Reaching Equivalency between PageRank and Patent Rank

Let $Q =$

	1	3	9	4	5	6	7	8	11	12	2	10	13	14	15
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
7	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
8	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
11	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	1	0	1	0	0	1	1	1	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0

and $Q' =$

PTO	PTO	1	3	9	4	5	6	7	8	11	12	2	10	13	14	15
PTO	**0**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
1	$\alpha_{\mathbf{1}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	$\alpha_{\mathbf{3}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	$\alpha_{\mathbf{9}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	$\alpha_{\mathbf{4}}$	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	$\alpha_{\mathbf{5}}$	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	$\alpha_{\mathbf{6}}$	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
7	$\alpha_{\mathbf{7}}$	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0
8	$\alpha_{\mathbf{8}}$	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
11	$\alpha_{\mathbf{11}}$	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	$\alpha_{\mathbf{12}}$	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
2	$\alpha_{\mathbf{2}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	$\alpha_{\mathbf{10}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	$\alpha_{\mathbf{13}}$	0	0	1	0	1	0	0	1	1	1	0	0	0	0	0
14	$\alpha_{\mathbf{14}}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	$\alpha_{\mathbf{15}}$	0	0	0	0	0	1	1	1	0	1	0	0	0	0	0

Let $\pi =$ the PageRank solution from Q with fixed damping factor α (e.g., 0.85).

Let $\pi' =$ the Patent Rank solution (without the super node) from Q' with super-node column associations $\alpha_i \forall$ sorted nodes $i \in Q'$.

If we assign $\alpha_i = \frac{\alpha}{(1-\alpha)}$ for the super-node column associations, then $\pi = (1+\alpha)\pi'$.