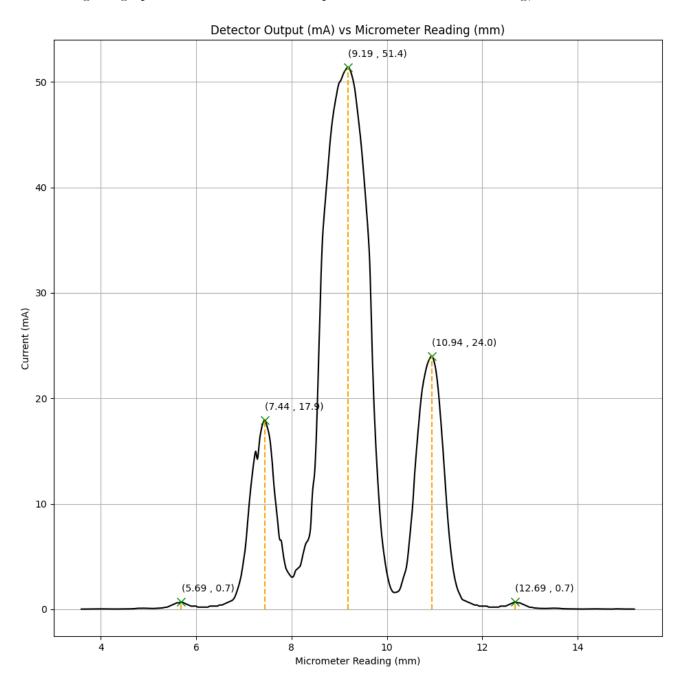
V. Observations, Graphs and Calculations:

We took around 230 readings and got the peak at the following values:

MSR	CSR	Micrometer reading (mm)	Detector Output
Mon	Con	Microffieter reading (film)	mA
5.5	19	5.69	0.7
7.0	44	7.44	17.9
9.0	19	9.19	51.4
10.5	44	10.94	24.0
12.5	19	12.69	0.7

(Images of these readings are in Section VIII)

Plotting the graph between the detector output and the micrometer reading,



From graph we get the following observations and calculations:

					Distance between			
Order		Distance f	rom central spot	to nth order spot	crystal and detector	Angle of ultrasonic diffraction	Wavelength (Λ)	V
n		D (L) (mm)	D (R) (mm)	D (Mean) (mm)	L (mm)	theta = tan^-1(D/L)	n*λ/sin(theta) (mm)	f*Λ (mm/s)
	1	1.75	1.75	1.75	1295	0.0013510375474717	0.48	1462580.08
	2	3.50	3.50	3.50	1321	0.0026505049632388	0.49	1491038.38
							v (mean) (m/s)	1476.80923
							σ (m/s)	14.22915

 $v = 1476.80923 \, m/s$

The standard deviation for velocity comes out to be $14.22915 \, m/s$.

The bulk modulus of liquid is given by, $\beta = \rho \times v^2$

Taking $\rho = 1000 \, kg/m^3$, β comes out to be,

$$\beta = 1000 \times (1476.80923)^2 Pa$$

 $\beta = 2.18097 \times 10^9 Pa$

The compressibility factor is given by $\kappa = \frac{1}{\beta}$,

$$\kappa = 0.45851 \times 10^{-9} Pa^{-1}$$

VI. Results, Discussion and Error Analysis:

To calculate the error,

$$\Lambda = \frac{n \times \lambda}{\sin(\theta)}$$

Taking magnitude of each term of log of both sides to compute maximum possible error,

$$ln(\Lambda) = ln(n) + ln(\lambda) + ln(sin(\theta))$$

Differentiating both sides,

$$\frac{\Delta\Lambda}{\Lambda} = \frac{\Delta\lambda}{\lambda} + \frac{\cos(\theta)}{\sin(\theta)} \Delta\theta$$

We know that,

$$\theta = tan^{-1} \left(\frac{D}{L} \right)$$
$$\Rightarrow tan(\theta) = \frac{D}{L}$$

Taking magnitude of each term of log of both sides,

$$ln(tan(\theta)) = ln(D) + ln(L)$$

Differentiating both sides,

$$\frac{sec^{2}(\theta)}{tan(\theta)}\Delta\theta = \frac{\Delta D}{D} + \frac{\Delta L}{L}$$

$$\Rightarrow \Delta\theta = \frac{tan(\theta)}{sec^{2}(\theta)} \times \left(\frac{\Delta D}{D} + \frac{\Delta L}{L}\right)$$

Substituting this in our equation for error in Λ ,

$$\frac{\Delta\Lambda}{\Lambda} = \frac{\Delta\lambda}{\lambda} + \frac{\cos(\theta)}{\sin(\theta)} \times \frac{\tan(\theta)}{\sec^2(\theta)} \times \left(\frac{\Delta D}{D} + \frac{\Delta L}{L}\right)$$
$$\Rightarrow \frac{\Delta\Lambda}{\Lambda} = \frac{\Delta\lambda}{\lambda} + \cos^2(\theta) \times \left(\frac{\Delta D}{D} + \frac{\Delta L}{L}\right)$$

The value of velocity of wave is given by the formula,

$$v = f \times \Lambda$$

Taking log and differentiating both sides, we get,

$$\Delta \mathbf{v} = \mathbf{v} \times \left(\frac{\Delta f}{f} + \frac{\Delta \Lambda}{\Lambda}\right)$$

Substituting the value of $\frac{\Delta\Lambda}{\Lambda}$ from above,

$$\Delta v = v \times \left(\frac{\Delta f}{f} + \frac{\Delta \lambda}{\lambda} + \cos^2(\theta) \times \left(\frac{\Delta D}{D} + \frac{\Delta L}{L} \right) \right)$$

Computing this Δv for both order 1 and order 2, by taking value of

$$\Delta f = 0.01 \, MHz$$
, $\Delta \lambda = 0 \, \text{nm}$ (given), $\Delta D = 0.01 \, mm$, $\Delta L = 1 \, mm$,

Frequency	Distance from central spot to nth order spot D (Mean) (mm)	•	Angle of ultrasonic diffraction theta = tan^-1(D/L)	ν f*Λ (mm/s)	Δv (m/s)
3.04	, ,, ,	,	,	1462580.08	14.29785
3.04	3.50	1321	0.0026505049632388	1491038.38	10.29395
				Δv (mean) (m/s)	12.29590

$$\Delta v = 12.29590 \, m/s$$

Similarly, computing the error for bulk modulus of liquid as well as the compressibility,

$$\beta = \rho \times v^{2}$$

$$ln(\beta) = ln(\rho) + 2ln(v)$$

$$\frac{\Delta\beta}{\beta} = \frac{\Delta\rho}{\rho} + 2\frac{\Delta v}{v}$$

Taking zero error in density,

$$\Delta\beta = 0.03632 \times 10^9 \ Pa$$

$$\kappa = \frac{1}{\beta}$$

Taking magnitude of log of each term,

$$ln(\kappa) = ln(\beta)$$

Differentiating,

$$\frac{\Delta \kappa}{\kappa} = \frac{\Delta \beta}{\beta}$$

$$\Delta \kappa = 0.00764 \times 10^{-9} \, Pa^{-1}$$

The velocity of ultrasonic wave comes out to be,

$$v = 1476.80923 \pm 12.29590 \, m/s$$

The bulk modulus of the liquid comes out to be,

$$\beta = (2.18097 \pm 0.03632) \times 10^9 Pa$$

The compressibility of the liquid comes out to be,

$$\kappa = (0.45851 \pm 0.00764) \times 10^{-9} Pa^{-1}$$

- Possible sources of errors:
 - 1. The velocity of ultrasonic wave depends on the temperature of the liquid, that could have led to a deviation from the expected value.
 - 2. We calculated the bulk modulus assuming density of the liquid to be constant, but density depends on temperature as well as pressure. And since compressibility of liquid is also dependent on the bulk modulus, both the values would have a slight error due to changes in temperature and pressure.
 - 3. Impurities in the liquid will cause scattering of the laser beam and could lead to false peaks.
 - 4. The value of wavelength of laser beam may have a range of wavelengths of light, that would introduce an error in wavelength, i.e., $\Delta \lambda \neq 0$ (which we assumed to be zero)

VII. Conclusion:

We conducted this experiment to determine the value of the velocity of ultrasonic wave in liquid, which comes out to be $v = 1476.80923 \pm 12.29590$ m/s, with a standard deviation of 14.22915 m/s, the value of bulk modulus of the liquid, which comes out to be $\beta = (2.18097 \pm 0.03632) \times 10^9 \, Pa$ and the value of compressibility of the liquid, which comes out to be $\kappa = (0.45851 \pm 0.00764) \times 10^{-9} \, Pa^{-1}$. We can repeat the experiment for different liquids at different temperatures to see the relation between the velocity of the ultrasonic wave and density and temperature of the liquid.

VIII. Readings Images:

MOD	000	Missassassassassassassassassassassassassa	Detector Ou	utput	
MSR	CSR	Micrometer reading (mm)	mA	<u>.</u> μΑ	
3.5	9	3.59	0.0131	13.1	
3.5	14	3.64	0.0153	15.3	
3.5	19	3.69	0.0193	19.3	
3.5	24	3.74	0.0210	21.0	
3.5	29	3.79	0.0224	22.4	
3.5	34	3.84	0.0209	20.9	
3.5	39	3.89	0.0285	28.5	
3.5	44	3.94	0.0317	31.7	
3.5	49	3.99	0.0374	37.4	
4.0	4	4.04	0.0383	38.3	
4.0	9	4.09	0.0302	30.2	
4.0	14	4.14	0.0274	27.4	
4.0	19	4.19	0.0235	23.5	
4.0	24	4.24	0.0214	21.4	
4.0	29	4.29	0.0154	15.4	
4.0	34	4.34	0.0207	20.7	
4.0	39	4.39	0.0232	23.2	
4.0	44	4.44	0.0286	28.6	
4.0	49	4.49	0.0297	29.7	
4.5	4	4.54	0.0271	27.1	
4.5	9	4.59	0.0287	28.7	
4.5	14	4.64	0.0427	42.7	
4.5	19	4.69	0.0499	49.9	
4.5	24	4.74	0.0734	73.4	
4.5	29	4.79	0.0927	92.7	
4.5	34	4.84	0.0961	96.1	
4.5	39	4.89	0.0990	99.0	
4.5	44	4.94	0.0904	90.4	
4.5	49	4.99	0.0776	77.6	
5.0	4	5.04	0.0715	71.5	
5.0	9	5.09	0.0696	69.6	
5.0	14	5.14	0.0804	80.4	
5.0	19	5.19	0.0865	86.5	
5.0	24	5.24	0.1083	108.3	
5.0	29	5.29	0.1277	127.7	
5.0	34	5.34	0.1851	185.1	
5.0	39	5.39	0.1993	199.3	
5.0	44	5.44	0.3		#we changed from µA to mA
5.0	49	5.49	0.4		
5.5	4	5.54	0.5		
5.5	9	5.59	0.6		
5.5	14	5.64	0.6		
5.5	19	5.69	0.7		
5.5	24	5.74	0.6		
5.5	29	5.79	0.5		
5.5	34	5.84	0.4		

				T
5.5	39	5.89	0.3	
5.5	44	5.94	0.3	
5.5	49	5.99	0.3	
6.0	4	6.04	0.2	
6.0	9	6.09	0.2	
6.0	14	6.14	0.2	
6.0	19	6.19	0.2	
6.0	24	6.24	0.2	
6.0	29	6.29	0.3	
6.0	34	6.34	0.3	
6.0	39	6.39	0.3	
6.0	44	6.44	0.3	
6.0	49	6.49	0.4	
6.5	4	6.54	0.4	
6.5	9	6.59	0.5	
6.5	14	6.64	0.6	
6.5	19	6.69	0.7	
6.5	24	6.74	0.8	
6.5	29	6.79	1.0	
6.5	34	6.84	1.5	
6.5	39	6.89	2.2	
6.5	44	6.94	3.1	
6.5	49	6.99	4.5	
7.0	49	7.04	6.2	
7.0	9	7.09	8.9	
	14		11.3	
7.0		7.14		
7.0	19	7.19	13.3	
7.0	24	7.24	14.9	
7.0		7.29	14.3	
7.0	34	7.34	16.4	
7.0	39	7.39	17.5	
7.0	44	7.44	17.9	
7.0	49	7.49	17.3	
7.5	4	7.54	16.3	
7.5	9	7.59	14.2	
7.5	14	7.64	11.3	
7.5	19	7.69	9.2	
7.5	24	7.74	6.9	
7.5	29	7.79	6.4	
7.5	34	7.84	4.8	
7.5	39	7.89	3.8	
7.5	44	7.94	3.4	
7.5	49	7.99	3.1	
8.0	4	8.04	3.1	
8.0	9	8.09	3.7	
8.0	14	8.14	3.9	
8.0	19	8.19	4.2	
8.0	24	8.24	5.2	

				T
8.0	29	8.29	6.1	
8.0	34	8.34	6.5	
8.0	39	8.39	7.3	
8.0	44	8.44	11.1	
8.0	49	8.49	13.3	
8.5	4	8.54	19.1	
8.5	9	8.59	27.1	
8.5	14	8.64	34.2	
8.5	19	8.69	37.7	
8.5	24	8.74	40.4	
8.5	29	8.79	43.3	
8.5	34	8.84	45.7	
8.5	39	8.89	47.4	
8.5	44	8.94	48.7	
8.5	49	8.99	49.8	
9.0	4	9.04	50.2	
9.0	9	9.09	50.8	
9.0	14	9.14	51.2	
9.0	19	9.19	51.4	
9.0	24	9.24	51.0	
9.0	29	9.29	50.2	
9.0	34	9.34	48.9	
9.0	39	9.39	47.0	
9.0	44	9.44	45.0	
9.0	49	9.49	42.6	
9.5	4	9.54	39.8	
9.5	9	9.59	36.8	
9.5	14	9.64	32.9	
9.5	19	9.69	24.7	
9.5	24	9.74	18.3	
9.5	29	9.79	13.7	
9.5	34	9.84	10.0	
9.5	39	9.89	7.1	
9.5	44	9.94	5.2	
9.5	49	9.99	3.7	
10.0	49	10.04	2.6	
10.0	9	10.04	1.9	
10.0	14	10.14	1.6	
10.0	19	10.19	1.6	
10.0	24	10.24	1.7	
10.0	29	10.29	2.1	
10.0	34	10.34	2.9	
10.0	39	10.39	3.6	
10.0	44	10.44	5.0	
10.0	49	10.49	7.3	
10.5	4	10.54	9.8	
10.5	9	10.59	13.3	
10.5	14	10.64	16.1	

10.5	19	10.69	18.7		
10.5	24	10.74	20.8		
10.5	29	10.79	22.1		
10.5	34	10.84	23.1		
10.5	39	10.89	23.7		
10.5	44	10.94	24.0		
10.5	49	10.99	23.5		
11.0	4	11.04	22.3		
11.0	9	11.09	20.2		
11.0	14		17.4		
11.0	19	11.19	14.3		
11.0	24		10.9		
11.0	29	11.29	7.9		
11.0	34	11.34	5.6		
11.0	39	11.39	3.8		
11.0	44	11.44	2.6		
11.0	49	11.49	1.8		
11.5	4	11.54	1.3		
11.5	9	11.59	0.9		
11.5	14	11.64	0.8		
11.5	19	11.69	0.7		
11.5	24		0.6		
11.5	29	11.79	0.5		
11.5	34	11.84	0.4		
11.5	39	11.89	0.4		
11.5	44	11.94	0.3		
11.5	49	11.99	0.3		
12.0	4	12.04	0.3		
12.0	9	12.09	0.3		
12.0	14	12.14	0.2		
12.0	19	12.19	0.2		
12.0	24	12.24	0.2		
12.0	29	12.29	0.2		
12.0	34	12.34	0.2		
12.0	39	12.39	0.3		
12.0	44	12.44	0.3		
12.0	49	12.49	0.3		
12.5	4	12.54	0.4		
12.5	9	12.59	0.5		
12.5	14	12.64	0.6		
12.5	19	12.69	0.7		
12.5	24		0.6		
12.5		12.79	0.6		
12.5	34		0.5		
12.5		12.89	0.4		
12.5	44	12.94	0.3		
12.5			0.1973	197.3	#we changed from mA to µA
13.0			0.1831	183.1	
15.0	1 4	15.04	0.1031	100.1	

13.0	9	13.09	0.1257	125.7	
13.0	14	13.14	0.1073	107.3	
13.0	19	13.19	0.0855	85.5	
13.0	24	13.24	0.0824	82.4	
13.0	29	13.29	0.0716	71.6	
13.0	34	13.34	0.0735	73.5	
13.0	39	13.39	0.0796	79.6	
13.0	44	13.44	0.0914	91.4	
13.0	49	13.49	0.1010	101.0	
13.5	4	13.54	0.0951	95.1	
13.5	9	13.59	0.0907	90.7	
13.5	14	13.64	0.0754	75.4	
13.5	19	13.69	0.0499	49.9	
13.5	24	13.74	0.0417	41.7	
13.5	29	13.79	0.0282	28.2	
13.5	34	13.84	0.0281	28.1	
13.5	39	13.89	0.0312	31.2	
13.5	44	13.94	0.0296	29.6	
13.5	49	13.99	0.0222	22.2	
14.0	4	14.04	0.0207	20.7	
14.0	9	14.09	0.0164	16.4	
14.0	14	14.14	0.0209	20.9	
14.0	19	14.19	0.0250	25.0	
14.0	24	14.24	0.0279	27.9	
14.0	29	14.29	0.0312	31.2	
14.0	34	14.34	0.0388	38.8	
14.0	39	14.39	0.0374	37.4	
14.0	44	14.44	0.0312	31.2	
14.0	49	14.49	0.0275	27.5	
14.5	4	14.54	0.0224	22.4	
14.5	9	14.59	0.0234	23.4	
14.5	14	14.64	0.0215	21.5	
14.5	19	14.69	0.0183	18.3	
14.5	24	14.74	0.0158	15.8	
14.5	29	14.79	0.0378	37.8	
14.5	34	14.84	0.0384	38.4	
14.5	39	14.89	0.0312	31.2	
14.5	44	14.94	0.0290	29.0	
14.5	49	14.99	0.0209	20.9	
15.0	4	15.04	0.0229	22.9	
15.0	9	15.09	0.0215	21.5	
15.0	14	15.14	0.0188	18.8	
15.0	19	15.19	0.0148	14.8	

And.

IX. Author Contribution:

Name	Roll Number	Contribution	Signature
Jaskirat Singh Maskeen	23110146	Plotting graph using Matplotlib, Error analysis, Finding possible sources of error and conclusion.	Jakint her
Nishchay Bhutoria	23110222	Index, Aim, Taking readings in lab, Cover page, Document structure, Theory.	Nach
Kavya Lavti	23110164	Document structure, Taking readings in lab, Procedure.	forthe
Kanhaiyalal	23110155	Noting readings, Apparatus Diagrams.	Kanhai Halal