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# SEISMIC BEHAVIOUR OF MULTI-STOREY BUILDINGS WITH ASYMMETRIC SETBACK – THE EFFECT OF THE LEVEL AND THE DEGREE OF SETBACK

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**Abstract:** Multi-storey buildings with asymmetric setbacks possess both vertical and in-plan irregularities. The former is due to the abrupt changes in the plan dimension at the level of the setback while the latter results from the asymmetry between the upper tower portion and the lower base portion. These irregularities introduce mass, stiffness and resistance eccentricities in the structure. As a result, the structural behaviour under lateral seismic excitation in the direction perpendicular to the setback is coupled in translation-torsion, which could increase the demand on the structural response. This coupling behaviour has not been studied in detail, and its effect is largely ignored by many conventional seismic design methods. In this research, the effects of both the degree and level of setback on the seismic response of multi-storey building frames were evaluated. The degree of setback refers to the ratio of the plan area of the lower base portion to the plan area of the upper tower. An analytical study was carried out to identify the governing parameters of the structural behaviour and their effects on the overall response of setback buildings. A total of nine different configurations were examined. The study showed that both the degree and level of setback have a considerable influence on the overall seismic behaviour of asymmetric setback buildings loaded in the direction perpendicular to the setback.

## 1 Introduction

There is a growing interest for building structures with asymmetric setbacks because of their architectural and practical advantages. The characteristic setback configuration introduces both vertical and in-plan irregularities. The former is due to the abrupt changes in the plan dimension at the level of the setback, while the latter results from the asymmetry between the upper tower portion and the lower base portion. These irregularities introduce mass, stiffness and resistance eccentricities between the tower and the base. The behaviour of this building type under lateral earthquake-induced forces in the direction perpendicular to the setback is coupled in translation-torsion and this could significantly affect deformations in the resisting elements. Recent observations of building structures with asymmetric setbacks (Lew et al. 2010) revealed the need for a thorough study to understand the behaviour of this irregular building type and to develop methods for a safe and satisfactory design.

Previous studies on the torsional behaviour of buildings focused mostly on single-storey shear configurations (Kan and Chopra 1976). Results and conclusions from these studies can be extended to multi-storey buildings only under two conditions: 1) the stiffness of the resisting elements from one storey to another must bear a constant ratio over the height of the building, and 2) the centers of mass of the floors must lie on the same vertical line, i.e. building must be regular in elevation. These conclusions have influenced subsequent studies (Hejal and Chopra 1987), which have focused on the response of single-storey and multi-storey irregular building configurations that are regular in elevation. The seismic behaviour of these structural systems is reasonably understood and many simplified analysis techniques have been developed for use in design. In general, irregular multi-storey buildings that are regular in elevation are represented by a multi-storey regular building and an associated torsionally-coupled one-storey system. In other words, the seismic response of an irregular multi-storey building that is regular in elevation is obtained from a combination of the responses of regular multi-storey and irregular one-storey systems.

Asymmetric setback buildings have not been studied in great detail. The majority of these studies focused on their behaviour loaded seismically in the direction parallel to the setback. These studies also used 2D plane frame models and analysis so that torsional effects were not of concern. The very few studies that looked at the coupled translational-torsional behaviour of setback buildings loaded in the direction perpendicular to the direction of the setback (Duan and Chandler 1995; Khoury et al. 2005) used specific setback configurations whose results could not be extended to generic setback configurations. Jhaveri (1967) conducted one of the few analytical studies on this subject but his hypotheses were simplistic and hardly practical. He used the shear beam idealisation model and considered that the base portion of the building was regular. From this study, there was some suggestion that both the degree and level of setback could affect the base shear, base torque and displacements of the building in a proportional manner. Here, and throughout the present paper, the degree of setback refers to the ratio of the plan area of the lower base to the plan area of the upper tower, and the level of setback refers to the ratio of the height of the lower base portion to the total height of the structure. In all of the above-mentioned studies on setback buildings, none accounted for in-plan mass irregularity due to eccentricity with respect to the centers of mass of the floors in the base and the tower portions, which characterizes an asymmetric setback building.

This paper aims at identifying the governing parameters in the torsional response of asymmetric setback buildings under lateral earthquake-induced forces in the direction perpendicular to the setback. A two-storey asymmetric setback building model is defined and analyzed through the governing equation of dynamic motion. Nine configurations having different degrees and levels of setback are analyzed and the influence of the varying degree and level of setback on the seismic behaviour of asymmetric setback buildings is examined.

## 2 Definition of Building Model

The building models selected for the present study are moment-resisting steel frames. The setback structure is modeled as a two-system structure representing building sections below and above the setback level. Each sub-system is considered as regular in elevation. Based on the understanding of the behavior of the irregular multi-story building configuration with regular elevation, and its representation by the combined behavior of an irregular single-story and regular multi-story systems, a setback steel building system can be idealized as a two-story structure. Figure 1 illustrates elevation views of the asymmetric setback building model used in this study.

The buildings have one axis of symmetry in the direction parallel to the setback. The bay dimensions between gridlines A and B (shown as b in Figure 1) and gridlines 1 and 2 (indicated by d in Figure 1) are kept constant. Only the B-C bay width dimension is varied to define the different degrees of setback (c). In addition, the total height of the building, h, is kept constant but the relative heights of the lower base portion and upper tower portion are varied to allow different levels of setback ($0 \leq p \geq 1$).

## 3 Governing Equations of Motion

In order to determine the parameters influencing the seismic response of asymmetric setback building frames and to be able to obtain results applicable to any generic configuration, the governing dynamic equation of motion for the study model presented in the previous section is developed.

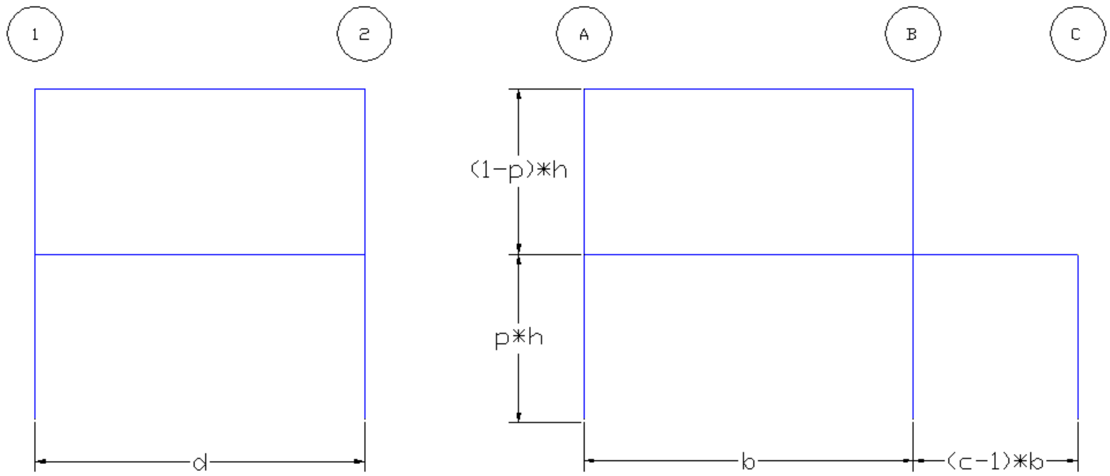

Figure 1 - Elevation views

Considering that the origin of the entire frame system is located at the center of mass of the first storey at ground level, and that the earthquake ground motion is applied in the *y*-direction, the system of the governing equations of motion can be obtained as:

[1] $$\begin{bmatrix} \mathbf{m} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{m} & \mathbf{e_m m} \\ \mathbf{0} & \mathbf{e_m m} & \mathbf{I_0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{u}}_\mathbf{x} \\ \ddot{\mathbf{u}}_\mathbf{y} \\ \ddot{\boldsymbol{\theta}}_\mathbf{z} \end{Bmatrix} + \begin{bmatrix} \mathbf{K_{xx}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{K_{yy}} & \mathbf{K_{yz}} \\ \mathbf{0} & \mathbf{K_{zy}} & \mathbf{K_{zz}} \end{bmatrix} \begin{Bmatrix} \mathbf{u_x} \\ \mathbf{u_y} \\ \boldsymbol{\theta}_\mathbf{z} \end{Bmatrix} = -\mathbf{m} \begin{Bmatrix} \mathbf{0} \\ \ddot{\mathbf{u}}_\mathbf{gy} \\ \mathbf{0} \end{Bmatrix}$$

where $\mathbf{m}$ is a mass matrix containing the floor masses, $\mathbf{e_m}$ is the mass eccentricity matrix between the centers of mass of the tower portion and the base portion, $\mathbf{I_0}$ is the matrix of the second moments of area of the floors about the origin of the system, $\mathbf{K_{ij}}$ are the stiffness matrices along the *x*-, *y*- and *z*-direction, and $\mathbf{0}$ is a null matrix. $\mathbf{u_x}$, $\mathbf{u_y}$, and $\boldsymbol{\theta}_\mathbf{z}$ are the deformation response vectors along the *x*-, *y*- and *z*-direction respectively, and $\ddot{\mathbf{u}}_\mathbf{gy}$ is the ground acceleration in the *y*-direction. All the displacements/rotation response quantities are measured from or about the origin of the system. Damping is ignored in this study. All matrices are of dimension *2x2* and the vectors *2x1*. Clearly, the behaviour in the *x*-direction is completely independent of the coupled behaviour in the *y*-*z*-plane and it is therefore ignored since only the coupled behaviour is of interest in this study.

The mass matrix of the system is composed of the masses and second moment area of each storey. The mass of the floors below and above the setback can be defined from the mass of the first floor m and the degree of setback, c, (Equation 2) while the second moment of area of the floors is computed as the product of the radius of gyration, r, squared and the mass of the floor (Equation 3).

[2] $$\mathbf{m} = \begin{bmatrix} m & 0 \\ 0 & \frac{1}{c} \cdot m \end{bmatrix}$$

[3] $$\mathbf{I}_0 = \begin{bmatrix} m \cdot r_1^2 & 0 \\ 0 & \frac{1}{c} \cdot m \cdot (r_2^2 + e_m^2) \end{bmatrix}$$

It can be observed from the governing equations of motion that the mass matrix of asymmetric setback structure is non-diagonal. This is due to the fact that the tower portion is not symmetrically located over the lower base portion, thereby introducing an in-plan mass eccentricity between the centers of mass of the tower and the base. To the best of the authors' knowledge, this unique characteristic has not been directly considered in previous studies. The complete mass matrix can therefore be assembled as

[4] $$\mathbf{M}=\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & \frac{1}{c}\cdot m & 0 & \frac{1}{c}\cdot m\cdot e_m \\ 0 & 0 & m\cdot r_1^2 & 0 \\ 0 & \frac{1}{c}\cdot m\cdot e_m & 0 & \frac{1}{c}\cdot m\cdot (r_2^2+e_m^2) \end{bmatrix}$$

The stiffness matrix is also coupled in the *y-z*-direction, and can be assembled as

[5] $$\mathbf{K}=\begin{bmatrix} K_{yy-11} & K_{yy-12} & K_{yz-11} & K_{yz-12} \\ K_{yy-21} & K_{yy-22} & K_{yz-21} & K_{yz-22} \\ K_{zy-11} & K_{zy-12} & K_{zz-11} & K_{zz-12} \\ K_{zy-21} & K_{zy-22} & K_{zz-21} & K_{zz-22} \end{bmatrix}$$

The subscript *ij-kl* for the various terms in the stiffness matrix represents the directions (ij) and the floor levels (kl) respectively. Previous studies concerning single-storey shear buildings (Tso and Dempsey 1980; Humar and Kumar 1998) concluded that the stiffness parameters for torsionally sensitive irregular one-storey building systems are function of, or influenced by the in-plan stiffness eccentricity and the torsional stiffness radius. The following sections present the definitions of these two parameters for the multi-storey asymmetric setback buildings studied.

### 3.1 In-plan stiffness eccentricity

The in-plan stiffness eccentricity is an important parameter in the understanding of the torsional behaviour of the system. In general, this refers to the distance between the center of mass and the center of rigidity of a floor. For a single-storey building configuration, the center of rigidity is defined as the point on the floor through which the application of lateral load will result in pure translation of the floor in the direction of the applied force without any rotation. This definition can be applied to a special class of multi-storey building configurations that are proportionate and regular in elevation (Hejal and Chopra 1987). For this case, the centers of rigidity of the structure lie on the same vertical line and their locations are independent of the external lateral loading. The building is also characterized by two principal axes through which lateral load action results in pure translational motion.

For multi-storey buildings that do not meet the above-mentioned requirements, the center of rigidity may be defined by a set of points located at floor levels such that when the given distribution of lateral loading passes through them, no rotational movement of the building about the vertical axis (z-direction) will occur (Cheung and Tso 1986). In this case, the locations of the centers of rigidity along the height of the building structure are dependent on the distribution of the applied lateral load and principal axes do not exist. This difference in the definition of the center of rigidity, and thus the stiffness eccentricity, makes the analytical problem of multi-story buildings complicated. Consequently, the torsional sensitivity criteria in many building codes, including the National Building Code of Canada (NBCC), were developed from the analysis of a single-storey shear building, making them inapplicable to asymmetric setback multi-storey buildings.

The selected models in the present study, composed uniquely of moment-resisting frames, are proportionate but irregular in elevation. The system thus has no principal axes and the centers of rigidity do not coincide on a single vertical line along the height of the building. This can be deduced from the eccentricity matrix, which consists of the eccentricity parameters defined by

[6] $$e=K_{yy}^{-1}\cdot K_{yz}$$

This above definition of the eccentricity parameter is applicable to single-storey buildings and proportionate multi-storey buildings that are regular in elevation. An attempt can, however, be made to define an eccentricity matrix for any structural configuration. It can be shown that the locations of the centers of rigidity for any structural system is given by

[7] $$x_{ri} = e_{xii} + \sum_{\substack{j=1 \\ j \neq i}}^{N} e_{xij} \frac{F_{yj}}{F_{yi}}, \quad y_{ri} = e_{yii} + \sum_{\substack{j=1 \\ j \neq i}}^{N} e_{yij} \frac{F_{xj}}{F_{xi}}$$

where $e_{xii}$ are the diagonal terms of the eccentricity matrix obtained from Equation (6), $e_{xij}$ are non-diagonal terms with subscripts ij corresponding to floor levels, and $F_{yj}$ and $F_{yi}$ are lateral forces at floors I and j in the y direction. Values for the x-direction are obtained by interchanging x and y. From Equation (7), it can be deduced that the off-diagonal terms influence the location of the center of rigidity depending on the ratio of the lateral loading at the storey of interest to the storeys below and above ($\frac{F_{xj}}{F_{xi}}$). For these reasons, the parameter *e* defined above is used to determine the location of the centers of rigidity in the structural system. The off-diagonal terms will be considered separately from the diagonal terms in order to assess their relative influence on the variation of the location of the center of rigidity. In this paper, the locations of the centers of rigidity directly identify the eccentricity parameter as the origin of the system is taken as the center of mass of the base.

### 3.2 Torsional stiffness radius

Another important parameter that can be defined from the stiffness matrix is the torsional stiffness radius. This parameter has been used to explain the dynamic behaviour of mono-symmetric single-storey buildings and to present code requirements and criteria to assess the torsional sensitivity of building structures. As for the in-plan stiffness eccentricity, the torsional stiffness radius is derived from the analysis of a single-storey shear building and cannot be directly applied to multi-storey systems, except if they meet the conditions of proportionality and regularity in elevation. It can, however, be shown that the square of the torsional stiffness radius (Equation 8)

[8] $$\rho_k^2 = K_{yy}^{-1} \cdot K_{Rzz}$$

can be defined for any structural system, where $K_{Rzz}$ is the torsional stiffness about the center of stiffness of the building. For multi-storey buildings that do not meet the conditions of proportionality and regularity in elevation, the square of the torsional stiffness radius obtained is a non-diagonal matrix. The torsional stiffness radius is a direct indicator of the response of an irregular building. For translational ground motion excitation, if the torsional stiffness radius is greater than the radius of gyration, the response will be predominantly in translation, whereas if the torsional stiffness radius is smaller than the radius of gyration, large torsional behaviour can be expected. According to Makarios (2008), this is irrespective of the stiffness eccentricity.

### 3.3 Governing parameters

Based on the two parameters defined above (i.e. the in-plan stiffness eccentricity and the torsional stiffness radius), the stiffness matrix can be simplified as

[9] $$\mathbf{K} = \begin{bmatrix} \mathbf{K_{yy}} & \mathbf{K_{yy} \cdot e} \\ \mathbf{(K_{yy} \cdot e)'} & \mathbf{K_{yy} \cdot (\rho_k^2 + e^2)} \end{bmatrix}$$

where $\mathbf{(e \cdot K_{yy})'}$ in the transpose of $\mathbf{e \cdot K_{yy}}$.The complete system of equations of dynamic motion for the asymmetric setback building structure in the *y*- and *z*-directions can therefore be expressed as

[10] $$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & \frac{1}{c} \cdot m & 0 & \frac{1}{c} \cdot m \cdot e_m \\ 0 & 0 & m \cdot r_1^2 & 0 \\ 0 & \frac{1}{c} \cdot m \cdot e_m & 0 & \frac{1}{c} \cdot m \cdot (r_2^2 + e_m^2) \end{bmatrix} \begin{Bmatrix} \ddot{u}_{y1} \\ \ddot{u}_{y2} \\ \ddot{\theta}_{z1} \\ \ddot{\theta}_{z2} \end{Bmatrix} + \begin{bmatrix} K_{yy-11} & K_{yy-12} & K_{yz-11} & K_{yz-12} \\ K_{yy-21} & K_{yy-22} & K_{yz-21} & K_{yz-22} \\ K_{zy-11} & K_{zy-12} & K_{zz-11} & K_{zz-12} \\ K_{zy-21} & K_{zy-22} & K_{zz-21} & K_{zz-22} \end{bmatrix} \begin{Bmatrix} u_{y1} \\ u_{y2} \\ \theta_{z1} \\ \theta_{z2} \end{Bmatrix} = - \begin{Bmatrix} m \cdot \ddot{u}_{gy1} \\ \frac{1}{c} \cdot m \cdot \ddot{u}_{gy2} \\ 0 \\ 0 \end{Bmatrix}$$

It can be deduced from this equation that the coupled seismic behaviour of asymmetric setback multi-storey building structures is governed by the degree of setback, c, the mass eccentricity, $e_m$, the mass of the floors, m, the stiffness eccentricity, *e,* the torsional stiffness radius squared, $\rho_k^2$, and the lateral rigidity in the *y*-direction, $K_{yy}$. While it is not explicitly shown in Equation (10), the response of the system is also a function of the level of setback, *p*, as it influences the stiffness matrices of the structural resisting elements as well as the mass distribution over the height of the building. Except for the in-plan mass eccentricity and the degree and level of setback, all the other governing parameters identified above are the same as those governing the behaviour of irregular single-storey buildings. The effects of the variation of these newly identified parameters on the behaviour on the model building configuration are discussed in this study.

## 4 Modeling and numerical study

Nine different geometric configurations of a two-storey asymmetric setback building structure are selected for analysis and parametric studies. An undamped free vibration system was considered and based on the equations of motion derived in the previous section, a MATLAB code was developed to assemble the stiffness and mass matrices, solve the eigenvalue problem and compute the modal quantities, such as the base shear and torque, shear and torque at the level of setback and displacements – for the irregular asymmetric setback systems as well as for corresponding equivalent regular structural systems. The equivalent regular models were developed using the uncoupled modal properties of the corresponding asymmetric setback systems. For the purpose of the parametric study, both a flat spectrum of constant spectral acceleration of magnitude 1 g, and a hyperbolic spectrum, for which the spectral acceleration is inversely proportional to the period, is used for the modal analysis. The hyperbolic spectrum was adjusted so that the smallest period is equivalent to a spectral acceleration of 1 g.

The ratio of the distance between gridlines 1 and 2 and gridlines A and B is considered equal to unity. Furthermore, the flexural rigidity of the beams and columns in the tower is half the flexural rigidities in the base. Since the study uses a steel structure, the ratio between the moment of inertia with respect to the strong axis and the weak axis for the columns is assumed as 10, and the beam to column inertia ratio, with respect to the strong axis, is equal to 0.5. The slenderness ratio, that is, the ratio of the total height of the building to the tower bay width (gridlines A-B) is taken as 4. A parameter $f_m$ is also introduced to further simplify the problem and corresponds to the ratio of the flexural rigidity of the columns of the first storey with respect to the strong axis to the mass of the first floor. The value for this parameter is obtained using the drift limit of *0,025h* proposed in the NBCC 2010 and the equation for the displacement of a single storey system under a static load. For the definition of this parameter, the spectral acceleration is considered to be constant and equal to 1g, and the response modification factors are selected as $R_D = 5$ and $R_0 = 1.5$, representing the ductility and overstrength related modification factors, respectively. Finally, the degrees of setback are set equal to 1.2,1.4 and 1.6, and the levels of setback equal to 0.25, 0.50 and 0.75 of the total height, allowing nine different configurations to be modeled and studied.

## 5 Results and discussion

The results presented in this paper include the normalized displacements of the stiff and flexible edges as well as the normalized eccentricity and torsional stiffness radius. The stiff edges are located on gridline C and B for the first and second floor respectively, while the flexible edge is located on gridline A (Figure 1). The normalized displacements are defined by the ratio of the displacements of the asymmetric setback building over the displacements of the regular equivalent building. The normalized eccentricity matrix is obtained by dividing the terms of the eccentricity matrix (Equation (6)) by the distance between gridlines A and C. To better assess the variation of the torsional stiffness radius, results from Equation (8) are normalized by the square of the radius of gyration of their respective floors. This quantity is also referred to as the torsional flexibility parameter. The upper terms – i.e. terms (1-1) and (1-2) – are divided by the radius of gyration of the first floor while the lower terms – terms (2-1) and (2-2) – are divided by the radius of gyration of the second floor. For the sake of brevity, only the displacements obtained from the hyperbolic spectrum are presented as they represent more realistic cases. The results for base shear and torque as well as shear and torque at the level of setback are not presented in this paper for the reason that displacements allow a better assessment of the torsional behaviour (Dempsey and Tso 1982).

### 5.1 Displacements

For the flexible edge (Figure 2a), the normalized displacements for both the first and second floor tend to decrease with increasing degree of setback. For these cases, the increase in rotational stiffness due to a greater lever arm for the peripheral resisting elements in the *y*-direction is greater than the increase in the mass of the first floor and therefore leads to the decrease in the normalized displacements. Figure 2a also shows that the level of setback has a significant effect on the flexible edge normalized displacements. The normalized displacements of the first and second floor are similar for p = 0.75 as the inter-storey height is small, and tend to become different as the level of setback decreases. The flexible edge normalized displacements in asymmetric setback building structures are significantly greater than the displacements that can be observed in regular buildings as indicated by the observed range of normalized displacements from 1.05 to 1.55.

For the stiff edge (Figure 2b), it is first important to remind that the displacements for the first and the second storey correspond to two different resisting elements. Therefore, the distance between the resisting elements of the first floor and the origin increases with an increasing degree of setback, while the distance resisting elements of the second floor and the origin increases with an increasing degree of setback, which explains why an increase in the degree of setback tends to increase the normalized displacements of the second floor, while decreasing those at the first floor. Furthermore, the level of setback does not seem to have a significant effect on the normalized displacements at the stiff edge. Finally, it is observed that the stiff edge displacements in an asymmetric setback building are smaller than those observed for a regular building (i.e. the normalized displacements are less than 1).

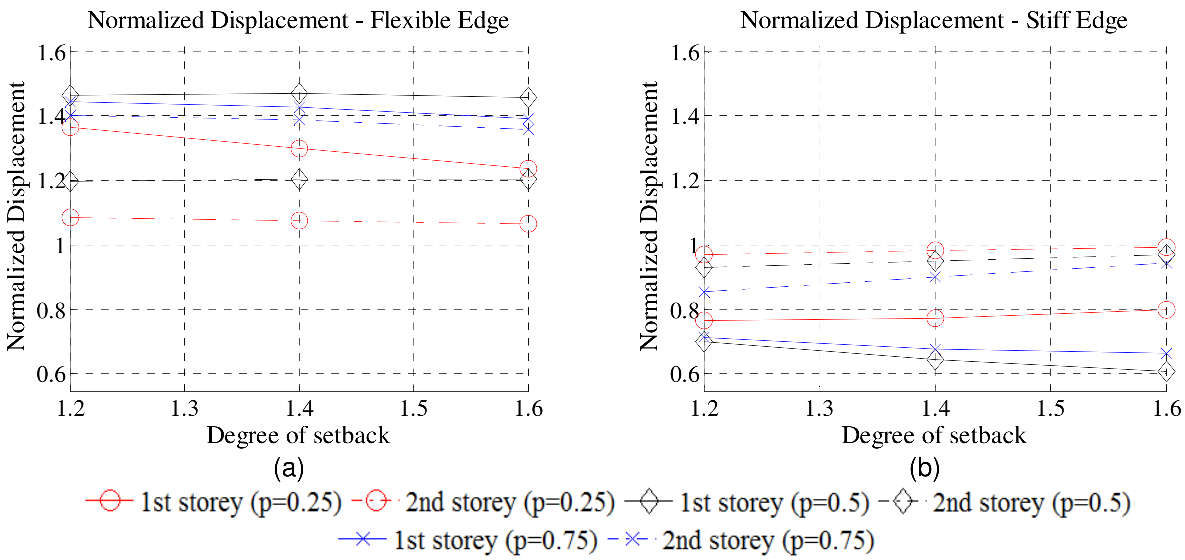


Figure 2 – (a) Normalized flexible edge displacements as a function of the level and degree of setback (hyperbolic spectrum), (b) stiff edge displacements as a function of the level and degree of setback (hyperbolic spectrum)

### 5.2 In-plan Stiffness Eccentricity

Figure 3 below illustrates the variation of three of the four terms of the in-plan stiffness eccentricity matrix (Equation (6)) as a function of varying degree and level of setback. Figure 3a presents the variation of the in-plan stiffness eccentricity of the first floor, Figure 3b presents the variation of the off-diagonal term of Equation (6) while Figure 3c presents the variation of the in-plan stiffness eccentricity of the second floor. The fourth term, the upper right term of the matrix, is not presented since it is always equal to zero for all configurations that are proportional. It is noteworthy that for concentric setbacks, the eccentricity matrix is diagonal, with the asymmetry in the configuration captured by only a non-zero lower left term in the matrix.

As can be seen, the upper left diagonal term (1-1) of the eccentricity matrix tends to decrease with increasing degree of setback because, as the degree of setback increases, the resisting element located on gridline C (Figure 1) moves away from the tower portion (gridlines A and B) of the structure, making the in-plan stiffness eccentricity of the first floor move towards the center of mass. On the other hand, the lower right diagonal term (2-2) of the eccentricity matrix tends to increase with increasing degree of setback. It must be mentioned here that values for this term are negative since these eccentricities are located on the left side of the origin of the system. They are presented in absolute term to better represent its behaviour. Since the resisting elements are arranged symmetrically in the upper tower portion, the location of the center of stiffness for that story also corresponds to the location of its center of mass. Therefore, term (2-2) of the in-plan stiffness eccentricity matrix also corresponds to the in-plan mass eccentricity. It can be observed from Figure 3a and Figure 3c that the level of setback has no influence on the diagonal terms of the in-plan stiffness eccentricity.

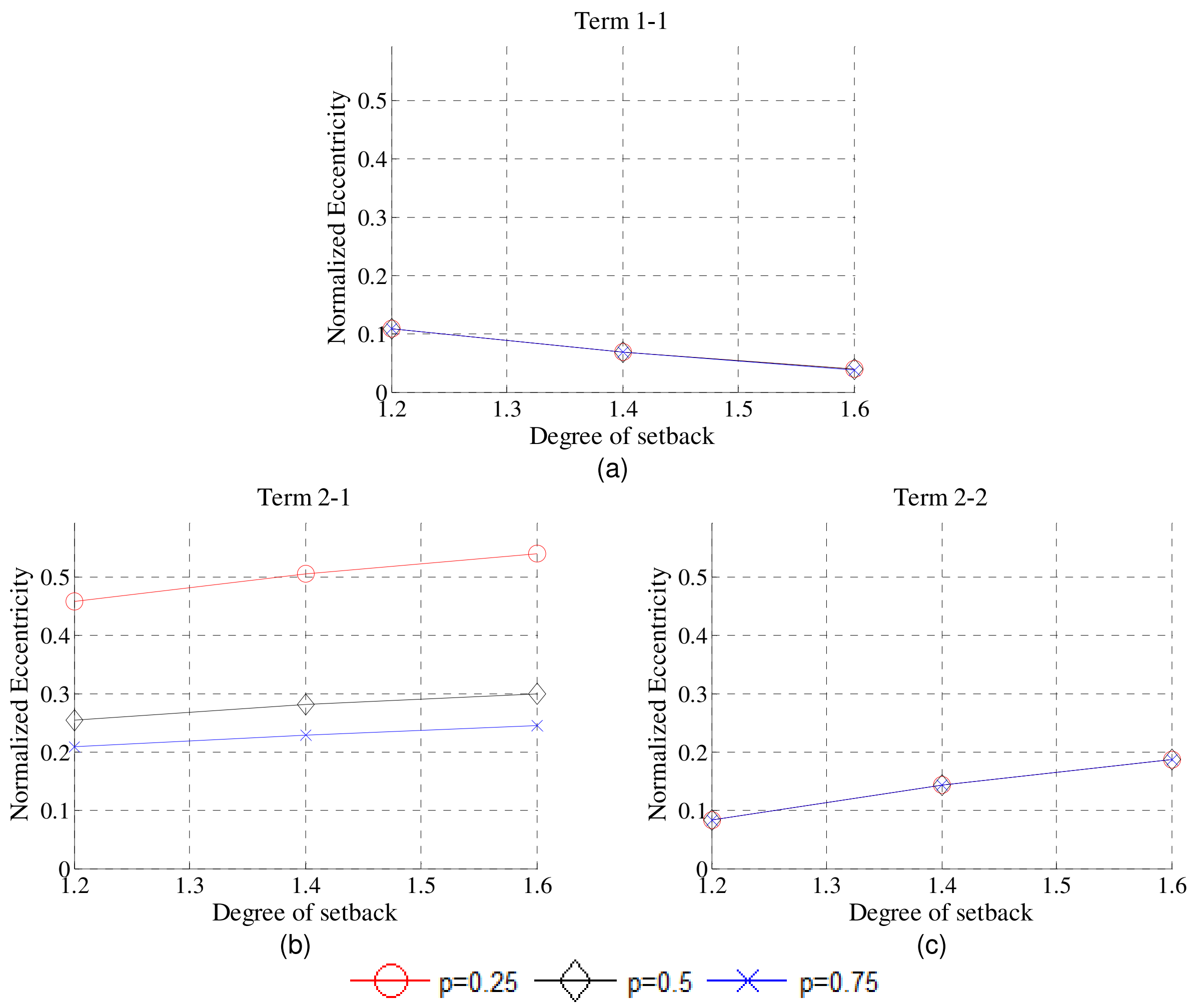


Figure 3 - In-plan stiffness eccentricities as a function of the level and degree of setback (a) Term (1-1), (b) Term (2-1), (c) Term (2-2)

As for the off-diagonal term which represents the relative eccentricity between the second floor and the first floor, the values decrease with increasing level of setback. This is because as the level of setback increases, the building becomes more regular in elevation with respect to the base portion. It is reminded that, as it was presented previously that the off-diagonal terms influence the location of the center of rigidity (Equation (7)). From the results presented in Figure 3, it can be concluded that the eccentricity term (1-1) of the first floor actually corresponds to the elastic center of the first floors as it is not influenced by the relative stiffness of the second floor (term (1-2)). Finally, as opposed to the diagonal terms, both the degree and level of setback significantly affect the off-diagonal term.

### 5.3 Torsional stiffness radius

The results obtained for the square of the torsional stiffness radius (Equation (8)) divided by the square of the radius of gyration of respective floors, are presented in this section. Each of the four terms of this matrix is plotted in Figure 4 as a function of the degree and level of setback. Terms presented in Figure 4a and Figure 4d correspond to the torsional flexibility parameter of the first and second floors respectively while the terms in Figure 4b and Figure 4c correspond to the relative torsional flexibility between the floors. As for the off-diagonal terms of the in-plan stiffness eccentricity matrix, they represent a relative relationship between the two floors.

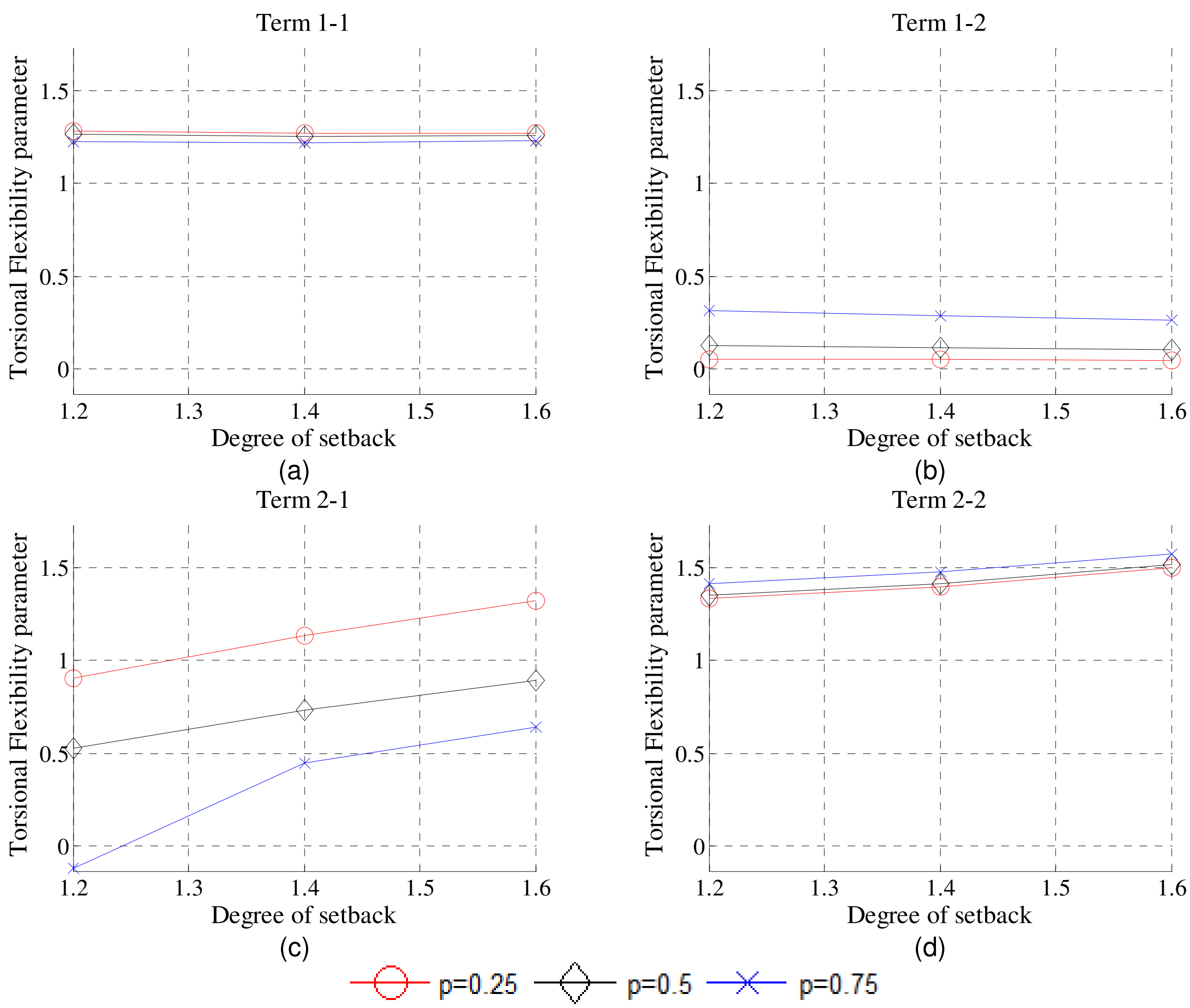


Figure 4 - Torsional flexibility parameters as a function of the level and degree of setback (a) Term(1-1), (b) Term (1-2), (c) Term (2-1), (d) Term (2-2)

If a torsional flexibility parameter is greater than unity, it means that the structural system is torsionally stiff and vice versa. From this definition, it can be deduced that the selected models are insensitive to torsion as the diagonal terms (Figure 4a and Figure 4d) are greater than 1 for all degrees and levels of setback.

The variation of the degree of setback has a significant effect on the lower left and right terms of the torsional flexibility parameter matrix (Figure 4c and Figure 4d). They increase as the degree of setback increases. On the opposite, terms (1-1) and (1-2) (Figure 4a and Figure 4b) are insensitive to the variation of the degree of setback. In addition, the level of setback has a significant effect on the variation of the values of the off-diagonal terms (Figure 4b and Figure 4c), while it is moderate on the diagonal terms. When the level of setback increases, the effects on the first floor will be less important as the building is becoming more regular with respect to this first floor. On the other hand, as the level of setback increases, the effects on the second floor will be greater as the first floor draws closer to the second floor. This observation is reflected as a decrease in the values obtained for the left-hand side terms (Figure 4a and Figure 4c), and increase for the right-hand side terms.

## 6 Conclusions

This paper has presented the results of a parametric study aimed at identifying the governing parameters in the torsional response of multi-storey asymmetric setback buildings under earthquake ground motions in the direction perpendicular to the setback. It also investigated the influence of varying level and degree of setback on the seismic behaviour of this type of building configurations. In total, nine different geometric configurations of a two-storey moment resisting steel frames were modeled and analysed. It was observed that the parameters governing the response of multi-storey asymmetric setback buildings are largely similar to those for irregular single-storey systems, with an additional effect from the in-plan mass eccentricity created by the asymmetric geometric configuration. The governing parameters are found to be dependent of the level and degree of setback. It was observed that the effects of increasing degree of setback lead to increased and decreased displacements of the flexible and stiff edges respectively in the asymmetric setback building structures, reduced in-plan stiffness eccentricity of the first floor and increased eccentricity of the second floor. An increasing degree of setback generally decreases the torsional flexibility of the first floor and increases the torsional flexibility of the second floor. On the other hand, an increasing level of setback has a significant effect on the flexible edge displacements while have a moderate effect on the stiff edge displacements, and also influence only the off-diagonal terms of both the eccentricity matrix and the torsional stiffness radius matrix. Finally, it was observed that the stiffness eccentricity of the second floor corresponds to the in-plan mass eccentricity between the upper and the base portions.

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