Leverage Effects of Hedge Funds

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Hedge funds can earn high profits by the use of leverage, but leverage also creates high risk. The simulations in this experiment explore the effects of leverage. Suppose a hedge fund owns $1,000,000 of stock and used $50,000 of its own capital and $950,000 in borrowed money for the purchase. If the value of the stock falls below $950,000 at the end of any trading day, then the hedge fund must sell all the stock and repay the loan. This will wipe out its $50,000 investment. **The hedge fund is said to be leveraged 20:1 since its position is 20 times the amount of its own capital invested.** Let R_t denote the daily simple return of the stock at time t. Then $r_t = log(1 + R_t)$ is the daily log/cc return of this stock. Suppose $\{\cdots, r_{t-1}, r_t, r_{t+1}, \ldots\}$ are independent normal random variables with mean 0.05/253 and variance $(0.23)^2/253$, respectively.

Suppose the hedge fund will sell the stock for a profit of at least $100,000 if the value of the stock rises to at least $1,100,000 at the end of one of the 45 trading days, sell it for a loss if the value falls below $950,000 at the end of one of the 45 trading days, or sell after 45 trading days if the closing price has stayed between $950,000 and $1,100,000.

Part I

What is the probability that the value of the stock will be below $950,000 at the close of at least one of the next 45 trading days?

We can solve for the probability of interest by monte carlo simulation.

```
mu.r <- 0.05 / 253
sigma.r <- 0.23 / sqrt(253)

n.day <- 100 # num of trading days
runs <- 5000
W0 <- 1000000
capital <- 50000

set.seed(2020)
r <- rnorm(runs * n.day, mu.r, sigma.r)
r <- matrix(r, nrow = n.day, byrow = TRUE)
```

```r
dim(r)
```

```
## [1]  100 5000
```

```r
head(r[, 1:5])
```

```
##              [,1]          [,2]          [,3]         [,4]          [,5]
## [1,]  0.005648636  0.0045580107 -0.0156797604 -0.016148014 -0.0402401910
## [2,] -0.016695046  0.0097503980  0.0198706334 -0.001293937 -0.0006259215
## [3,]  0.026461512 -0.0058546261 -0.0050162223  0.001259594 -0.0010743811
## [4,] -0.009133340 -0.0104373663 -0.0003404587 -0.017173935  0.0119585092
## [5,] -0.020409106 -0.0202588585  0.0044916353 -0.003741937 -0.0163901205
## [6,] -0.023777086  0.0005236673  0.0066349239  0.009124486  0.0233123696
```

After generating the simulated data of cc returns, we can determine the simulated stock values.

```r
# compute the value of stock at the end of each day
# W_func <- function(r.col, w0 = W0, THRES = thres) {
#   res <- rep(-1, n.day) # a seq for storing results
#   res[1] <- w0 * exp(r.col[1])
#   hold <- n.day # by default, hold the stock till the last day
#   for (i in 2:n.day) {
#     if (res[i-1] >= THRES) {
#       res[i] <- res[i-1] * exp(r.col[i])
#     } else {
#       hold <- i-1
#       res[i:n.day] <- 0
#       break
#     }
#   }
#   list(res.col = res, hold.n = hold)
# }

W_func <- function(r.col, lb = W0 - capital, ub = W0 + 100000) {
  res <- rep(-1, n.day) # a seq for storing results
  res[1] <- W0 * exp(r.col[1])
  hold <- n.day # by default, hold the stock till the last day
  for (i in 2:n.day) {
    if (res[i-1] >= lb & res[i-1] <= ub) {
      res[i] <- res[i-1] * exp(r.col[i])
    } else if (res[i-1] > ub ) {
      hold <-  i - 1
      res[i:n.day] <- res[i-1]
      break
    } else {
      hold <- i - 1
      res[i:n.day] <- 0
      break
    }
  }
  list(res.col = res, hold.n = hold)
}

W <- matrix(-1, nrow = n.day, ncol = runs)
hold <- rep(-1, runs)
```

```r
for (i in 1:runs) {
  temp <- W_func(r[,i])
  W[,i] <- temp$res.col
  hold[i] <- temp$hold.n
}

head(W[, 1:5])
```

```
##            [,1]      [,2]      [,3]     [,4]     [,5]
## [1,] 1005664.6 1004568.4  984442.5 983981.7 960558.7
## [2,]  989014.4 1014411.3 1004199.7 982709.3 959957.6
## [3,] 1015534.5 1008489.6  999175.0 983947.9 958926.8
## [4,] 1006301.5  998018.4  998834.9 967193.9 970463.0
## [5,]  985972.0  978003.1 1003331.4 963581.5 954686.7
## [6,]  962805.0  978515.4 1010010.5 972413.9 977204.1
```

Through monte carlo simulation, we can first calculate the probability that the stock value will be no less than $950,000 at the close of each day. Then, subtracting this probability from 1 yields the probability of interest.

```r
ans1 <- 1 - sum(W[n.day,] >= W0 - capital) / runs
ans1
```

```
## [1] 0.5858
```

Moreover, we can consider the probability that the stock value will be below $950,000 at the close of each day.

```r
P <- rep(0, n.day)
for (i in 1:n.day) {
  P[i] <- sum(W[i,] <  W0 - capital)/runs
}

P
```

```
##   [1] 0.0002 0.0058 0.0232 0.0452 0.0720 0.0956 0.1196 0.1436 0.1718 0.1940
##  [11] 0.2136 0.2312 0.2476 0.2652 0.2796 0.2932 0.3068 0.3210 0.3310 0.3422
##  [21] 0.3530 0.3630 0.3720 0.3824 0.3912 0.3992 0.4078 0.4148 0.4214 0.4286
##  [31] 0.4342 0.4402 0.4468 0.4522 0.4596 0.4664 0.4712 0.4762 0.4818 0.4852
##  [41] 0.4896 0.4950 0.5004 0.5030 0.5062 0.5108 0.5140 0.5170 0.5210 0.5236
##  [51] 0.5270 0.5294 0.5312 0.5336 0.5360 0.5380 0.5406 0.5426 0.5444 0.5460
##  [61] 0.5482 0.5494 0.5508 0.5526 0.5546 0.5556 0.5568 0.5598 0.5610 0.5618
##  [71] 0.5630 0.5638 0.5658 0.5672 0.5682 0.5692 0.5708 0.5714 0.5730 0.5744
##  [81] 0.5748 0.5750 0.5754 0.5764 0.5772 0.5786 0.5790 0.5800 0.5802 0.5812
##  [91] 0.5820 0.5824 0.5826 0.5838 0.5844 0.5844 0.5848 0.5852 0.5852 0.5858
```

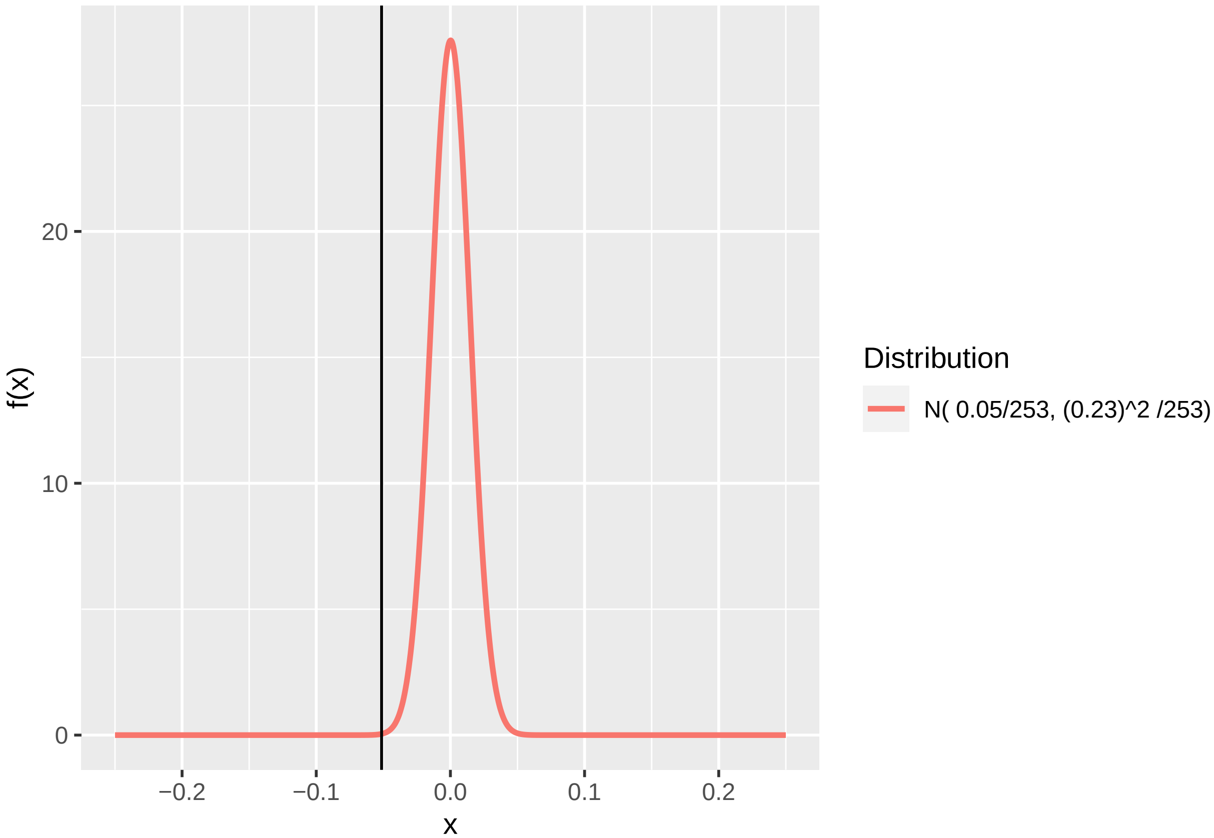
Though the probability $Pr(W_1 < 95,000)$ is nearly 1 as shown below. The probabilities of losing own capital grow quickly with trading days.

```r
x.vals <- seq(-0.25, 0.25, length = 1000)
df <- data.frame(x.vals, dnorm(x.vals, mu.r, sigma.r))
colnames(df) <- c("x", "y")
head(df$y)
```

```
## [1] 2.691149e-64 4.895256e-64 8.893913e-64 1.613950e-63 2.925276e-63
## [6] 5.295701e-63
```

```r
library(ggplot2)
ggplot(data = df) +
```

```r
  geom_line(mapping = aes(y=y,x= x, color = "N( 0.05/253, (0.23)^2 /253)"), size = 1) +
  labs(color = "Distribution", x = "x", y = "f(x)") +
  geom_vline(xintercept=log(0.95))
```



Part II

What is the probability that the hedge fund will make a profit of at least $100,000?

```r
sum(W[n.day,] > W0 + 100000) / runs
```

```
## [1] 0.3874
```

Part III

What is the probability the hedge fund will suffer a loss?

```r
sum(W[n.day, ] < W0) / runs
```

```
## [1] 0.5926
```

The probability mirrors two possible events:

- lose the stock at the end of any trading day
- the stock value at the end of day 45 is between $950,000 and $1,000,000.

```r
ans2 <- sum(W[n.day, ] <= W0 & W[n.day, ] > W0 - capital) / runs
ans2 + ans1
```

```
## [1] 0.5926
```

Part IV

What is the expected profit from this trading strategy?

```
# rm(profit)
profit <- rep(0, runs)
for (i in 1:runs) {
  profit[i] <- W[hold[i], i] - W0
}
head(profit)
```

```
## [1] -84901.32 -57939.53 -51458.38 -55148.15  43357.01 -54590.76
```

```
head(W[,1:6])
```

```
##           [,1]      [,2]      [,3]     [,4]     [,5]      [,6]
## [1,] 1005664.6 1004568.4  984442.5 983981.7 960558.7 1010673.7
## [2,]  989014.4 1014411.3 1004199.7 982709.3 959957.6 1006190.8
## [3,] 1015534.5 1008489.6  999175.0 983947.9 958926.8 1007490.8
## [4,] 1006301.5  998018.4  998834.9 967193.9 970463.0  988117.5
## [5,]  985972.0  978003.1 1003331.4 963581.5 954686.7  979348.7
## [6,]  962805.0  978515.4 1010010.5 972413.9 977204.1  975388.3
```

```
sprintf("The expected profit is %#.3f", mean(profit))
```

```
## [1] "The expected profit is 9293.564"
```

Part V

What is the expected averaging return? When answering this question, remember that only $50,000 was invested. Also, the units of return are time, e.g., one can express a return as a daily averaging return. Therefore, one must keep track of how long the hedge fund holds its position before selling.

The geometric average will be used to compute the average return.

```
ret.avg <- rep(0, runs)
for (i in 1:runs) {
  ret.avg[i] <- ((profit[i]) / 50000) / hold[i]
}

head(ret.avg)
```

```
## [1] -0.154366036 -0.019313178 -0.068611175 -0.137870385  0.008671401
## [6] -0.054590764
```

```
sprintf("The expected (arithmetic) average return is %#.4f", mean(ret.avg))
```

```
## [1] "The expected (arithmetic) average return is -0.0238"
```

```
sprintf("Given the hedge fund makes a profit, the expected average return is %#.4f",
        mean(ret.avg * (profit >= 0)) / mean(profit >= 0))
```

```
## [1] "Given the hedge fund makes a profit, the expected average return is 0.0902"
```

```
sprintf("The probability of making a profit is %#.4f",
        mean(profit >= 0))
```

```
## [1] "The probability of making a profit is 0.4074"
```

```
sprintf("Accordingly, the conditional profit is %f",
        mean(profit * (profit >= 0)) / mean(profit >= 0))
```

```
## [1] "Accordingly, the conditional profit is 106267.159007"
```

```r
sprintf("Given the hedge fund suffers a loss, the expected average return is %#.4f",
        mean(ret.avg * (profit < 0)) / mean(profit < 0))
```

```
## [1] "Given the hedge fund suffers a loss, the expected average return is -0.1022"
```

```r
sprintf("The probability of making a profit is %#.4f",
        mean(profit < 0))
```

```
## [1] "The probability of making a profit is 0.5926"
```

```r
sprintf("Accordingly, the conditional profit is %f",
        mean(profit * (profit < 0)) / mean(profit < 0))
```

```
## [1] "Accordingly, the conditional profit is -57373.736830"
```