Optimization Methods

Until now, you've always used Gradient Descent to update the parameters and minimize the cost. In this notebook, you will learn more advanced optimization methods that can speed up learning and perhaps even get you to a better final value for the cost function. Having a good optimization algorithm can be the difference between waiting days vs. just a few hours to get a good result.

Gradient descent goes "downhill" on a cost function J. Think of it as trying to do this:

Figure 1 : **Minimizing the cost is like finding the lowest point in a hilly landscape**
At each step of the training, you update your parameters following a certain direction to try to get to the lowest possible point.

Notations: As usual, $\frac{\partial J}{\partial a} = $ `da` for any variable `a` .

To get started, run the following code to import the libraries you will need.

Updates to Assignment

If you were working on a previous version

- The current notebook filename is version "Optimization_methods_v1b".
- You can find your work in the file directory as version "Optimization methods'.
- To see the file directory, click on the Coursera logo at the top left of the notebook.

List of Updates

- op_utils is now opt_utils_v1a. Assertion statement in `initialize_parameters` is fixed.
- opt_utils_v1a: `compute_cost` function now accumulates total cost of the batch without taking the average (average is taken for entire epoch instead).
- In `model` function, the total cost per mini-batch is accumulated, and the average of the entire epoch is taken as the average cost. So the plot of the cost function over time is now a smooth downward curve instead of an oscillating curve.
- Print statements used to check each function are reformatted, and 'expected output` is reformatted to match the format of the print statements (for easier visual comparisons).

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        import scipy.io
        import math
        import sklearn
        import sklearn.datasets

        from opt_utils_v1a import load_params_and_grads, initialize_parameters,
        forward_propagation, backward_propagation
        from opt_utils_v1a import compute_cost, predict, predict_dec, plot_decis
        ion_boundary, load_dataset
        from testCases import *

        %matplotlib inline
        plt.rcParams['figure.figsize'] = (7.0, 4.0) # set default size of plots
        plt.rcParams['image.interpolation'] = 'nearest'
        plt.rcParams['image.cmap'] = 'gray'
```

1 - Gradient Descent

A simple optimization method in machine learning is gradient descent (GD). When you take gradient steps with respect to all m examples on each step, it is also called Batch Gradient Descent.

Warm-up exercise: Implement the gradient descent update rule. The gradient descent rule is, for $l = 1, \ldots, L$:

$$W^{[l]} = W^{[l]} - \alpha \, dW^{[l]} \tag{1}$$
$$b^{[l]} = b^{[l]} - \alpha \, db^{[l]} \tag{2}$$

where L is the number of layers and α is the learning rate. All parameters should be stored in the `parameters` dictionary. Note that the iterator `l` starts at 0 in the `for` loop while the first parameters are $W^{[1]}$ and $b^{[1]}$. You need to shift `l` to `l+1` when coding.

```
In [2]: # GRADED FUNCTION: update_parameters_with_gd

        def update_parameters_with_gd(parameters, grads, learning_rate):
            """
            Update parameters using one step of gradient descent

            Arguments:
            parameters -- python dictionary containing your parameters to be upd
            ated:
                            parameters['W' + str(l)] = Wl
                            parameters['b' + str(l)] = bl
            grads -- python dictionary containing your gradients to update each
             parameters:
                            grads['dW' + str(l)] = dWl
                            grads['db' + str(l)] = dbl
            learning_rate -- the learning rate, scalar.

            Returns:
            parameters -- python dictionary containing your updated parameters
            """

            L = len(parameters) // 2 # number of layers in the neural networks

            # Update rule for each parameter
            for l in range(L):
                ### START CODE HERE ### (approx. 2 lines)
                parameters["W" + str(l+1)] = parameters["W" + str(l+1)] - np.mul
        tiply(learning_rate, grads['dW' + str(l+1)])
                parameters["b" + str(l+1)] = parameters["b" + str(l+1)] - np.mul
        tiply(learning_rate, grads['db' + str(l+1)])
                ### END CODE HERE ###

            return parameters
```

```
In [3]: parameters, grads, learning_rate = update_parameters_with_gd_test_case()

        parameters = update_parameters_with_gd(parameters, grads, learning_rate)
        print("W1 =\n" + str(parameters["W1"]))
        print("b1 =\n" + str(parameters["b1"]))
        print("W2 =\n" + str(parameters["W2"]))
        print("b2 =\n" + str(parameters["b2"]))
```

```
W1 =
[[ 1.63535156 -0.62320365 -0.53718766]
 [-1.07799357  0.85639907 -2.29470142]]
b1 =
[[ 1.74604067]
 [-0.75184921]]
W2 =
[[ 0.32171798 -0.25467393  1.46902454]
 [-2.05617317 -0.31554548 -0.3756023 ]
 [ 1.1404819  -1.09976462 -0.1612551 ]]
b2 =
[[-0.88020257]
 [ 0.02561572]
 [ 0.57539477]]
```

Expected Output:

```
W1 =
[[ 1.63535156 -0.62320365 -0.53718766]
 [-1.07799357  0.85639907 -2.29470142]]
b1 =
[[ 1.74604067]
 [-0.75184921]]
W2 =
[[ 0.32171798 -0.25467393  1.46902454]
 [-2.05617317 -0.31554548 -0.3756023 ]
 [ 1.1404819  -1.09976462 -0.1612551 ]]
b2 =
[[-0.88020257]
 [ 0.02561572]
 [ 0.57539477]]
```

A variant of this is Stochastic Gradient Descent (SGD), which is equivalent to mini-batch gradient descent where each mini-batch has just 1 example. The update rule that you have just implemented does not change. What changes is that you would be computing gradients on just one training example at a time, rather than on the whole training set. The code examples below illustrate the difference between stochastic gradient descent and (batch) gradient descent.

- **(Batch) Gradient Descent**:

```
X = data_input
Y = labels
parameters = initialize_parameters(layers_dims)
for i in range(0, num_iterations):
    # Forward propagation
    a, caches = forward_propagation(X, parameters)
    # Compute cost.
    cost += compute_cost(a, Y)
    # Backward propagation.
    grads = backward_propagation(a, caches, parameters)
    # Update parameters.
    parameters = update_parameters(parameters, grads)
```

- **Stochastic Gradient Descent**:

```
X = data_input
Y = labels
parameters = initialize_parameters(layers_dims)
for i in range(0, num_iterations):
    for j in range(0, m):
        # Forward propagation
        a, caches = forward_propagation(X[:,j], parameters)
        # Compute cost
        cost += compute_cost(a, Y[:,j])
        # Backward propagation
        grads = backward_propagation(a, caches, parameters)
        # Update parameters.
        parameters = update_parameters(parameters, grads)
```

In Stochastic Gradient Descent, you use only 1 training example before updating the gradients. When the training set is large, SGD can be faster. But the parameters will "oscillate" toward the minimum rather than converge smoothly. Here is an illustration of this:

Figure 1 : **SGD vs GD**

"+" denotes a minimum of the cost. SGD leads to many oscillations to reach convergence. But each step is a lot faster to compute for SGD than for GD, as it uses only one training example (vs. the whole batch for GD).

Note also that implementing SGD requires 3 for-loops in total:
1. Over the number of iterations
2. Over the m training examples
3. Over the layers (to update all parameters, from $(W^{[1]}, b^{[1]})$ to $(W^{[L]}, b^{[L]})$)

In practice, you'll often get faster results if you do not use neither the whole training set, nor only one training example, to perform each update. Mini-batch gradient descent uses an intermediate number of examples for each step. With mini-batch gradient descent, you loop over the mini-batches instead of looping over individual training examples.
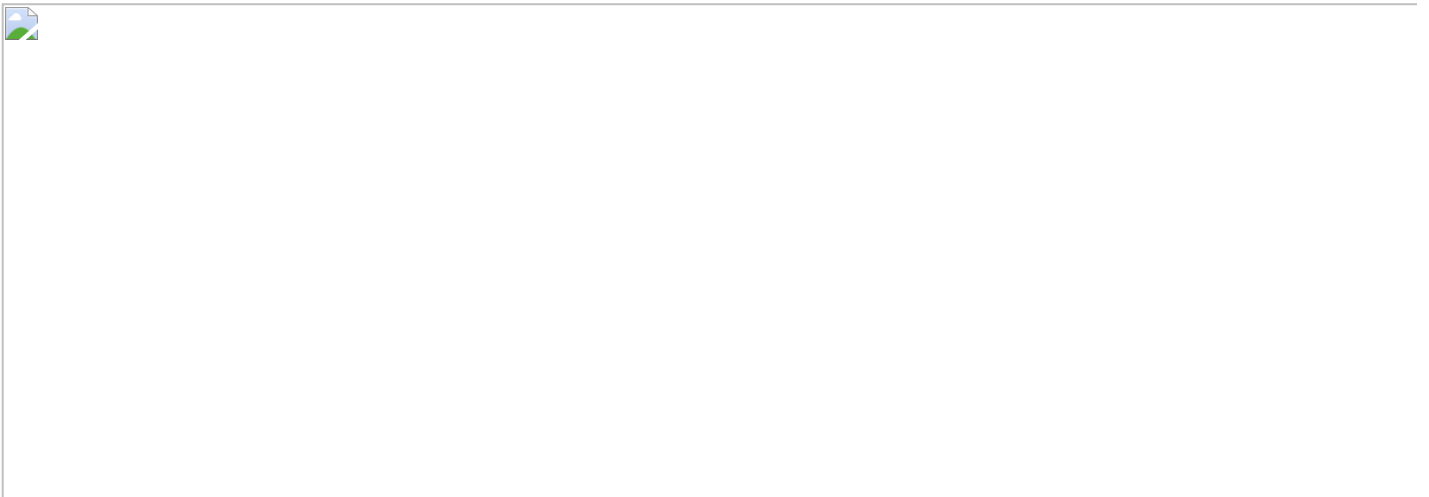

Figure 2 : **SGD vs Mini-Batch GD**

"+" denotes a minimum of the cost. Using mini-batches in your optimization algorithm often leads to faster optimization.

What you should remember:

- The difference between gradient descent, mini-batch gradient descent and stochastic gradient descent is the number of examples you use to perform one update step.
- You have to tune a learning rate hyperparameter α.
- With a well-turned mini-batch size, usually it outperforms either gradient descent or stochastic gradient descent (particularly when the training set is large).

2 - Mini-Batch Gradient descent

Let's learn how to build mini-batches from the training set (X, Y).

There are two steps:

- **Shuffle**: Create a shuffled version of the training set (X, Y) as shown below. Each column of X and Y represents a training example. Note that the random shuffling is done synchronously between X and Y. Such that after the shuffling the ith column of X is the example corresponding to the ith label in Y. The shuffling step ensures that examples will be split randomly into different mini-batches.

- **Partition**: Partition the shuffled (X, Y) into mini-batches of size `mini_batch_size` (here 64). Note that the number of training examples is not always divisible by `mini_batch_size`. The last mini batch might be smaller, but you don't need to worry about this. When the final mini-batch is smaller than the full `mini_batch_size`, it will look like this:

Exercise: Implement `random_mini_batches` . We coded the shuffling part for you. To help you with the partitioning step, we give you the following code that selects the indexes for the 1st and 2nd mini-batches:

```
first_mini_batch_X = shuffled_X[:, 0 : mini_batch_size]
second_mini_batch_X = shuffled_X[:, mini_batch_size : 2 * mini_batch_size]
...
```

Note that the last mini-batch might end up smaller than `mini_batch_size=64` . Let $\lfloor s \rfloor$ represents s rounded down to the nearest integer (this is `math.floor(s)` in Python). If the total number of examples is not a multiple of `mini_batch_size=64` then there will be $\lfloor \frac{m}{mini\_batch\_size} \rfloor$ mini-batches with a full 64 examples, and the number of examples in the final mini-batch will be (
$m - mini\_batch\_size \times \lfloor \frac{m}{mini\_batch\_size} \rfloor$).

```python
In [4]:  # GRADED FUNCTION: random_mini_batches

         def random_mini_batches(X, Y, mini_batch_size = 64, seed = 0):
             """
             Creates a list of random minibatches from (X, Y)

             Arguments:
             X -- input data, of shape (input size, number of examples)
             Y -- true "label" vector (1 for blue dot / 0 for red dot), of shape
          (1, number of examples)
             mini_batch_size -- size of the mini-batches, integer

             Returns:
             mini_batches -- list of synchronous (mini_batch_X, mini_batch_Y)
             """

             np.random.seed(seed)            # To make your "random" minibatches
          the same as ours
             m = X.shape[1]                  # number of training examples
             mini_batches = []

             # Step 1: Shuffle (X, Y)
             permutation = list(np.random.permutation(m))
             shuffled_X = X[:, permutation]
             shuffled_Y = Y[:, permutation].reshape((1,m))

             # Step 2: Partition (shuffled_X, shuffled_Y). Minus the end case.
             num_complete_minibatches = math.floor(m/mini_batch_size) # number of
         mini batches of size mini_batch_size in your partitionning
             for k in range(0, num_complete_minibatches):
                 ### START CODE HERE ### (approx. 2 lines)
                 mini_batch_X = shuffled_X[:, k * mini_batch_size : np.multiply(n
         p.add(k,1), mini_batch_size)]
                 mini_batch_Y = shuffled_Y[:, k * mini_batch_size : np.multiply(n
         p.add(k,1), mini_batch_size)]
                 ### END CODE HERE ###
                 mini_batch = (mini_batch_X, mini_batch_Y)
                 mini_batches.append(mini_batch)

             # Handling the end case (last mini-batch < mini_batch_size)
             if m % mini_batch_size != 0:
                 ### START CODE HERE ### (approx. 2 lines)
                 remaining_batch_size = np.floor((np.divide(m, mini_batch_size) -
         num_complete_minibatches) * mini_batch_size)
                 mini_batch_X = shuffled_X[:,  : remaining_batch_size ]
                 mini_batch_Y = shuffled_Y[:,  : remaining_batch_size ]
                 ### END CODE HERE ###
                 mini_batch = (mini_batch_X, mini_batch_Y)
                 mini_batches.append(mini_batch)

             return mini_batches
```

```
In [5]: X_assess, Y_assess, mini_batch_size = random_mini_batches_test_case()
        mini_batches = random_mini_batches(X_assess, Y_assess, mini_batch_size)

        print ("shape of the 1st mini_batch_X: " + str(mini_batches[0][0].shape
        ))
        print ("shape of the 2nd mini_batch_X: " + str(mini_batches[1][0].shape
        ))
        print ("shape of the 3rd mini_batch_X: " + str(mini_batches[2][0].shape
        ))
        print ("shape of the 1st mini_batch_Y: " + str(mini_batches[0][1].shape
        ))
        print ("shape of the 2nd mini_batch_Y: " + str(mini_batches[1][1].shape
        ))
        print ("shape of the 3rd mini_batch_Y: " + str(mini_batches[2][1].shape
        ))
        print ("mini batch sanity check: " + str(mini_batches[0][0][0][0:3]))
```

```
shape of the 1st mini_batch_X: (12288, 64)
shape of the 2nd mini_batch_X: (12288, 64)
shape of the 3rd mini_batch_X: (12288, 20)
shape of the 1st mini_batch_Y: (1, 64)
shape of the 2nd mini_batch_Y: (1, 64)
shape of the 3rd mini_batch_Y: (1, 20)
mini batch sanity check: [ 0.90085595 -0.7612069   0.2344157 ]

/opt/conda/lib/python3.6/site-packages/ipykernel/__main__.py:39: Visibl
eDeprecationWarning: using a non-integer number instead of an integer w
ill result in an error in the future
/opt/conda/lib/python3.6/site-packages/ipykernel/__main__.py:40: Visibl
eDeprecationWarning: using a non-integer number instead of an integer w
ill result in an error in the future
```

Expected Output:

shape of the 1st mini_batch_X	(12288, 64)
shape of the 2nd mini_batch_X	(12288, 64)
shape of the 3rd mini_batch_X	(12288, 20)
shape of the 1st mini_batch_Y	(1, 64)
shape of the 2nd mini_batch_Y	(1, 64)
shape of the 3rd mini_batch_Y	(1, 20)
mini batch sanity check	[0.90085595 -0.7612069 0.2344157]

What you should remember:
- Shuffling and Partitioning are the two steps required to build mini-batches
- Powers of two are often chosen to be the mini-batch size, e.g., 16, 32, 64, 128.

3 - Momentum

Because mini-batch gradient descent makes a parameter update after seeing just a subset of examples, the direction of the update has some variance, and so the path taken by mini-batch gradient descent will "oscillate" toward convergence. Using momentum can reduce these oscillations.

Momentum takes into account the past gradients to smooth out the update. We will store the 'direction' of the previous gradients in the variable v. Formally, this will be the exponentially weighted average of the gradient on previous steps. You can also think of v as the "velocity" of a ball rolling downhill, building up speed (and momentum) according to the direction of the gradient/slope of the hill.
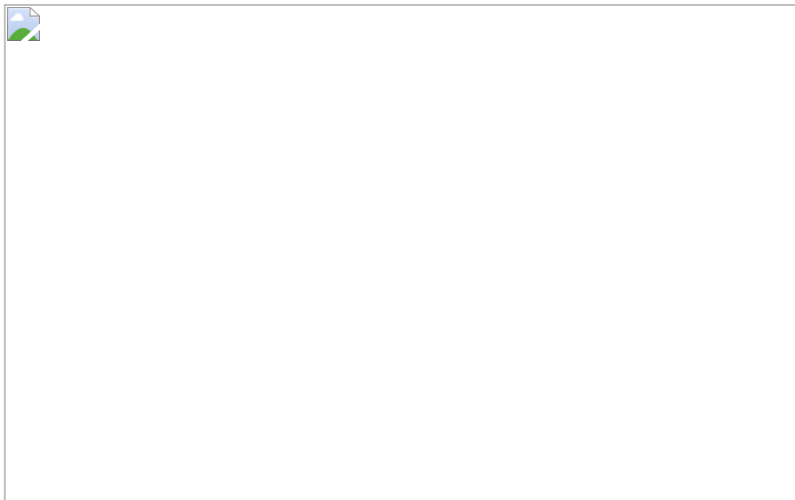

Figure 3: The red arrows shows the direction taken by one step of mini-batch gradient descent with momentum. The blue points show the direction of the gradient (with respect to the current mini-batch) on each step. Rather than just following the gradient, we let the gradient influence v and then take a step in the direction of v.

Exercise: Initialize the velocity. The velocity, v, is a python dictionary that needs to be initialized with arrays of zeros. Its keys are the same as those in the `grads` dictionary, that is: for $l = 1, \ldots, L$:

```
v["dW" + str(l+1)] = ... #(numpy array of zeros with the same shape as param
eters["W" + str(l+1)])
v["db" + str(l+1)] = ... #(numpy array of zeros with the same shape as param
eters["b" + str(l+1)])
```

Note that the iterator l starts at 0 in the for loop while the first parameters are v["dW1"] and v["db1"] (that's a "one" on the superscript). This is why we are shifting l to l+1 in the `for` loop.

```
In [29]:  # GRADED FUNCTION: initialize_velocity

          def initialize_velocity(parameters):
              """
              Initializes the velocity as a python dictionary with:
                          - keys: "dW1", "db1", ..., "dWL", "dbL"
                          - values: numpy arrays of zeros of the same shape as the
              corresponding gradients/parameters.
              Arguments:
              parameters -- python dictionary containing your parameters.
                              parameters['W' + str(l)] = Wl
                              parameters['b' + str(l)] = bl

              Returns:
              v -- python dictionary containing the current velocity.
                              v['dW' + str(l)] = velocity of dWl
                              v['db' + str(l)] = velocity of dbl
              """

              L = len(parameters) // 2 # number of layers in the neural networks
              v = {}

              # Initialize velocity
              for l in range(L):
                  ### START CODE HERE ### (approx. 2 lines)
                  v["dW" + str(l+1)] = np.zeros(parameters["W" + str(l+1)].shape)
                  v["db" + str(l+1)] = np.zeros(parameters["b" + str(l+1)].shape)
                  ### END CODE HERE ###

              return v
```

```
In [30]:  parameters = initialize_velocity_test_case()

          v = initialize_velocity(parameters)
          print("v[\"dW1\"] =\n" + str(v["dW1"]))
          print("v[\"db1\"] =\n" + str(v["db1"]))
          print("v[\"dW2\"] =\n" + str(v["dW2"]))
          print("v[\"db2\"] =\n" + str(v["db2"]))
```

```
v["dW1"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db1"] =
[[ 0.]
 [ 0.]]
v["dW2"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db2"] =
[[ 0.]
 [ 0.]
 [ 0.]]
```

Expected Output:

```
v["dW1"] =
[[ 0.   0.   0.]
 [ 0.   0.   0.]]
v["db1"] =
[[ 0.]
 [ 0.]]
v["dW2"] =
[[ 0.   0.   0.]
 [ 0.   0.   0.]
 [ 0.   0.   0.]]
v["db2"] =
[[ 0.]
 [ 0.]
 [ 0.]]
```

Exercise: Now, implement the parameters update with momentum. The momentum update rule is, for $l = 1, \ldots, L$:

$$\begin{cases} v_{dW^{[l]}} = \beta v_{dW^{[l]}} + (1 - \beta)dW^{[l]} \\ W^{[l]} = W^{[l]} - \alpha v_{dW^{[l]}} \end{cases} \tag{3}$$

$$\begin{cases} v_{db^{[l]}} = \beta v_{db^{[l]}} + (1 - \beta)db^{[l]} \\ b^{[l]} = b^{[l]} - \alpha v_{db^{[l]}} \end{cases} \tag{4}$$

where L is the number of layers, β is the momentum and α is the learning rate. All parameters should be stored in the `parameters` dictionary. Note that the iterator `l` starts at 0 in the `for` loop while the first parameters are $W^{[1]}$ and $b^{[1]}$ (that's a "one" on the superscript). So you will need to shift `l` to `l+1` when coding.

```python
In [32]: # GRADED FUNCTION: update_parameters_with_momentum

         def update_parameters_with_momentum(parameters, grads, v, beta, learning
         _rate):
             """
             Update parameters using Momentum

             Arguments:
             parameters -- python dictionary containing your parameters:
                             parameters['W' + str(l)] = Wl
                             parameters['b' + str(l)] = bl
             grads -- python dictionary containing your gradients for each parame
         ters:
                             grads['dW' + str(l)] = dWl
                             grads['db' + str(l)] = dbl
             v -- python dictionary containing the current velocity:
                             v['dW' + str(l)] = ...
                             v['db' + str(l)] = ...
             beta -- the momentum hyperparameter, scalar
             learning_rate -- the learning rate, scalar

             Returns:
             parameters -- python dictionary containing your updated parameters
             v -- python dictionary containing your updated velocities
             """

             L = len(parameters) // 2 # number of layers in the neural networks

             # Momentum update for each parameter
             for l in range(L):

                 ### START CODE HERE ### (approx. 4 lines)
                 # compute velocities
                 v["dW" + str(l+1)] = beta*v["dW" + str(l+1)] + (1-beta)*grads['d
         W' + str(l+1)]
                 v["db" + str(l+1)] = beta*v["db" + str(l+1)] + (1-beta)*grads['d
         b' + str(l+1)]

                 # update parameters
                 parameters["W" + str(l+1)] = np.subtract(parameters["W" + str(l+
         1)], np.multiply(learning_rate, v["dW" + str(l+1)]))
                 parameters["b" + str(l+1)] = np.subtract(parameters["b" + str(l+
         1)], np.multiply(learning_rate, v["db" + str(l+1)]))
                 ### END CODE HERE ###

             return parameters, v
```

```
parameters, grads, v = update_parameters_with_momentum_test_case()

parameters, v = update_parameters_with_momentum(parameters, grads, v, be
ta = 0.9, learning_rate = 0.01)
print("W1 = \n" + str(parameters["W1"]))
print("b1 = \n" + str(parameters["b1"]))
print("W2 = \n" + str(parameters["W2"]))
print("b2 = \n" + str(parameters["b2"]))
print("v[\"dW1\"] = \n" + str(v["dW1"]))
print("v[\"db1\"] = \n" + str(v["db1"]))
print("v[\"dW2\"] = \n" + str(v["dW2"]))
print("v[\"db2\"] = v" + str(v["db2"]))
```

```
W1 =
[[ 1.62544598 -0.61290114 -0.52907334]
 [-1.07347112  0.86450677 -2.30085497]]
b1 =
[[ 1.74493465]
 [-0.76027113]]
W2 =
[[ 0.31930698 -0.24990073  1.4627996 ]
 [-2.05974396 -0.32173003 -0.38320915]
 [ 1.13444069 -1.0998786  -0.1713109 ]]
b2 =
[[-0.87809283]
 [ 0.04055394]
 [ 0.58207317]]
v["dW1"] =
[[-0.11006192  0.11447237  0.09015907]
 [ 0.05024943  0.09008559 -0.06837279]]
v["db1"] =
[[-0.01228902]
 [-0.09357694]]
v["dW2"] =
[[-0.02678881  0.05303555 -0.06916608]
 [-0.03967535 -0.06871727 -0.08452056]
 [-0.06712461 -0.00126646 -0.11173103]]
v["db2"] = v[[ 0.02344157]
 [ 0.16598022]
 [ 0.07420442]]
```

Expected Output:

```
W1 =
[[ 1.62544598 -0.61290114 -0.52907334]
 [-1.07347112  0.86450677 -2.30085497]]
b1 =
[[ 1.74493465]
 [-0.76027113]]
W2 =
[[ 0.31930698 -0.24990073  1.4627996 ]
 [-2.05974396 -0.32173003 -0.38320915]
 [ 1.13444069 -1.0998786  -0.1713109 ]]
b2 =
[[-0.87809283]
 [ 0.04055394]
 [ 0.58207317]]
v["dW1"] =
[[-0.11006192  0.11447237  0.09015907]
 [ 0.05024943  0.09008559 -0.06837279]]
v["db1"] =
[[-0.01228902]
 [-0.09357694]]
v["dW2"] =
[[-0.02678881  0.05303555 -0.06916608]
 [-0.03967535 -0.06871727 -0.08452056]
 [-0.06712461 -0.00126646 -0.11173103]]
v["db2"] = v[[ 0.02344157]
 [ 0.16598022]
 [ 0.07420442]]
```

Note that:

- The velocity is initialized with zeros. So the algorithm will take a few iterations to "build up" velocity and start to take bigger steps.
- If $\beta = 0$, then this just becomes standard gradient descent without momentum.

How do you choose β?

- The larger the momentum β is, the smoother the update because the more we take the past gradients into account. But if β is too big, it could also smooth out the updates too much.
- Common values for β range from 0.8 to 0.999. If you don't feel inclined to tune this, $\beta = 0.9$ is often a reasonable default.
- Tuning the optimal β for your model might need trying several values to see what works best in term of reducing the value of the cost function J.

What you should remember:
- Momentum takes past gradients into account to smooth out the steps of gradient descent. It can be applied with batch gradient descent, mini-batch gradient descent or stochastic gradient descent.
- You have to tune a momentum hyperparameter β and a learning rate α.

4 - Adam

Adam is one of the most effective optimization algorithms for training neural networks. It combines ideas from RMSProp (described in lecture) and Momentum.

How does Adam work?

1. It calculates an exponentially weighted average of past gradients, and stores it in variables v (before bias correction) and $v^{corrected}$ (with bias correction).
2. It calculates an exponentially weighted average of the squares of the past gradients, and stores it in variables s (before bias correction) and $s^{corrected}$ (with bias correction).
3. It updates parameters in a direction based on combining information from "1" and "2".

The update rule is, for $l = 1, \ldots, L$:

$$
\begin{cases}
v_{dW^{[l]}} = \beta_1 v_{dW^{[l]}} + (1 - \beta_1)\frac{\partial J}{\partial W^{[l]}} \\
v_{dW^{[l]}}^{corrected} = \frac{v_{dW^{[l]}}}{1-(\beta_1)^t} \\
s_{dW^{[l]}} = \beta_2 s_{dW^{[l]}} + (1 - \beta_2)(\frac{\partial J}{\partial W^{[l]}})^2 \\
s_{dW^{[l]}}^{corrected} = \frac{s_{dW^{[l]}}}{1-(\beta_2)^t} \\
W^{[l]} = W^{[l]} - \alpha \frac{v_{dW^{[l]}}^{corrected}}{\sqrt{s_{dW^{[l]}}^{corrected}}+\varepsilon}
\end{cases}
$$

where:

- t counts the number of steps taken of Adam
- L is the number of layers
- β_1 and β_2 are hyperparameters that control the two exponentially weighted averages.
- α is the learning rate
- ε is a very small number to avoid dividing by zero

As usual, we will store all parameters in the `parameters` dictionary

Exercise: Initialize the Adam variables v, s which keep track of the past information.

Instruction: The variables v, s are python dictionaries that need to be initialized with arrays of zeros. Their keys are the same as for `grads`, that is: for $l = 1, \ldots, L$:

```
v["dW" + str(l+1)] = ... #(numpy array of zeros with the same shape as param
eters["W" + str(l+1)])
v["db" + str(l+1)] = ... #(numpy array of zeros with the same shape as param
eters["b" + str(l+1)])
s["dW" + str(l+1)] = ... #(numpy array of zeros with the same shape as param
eters["W" + str(l+1)])
s["db" + str(l+1)] = ... #(numpy array of zeros with the same shape as param
eters["b" + str(l+1)])
```

```
In [34]:  # GRADED FUNCTION: initialize_adam

          def initialize_adam(parameters) :
              """
              Initializes v and s as two python dictionaries with:
                          - keys: "dW1", "db1", ..., "dWL", "dbL"
                          - values: numpy arrays of zeros of the same shape as the
              corresponding gradients/parameters.

              Arguments:
              parameters -- python dictionary containing your parameters.
                              parameters["W" + str(l)] = Wl
                              parameters["b" + str(l)] = bl

              Returns:
              v -- python dictionary that will contain the exponentially weighted
           average of the gradient.
                              v["dW" + str(l)] = ...
                              v["db" + str(l)] = ...
              s -- python dictionary that will contain the exponentially weighted
           average of the squared gradient.
                              s["dW" + str(l)] = ...
                              s["db" + str(l)] = ...

              """

              L = len(parameters) // 2 # number of layers in the neural networks
              v = {}
              s = {}

              # Initialize v, s. Input: "parameters". Outputs: "v, s".
              for l in range(L):
              ### START CODE HERE ### (approx. 4 lines)
                  v["dW" + str(l+1)] = np.zeros((parameters["W" + str(l+1)]).shape
          )
                  v["db" + str(l+1)] = np.zeros((parameters["b" + str(l+1)]).shape
          )
                  s["dW" + str(l+1)] = np.zeros((parameters["W" + str(l+1)]).shape
          )
                  s["db" + str(l+1)] = np.zeros((parameters["b" + str(l+1)]).shape
          )
              ### END CODE HERE ###

              return v, s
```

```
In [35]: parameters = initialize_adam_test_case()

         v, s = initialize_adam(parameters)
         print("v[\"dW1\"] = \n" + str(v["dW1"]))
         print("v[\"db1\"] = \n" + str(v["db1"]))
         print("v[\"dW2\"] = \n" + str(v["dW2"]))
         print("v[\"db2\"] = \n" + str(v["db2"]))
         print("s[\"dW1\"] = \n" + str(s["dW1"]))
         print("s[\"db1\"] = \n" + str(s["db1"]))
         print("s[\"dW2\"] = \n" + str(s["dW2"]))
         print("s[\"db2\"] = \n" + str(s["db2"]))
```

```
v["dW1"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db1"] =
[[ 0.]
 [ 0.]]
v["dW2"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db2"] =
[[ 0.]
 [ 0.]
 [ 0.]]
s["dW1"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]]
s["db1"] =
[[ 0.]
 [ 0.]]
s["dW2"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]
s["db2"] =
[[ 0.]
 [ 0.]
 [ 0.]]
```

Expected Output:

```
v["dW1"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db1"] =
[[ 0.]
 [ 0.]]
v["dW2"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]
v["db2"] =
[[ 0.]
 [ 0.]
 [ 0.]]
s["dW1"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]]
s["db1"] =
[[ 0.]
 [ 0.]]
s["dW2"] =
[[ 0.  0.  0.]
 [ 0.  0.  0.]
 [ 0.  0.  0.]]
s["db2"] =
[[ 0.]
 [ 0.]
 [ 0.]]
```

Exercise: Now, implement the parameters update with Adam. Recall the general update rule is, for $l = 1, \ldots, L$:

$$
\begin{cases}
v_{W^{[l]}} = \beta_1 v_{W^{[l]}} + (1 - \beta_1) \frac{\partial J}{\partial W^{[l]}} \\[2mm]
v_{W^{[l]}}^{corrected} = \frac{v_{W^{[l]}}}{1 - (\beta_1)^t} \\[2mm]
s_{W^{[l]}} = \beta_2 s_{W^{[l]}} + (1 - \beta_2)(\frac{\partial J}{\partial W^{[l]}})^2 \\[2mm]
s_{W^{[l]}}^{corrected} = \frac{s_{W^{[l]}}}{1 - (\beta_2)^t} \\[2mm]
W^{[l]} = W^{[l]} - \alpha \frac{v_{W^{[l]}}^{corrected}}{\sqrt{s_{W^{[l]}}^{corrected}} + \varepsilon}
\end{cases}
$$

Note that the iterator `l` starts at 0 in the `for` loop while the first parameters are $W^{[1]}$ and $b^{[1]}$. You need to shift `l` to `l+1` when coding.

```
In [36]:  # GRADED FUNCTION: update_parameters_with_adam

          def update_parameters_with_adam(parameters, grads, v, s, t, learning_rat
          e = 0.01,
                                          beta1 = 0.9, beta2 = 0.999,  epsilon = 1
          e-8):
              """
              Update parameters using Adam

              Arguments:
              parameters -- python dictionary containing your parameters:
                              parameters['W' + str(l)] = Wl
                              parameters['b' + str(l)] = bl
              grads -- python dictionary containing your gradients for each parame
          ters:
                              grads['dW' + str(l)] = dWl
                              grads['db' + str(l)] = dbl
              v -- Adam variable, moving average of the first gradient, python dic
          tionary
              s -- Adam variable, moving average of the squared gradient, python d
          ictionary
              learning_rate -- the learning rate, scalar.
              beta1 -- Exponential decay hyperparameter for the first moment estim
          ates
              beta2 -- Exponential decay hyperparameter for the second moment esti
          mates
              epsilon -- hyperparameter preventing division by zero in Adam update
          s

              Returns:
              parameters -- python dictionary containing your updated parameters
              v -- Adam variable, moving average of the first gradient, python dic
          tionary
              s -- Adam variable, moving average of the squared gradient, python d
          ictionary
```

```python
    """

    L = len(parameters) // 2                    # number of layers in the n
eural networks
    v_corrected = {}                            # Initializing first moment
estimate, python dictionary
    s_corrected = {}                            # Initializing second momen
t estimate, python dictionary

    # Perform Adam update on all parameters
    for l in range(L):
        # Moving average of the gradients. Inputs: "v, grads, beta1". Ou
tput: "v".
        ### START CODE HERE ### (approx. 2 lines)
        v["dW" + str(l+1)] = beta1 * v["dW" + str(l+1)] + (1 - beta1) *
grads["dW" + str(l+1)]
        v["db" + str(l+1)] = beta1 * v["db" + str(l+1)] + (1 - beta1) *
grads["db" + str(l+1)]
        ### END CODE HERE ###

        # Compute bias-corrected first moment estimate. Inputs: "v, beta
1, t". Output: "v_corrected".
        ### START CODE HERE ### (approx. 2 lines)
        v_corrected["dW" + str(l+1)] = v["dW" + str(l+1)] / (1 - beta1**
t)
        v_corrected["db" + str(l+1)] = v["db" + str(l+1)] / (1 - beta1**
t)
        ### END CODE HERE ###

        # Moving average of the squared gradients. Inputs: "s, grads, be
ta2". Output: "s".
        ### START CODE HERE ### (approx. 2 lines)
        s["dW" + str(l+1)] = beta2 * s["dW" + str(l+1)] + (1 - beta2) *
(grads["dW" + str(l+1)] ** 2)
        s["db" + str(l+1)] = beta2 * s["db" + str(l+1)] + (1 - beta2) *
(grads["db" + str(l+1)] ** 2)
        ### END CODE HERE ###

        # Compute bias-corrected second raw moment estimate. Inputs: "s,
beta2, t". Output: "s_corrected".
        ### START CODE HERE ### (approx. 2 lines)
        s_corrected["dW" + str(l+1)] = s["dW" + str(l+1)] / (1 - beta2 *
* t)
        s_corrected["db" + str(l+1)] = s["db" + str(l+1)] / (1 - beta2 *
* t)
        ### END CODE HERE ###

        # Update parameters. Inputs: "parameters, learning_rate, v_corre
cted, s_corrected, epsilon". Output: "parameters".
        ### START CODE HERE ### (approx. 2 lines)
        parameters["W" + str(l+1)] = parameters["W" + str(l+1)] - learni
ng_rate * v_corrected["dW" + str(l+1)] / np.sqrt(s_corrected["dW" + str(
l+1)] + epsilon)
        parameters["b" + str(l+1)] = parameters["b" + str(l+1)] - learni
ng_rate * v_corrected["db" + str(l+1)] / np.sqrt(s_corrected["db" + str(
l+1)] + epsilon)
        ### END CODE HERE ###
```

```python
    return parameters, v, s
```

```python
parameters, grads, v, s = update_parameters_with_adam_test_case()
parameters, v, s  = update_parameters_with_adam(parameters, grads, v, s,
t = 2)

print("W1 = " + str(parameters["W1"]))
print("b1 = " + str(parameters["b1"]))
print("W2 = " + str(parameters["W2"]))
print("b2 = " + str(parameters["b2"]))
print("v[\"dW1\"] = \n" + str(v["dW1"]))
print("v[\"db1\"] = \n" + str(v["db1"]))
print("v[\"dW2\"] = \n" + str(v["dW2"]))
print("v[\"db2\"] = \n" + str(v["db2"]))
print("s[\"dW1\"] = \n" + str(s["dW1"]))
print("s[\"db1\"] = \n" + str(s["db1"]))
print("s[\"dW2\"] = \n" + str(s["dW2"]))
print("s[\"db2\"] = \n" + str(s["db2"]))
```

```
W1 = [[ 1.63178673 -0.61919778 -0.53561312]
 [-1.08040999  0.85796626 -2.29409733]]
b1 = [[ 1.75225313]
 [-0.75376553]]
W2 = [[ 0.32648046 -0.25681174  1.46954931]
 [-2.05269934 -0.31497584 -0.37661299]
 [ 1.14121081 -1.09245036 -0.16498684]]
b2 = [[-0.88529978]
 [ 0.03477238]
 [ 0.57537385]]
v["dW1"] =
[[-0.11006192  0.11447237  0.09015907]
 [ 0.05024943  0.09008559 -0.06837279]]
v["db1"] =
[[-0.01228902]
 [-0.09357694]]
v["dW2"] =
[[-0.02678881  0.05303555 -0.06916608]
 [-0.03967535 -0.06871727 -0.08452056]
 [-0.06712461 -0.00126646 -0.11173103]]
v["db2"] =
[[ 0.02344157]
 [ 0.16598022]
 [ 0.07420442]]
s["dW1"] =
[[ 0.00121136  0.00131039  0.00081287]
 [ 0.0002525   0.00081154  0.00046748]]
s["db1"] =
[[  1.51020075e-05]
 [  8.75664434e-04]]
s["dW2"] =
[[  7.17640232e-05   2.81276921e-04   4.78394595e-04]
 [  1.57413361e-04   4.72206320e-04   7.14372576e-04]
 [  4.50571368e-04   1.60392066e-07   1.24838242e-03]]
s["db2"] =
[[  5.49507194e-05]
 [  2.75494327e-03]
 [  5.50629536e-04]]
```

Expected Output:

```
W1 =
[[ 1.63178673 -0.61919778 -0.53561312]
 [-1.08040999  0.85796626 -2.29409733]]
b1 =
[[ 1.75225313]
 [-0.75376553]]
W2 =
[[ 0.32648046 -0.25681174  1.46954931]
 [-2.05269934 -0.31497584 -0.37661299]
 [ 1.14121081 -1.09245036 -0.16498684]]
b2 =
[[-0.88529978]
 [ 0.03477238]
 [ 0.57537385]]
v["dW1"] =
[[-0.11006192  0.11447237  0.09015907]
 [ 0.05024943  0.09008559 -0.06837279]]
v["db1"] =
[[-0.01228902]
 [-0.09357694]]
v["dW2"] =
[[-0.02678881  0.05303555 -0.06916608]
 [-0.03967535 -0.06871727 -0.08452056]
 [-0.06712461 -0.00126646 -0.11173103]]
v["db2"] =
[[ 0.02344157]
 [ 0.16598022]
 [ 0.07420442]]
s["dW1"] =
[[ 0.00121136  0.00131039  0.00081287]
 [ 0.0002525   0.00081154  0.00046748]]
s["db1"] =
[[  1.51020075e-05]
 [  8.75664434e-04]]
s["dW2"] =
[[  7.17640232e-05   2.81276921e-04   4.78394595e-04]
 [  1.57413361e-04   4.72206320e-04   7.14372576e-04]
 [  4.50571368e-04   1.60392066e-07   1.24838242e-03]]
s["db2"] =
[[  5.49507194e-05]
 [  2.75494327e-03]
 [  5.50629536e-04]]
```

You now have three working optimization algorithms (mini-batch gradient descent, Momentum, Adam). Let's implement a model with each of these optimizers and observe the difference.

5 - Model with different optimization algorithms

Lets use the following "moons" dataset to test the different optimization methods. (The dataset is named "moons" because the data from each of the two classes looks a bit like a crescent-shaped moon.)

```
In [38]:  train_X, train_Y = load_dataset()
```
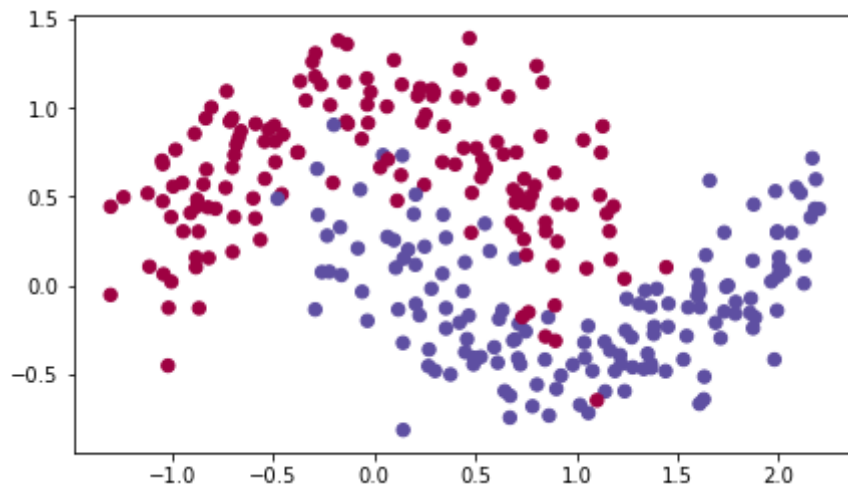

We have already implemented a 3-layer neural network. You will train it with:

- Mini-batch **Gradient Descent**: it will call your function:
    - `update_parameters_with_gd()`
- Mini-batch **Momentum**: it will call your functions:
    - `initialize_velocity()` and `update_parameters_with_momentum()`
- Mini-batch **Adam**: it will call your functions:
    - `initialize_adam()` and `update_parameters_with_adam()`

In [39]:
```python
def model(X, Y, layers_dims, optimizer, learning_rate = 0.0007, mini_bat
ch_size = 64, beta = 0.9,
          beta1 = 0.9, beta2 = 0.999,  epsilon = 1e-8, num_epochs = 1000
0, print_cost = True):
    """
    3-layer neural network model which can be run in different optimizer
modes.

    Arguments:
    X -- input data, of shape (2, number of examples)
    Y -- true "label" vector (1 for blue dot / 0 for red dot), of shape
 (1, number of examples)
    layers_dims -- python list, containing the size of each layer
    learning_rate -- the learning rate, scalar.
```

```python
    mini_batch_size -- the size of a mini batch
    beta -- Momentum hyperparameter
    beta1 -- Exponential decay hyperparameter for the past gradients est
imates
    beta2 -- Exponential decay hyperparameter for the past squared gradi
ents estimates
    epsilon -- hyperparameter preventing division by zero in Adam update
s
    num_epochs -- number of epochs
    print_cost -- True to print the cost every 1000 epochs

    Returns:
    parameters -- python dictionary containing your updated parameters
    """

    L = len(layers_dims)                    # number of layers in the neural ne
tworks
    costs = []                              # to keep track of the cost
    t = 0                                   # initializing the counter required
for Adam update
    seed = 10                               # For grading purposes, so that you
r "random" minibatches are the same as ours
    m = X.shape[1]                          # number of training examples

    # Initialize parameters
    parameters = initialize_parameters(layers_dims)

    # Initialize the optimizer
    if optimizer == "gd":
        pass # no initialization required for gradient descent
    elif optimizer == "momentum":
        v = initialize_velocity(parameters)
    elif optimizer == "adam":
        v, s = initialize_adam(parameters)

    # Optimization loop
    for i in range(num_epochs):

        # Define the random minibatches. We increment the seed to reshuf
fle differently the dataset after each epoch
        seed = seed + 1
        minibatches = random_mini_batches(X, Y, mini_batch_size, seed)
        cost_total = 0

        for minibatch in minibatches:

            # Select a minibatch
            (minibatch_X, minibatch_Y) = minibatch

            # Forward propagation
            a3, caches = forward_propagation(minibatch_X, parameters)

            # Compute cost and add to the cost total
            cost_total += compute_cost(a3, minibatch_Y)

            # Backward propagation
            grads = backward_propagation(minibatch_X, minibatch_Y, cache
```

```
s)

            # Update parameters
            if optimizer == "gd":
                parameters = update_parameters_with_gd(parameters, grads
, learning_rate)
            elif optimizer == "momentum":
                parameters, v = update_parameters_with_momentum(paramete
rs, grads, v, beta, learning_rate)
            elif optimizer == "adam":
                t = t + 1 # Adam counter
                parameters, v, s = update_parameters_with_adam(parameter
s, grads, v, s,
                                                               t, learni
ng_rate, beta1, beta2,  epsilon)
        cost_avg = cost_total / m

        # Print the cost every 1000 epoch
        if print_cost and i % 1000 == 0:
            print ("Cost after epoch %i: %f" %(i, cost_avg))
        if print_cost and i % 100 == 0:
            costs.append(cost_avg)

    # plot the cost
    plt.plot(costs)
    plt.ylabel('cost')
    plt.xlabel('epochs (per 100)')
    plt.title("Learning rate = " + str(learning_rate))
    plt.show()

    return parameters
```

You will now run this 3 layer neural network with each of the 3 optimization methods.

5.1 - Mini-batch Gradient descent

Run the following code to see how the model does with mini-batch gradient descent.
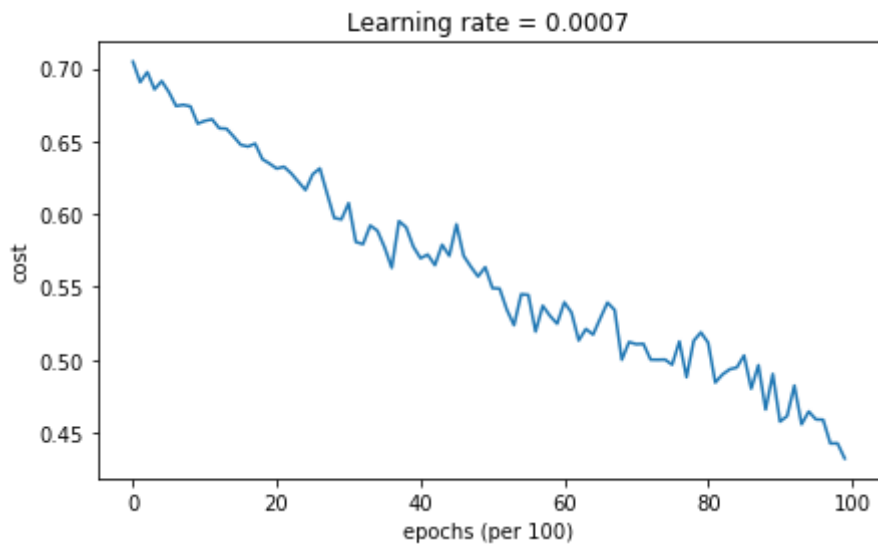
```
In [40]:  # train 3-layer model
          layers_dims = [train_X.shape[0], 5, 2, 1]
          parameters = model(train_X, train_Y, layers_dims, optimizer = "gd")

          # Predict
          predictions = predict(train_X, train_Y, parameters)

          # Plot decision boundary
          plt.title("Model with Gradient Descent optimization")
          axes = plt.gca()
          axes.set_xlim([-1.5,2.5])
          axes.set_ylim([-1,1.5])
          plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X,
          train_Y)
```

```
/opt/conda/lib/python3.6/site-packages/ipykernel/__main__.py:39: Visibl
eDeprecationWarning: using a non-integer number instead of an integer w
ill result in an error in the future
/opt/conda/lib/python3.6/site-packages/ipykernel/__main__.py:40: Visibl
eDeprecationWarning: using a non-integer number instead of an integer w
ill result in an error in the future

Cost after epoch 0: 0.704583
Cost after epoch 1000: 0.664001
Cost after epoch 2000: 0.631245
Cost after epoch 3000: 0.607541
Cost after epoch 4000: 0.569744
Cost after epoch 5000: 0.549242
Cost after epoch 6000: 0.539490
Cost after epoch 7000: 0.510797
Cost after epoch 8000: 0.511862
Cost after epoch 9000: 0.457633
```



```
Accuracy: 0.796666666667
```
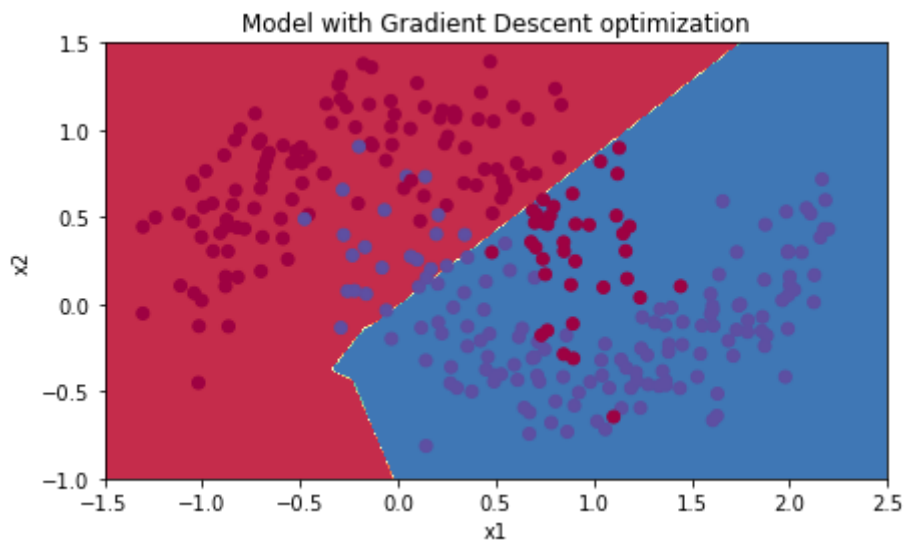
5.2 - Mini-batch gradient descent with momentum

Run the following code to see how the model does with momentum. Because this example is relatively simple, the gains from using momemtum are small; but for more complex problems you might see bigger gains.
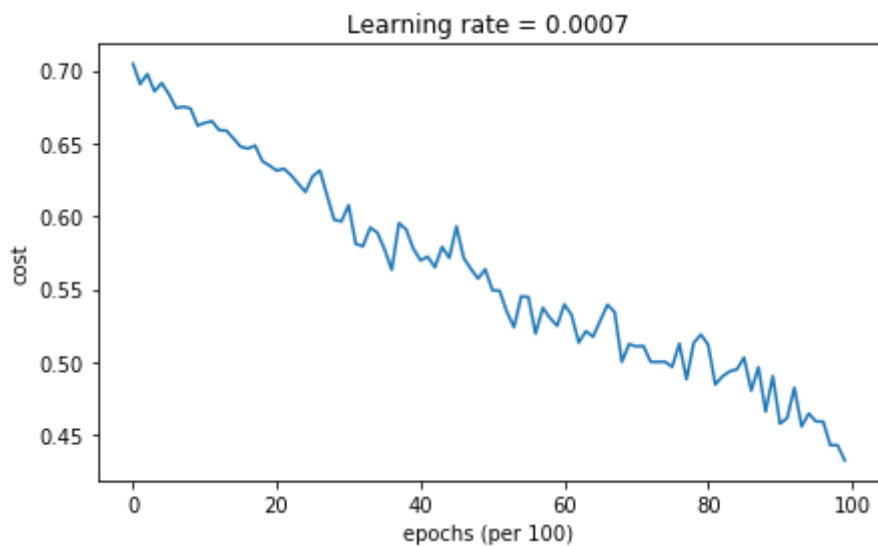
```python
In [41]:  # train 3-layer model
          layers_dims = [train_X.shape[0], 5, 2, 1]
          parameters = model(train_X, train_Y, layers_dims, beta = 0.9, optimizer
          = "momentum")

          # Predict
          predictions = predict(train_X, train_Y, parameters)

          # Plot decision boundary
          plt.title("Model with Momentum optimization")
          axes = plt.gca()
          axes.set_xlim([-1.5,2.5])
          axes.set_ylim([-1,1.5])
          plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X,
          train_Y)
```
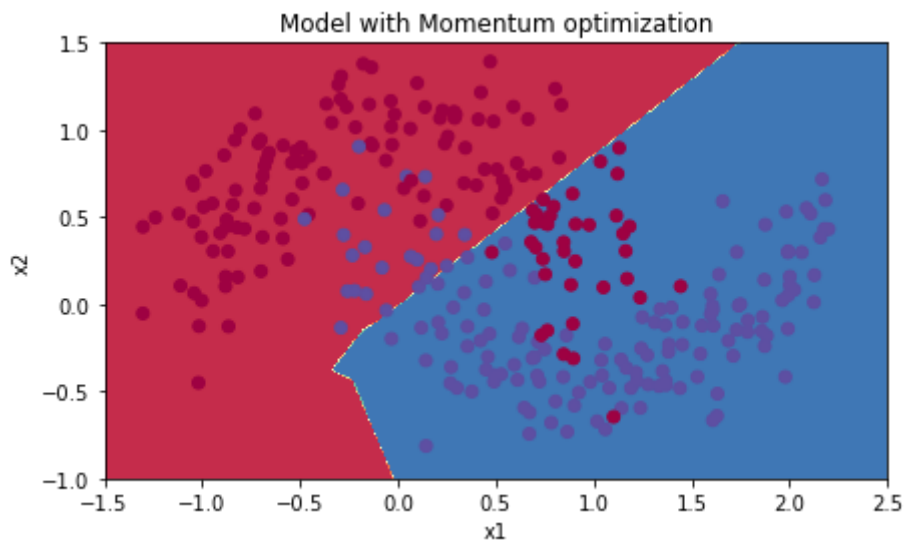
```
/opt/conda/lib/python3.6/site-packages/ipykernel/__main__.py:39: Visibl
eDeprecationWarning: using a non-integer number instead of an integer w
ill result in an error in the future
/opt/conda/lib/python3.6/site-packages/ipykernel/__main__.py:40: Visibl
eDeprecationWarning: using a non-integer number instead of an integer w
ill result in an error in the future
```

```
Cost after epoch 0: 0.704595
Cost after epoch 1000: 0.664071
Cost after epoch 2000: 0.631347
Cost after epoch 3000: 0.607646
Cost after epoch 4000: 0.569818
Cost after epoch 5000: 0.549311
Cost after epoch 6000: 0.539577
Cost after epoch 7000: 0.510867
Cost after epoch 8000: 0.512012
Cost after epoch 9000: 0.457824
```



```
Accuracy: 0.796666666667
```

5.3 - Mini-batch with Adam mode

Run the following code to see how the model does with Adam.

```
In [42]:   # train 3-layer model
           layers_dims = [train_X.shape[0], 5, 2, 1]
           parameters = model(train_X, train_Y, layers_dims, optimizer = "adam")

           # Predict
           predictions = predict(train_X, train_Y, parameters)

           # Plot decision boundary
           plt.title("Model with Adam optimization")
           axes = plt.gca()
           axes.set_xlim([-1.5,2.5])
           axes.set_ylim([-1,1.5])
           plot_decision_boundary(lambda x: predict_dec(parameters, x.T), train_X,
           train_Y)
```
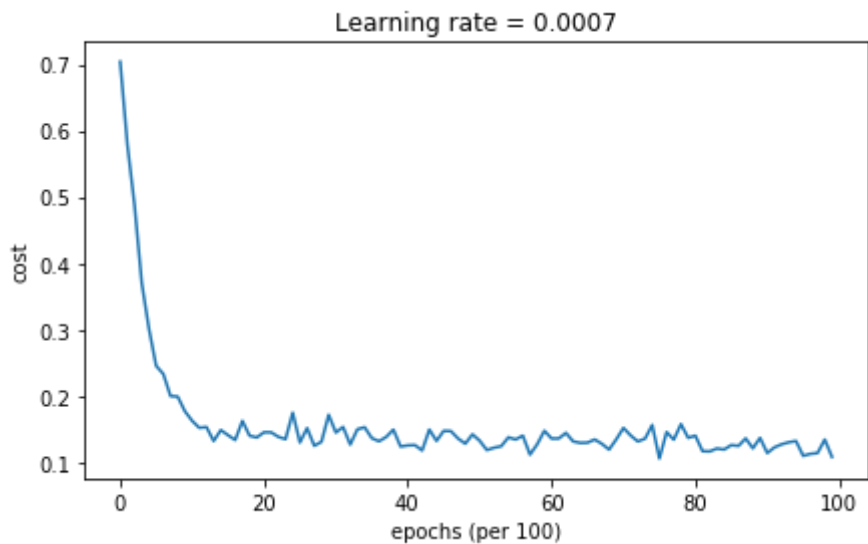
```
Cost after epoch 0: 0.704177
Cost after epoch 1000: 0.162439
Cost after epoch 2000: 0.144940
Cost after epoch 3000: 0.144516
Cost after epoch 4000: 0.125187
Cost after epoch 5000: 0.132232
Cost after epoch 6000: 0.135526
Cost after epoch 7000: 0.151504
Cost after epoch 8000: 0.139858
Cost after epoch 9000: 0.113703
```


Learning rate = 0.0007

```
Accuracy: 0.94
```


Model with Adam optimization

5.4 - Summary

optimization method	**accuracy**	**cost shape**
Gradient descent	79.7%	oscillations
Momentum	79.7%	oscillations
Adam	94%	smoother

Momentum usually helps, but given the small learning rate and the simplistic dataset, its impact is almost negligeable. Also, the huge oscillations you see in the cost come from the fact that some minibatches are more difficult thans others for the optimization algorithm.

Adam on the other hand, clearly outperforms mini-batch gradient descent and Momentum. If you run the model for more epochs on this simple dataset, all three methods will lead to very good results. However, you've seen that Adam converges a lot faster.

Some advantages of Adam include:

- Relatively low memory requirements (though higher than gradient descent and gradient descent with momentum)
- Usually works well even with little tuning of hyperparameters (except α)

References:

- Adam paper: https://arxiv.org/pdf/1412.6980.pdf (https://arxiv.org/pdf/1412.6980.pdf)