Validity, Paradox, and the Ideal of Deductive Logic

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Paper in short

- Our conception of deductive logic as aiming for truth preservation is a philosopher's dream. We ought to adopt default reasoning to deal with semantic paradoxes instead.
- If so, an inference rule is valid if and only if it generally (that is, not necessarily in every instance) preserves truth.
- Likewise, we should not attack the validity or applicability of the 3 'ingredients' from which the semantic paradoxes arise. Rather, an alternative for semantic paradoxes should be introduced.

Thinking about plan B

Three[1] Ingredients that together give rise to semantic paradoxes:

- Logic (Inference rules associated with certain logically special expressions; e.g., classical logic).
- A Truth Predicate (Tarski T-schema or introduction and elimination rules).
- Expressive resources that allow for the formulation of trouble inducing sentences (with the liar sentence and curry's paradox [C: if C $\rightarrow$ F; with F arbitrary] in particular).

[1] Necessary and sufficient

Thinking about plan B

Plan A involves adhering to any of the following:

- weakening classical logic.
- weakening the rules governing the truth predicate.
- deriving that the problematic sentences are not of the kind that the rules governing the truth predicate properly apply to.

Thinking about plan B

Plan A

These solutions merely attack the validity of the ‘ingredients’ or they argue that the ‘ingredients’ do not apply to the problematic sentences.

We learn from plan A where our reasoning that leads us into trouble went wrong. In doing this, it becomes clear which inference rule is not valid.

Do the solutions of plan A resist revenge paradoxes?

Thinking about plan B

"*It appears that even the most sophisticated solutions [of plan A] face further paradoxes that can be formulated using the terms of the solution.*" (p.146, Hofweber 2007)

"*To reject invalid rules will use new semantic terminology that then can be used to state new semantic paradoxes, ones especially tailored to the solution of the old paradoxes in question.*" (Ibid)

Problem: Paradox not solved only pushed around.

Dilemma of pushing around philosophical problems

Push problem somewhere else, or discard solution as not being applicable to our natural languages.

Thinking about plan B

Hofweber's stance

Don't delve into the merit of plan A too deeply. Rather, proceed with the assumption that plan A does not suffice for said paradoxes, in order to explore other options.

The great collapse

If no solutions to said paradoxes, then conceptual schema' of describing truth and falsity collapses. (e.g., Curry's paradox entails every statement being both true and false; every statement is equally warranted).

Thinking about plan B

Plan B

To not adopt a plan of the type of plan A and to avoid the great collapse.

In other words: a solution where the ingredients are always considered valid and where they apply to the problematic sentences.

Deductive Logic, Default Reasoning, and Generics

The real culprit: our conception of deductive logic is mistaken (it is in fact dispensable).

Truth preservation

Any inference that is licensed is valid and thus truth preserving in classical (deductive) logic and the inference rules governing the truth predicate.

Deductive Logic, Default Reasoning, and Generics

Ordinary, everyday reasoning is mostly not strictly deductive.

It primarily consists of generic statements that allow for exceptions.

(1) Bears are dangerous [Generic; true].

(2) *t* is a bear.

(3) *t* is dangerous.

(4) In general, a bear is dangerous [generic; expresses meaning of (1)].

(5) All bears are dangerous [Universally qualified; false].

Non-monotonic/default reasoning

More information can make an otherwise appropriate inference inappropriate.

Deductive Logic, Default Reasoning, and Generics

Outline for plan B

Adopt default reasoning such that we do not have to preserve truth in all instances (given non-monotonic inference rules).

Generic validity

In general, instances of inference rules are truth preserving

Strict validity

Each and every instance of an inference rule strictly is truth preserving

How is this a solution?

Solution to the paradox. *"We can accept the reasoning that leads us into paradox, but not accept its conclusions, or the Great Collapse".* (p.151, Hofweber 2007)

With plan B we have a justification for rationally rejecting the conclusions drawn with the particular cases of the (generically) valid inference rules.

Hofweber's (subtle) disclaimer

Plan B's solution only applies to semantic paradoxes like the liar and Curry's paradox. It does not simply carry over to other paradoxes like, say, the Sorites paradox, or the paradoxes of motion, and so on.

What are the exceptions?

Conditions for strict validity

The exceptions to the generically valid rules are simply the instances that don’t preserve truth.

The exceptions — just like the bear that is not dangerous — have to be examined case by case for truth-preservation.

What about Revenge?

"*This [plan B] solution already accepts that there are instances of the (generically) valid inference rules that lead to contradictions or arbitrary conclusions. But these instances are the exceptions to the (generically) valid inference rules. Any revenge paradox can only lead to more of these cases.*" (p.153, Hofweber 2007)

Conclusion

Hofweber advocates for truth preserving in the generic sense.

How Hofweber characterizes Plan A

Plan A solutions try to say where the reasoning goes wrong. One of the rules, or the sentence, will have to go.

Why Hofweber advocates in favor or plan B

1. It captures the wisdom in the natural reaction to the paradoxes
2. It doesn't seem to be threatened by revenge paradoxes
3. it explicitly affirms out front what otherwise seems like a counterintuitive consequence, namely that our inference rules are not (strictly) truth preserving

Discussion

1. Is Hofweber's justification for a distinction between strict and generic validity ad-hoc[2]?
2. Can we solve the liar paradox if the law of non-contradiction is itself not truth preserving[3]?
3. Is generic validity more attractive than strict validity[4]?
4. Does Hofweber sufficiently confront the paradoxes when he advocates for not accepting a conclusion/great collapse following a line or reasoning[5]?

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[2] i.e., it is not a priori but either empirical or tautological.

[3] i.e., how does Hofweber confront dialetheism?

[4] e.g., unknown/incomplete facts preventing absolute certainty?

[5] i.e., the liar paradox is now evaded by virtue of being an exception rather than by reconsidering some logical elimination rules. So is this not pushing the problem around to the realm of exceptions?