Assessing two-stage Approximation Algorithms for the Routing and Wavelength Assignment Problem

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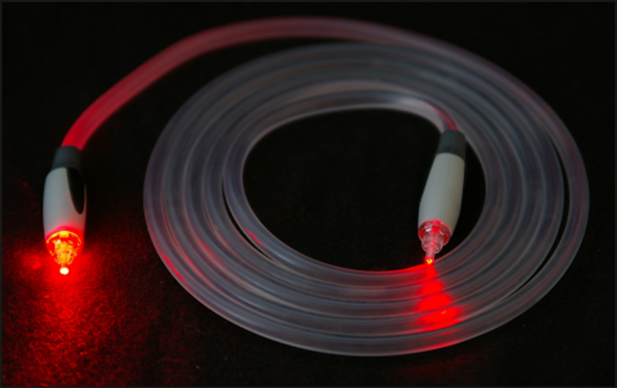
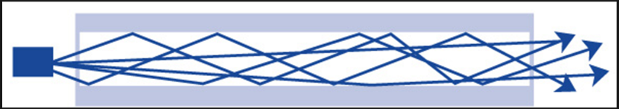
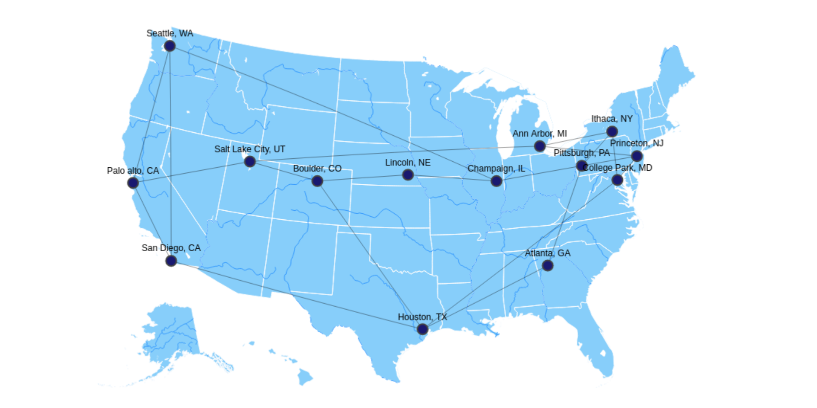
`GitLab [RL4RWA]`

The Routing and Wavelength Assignment Problem

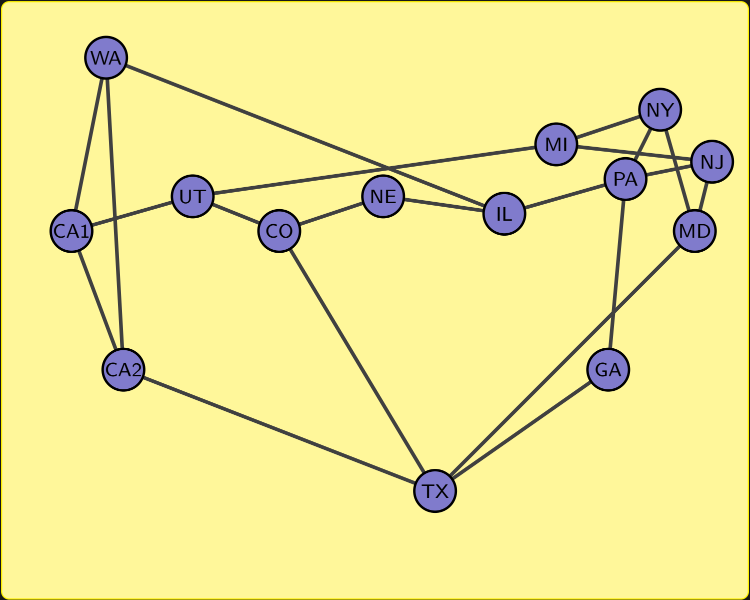




The National Science Foundation Network (NSFNET)



NSFNET



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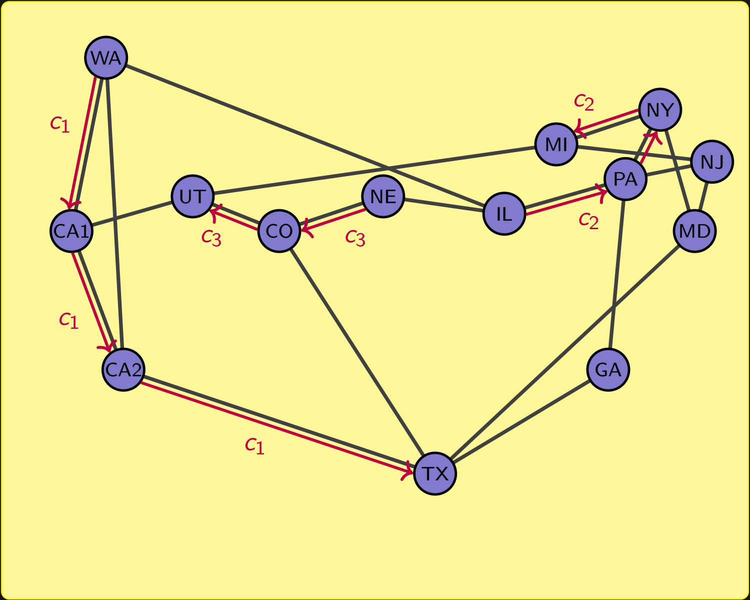
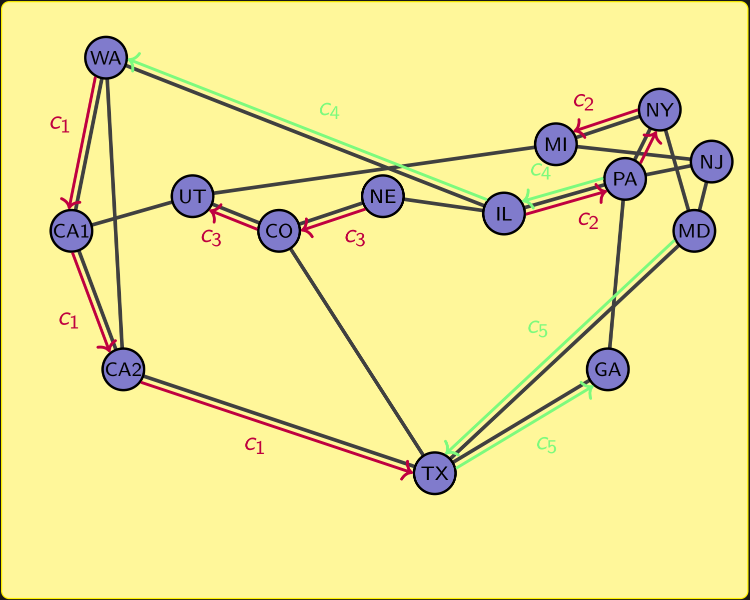


NSFNET



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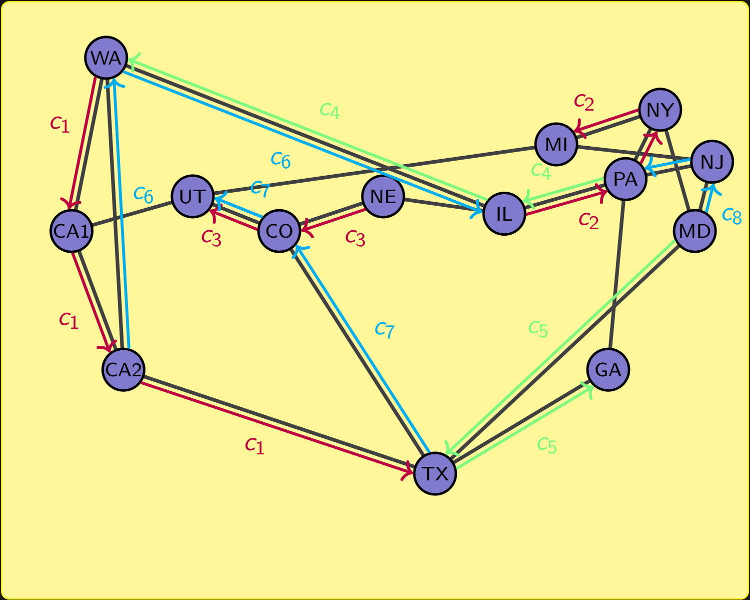
NSFNET



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Definition RWA

The Routing and Wavelength Assignment Problem

1. Multicommodity flow problem
2. Distinct wavelength assignment (clash)
3. Wavelength continuity

Definition RWA

Multicommodity Flow Problem

Graph $G = (V, E)$

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Demands $C = \{c_1, \dots, c_n\}$ with $c = (s_c, d_c) \forall c$

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Definition RWA

Multicommodity Flow Problem

Graph $G = (V, E)$

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Paths $P = \{p_{c_1}^1, \ldots, p_{c_1}^k, \ldots, p_{c_n}^1, \ldots, p_{c_n}^k\}$

Wavelengths $\Lambda = \{\lambda_1, \ldots, \lambda_l\}$

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Multicommodity Flow Problem

Graph $G = (V, E)$

Demands $C = \{c_1, \dots, c_n\}$ with $c = (s_c, d_c) \forall c$

Paths $P = \{p_{c_1}^1, \dots, p_{c_1}^k, \dots, p_{c_n}^1, \dots, p_{c_n}^k\}$

Wavelengths $\Lambda = \{\lambda_1, \dots, \lambda_l\}$

Goal: Given above resources, maximize the amount of concurrent demands that can be supported in the network

Definition RWA

Problem Formulation

* Distinct wavelength assignment (clash)

  $\forall\, e \in E$, for each $\lambda \in \Lambda$ at most one path is supported
* Wavelength continuity

  Every p_c with demand c is assigned to a single $\lambda \in \Lambda$ or to ε

Lightpath: a tuple (p_c, λ) comprising a path p_c and a wavelength λ

Definition RWA

Optimization Function

Let $P^+ \subseteq P$ be the set of accepted paths, where a connection c is provisioned by at most a single p_c^+ or not at all

$$f(P^+) = |P^+| + \frac{1}{1 + \sum_{p^+ \in P^+} |p^+|}$$

Then optimize $Z = \max f(P^+)$

Definition RWA

Research Question

“What is the approximation ratio of the best performing reinforcement learning algorithms compared to the best performing non-reinforcement learning algorithms?”

Implementation Model

Model Assumptions

Vertices can be endpoint or transit to multiple connections

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Paths $k = 25$

Implementation Model

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Time Complexity RWA $\in$ NP-Complete and P $\neq$ NP

Implementation Model

Simulation Model Parameters

$|V| = 14, |E| = 21$

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$$|C| \sim U\left(\left\lceil \sqrt{|V| \cdot |\Lambda|} \right\rceil, 2\left\lceil \sqrt{|V| \cdot |\Lambda|} \right\rceil\right) = U(15, 30)$$

Implementation Model

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$$|\Lambda| = \left\lceil \frac{|V|}{\sqrt{|E|}} \right\rceil = 4$$

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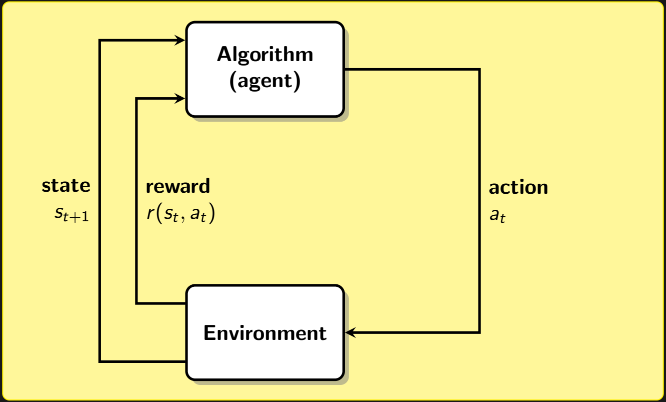
C using random sampling pairs from V

Implementation Model

Residual WA	Assignment of unassigned wavelengths
RL WA Algorithms	Q-Learning algorithm, Deep Q-Learning algorithm, A2C Algorithm, PPO Algorithm
Base WA Algorithms	Random Base, Shortest-Path Base
Routing	Yen's K-shortest Paths Algorithm

Implementation Model



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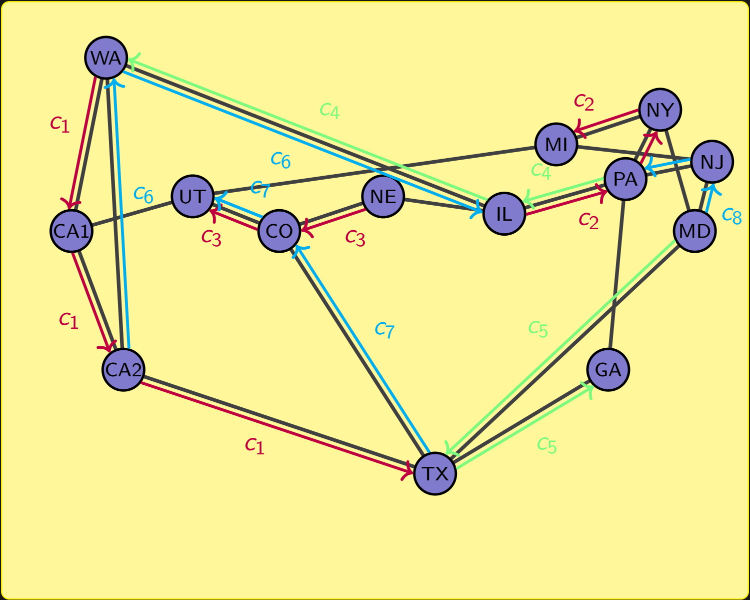

Implementation Model

State space S

Short: $\{\lambda, \lambda, \lambda, \lambda, \lambda, \lambda, \lambda, \lambda, \varepsilon, \varepsilon\}$ with $\dim(S) = (|\Lambda| + 1)^{|C|}$

Binary: $\{1, 1, 1, 1, 1, 1, 1, 1, 0, 0\}$ with $\dim(S) = 2^{|C|}$

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Binary: $\{1, 1, 1, 1, 1, 1, 1, 1, 0, 0\}$ with $\dim(S) = 2^{|C|}$

Action space A

$\forall\ c \in C$

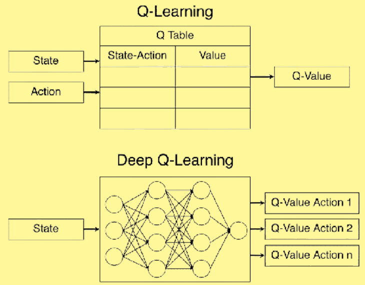
Add path of c opt

Add path of c prob

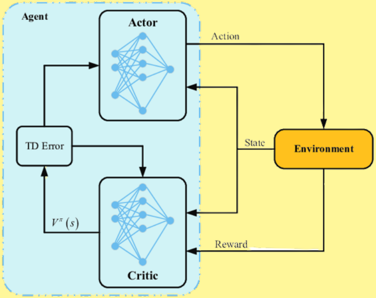
Remove any path of c

with $\dim(A) = 3 \cdot |C|$

Q-learning & Deep Q-learning



Advantage Actor Critic & Proximal Policy Optimization



Other algorithms and the MCF Upper Bound

Multicommodity flow upper bound

$|C| + \frac{1}{1+\sum_{c\in C}|p_c^1|}$

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Integer Linear Programming

Solving ILP with GEKKO's Advanced Process OPTimizer (APOPT) solver

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Integer Linear Programming

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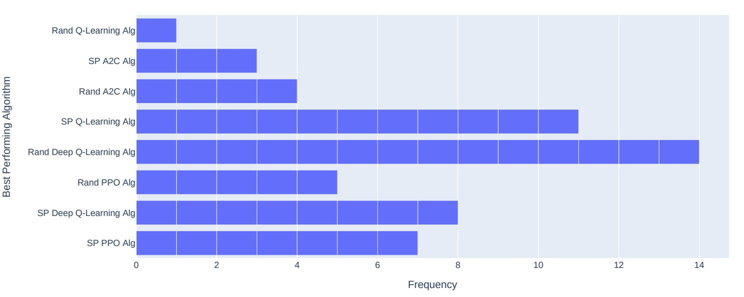
Dependency Graph Algorithm

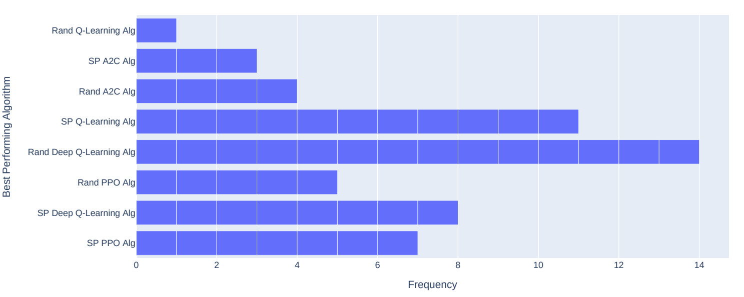
Combination of identifying rewarding paths and random sampling

Results



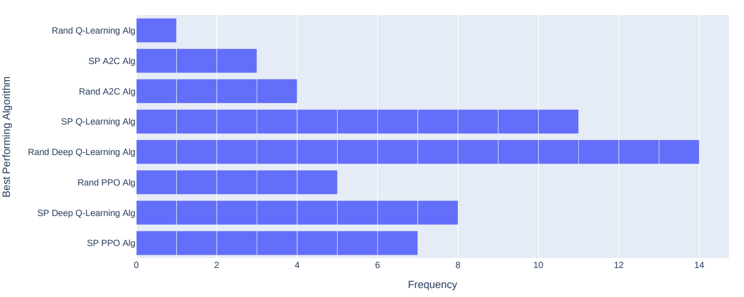
Rand Q-Learning Alg
SP A2C Alg
Rand A2C Alg
SP Q-Learning Alg
Rand Deep Q-Learning Alg
Rand PPO Alg
SP Deep Q-Learning Alg
SP PPO Alg
Best Performing Algorithm
0
2
4
6
8
10
12
14
Frequency

Results



1. SP Q-learning algorithm 18.938
2. SP Deep Q-learning algorithm 18.799
3. Rand Deep Q-learning algorithm 18.779

Results



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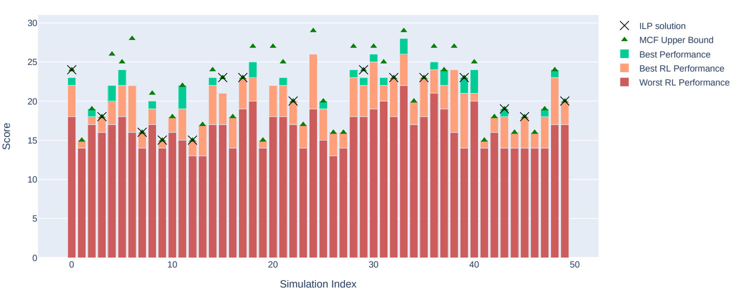
MCF upper bound 21.382

SP dependency graph algorithm 21.256

Results



Score
Simulation Index
30
25
20
15
10
5
0
0
10
20
30
40
50
ILP solution
MCF Upper Bound
Best Performance
Best RL Performance
Worst RL Performance

Conclusion

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Average Approximation Ratios

Best RL to best dependency graph algorithm: 0.984 (0.902)

Best RL to ILP is: 0.974 (0.899)

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Best Performing RL Algorithms

1. Shortest-path Q-learning Algorithm
2. Shortest-path Deep Q-Learning Algorithm
3. Random Deep Q-Learning Algorithm

Discussion

Reflection on Limitations

* Only considered NSFNET & a single wavelength intensity
* Reliance on base algorithms and residual wavelength assignment, imprecise state representation
* Static perfectly predictable, stable discrete-time connections
* Graph edge uniformity

Thank you for your attention!

Thesis' Contributions

* Created a comprehensive model for RWA benchmarking
* Demonstrated how RWA can be solved using RL
* Introduced methods to assess the algorithmic performance for RWA