The n-class model for the US(PTO) patent network

Below we outline the general block strategy for creating an adjacency matrix A for an n-class model. Below, we specify what we believe to be the most important actors: patents, inventors, and firms. We propose to also include other classes such as: technologies, examiners, lawyers, country-inventors.

[PTO	1	;]
	(patents)	1
	(inventors]
		$[\overline{\text{(firms)}}]$

From this block matrix above, we need to determine (1) how to form the matrix, (2) how to address reducibility (e.g., super-node STATIC), (3) how to block normalize, (4) how to address PROPEN-SITY/TEMPORAL CONSTRAINTS/SPAMMING BIAS

To be explicit in our understanding of the problem and not have concerns with conformability, we will describe the block matrices as follows:

- (PP) Core patent-citation matrix, square, $pp_{r,c}$, rows represent backward citations, columns represent forward citations.
- (IP) Inventor-Patent structure, not square, $ip_{r,c}$, rows represent inventors, columns represent patents, and the entries represent a truth table of a link structure based on inventor authorship. (By construction, every patent needs at least one inventor, so if the data is missing, what should we do? Possibly create a MISSING node?)
- (FP) Firm-Patent structure, not square, $\operatorname{fp}_{r,c}$, rows represent firms, columns represent patents, and the entries represent a truth table of a link structure based on firm assignment. By construction, a patent requires at least one firm, but does not require a firm (the inventor can maintain assignment/ownership). For this reason, we will introduce a NULL node to capture the information that no firm was assigned, but the data is not MISSING (again, possibly create a MISSING node as well?)

We emphasize this nomenclature by updating the matrix:

[PTO	1 1 1 1	-
	(\mathbf{PP})	
	<u> </u>	
	dash)

From these basic premises, we have options on creating the diagonal block structures:

- (PP) Anchored to our traditional Patent Rank model, we have several methods to form the network, include citations, and possibly their strength (combined model as structure + class match). We may want to readdress some of these choices if we include technology as its own class with nodes/edges.
- (II) = 0 We could just say we have no network information about inventor collaborations.
- (II) = (IP)(PI) We could multiple the inventor-patent network structure by its transpose to identify the collaboration among inventors $(i_1 \text{ worked with } i_3)$.
- (II) = (IP)(PP)(PI) We could create a quadratic form to weight the associations to include the patent-citation structure.

 This would capture "influence" but not direct collaboration.
- (II) = (IP)(PI) + (IP)(PP)(PI) We could create a combined model where we identify the "structure" and "influence".

We could respectively define (FF) following analogous structures, or we could create additional nested structures: patents are created by inventors AND assigned to firms.

$$(FF) = (\mathbf{FP})(\mathbf{PI})(\mathbf{IP})(\mathbf{PF})$$
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 $(FF) = (\mathbf{FP})(\mathbf{PI})(\mathbf{PP})(\mathbf{IP})(\mathbf{PF})$????
 $(FF) = (\mathbf{FI})(\mathbf{IP})(\mathbf{PI})(\mathbf{IF})$???????
 $(FF) = (\mathbf{FI})(\mathbf{IP})(\mathbf{PP})(\mathbf{PI})(\mathbf{IF})$????????

Alternatively, we could argue that patents are owned by firms who hire inventors to create them, altering the form of the construction of (II) in a nested manner. We need to think carefully about such constructions both mathematically and conceptually.

Additionally, determining the off-diagonals need to be considered in the construction of the entire matrix.

Once we consider the construction, we then would need to consider the SUPER-NODE. I prefer the single (PTO) supernode at the highest level (the STATIC model) with possible NULL and/or MISSING nodes within specific blocks (but not true "dummy nodes").

In terms of normalization, I also intuitively prefer the row-normalization within a block but there are other possible methods.

A proposed consideration:

Γ	PTO		i I			0	1	1	1
ı		(patents)			_	1	(\mathbf{PP})	(IP)	(FP)
			(inventors)		_	$ \bar{1} $	$\bar{P}\bar{I}$	$\bar{\mathbf{P}}[\bar{\mathbf{I}}\bar{\mathbf{I}}] = \bar{\mathbf{I}}\bar{\mathbf{P}}[\bar{\mathbf{P}}\bar{\mathbf{I}}] + \bar{\mathbf{I}}\bar{\mathbf{P}}[\bar{\mathbf{P}}\bar{\mathbf{P}}][\bar{\mathbf{P}}\bar{\mathbf{I}}]$	$(\bar{F}\bar{I}) = (\bar{F}\bar{P})(\bar{P}\bar{I})$
				$\overline{(\mathbf{firms})}$		$\lfloor ar{1} floor$	\bar{PF}	(IF) = (IP)(PF)	$\mathbf{F}(\mathbf{F}) = \mathbf{FP}(\mathbf{PI})(\mathbf{II})(\mathbf{IP})(\mathbf{PF})$