

Econ 101 Honors Section

Pricing in Many Market Segments

A monopolist faces n market segments, each with demand curves:

$$p_i(q_i) = a_i - b_i q_i$$

The monopolist's total cost curve is:

$$C(q) = cq + dq^2$$

The list of a_i , b_i , c and d are given in `data1.txt`. The first line in `data1.txt` tells us n . The next n lines tells us the n values of a_i . The next n lines tells us the values of b_i . The last two lines tell us c and d .

Write a python program that solves for the profit-maximizing price in each market.

Step 1: Derive a system of equations that solves the profit-maximization problem

$$\begin{aligned}\pi &= \sum_{i=1}^n p_i(q_i)q_i - C\left(\sum_{i=1}^n q_i\right) \\ &= \sum_{i=1}^n (a_i - b_i q_i)q_i - c\left(\sum_{i=1}^n q_i\right) - d\left(\sum_{i=1}^n q_i\right)^2 \\ FOC[q_i]: a_i - 2b_i q_i - c - 2d\left(\sum_{j=1}^n q_j\right) &= 0 \\ (2b_i + 2d)q_i + 2d\left(\sum_{j \neq i} q_j\right) &= a_i - c\end{aligned}$$

This gives us n equations in n unknowns, which we can write as a matrix equation:

$$\begin{pmatrix} 2b_1 + 2d & 2d & \dots & 2d & 2d \\ 2d & 2b_2 + 2d & \dots & 2d & 2d \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 2d & 2d & \dots & 2b_{n-1} + 2d & 2d \\ 2d & 2d & \dots & 2d & 2b_n + 2d \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_{n-1} \\ q_n \end{pmatrix} = \begin{pmatrix} a_1 - c \\ a_2 - c \\ \vdots \\ a_{n-1} - c \\ a_n - c \end{pmatrix}$$

Step 2: Write a program that reads in the data and solves the problem

```

import numpy

# Open the data file for reading
myfile = open('data1.txt','r')

# First line is n
n = int(myfile.readline().strip())

# Note: readline() reads the next line in the file
#       strip() removes line breaks
#       int(...) transforms the string to an integer

# Next n lines are a
a = numpy.zeros(n)
for i in range(n):
    a[i] = float(myfile.readline().strip())
# float(...) transforms the string to a float (a numeric data type)

# Next n lines are b
b = numpy.zeros(n)
for i in range(n):
    b[i] = float(myfile.readline().strip())

# Next line is c
c = float(myfile.readline().strip())

# Next line is d
d = float(myfile.readline().strip())

myfile.close() # close the file

## Solving the model
#
# FOC's give us  $Mq = y$ 

# You fill in the rest

```