Econometrics ECON 662D1

Assignment 2

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The estimates of the covariance matrices are presented below. Regression outputs, and a detailed code of how all estimates were obtained can be found in the notebook attached.

Standard OLS estimate

$$\widehat{\text{Var}}(\hat{\beta}) = \begin{pmatrix} 1.37412989e+04 & -1.14577856e+02 & 8.04444087e+00 & 6.76760297e+01 \\ -1.14577856e+02 & 1.10947664e+00 & -1.64873513e-01 & -4.22071133e-01 \\ 8.04444087e+00 & -1.64873513e-01 & 8.34828446e-01 & -1.68916430e-02 \\ 6.76760297e+01 & -4.22071133e-01 & -1.68916430e-02 & 5.74773180e-01 \end{pmatrix}$$

HC_0

$$\widehat{\text{Var}}(\hat{\beta}) = \begin{pmatrix} 1.24530797e+04 & -9.57197205e+01 & -3.72882783e+00 & 8.31958443e+01 \\ -9.57197205e+01 & 8.66960180e-01 & 6.63431413e-02 & -4.32988909e-01 \\ -3.72882783e+00 & 6.63431413e-02 & 5.68223212e-01 & 1.48052795e-01 \\ 8.31958443e+01 & -4.32988909e-01 & 1.48052795e-01 & 1.03751374e+00 \end{pmatrix}$$

HC_1

$$\widehat{\text{Var}}(\hat{\beta}) = \begin{pmatrix} 1.29719580e+04 & -9.97080422e+01 & -3.88419566e+00 & 8.66623378e+01 \\ -9.97080422e+01 & 9.03083521e-01 & 6.91074389e-02 & -4.51030114e-01 \\ -3.88419566e+00 & 6.91074389e-02 & 5.91899179e-01 & 1.54221662e-01 \\ 8.66623378e+01 & -4.51030114e-01 & 1.54221662e-01 & 1.08074348e+00 \end{pmatrix}$$

HC_2

$$\widehat{\text{Var}}(\hat{\beta}) = \begin{pmatrix} 1.31512097e+04 & -1.00699376e+02 & -3.45217061e+00 & 8.87534625e+01 \\ -1.00699376e+02 & 9.11276205e-01 & 7.00157549e-02 & -4.56216877e-01 \\ -3.45217061e+00 & 7.00157549e-02 & 6.06033330e-01 & 1.68856411e-01 \\ 8.87534625e+01 & -4.56216877e-01 & 1.68856411e-01 & 1.11947925e+00 \end{pmatrix}$$

HC_3

$$\widehat{\text{Var}}(\hat{\beta}) = \begin{pmatrix} 1.38970463e+04 & -1.05998019e+02 & -3.11952648e+00 & 9.47429613e+01 \\ -1.05998019e+02 & 9.58431782e-01 & 7.39758838e-02 & -4.80899607e-01 \\ -3.11952648e+00 & 7.39758838e-02 & 6.46927965e-01 & 1.92074060e-01 \\ 9.47429613e+01 & -4.80899607e-01 & 1.92074060e-01 & 1.20863960e+00 \end{pmatrix}$$

Newey-West HAC (p=2)

$$\widehat{\text{Var}}(\hat{\beta}) = \begin{pmatrix} 1.59509096e+04 & -1.22007262e+02 & -2.11055587e+01 & 1.03853863e+02 \\ -1.22007262e+02 & 1.08197167e+00 & 2.07669560e-01 & -5.89471279e-01 \\ -2.11055587e+01 & 2.07669560e-01 & 4.82978011e-01 & 2.08198238e-02 \\ 1.03853863e+02 & -5.89471279e-01 & 2.08198238e-02 & 1.15960685e+00 \end{pmatrix}$$

Newey-West HAC (p=3)

$$\widehat{\text{Var}}(\hat{\beta}) = \begin{pmatrix} 1.75247862e+04 & -1.34764371e+02 & -2.15287723e+01 & 1.13868420e+02 \\ -1.34764371e+02 & 1.19271959e+00 & 2.04323007e-01 & -6.73921285e-01 \\ -2.15287723e+01 & 2.04323007e-01 & 4.30557868e-01 & -1.02780512e-02 \\ 1.13868420e+02 & -6.73921285e-01 & -1.02780512e-02 & 1.20728311e+00 \end{pmatrix}$$

Newey-West HAC (p=4)

$$\widehat{\text{Var}}(\hat{\beta}) = \begin{pmatrix} 1.88068237e+04 & -1.45722111e+02 & -2.41397044e+01 & 1.21547593e+02 \\ -1.45722111e+02 & 1.28691700e+00 & 2.29229787e-01 & -7.48750302e-01 \\ -2.41397044e+01 & 2.29229787e-01 & 4.07763454e-01 & -2.93807103e-02 \\ 1.21547593e+02 & -7.48750302e-01 & -2.93807103e-02 & 1.23628549e+00 \end{pmatrix}$$

Newey-West HAC (p=5)

$$\widehat{\text{Var}}(\hat{\beta}) = \begin{pmatrix} 1.94330580e+04 & -1.52904486e+02 & -2.60800872e+01 & 1.22972767e+02 \\ -1.52904486e+02 & 1.35642975e+00 & 2.47375315e-01 & -7.89429377e-01 \\ -2.60800872e+01 & 2.47375315e-01 & 3.90543527e-01 & -4.47005969e-02 \\ 1.22972767e+02 & -7.89429377e-01 & -4.47005969e-02 & 1.20894407e+00 \end{pmatrix}$$

Newey-West HAC (p=6)

$$\widehat{\text{Var}}(\hat{\beta}) = \begin{pmatrix} 1.95867191e+04 & -1.56910995e+02 & -2.54800401e+01 & 1.22319317e+02 \\ -1.56910995e+02 & 1.40329499e+00 & 2.49113377e-01 & -8.16323658e-01 \\ -2.54800401e+01 & 2.49113377e-01 & 3.89521239e-01 & -4.05408728e-02 \\ 1.22319317e+02 & -8.16323658e-01 & -4.05408728e-02 & 1.17744467e+00 \end{pmatrix}$$

Newey-West HAC (p=20)

$$\widehat{\text{Var}}(\hat{\beta}) = \begin{pmatrix} 2.44866026e+04 & -2.10368186e+02 & -2.44172328e+01 & 1.61685511e+02 \\ -2.10368186e+02 & 1.88936709e+00 & 2.96454826e-01 & -1.27007475e+00 \\ -2.44172328e+01 & 2.96454826e-01 & 2.94300602e-01 & -6.61932821e-02 \\ 1.61685511e+02 & -1.27007475e+00 & -6.61932821e-02 & 1.40970965e+00 \end{pmatrix}$$

The estimates for the covariance matrix of OLS parameter estimate $\hat{\beta}$ presented above were calculated using different methods. The first 5 estimates—standard OLS, and the four HCCME estimates—used results of the standard OLS regression model, and were calculated directly using the following equations:

Standard OLS:	$s^2(X^TX)^{-1}$
HC_0:	$(X^TX)^{-1}X^T\hat{\Omega}X(X^TX)^{-1}$
HC_1:	$\frac{n}{n-k}(X^TX)^{-1}X^T\hat{\Omega}X(X^TX)^{-1}$, $n = 100, k = 4$
HC_2:	$(X^TX)^{-1}X^T\hat{\Omega}X(X^TX)^{-1}$
HC_3:	$(X^TX)^{-1}X^T\hat{\Omega}X(X^TX)^{-1}$

in which $\hat{\Omega}$ is appropriately defined for each of the HCCMEs. For those first 5 estimates, results were also compared with output from the statistical package used, and yield exactly the same results. Further note that, for the HCCMEs, separate regression models specifying the type of covariance could have been defined, and yield the exact same results.

For the Newey-West HAC estimates, one regression model was defined for each lag parameter considered. Results were obtained directly from the statistical package.

Standard error estimates for each of the parameters of $\hat{\beta}$ are obtained by taking the square root of the elements of the diagonal of the covariance matrix. Table 1 below summarizes the results of the standard erros for the estimate of the coefficient of x_3. Note that a detailed code of how the standard errors were obtained

can be found in the notebook attached.

Model Type	**Estimate of Standard Error**
Standard OLS	0.75813797
HCCME type 0 (HC_0)	1.01858418
HCCME type 1 (HC_0)	1.03958813
HCCME type 2 (HC_0)	1.05805447
HCCME type 3 (HC_0)	1.09938146
Newey-West HAC (p=2)	1.07685043
Newey-West HAC (p=3)	1.09876436
Newey-West HAC (p=4)	1.11188376
Newey-West HAC (p=5)	1.09951993
Newey-West HAC (p=6)	1.08510123
Newey-West HAC (p=20)	1.18731194

Table 1: Summary of Results