Econ 101 Honors Section

Using Random Numbers for Numerical Integration and Simulation, with an Application to Insurance

In this lesson we will learn how to generate random numbers in Python, which will let us simulate the outcomes of situations with uncertainty, and thus help in calculating things like expected value, certainty equivalents, and insurance pricing. Consider the following scenario:

A driver with a wealth of w_0 has a chance θ of getting into an accident. If there is an accident, then the amount of damage is a random variable given by L, where L is log-normally distributed with a location parameter μ and scale parameter σ.[1] An insurance contract with deductible D says that if there is an accident, the driver will have to pay up to D in damages, but if the damages exceeds D, then the insurance company will cover the rest. The driver has a CARA utility function over final wealth: $u(W) = -e^{-\gamma W}$.

This is a difficult problem to solve by hand because it involves complicated integral equations. However, it is quite simple to do using a computer program. The reason is that the expected value of any random variable can be approximated simply by simulating the outcome of the random variable many times, and then taking the average.[2] Thus, we can write a program that simulates the outcomes and the utility that results from it many times, and then averages over the utilities. So let's write a program that helps us answer the following questions:

1. What is the certainty equivalent of the driver's risky situation without insurance?
2. What is the certainty equivalent of the driver's risky situation *with* insurance? (Note: There is still some risk because of the deductible.)
3. What is the driver's willingness to pay for an insurance contract with deductible D? (Hint: It is equal to the difference in the two answers above.)
4. What is the actuarially fair premium for an insurance contract with deductible D?

[1] This means that we can write $L = e^X$ where X is a normal random variable with mean μ and standard deviation σ. See https://en.wikipedia.org/wiki/Log-normal_distribution.

[2] This is known as the Law of Large Numbers.