

## ch3 part 13 wrapup

# The Mechatronic Engineer's Python Study Bible

## Introduction: From Textbook Problems to Engineering Principles

### Prologue

Welcome. The document before you is not merely a set of answers to textbook exercises. It is a study bible, a guide designed to translate the fundamental concepts of programming into the practical language of a mechatronics engineer. The problems presented in your course materials are not isolated academic challenges; they are carefully constructed microcosms of larger, more complex engineering principles that govern the worlds of robotics, control systems, and data-driven industrial automation.<sup>1</sup> We will treat each problem as a launchpad—a starting point from which we can explore the real-world thinking, the trade-offs, and the profound responsibilities of an engineer who builds systems that interact with the physical world.

### Core Theme

This journey is about making a critical transition: moving from simply writing code that *works* to engineering solutions that are *robust*, *efficient*, and *reliable*. The simple logic required to convert a binary number or calculate change forms the very bedrock of the complex systems you will one day design, build, and maintain. The core theme that will guide us is the

development of an engineering mindset, where every line of code is written with an awareness of its potential physical consequences.

This study bible is built upon a central philosophy: the most critical skill for a modern engineer is the ability to bridge the gap between simplified, abstract models—like the problems in a textbook—and the complex, messy, and constrained reality of the physical world. The exercises from your book represent the abstract model. Our exploration will build the bridge to concrete applications in robotics, manufacturing, and data science. This process is not just about learning Python; it is about learning how to think like an engineer. It is about understanding how to take a clean, theoretical concept and apply it within a system of noisy sensors, finite processing power, and immovable physical laws. This is the art and science of mechatronics, and this is the journey we are about to begin.

## **Chapter 1: The Language of Machines — Deconstructing Binary**

### **Foundation: The Binary-to-Decimal Algorithm**

Your textbook presents the challenge of converting a binary integer, an integer composed of only zeros and ones, into its decimal equivalent.<sup>1</sup> The hint provided suggests a beautifully elegant software algorithm that uses the modulus and division operators. Let's walk through this logic as if we were the computer, step by step, without looking at any written code.

Imagine we are given the binary number one-one-zero-one, represented as the integer 1101. Our goal is to convert this to its decimal form. We'll need two variables to keep track of our progress. The first is our final result, which we can call `decimal_value`, and we'll initialize it to zero. The second is what we'll call the `base`, which represents the positional value of each digit in the binary number. In the binary system, the rightmost digit has a value of one (which is two to the power of zero), the next has a value of two, then four, then eight, and so on. So, we'll initialize our `base` variable to one.<sup>2</sup>

Now, we begin a loop that will continue as long as our binary number is not zero.

In the first step of the loop, we need to isolate the rightmost digit of 1101. We can do this using the modulus operator, specifically, 1101 modulus 10. This operation gives us the remainder of a division, which is a clever way to "peel off" the last digit of a number

represented in base ten. The result is one. This is our `last_digit`.

Next, we update our `decimal_value`. We take the `last_digit` (which is one), multiply it by our current base (which is also one), and add the result to `decimal_value`. So, `decimal_value` is now zero plus one times one, which equals one.

After that, we need to prepare for the next digit. We update our base by multiplying it by two. So, our base is now two. Finally, we must remove the digit we just processed from our binary number. We can do this with integer division.  $1101$  integer-divided by  $10$  gives us  $110$ . Our binary number is now  $110$ , and the first iteration of the loop is complete.

The loop continues. Our binary number is  $110$ . We take  $110$  modulus  $10$  to get the `last_digit`, which is zero. We update `decimal_value` by adding the `last_digit` (zero) times the current base (two). Zero times two is zero, so `decimal_value` remains one. We update our base by multiplying it by two, so it becomes four. We update our binary number by integer-dividing  $110$  by  $10$ , which gives us  $11$ .

The loop runs again. Our binary number is  $11$ .  $11$  modulus  $10$  gives us a `last_digit` of one. We update `decimal_value` by adding the `last_digit` (one) times the current base (four). One times four is four. We add this to our `decimal_value` of one, so it now becomes five. We update the base to eight and the binary number to one.

One final iteration. Our binary number is one. One modulus ten is one. We update `decimal_value` by adding the `last_digit` (one) times the current base (eight). One times eight is eight. We add this to our `decimal_value` of five, making it thirteen. We update the base to sixteen and the binary number to zero.

Now, the loop's condition—that the binary number must be greater than zero—is false. The loop terminates. The final value stored in our `decimal_value` variable is thirteen. And so, the decimal equivalent of the binary number  $1101$  is indeed thirteen, which is calculated as one times eight, plus one times four, plus zero times two, plus one times one.<sup>1</sup>

## Application: The Microcontroller's Perspective

This algorithm is more than an academic exercise; it is the fundamental language of mechatronics. A microcontroller at the heart of a robotic arm or an industrial sensor node does not perceive the world in human-readable terms like "25.5 degrees Celsius" or "3.7 meters per second." It receives raw, electrical signals.<sup>4</sup>

Consider a temperature sensor connected to a microcontroller. The sensor's analog voltage, which varies with temperature, is fed into an Analog-to-Digital Converter, or ADC. The ADC's

job is to convert this continuous voltage into a discrete binary number. For a 12-bit ADC, this might be a value like 000110011001.<sup>5</sup> This binary string is all the microcontroller receives. It is just a pattern of high and low voltages on its input pins.

The firmware running on that microcontroller—the C or Python code you will write—must perform this exact binary-to-decimal conversion to turn that raw binary pattern into a usable integer. In this case, 000110011001 becomes the decimal number 409.<sup>6</sup>

But this integer, 409, is still not a temperature. It is a raw ADC reading. The final step is to use the sensor's datasheet to scale this value. The datasheet might specify that the ADC's range of 0 to 4095 corresponds to a temperature range of -50 to 150 degrees Celsius. Your code must then apply a mathematical formula to map the integer 409 to its corresponding real-world temperature. This entire process—from binary input to scaled decimal output—is a constant, critical task happening thousands of times per second inside every digital control system. It is the bridge between the digital brain and the analog world. This fundamental skill is also the gateway to understanding more advanced topics you will encounter, such as designing digital logic circuits, performing low-level bitwise operations in C for embedded systems, and configuring industrial networks using IP addresses and subnet masks.<sup>7</sup>

## Broader Context: Beyond Simple Conversion

This simple conversion task reveals two profound principles at the heart of engineering. The first is the distinction between a software abstraction and the underlying hardware reality. The algorithm we just narrated, using modulus and division by ten, is a clever software trick. It treats the binary number as if it were a decimal integer for the sole purpose of easily extracting its digits. This is a high-level abstraction. In the actual silicon of the processor, a far more efficient operation occurs. The hardware would use bit-shifting instructions to move the bits to the right and bitwise AND operations to isolate them. This is orders of magnitude faster. As a mechatronics engineer working at the hardware-software interface, understanding these layers of abstraction is vital for writing high-performance code and debugging low-level problems.

The second principle is that data is meaningless without a protocol. The binary string 1101 has no inherent meaning. Is it the unsigned integer thirteen? Is it a command for a stepper motor to move to position thirteen? Is it part of a network packet? For data to be useful, both the sender (the sensor) and the receiver (the microcontroller) must agree on its format, its length, and its meaning. This concept of a "contract" or protocol governs everything from simple I2C communication between chips on a circuit board to complex industrial Ethernet protocols that orchestrate entire factories.<sup>4</sup> The simple binary number is the starting point for a deep

understanding of system communication and interoperability.

## Chapter 2: The Art of Optimal Choices — Greedy Algorithms and Their Limits

### Foundation: The Change-Making Problem

Your textbook presents another classic problem: calculating change using the fewest number of coins.<sup>1</sup> This problem is a perfect vehicle for exploring a powerful and intuitive algorithmic strategy known as the "greedy algorithm." The philosophy of a greedy algorithm is simple: at every step, make the choice that seems best at that moment, without looking ahead or considering the downstream consequences.<sup>10</sup>

For the change-making problem, the greedy strategy is to always take the largest denomination coin that is less than or equal to the remaining amount you need to make.<sup>11</sup> If you need to make 73 cents in change, the greedy approach is as follows:

1. The largest coin you can take is a quarter (25 cents). You take one. 48 cents remain.
2. The largest coin you can take is another quarter. You take it. 23 cents remain.
3. You can't take another quarter. The next largest is a dime (10 cents). You take one. 13 cents remain.
4. You take another dime. 3 cents remain.
5. You can't take a dime or a nickel. You must take a penny. 2 cents remain.
6. You take another penny. 1 cent remains.
7. You take a final penny. 0 cents remain.

The result is two quarters, two dimes, and three pennies. For the standard US currency system, this greedy approach is not only fast and simple to implement, it is also *optimal*—it is guaranteed to yield the fewest number of coins.<sup>12</sup>

### The Cautionary Tale: When Greed Fails

To truly understand the power and peril of a tool, one must understand its breaking point. The

intuitive appeal of the greedy algorithm is also its greatest danger. Let's consider a different, hypothetical currency system with denominations of  $\{1, 4, 5\}$  cents. Now, let's try to make change for 8 cents using the same greedy strategy.

1. The amount to make is 8 cents. The largest coin we can take is 5 cents. We take it. 3 cents remain.
2. We cannot take a 5 or a 4. We must take a 1-cent coin. We take it. 2 cents remain.
3. We take another 1-cent coin. 1 cent remains.
4. We take a final 1-cent coin. 0 cents remain.

The greedy algorithm gives us a solution of four coins: one 5-cent piece and three 1-cent pieces. However, a moment's thought reveals a better solution: two 4-cent pieces. The optimal solution uses only two coins. In this system, the greedy algorithm fails. It produces a correct result, but not an optimal one.<sup>13</sup> This simple counterexample is a powerful and memorable lesson: an algorithm that is perfect in one context can be suboptimal in another.

## Application 1: Robotics Path Planning

This abstract concept of algorithmic failure has direct, physical consequences in robotics. Consider a simple, greedy path-planning algorithm for a mobile robot: "From my current position, always move to the adjacent grid square that is geometrically closest to the final goal." In a wide-open field with no obstacles, this strategy works perfectly and finds the most direct path.

Now, let's introduce a large, U-shaped obstacle between the robot and its goal, with the goal located directly across from the opening of the 'U'. The greedy robot, starting its journey, will move forward, always choosing the square that minimizes its straight-line distance to the target. It will walk directly into the bottom of the 'U'.<sup>15</sup> Once there, every possible move—left, right, or backward—takes it

further from the goal. The robot is trapped. It has found a "local optimum" (the point inside the U closest to the goal) but has failed to find the "global optimum" (the longer path that goes around the obstacle). This physical trap is a perfect manifestation of the same logical flaw we saw in our  $\{1, 4, 5\}$  coin problem. The algorithm's short-sighted, "greedy" decisions prevented it from seeing the better, non-obvious solution.

## Application 2: Financial Portfolio Optimization

To demonstrate the breadth of this concept, let's shift from the physical world of robotics to the abstract world of finance. Imagine you have a target portfolio allocation, for example, 50% in Stock A, 30% in Stock B, and 20% in Stock C, with a total of \$10,000 to invest. You cannot buy fractions of shares, so you must find an integer number of shares for each stock that comes as close as possible to your target.

A greedy approach to this problem is computationally cheap and surprisingly effective. You would iterate through your list of stocks. For Stock A, you would buy as many shares as you can without exceeding its allocated \$5,000. Then, with the remaining money, you do the same for Stock B, and finally for Stock C.<sup>17</sup> This approach is not guaranteed to find the absolute perfect allocation that minimizes the leftover cash, but it gets very close, very quickly.<sup>18</sup> In a world where market prices are changing every millisecond, a "good enough" solution calculated instantly is often far more valuable than a "perfect" solution that takes too long to compute.<sup>19</sup>

This reveals a fundamental engineering trade-off: the balance between optimality and computational cost. In many real-world mechatronic systems, time is a critical constraint. A greedy algorithm might calculate a path for a robot arm that is 95% optimal but does so in 5 milliseconds. A more complex algorithm like A\* search might find the 100% optimal path but take 500 milliseconds to do so.<sup>20</sup> If the robot is trying to pick an item from a moving conveyor belt, the "perfect" solution is useless because it arrives too late. The engineer must recognize that "good enough, right now" is often superior to "perfect, too late".<sup>10</sup>

The failure of the greedy algorithm in our coin problem reveals a deeper truth: optimality is a property of the problem, not the algorithm. The greedy algorithm is not inherently "good" or "bad." Its success depends entirely on the structure of the problem space. Standard currency systems, like that of the US, are what mathematicians call "canonical coin systems." They possess specific mathematical properties that guarantee a greedy choice will always lead to a globally optimal solution. Our hypothetical {1, 4, 5} system lacks these properties. The lesson for an engineer is profound: you cannot simply choose an algorithm from a textbook and assume it will work. You must first analyze the unique structure and constraints of your specific problem to validate that your chosen algorithmic approach is appropriate. This is the shift from rote application to critical analysis.

## **Chapter 3: The Unbroken Loop — Mastering Python's else Clause**

## Foundation: The for...else and while...else Syntax

Your textbook introduces one of Python's most distinctive, powerful, and frequently misunderstood control flow features: the optional else clause on a loop.<sup>1</sup> This syntax often confuses programmers coming from other languages because it seems to defy the conventional logic of an

if...else statement.

To demystify this construct, we can use a simple, powerful mnemonic: think of the else on a loop as meaning "**no break**".<sup>21</sup> The block of code inside the

else clause will execute only if the loop it's attached to completes its entire sequence naturally. For a for loop, this means it has iterated over every single item in the sequence. For a while loop, this means its controlling condition has become false.<sup>22</sup> If, however, the loop is terminated prematurely by a

break statement, the else block is skipped entirely. The same is true if the loop is exited by other means, such as a return statement or an exception.<sup>22</sup>

## The Canonical Use Case: The Search Loop

The primary and most elegant use case for this construct is the implementation of search loops.<sup>23</sup> It allows a programmer to handle the two possible outcomes of a search—success or failure—in a clean and highly readable way.

Let's narrate a classic example: searching for a specific number within a list of numbers. The traditional approach in many languages would involve a "flag" variable. You might create a variable called found and initialize it to False. Then, you would loop through the list. If you find the number, you set found to True and break out of the loop. After the loop is finished, you need a separate if statement to check the value of the found flag to determine what to do next.

Python's for...else structure makes this much cleaner. We can narrate the code as follows:

We begin a for loop, iterating through each number in our list\_of\_numbers. Inside the loop, we have an if statement that checks if the current number is equal to our target\_number. If it is, we execute two commands. First, we print a message to the screen, like "Found the target!".

Second, we issue a break command, which immediately terminates the loop.

Now, attached to this for loop, not indented under it but at the same level, we add an else clause. Inside this else block, we have a single command: print a message like "Target not found in the list."

Let's trace the execution. If the target number is in the list, the if condition will eventually be true, the "Found" message will be printed, and the break will execute. Because the loop was terminated by a break, the else block is completely ignored.<sup>24</sup> If the

for loop iterates through every single number in the list and never finds the target, the break statement is never reached. The loop completes naturally, and because there was "no break," the else block is executed, printing the "Not found" message.<sup>21</sup> This structure is more concise and more clearly expresses the programmer's intent than the flag-based method.

## Application: Mechatronics System Health Check

This elegant control structure is perfectly suited for building robust diagnostic and safety checks in mechatronics systems. Imagine the pre-flight check sequence for an autonomous drone. The control software has a list of critical components that must be verified before takeoff: the GPS, the inertial measurement unit (IMU), the battery management system, and each of the motor controllers.

We can program this diagnostic check using a for...else loop. The code would read like this:

We start a for loop that iterates through each component in our list of critical\_components. Inside the loop, we poll the component to get its current status. We then have an if statement: if the component.status is equal to the string 'FAULT', we print a critical error message to the system log, such as "CRITICAL FAULT DETECTED IN [component name]. ABORTING MISSION.", and then we break out of the loop.

Attached to this for loop is the else clause. This else block will only run if the loop completes without a single component reporting a fault. Inside this block, we print the message "All systems nominal. Cleared for takeoff." and then proceed with the mission.

This code is not only functional but also highly readable and safe. It clearly separates the failure case (handled by the if and break) from the success case (handled by the else).

The for...else construct is a prime example of a language feature designed to promote a specific, cleaner coding pattern. It is about expressing *intent*. Using this structure immediately communicates to another programmer reading the code: "This is a search loop that has two

distinct outcomes: finding the item of interest, or exhausting the search." A flag-based approach is more generic and forces the reader to analyze the code after the loop to deduce the programmer's original intent. Writing code that leverages the unique, expressive strengths of a language is known as writing "idiomatic" code. It is a hallmark of a proficient programmer, as it leads to code that is more self-documenting and easier to maintain.

Furthermore, this concept of a "completion clause" provides a deeper, more unified mental model of Python's control flow. The behavior of a loop's else clause is conceptually identical to the else clause in a try...except block.<sup>23</sup> In a

try block, the else clause runs when "no exception" is raised. In a loop, the else clause runs when "no break" occurs.<sup>22</sup> Both are blocks of code that execute only when the preceding block finishes its work without an exceptional or premature exit. Recognizing this parallel structure helps build a more robust and intuitive understanding of the language's design philosophy.

## **Chapter 4: The Engineer's Conscience — Ensuring Code Quality with Static Analysis**

### **Foundation: The Significance of Indentation**

Your textbook begins its exploration of code quality with a topic that is fundamental to Python: indentation.<sup>1</sup> In many programming languages, indentation is a matter of style, used to make code more readable for humans. In Python, however, indentation is syntax. It is as meaningful as a parenthesis or a comma. Incorrect indentation will not just look messy; it will either prevent the code from running at all, resulting in an

IndentationError, or, more insidiously, it will cause the code to run with incorrect logic, leading to subtle, hard-to-find bugs.

The textbook introduces a tool from Python's standard library called tabnanny.<sup>1</sup> This tool addresses a very specific and notorious indentation problem: the ambiguous mixing of tabs and spaces. In one programmer's text editor, a tab might be configured to look like four spaces. In another's, it might look like eight spaces. If a programmer mixes these in the same block of code, the code might look perfectly aligned on their screen but be syntactically

incorrect to the Python interpreter.<sup>25</sup>

tabnanny is a specialized "nanny" that scans your code and raises an alarm if it finds this dangerous mixture, ensuring that indentation is consistent and unambiguous.<sup>26</sup>

## The Power Suite: prospector

While tabnanny is a useful specialist, your textbook quickly graduates you to a far more powerful and comprehensive tool: prospector.<sup>1</sup> It is best to think of

prospector not as a single tool, but as an automated code review suite. It is like having a team of virtual senior engineers who meticulously scrutinize your code for a wide range of potential issues before you even attempt to run it.<sup>28</sup>

prospector bundles the functionality of several industry-standard static analysis tools, each with its own specialty. Let's meet the members of this virtual quality assurance team<sup>30</sup>:

- **Pylint:** This is the meticulous generalist of the group. It is the most comprehensive tool and is the source of most of prospector's feedback. It checks for everything from potential runtime errors and unused variables to violations of coding conventions and even offers suggestions for refactoring your code.
- **pycodestyle:** This is the strict style guide enforcer. Its sole job is to ensure that your code adheres to PEP 8, the official style guide for Python code. It flags issues like lines that are too long, inconsistent spacing around operators, or improper naming conventions. Adhering to a common style makes code easier for everyone on a team to read and understand.
- **McCabe:** This is the complexity analyst. It measures something called "cyclomatic complexity," which is essentially a count of the number of different paths of execution through a function. If a function becomes too complex—with too many nested if statements and loops—the McCabe tool will flag it. Overly complex functions are a strong indicator of code that is difficult to test, hard to debug, and prone to logical errors.
- **Dodgy:** This is the security officer. Its simple but vital task is to scan your code for sensitive information that should never be stored in a source file. It looks for things like hardcoded passwords, API keys, or secret tokens. Accidentally committing such secrets to a public code repository is a common and serious security breach.
- **Pydocstyle:** This is the documentation expert. It checks that your functions, classes, and modules have well-formed documentation strings, or "docstrings," that conform to official conventions. Good documentation is essential for creating maintainable and usable code.

## The "Why": Preventing Engineering Disasters

Why is this level of automated scrutiny so important? Because in mechatronics and other engineering disciplines, software errors are not abstract. They have physical consequences, and those consequences can be catastrophic. Static analysis is our first and best line of defense against the kinds of simple human errors that can lead to disaster.<sup>32</sup>

Consider the cautionary tale of NASA's **Mars Climate Orbiter**. In 1999, the \$125 million spacecraft was lost because it burned up in the Martian atmosphere. The investigation revealed a shocking and simple cause: a software module on the ground was performing calculations using imperial units (pounds of force), while the spacecraft's onboard software expected those values in metric units (Newtons).<sup>34</sup> This unit mismatch, a type of error that modern static analysis tools can be configured to detect, led to a total mission failure.

An even more chilling example is the **Therac-25** radiation therapy machine from the 1980s. A series of subtle software bugs, including a "race condition" where the order of events could affect the outcome, allowed the machine to deliver massive overdoses of radiation to patients, resulting in multiple deaths and severe injuries.<sup>34</sup> These were not complex algorithmic failures, but fundamental errors in software design and safety checking.

Static analysis tools build a culture of discipline, quality, and automated verification. They act as a safety net, catching potential problems early in the development process, long before the code is ever deployed to a physical system where its failure could have devastating consequences.<sup>37</sup>

As a mechatronics engineer, you are familiar with the concept of preventative maintenance for a physical machine—regularly checking oil levels, monitoring for vibrations, and replacing worn parts to prevent a catastrophic failure. Static analysis is the software equivalent of preventative maintenance.<sup>37</sup> It is a set of diagnostic tools that you run on your codebase to detect "wear and tear" in the form of technical debt, potential "cracks" in the form of bugs, and "safety hazards" in the form of security vulnerabilities. It is a proactive, not reactive, approach to ensuring the health and reliability of your software systems.

Furthermore, these tools serve to institutionalize knowledge and automate mentorship. A senior engineer knows from years of experience not to write a 300-line function or to hardcode a password. A junior engineer may not. A well-configured static analysis tool, embodying the collective best practices of a team, acts as an automated mentor. It catches these common mistakes instantly, providing immediate feedback and enforcing a consistent quality standard across the entire team, regardless of individual experience levels.<sup>33</sup> This

process embeds collective wisdom directly into the development workflow, raising the quality of the entire project.

However, it is also important to understand the nuances of using these tools in the real world. Static analysis tools are not infallible; they can sometimes flag issues that are not actually problems, a phenomenon known as a "false positive." In these cases, developers may need to explicitly "suppress" a warning to tell the tool to ignore it.<sup>40</sup> This demonstrates that these tools are aids to, not replacements for, human judgment. It also introduces a new layer of engineering responsibility: a suppressed warning might unintentionally hide a real bug that is introduced later in the same section of code. This highlights the ongoing, dynamic challenge of maintaining code quality in large, evolving projects.

## **Chapter 5: The Data Scientist's Warning — Lessons from Anscombe's Quartet**

### **Foundation: The Story of the Quartet**

Your textbook now directs your attention to a famous and profoundly important case study in the world of data science: Anscombe's Quartet.<sup>1</sup> To understand its significance, we must first understand the story behind it. In 1973, the statistician Francis Anscombe grew concerned that his colleagues were becoming overly reliant on numerical calculations and statistical summaries. They were forgetting the most fundamental step of any data analysis: to simply

*look at the data.*<sup>41</sup> To demonstrate the dangers of this oversight, he constructed a brilliant statistical trap: four distinct datasets, each containing eleven (x, y) points.<sup>42</sup>

### **The Deception: Identical Statistics**

The genius of Anscombe's Quartet lies in its deceptive summary statistics. If you were to analyze these four datasets without visualizing them, you would be led to believe they are nearly identical. As we narrate the key statistical properties, notice how similar they are across all four sets<sup>42</sup>:

- The mean, or average, of the x-values is almost exactly 9.0 for all four datasets.
- The mean of the y-values is almost exactly 7.50 for all four datasets.
- The variance for both x and y is nearly identical for all four datasets.
- The correlation coefficient between x and y, a measure of how strongly they are linearly related, is approximately 0.816 for all four.
- Most strikingly, if you calculate the best-fit linear regression line—the straight line that best summarizes the relationship between x and y—you get virtually the same equation,  $y=0.5x+3$ , for all four datasets.

Based on these numbers alone, any data analyst would confidently conclude that these four datasets tell the same story.

## The Revelation: Drastically Different Graphs

The deception is shattered the moment you plot the data. The visual story told by each dataset is dramatically and fundamentally different from the others<sup>45</sup>:

1. **Dataset I:** This plot looks exactly as you would expect from the statistics. It is a simple, slightly scattered cloud of points with a clear, positive linear trend. The regression line fits the data well. This is a well-behaved, "normal" dataset.
2. **Dataset II:** This plot shows no linear relationship at all. Instead, the points form a perfect, clean, non-linear curve, like a parabola. The straight regression line is a completely inappropriate and misleading summary of this data. The relationship is strong, but it is not linear.
3. **Dataset III:** This plot shows an almost perfectly straight line of points, but with one dramatic outlier. A single data point is far removed from the others. This one outlier is powerful enough to skew the entire regression line, pulling it off course and reducing the correlation from a perfect 1.0 to the misleading 0.816.
4. **Dataset IV:** This plot is perhaps the most bizarre. All of the data points, except for one, have the exact same x-value, forming a vertical line. The final point is a single, extreme "high-leverage" point far to the right. This one point single-handedly creates the illusion of a strong linear trend where, for the vast majority of the data, no such trend exists.

## The Mechatronics Moral: Trust, but Verify Your Sensors

The lesson of Anscombe's Quartet is not an abstract statistical curiosity; it is a paramount principle for mechatronics engineering. Imagine you are monitoring the vibration data from a

critical industrial motor. You are collecting thousands of data points every hour, and you summarize them with descriptive statistics. For weeks, the mean, median, and standard deviation of the vibration might look perfectly normal, suggesting the motor is healthy.

However, if you were to plot that data over time, you might discover a pattern identical to Dataset III: a stable, low level of vibration with a single, massive spike that occurs for just a few seconds every 24 hours. This brief, extreme event, an outlier, might be completely washed out and hidden by the daily averages. But that spike could be the signal of a critical failure—perhaps a bearing seizing up during a specific high-load operational cycle. The summary statistics would lie to you, telling you everything is fine. The graph would tell you the truth, warning you of an impending catastrophic failure.<sup>47</sup>

This parable is the foundational argument for the entire field of Exploratory Data Analysis (EDA). It proves that summary statistics are a form of information compression, and in that compression, the essential truth can be lost.<sup>43</sup> Visualization provides a high-bandwidth channel directly to the human brain's incredibly powerful pattern-recognition centers. Therefore, the first commandment of data science, and of any data-driven engineering, must be:

**Thou Shalt Plot Thy Data.**<sup>41</sup> The first step of any analysis is not to calculate, but to see.

Anscombe's Quartet does more than just issue a generic warning. It provides a memorable, visual vocabulary for distinct types of data problems that engineers encounter every day. Dataset II represents a **model misspecification**—the error of using the wrong type of model (linear) to describe a system that is inherently non-linear. Dataset III represents the problem of an **outlier**, a single bad reading or a rare event that can corrupt an analysis. Dataset IV represents the problem of a **high-leverage point**, a single data point that has an undue influence on the entire model. These are fundamentally different problems that require different engineering solutions. Learning to recognize these visual patterns is a key diagnostic skill.

The spirit of Anscombe's work continues to be relevant. Modern researchers, using computational techniques, have created the "Datasaurus Dozen"—a collection of thirteen datasets that includes one which, when plotted, forms the outline of a dinosaur. All thirteen of these datasets share the exact same summary statistics as Anscombe's original four.<sup>42</sup> This serves as a powerful reminder that the potential for statistics to mislead is even more profound than Anscombe demonstrated, highlighting the enduring and critical importance of his original warning.

## Chapter 6: Modeling Our World — Population Growth

# and Engineering Assumptions

## Foundation: Building a Simple Growth Model

Your textbook now presents an exercise in modeling a dynamic system: calculating world population growth over the next 100 years.<sup>1</sup> This task serves as a powerful case study in the art and science of systems modeling, a core competency for any engineer.

First, as with any model, we must establish our initial parameters. Based on current demographic data, we can set our starting world population for the year 2025 at approximately 8.2 billion people, and the current annual growth rate at roughly 0.9%.<sup>50</sup>

With these parameters, we can narrate the logic of the Python script. We would initialize a variable, let's call it `current_population`, to 8.2 billion. We would also define our `growth_rate` as 0.009. Then, we would construct a `for` loop that iterates 100 times, once for each year we want to project. Inside the loop, in each iteration, we would calculate the population increase for that year by multiplying the `current_population` by the `growth_rate`. We would then update the `current_population` by adding this increase to it. This is equivalent to the formula: `new_population = current_population * (1 + growth_rate)`. The script would print the year, the new population, and the numerical increase for each of the 100 years.

## The Core Lesson: The "Simplifying Assumption"

The most important phrase in this entire exercise is buried in the problem description: "...using the simplifying assumption that the current growth rate will stay constant".<sup>1</sup> This single phrase is the heart of the lesson. Our entire model is built upon this one, powerful, and ultimately incorrect assumption. Because the growth rate is a constant percentage, the numerical increase in population will get larger each year, leading to unchecked, exponential growth. Our model will predict a future of ever-accelerating population increase.

## Confronting the Model with Reality

Now, we must act as engineers and do the most critical part of any modeling exercise: validate our model against real-world data and expert analysis. We will compare the explosive, unending growth curve predicted by our simple model with the far more nuanced projections from organizations like the United Nations Population Division.<sup>53</sup>

These expert models, which are built on vast amounts of data and complex demographic science, tell a very different story. They show that the global population growth rate is, in fact, *slowing down*. Their projections indicate that the world population is likely to peak at around 10.3 billion people in the mid-2080s and then begin a slow and steady decline by the end of the century.<sup>54</sup>

## Deconstructing the Difference: What Our Model Missed

Why is our simple model so spectacularly wrong in its long-term prediction? The answer lies in the complexities it ignores. Our "simplifying assumption" papers over the crucial real-world variables that govern population dynamics. These include<sup>54</sup>:

- **Declining Fertility Rates:** As nations develop economically and education levels rise, particularly for women, average family sizes tend to decrease.
- **Increasing Life Expectancy:** Advances in healthcare and nutrition mean people are living longer, which contributes to population size, but this effect is being overtaken by falling birth rates.
- **Resource Limitations:** While a controversial topic, the finite nature of resources like fresh water and arable land will inevitably constrain unlimited growth.

By confronting our simple model with this complex reality, the exercise is transformed. It is no longer a simple programming loop; it is a profound lesson on the difference between a first-order approximation and a sophisticated, multi-variable system model.

This process exemplifies a famous aphorism in science and engineering, often attributed to the statistician George Box: "All models are wrong, but some are useful." Our simple population model is "wrong" in the sense that its long-term predictions are inaccurate. But it is "useful" in two critical ways. First, it provides a powerful and intuitive demonstration of the principle of exponential compounding. Second, and more importantly, it serves as a vital baseline. The real learning occurs when we analyze the gap between our simple model's output and the complex reality. The model's failure forces us to ask "Why is it wrong?" and in answering that question, we uncover the deeper, more complex drivers of the system we are trying to understand.

This exercise also provides a tangible example of a core engineering practice: sensitivity analysis. The entire long-term behavior of our model hinges on that single assumption of a constant growth rate. An engineer must always ask, "How much does my final result change if my initial assumptions are slightly off?" In our model, a tiny change in the assumed growth rate leads to a difference of billions of people over the 100-year timeline. This dramatically illustrates how critically important it is to identify, question, and validate the foundational assumptions of any engineering model before trusting its output. The assumption is often the most sensitive and important part of the entire analysis.

## Chapter 7: The Heart of Data — Mean, Median, Mode, and the Problem of Outliers

### Foundation: Defining the Measures of Central Tendency

The final section of your textbook chapter introduces the foundational tools of descriptive statistics: the measures of central tendency.<sup>1</sup> These are single numbers that attempt to describe the "center" or "typical" value of a dataset. Let's define the three core measures in a way that is clear and intuitive<sup>58</sup>:

- **The Mean:** This is the most familiar measure, what we commonly call the arithmetic average. It is calculated by summing all the values in a dataset and dividing by the count of those values. Conceptually, the mean is the "balance point" of the data; if you were to place the data points on a seesaw, the mean is the point where you would place the fulcrum to make it balance.
- **The Median:** This is the middle value of a dataset *after* it has been sorted in numerical order. If there is an odd number of values, the median is the single value right in the middle. If there is an even number of values, the median is the average of the two middle values. The median's defining characteristic is that it splits the data into two equal halves: 50% of the data points are below the median, and 50% are above it.
- **The Mode:** This is the most frequently occurring value in a dataset. It is the "most popular" or most common data point. A dataset can have one mode, more than one mode (if multiple values have the same highest frequency), or no mode at all (if every value occurs only once).

## The Outlier Gauntlet: A Practical Demonstration

Your textbook asks a critical question: which of these measures is most affected by outliers?<sup>1</sup> An outlier is a value that is out of the ordinary, far removed from the rest of the data. Let's run a simple thought experiment to find the answer.

Consider the dataset from your book: 9, 11, 22, 34, 17, 22, 34, 22, and 40. The mean is approximately 23.4. The median, after sorting the data, is 22. The mode, the most frequent value, is also 22.

Now, let's introduce an outlier. Suppose one of the measurements was a data entry error, and instead of 40, the value was 400. Let's see what happens to our three measures.

The **mode** is unchanged; it is still 22. The **median** is also unchanged; 22 is still the middle value of the sorted list. But the **mean** is drastically affected. The sum of the values is now much larger, and the new mean is dragged all the way up to 63.4. This single outlier has completely distorted the mean, making it a poor representation of the "typical" value in the dataset. This provides a definitive, audible proof: the mean is highly sensitive to outliers, while the median and mode are "robust" to them.<sup>58</sup>

## Application 1: Mechatronics and Robust Sensor Fusion

This statistical concept has direct and critical applications in robotics and control systems. A stream of data coming from a sensor on a robot is rarely perfect. It will contain noise and, occasionally, outliers.<sup>61</sup> An outlier in a sensor reading could be a meaningless glitch caused by electronic interference, or it could be a vital signal of a real-world event, like a sudden collision or a critical system failure.<sup>62</sup>

When designing a filter to clean up noisy sensor data, an engineer must make a choice. If they use the **mean** of the last ten readings to smooth the data, a single spurious outlier can throw off the estimate significantly. A more robust approach is to use the **median** of the last ten readings. The median filter will effectively ignore the single outlier, providing a much more stable and reliable estimate of the true state of the system.

However, this choice comes with a trade-off. If you design a system that *always* filters out outliers as noise, you run the risk of ignoring the one reading that signals a true emergency. This introduces the core engineering challenge of anomaly detection: designing algorithms that can intelligently distinguish between meaningless noise and a meaningful, critical event.

## Application 2: Manufacturing and Statistical Process Control (SPC)

Let's journey to the factory floor to see these measures in action in the field of quality control. Statistical Process Control, or SPC, is a methodology used to monitor and control a manufacturing process to ensure it operates within its desired limits.<sup>63</sup>

A common tool in SPC is the control chart. Imagine a machine that fills bottles with a liquid. To monitor the process, an operator periodically takes a small sample of bottles (say, five bottles) and measures their fill volume. The **mean** of this sample is then plotted on a time-series chart. This chart has a center line (the target mean) and an upper and lower control limit, typically set at three standard deviations from the mean.<sup>65</sup> As long as the sample means fall within these limits, the process is considered "in control." A point that falls outside these limits is an "outlier" that signals a "special cause" of variation—perhaps a nozzle has become clogged or a pressure regulator has failed. This signals to the operator that the process requires immediate investigation and correction.<sup>66</sup>

In this context, the mean is often preferred precisely because of its sensitivity. It can provide an early warning of small shifts in the process average. However, in cases where the manufacturing data is known to be skewed or prone to occasional, explainable outliers, engineers will specifically use control charts based on the **median** to prevent false alarms and create a more robust monitoring system.<sup>67</sup>

## Handling Categorical Data

Finally, your textbook asks which of these statistics are appropriate for categorical data.<sup>1</sup> Categorical data consists of non-numerical labels or groups. For example, the operational state of a factory machine could be a categorical variable with the possible values: 'Running', 'Idle', 'In Maintenance', or 'Fault'.<sup>68</sup>

It is nonsensical to calculate the mean or median of these text labels. You cannot average 'Running' and 'Idle'. However, the **mode** is extremely useful. Calculating the mode of the machine's state over a 24-hour period tells you its most common state. This is a critical piece of information for analyzing operational efficiency. If the mode is 'Idle', it may indicate a bottleneck elsewhere in the production line. This kind of categorical analysis is also a key input for predictive maintenance algorithms, which use patterns in machine states and other

data to forecast when a failure is likely to occur.<sup>70</sup>

This discussion reveals that there is no universally "best" measure of central tendency. The choice of which statistic to use is a deliberate engineering decision based on the nature of the data and the specific question being asked. For highly skewed data like personal income or housing prices, economists and journalists report the **median** because it better represents the "typical" individual, as it is not skewed by the extremely high values of billionaires.<sup>60</sup> In quality control, the

**mean** is often preferred specifically because its sensitivity to small shifts makes it a good early warning indicator.<sup>63</sup> The engineer's job is to understand these trade-offs and select the right tool for the job.

Most importantly, in the context of mechatronics and industrial systems, an outlier should never be dismissed as a mere nuisance to be cleaned away. An outlier is a signal. It represents a deviation from the expected model of the system's behavior. The critical engineering task is to classify that deviation. Is it **noise** from a faulty sensor that needs to be replaced? Is it a true but rare **event**, like a once-a-day pressure spike that is causing long-term wear on a component? Or, in a security-critical system, could it be a **malicious attack**—a deliberately falsified signal meant to disrupt or damage the system?<sup>62</sup> This reframes outliers from a statistical problem into a rich and critical source of diagnostic information about a system's health, safety, and security.

## Works cited

1. ch3 part 13 wrapup.docx
2. Program for Binary To Decimal Conversion - GeeksforGeeks, accessed September 26, 2025,  
<https://www.geeksforgeeks.org/program-binary-decimal-conversion/>
3. Program for Binary To Decimal Conversion - GeeksforGeeks, accessed September 26, 2025,  
<https://www.geeksforgeeks.org/dsa/program-binary-decimal-conversion/>
4. Convert I2C Sensor (DS1624) reading into number - Stack Overflow, accessed September 26, 2025,  
<https://stackoverflow.com/questions/18504753/convert-i2c-sensor-ds1624-reading-into-number>
5. How to Read and Interpret Digital Temperature Sensor Output Data (Rev. A) - Texas Instruments, accessed September 26, 2025,  
<https://www.ti.com/lit/pdf/sbaa588>
6. Binary to Decimal Converter using Arduino and OLED Display, accessed September 26, 2025,  
[https://projecthub.arduino.cc/shreyas\\_arbatti/binary-to-decimal-converter-using-arduino-and-oled-display-783d87](https://projecthub.arduino.cc/shreyas_arbatti/binary-to-decimal-converter-using-arduino-and-oled-display-783d87)
7. Binary to Decimal Conversion and Subnetting Calculations using Augmented

- Reality | Request PDF - ResearchGate, accessed September 26, 2025,  
[https://www.researchgate.net/publication/378198187\\_Binary\\_to\\_Decimal\\_Conversion\\_and\\_Subnetting\\_Calculations\\_using\\_Augmented\\_Reality](https://www.researchgate.net/publication/378198187_Binary_to_Decimal_Conversion_and_Subnetting_Calculations_using_Augmented_Reality)
8. DEPARTMET OF MECHATRONICS, accessed September 26, 2025,  
<https://eopcw.com/find/downloadLectureNote/1306>
  9. Real life applications for binary? : r/math - Reddit, accessed September 26, 2025,  
[https://www.reddit.com/r/math/comments/1e9ngg/real\\_life\\_applications\\_for\\_binary/](https://www.reddit.com/r/math/comments/1e9ngg/real_life_applications_for_binary/)
  10. What is Greedy Algorithm: Example, Applications, Limitations and More - Simplilearn.com, accessed September 26, 2025,  
<https://www.simplilearn.com/tutorials/data-structure-tutorial/greedy-algorithm>
  11. Minimum number of Coins - GeeksforGeeks, accessed September 26, 2025,  
<https://www.geeksforgeeks.org/greedy-algorithm-to-find-minimum-number-of-coins/>
  12. Greedy Algorithms: Concept, Examples, and Applications - Codecademy, accessed September 26, 2025,  
<https://www.codecademy.com/article/greedy-algorithm-explained>
  13. Greed can fail for American money - 11011110, accessed September 26, 2025,  
<https://11011110.github.io/blog/2009/07/27/greed-can-fail.html>
  14. Coin Change Problem Greedy Approach - HeyCoach, accessed September 26, 2025, <https://heycoach.in/blog/coin-change-problem-greedy-approach/>
  15. (PDF) Combined improved A\* and greedy algorithm for path planning of multi-objective mobile robot - ResearchGate, accessed September 26, 2025, [https://www.researchgate.net/publication/362429029\\_Combined\\_improved\\_A\\_and\\_greedy\\_algorithm\\_for\\_path\\_planning\\_of\\_multi-objective\\_mobile\\_robot](https://www.researchgate.net/publication/362429029_Combined_improved_A_and_greedy_algorithm_for_path_planning_of_multi-objective_mobile_robot)
  16. Combined improved A\* and greedy algorithm for path planning of multi-objective mobile robot - PMC, accessed September 26, 2025, <https://pmc.ncbi.nlm.nih.gov/articles/PMC9345932/>
  17. Post-processing weights — PyPortfolioOpt 1.5.4 documentation, accessed September 26, 2025, <https://pyportfolioopt.readthedocs.io/en/latest/Postprocessing.html>
  18. Detailed Explanation of Greedy Algorithm | Sapien's AI Glossary, accessed September 26, 2025, <https://www.sapien.io/glossary/definition/greedy-algorithm>
  19. Tag Archives: greedy algorithm - Portfolio Probe, accessed September 26, 2025, <https://www.portfolioprobe.com/tag/greedy-algorithm/>
  20. The Greedy Algorithm and A\* | Path Planning - YouTube, accessed September 26, 2025, <https://www.youtube.com/watch?v=XTJOioKI-2I>
  21. How and When To Use For Else in Python Loop - Jerry Ng, accessed September 26, 2025, <https://jerrynsh.com/python-for-else-construct-a-deep-dive/>
  22. 4. More Control Flow Tools — Python 3.13.7 documentation, accessed September 26, 2025, <https://docs.python.org/3/tutorial/controlflow.html#break-and-continue-statements-and-else-clauses-on-loops>
  23. Else Clauses on Loop Statements - Alyssa Coghlan's Python Notes, accessed September 26, 2025,

[https://python-notes.curousefficiency.org/en/latest/python\\_concepts/break\\_else.html](https://python-notes.curousefficiency.org/en/latest/python_concepts/break_else.html)

24. How Does Python's For-Else Loop Construct Work? - freeCodeCamp, accessed September 26, 2025,  
<https://www.freecodecamp.org/news/for-else-loop-in-python/>
25. tabnanny – Indentation validator - Python 3 Module of the Week, accessed September 26, 2025, <https://pymotw.com/2/tabnanny/>
26. tabnanny — Detection of ambiguous indentation — Python 3.13.7 documentation, accessed September 26, 2025, <https://docs.python.org/3/library/tabnanny.html>
27. tabnanny — Indentation validator — PyMOTW 3 - Python 3 Module of the Week, accessed September 26, 2025, <https://pymotw.com/3/tabnanny/index.html>
28. 1. Prospector - Python Static Analysis — prospector documentation, accessed September 26, 2025, <https://prospector.landscape.io/>
29. prospector - PyPI, accessed September 26, 2025,  
<https://pypi.org/project/prospector/>
30. 5. Supported Tools — prospector documentation, accessed September 26, 2025, [https://prospector.landscape.io/en/master/supported\\_tools.html](https://prospector.landscape.io/en/master/supported_tools.html)
31. Python Static Analysis tools - Shubhendra Singh Chauhan, accessed September 26, 2025,  
<https://camelcaseguy.medium.com/python-static-analysis-tools-fe5960d8035>
32. What Is Static Analysis? | Datadog, accessed September 26, 2025,  
<https://www.datadoghq.com/knowledge-center/static-analysis/>
33. Boost Security and Performance with Static Code Analysis - CoStrategix, accessed September 26, 2025,  
<https://www.costrategix.com/insight/boost-security-and-performance-with-static-code-analysis/>
34. Top 5 Most Infamous Software Bugs In The World - Better QA, accessed September 26, 2025,  
<https://betterqa.co/top-five-most-infamous-bugs-in-the-world/>
35. A Collection of Well-Known Software Failures, accessed September 26, 2025, <https://www.cse.psu.edu/~gxt29/bug/softwarebug.html>
36. 20 Famous Software Disasters - DevTopics, accessed September 26, 2025, <https://www.devtopics.com/20-famous-software-disasters/>
37. Static Code Analysis: Top 7 Methods, Pros/Cons and Best Practices - Oligo Security, accessed September 26, 2025,  
<https://www.oligo.security/academy/static-code-analysis>
38. Benefits of Using Static Code Analysis Tools for Software Testing | StickyMinds, accessed September 26, 2025,  
<https://www.stickyminds.com/article/benefits-using-static-code-analysis-tools-software-testing>
39. Static Code Analysis Best Practices for Developers - ACCELQ, accessed September 26, 2025,  
<https://www.accelq.com/blog/static-code-analysis-best-practices/>
40. An Empirical Study of Suppressed Static Analysis Warnings - Software Lab, accessed September 26, 2025,

[https://software-lab.org/publications/fse2025\\_suppressions.pdf](https://software-lab.org/publications/fse2025_suppressions.pdf)

41. Anscombe's Quartet of Identical Simple Linear Regressions - AWS, accessed September 26, 2025,  
[https://rstudio-pubs-static.s3.amazonaws.com/52381\\_36ec82827e4b476fb968d9143aec7c4f.html](https://rstudio-pubs-static.s3.amazonaws.com/52381_36ec82827e4b476fb968d9143aec7c4f.html)
42. Anscombe's quartet - Wikipedia, accessed September 26, 2025,  
[https://en.wikipedia.org/wiki/Anscombe%27s\\_quartet](https://en.wikipedia.org/wiki/Anscombe%27s_quartet)
43. Anscombe's quartet - GeeksforGeeks, accessed September 26, 2025,  
<https://www.geeksforgeeks.org/machine-learning/anscombes-quartet/>
44. Anscombe's Quartet: What Is It and Why Do We Care? - Built In, accessed September 26, 2025, <https://builtin.com/data-science/anscombes-quartet>
45. Importance of Data Visualization — Anscombe's Quartet Way - DEV Community, accessed September 26, 2025,  
<https://dev.to/imspars/importance-of-data-visualization-anscombe-s-quartet-way-5693>
46. Anscombe's Quartet – the importance of graphs! - IB Maths Resources, accessed September 26, 2025,  
<https://ibmathsresources.com/2021/06/15/anscombes-quartet-the-importance-of-graphs-2/>
47. builtin.com, accessed September 26, 2025,  
<https://builtin.com/data-science/anscombes-quartet#:~:text=Before%20analyzing%20your%20data%20and,Anscombe's%20quartet%20shows%20us%20why.&text=Anscombe's%20quartet%20is%20a%20group,you%20plot%20each%20data%20set.>
48. What Lessons Can We Learn From Anscombe's Quartet? -- QuantHub, accessed September 26, 2025,  
<https://www.quanhub.com/what-lessons-can-we-learn-from-anscombes-quartet/>
49. Same Stats, Different Graphs: Generating Datasets with Varied Appearance and Identical Statistics through Simulated Annealing - Autodesk Research, accessed September 26, 2025,  
<https://www.research.autodesk.com/publications/same-stats-different-graphs/>
50. World Population Day: trends and demographic changes - World Bank Blogs, accessed September 26, 2025,  
<https://blogs.worldbank.org/en/opendata/world-population-day--trends-and-demographic-changes>
51. World Population by Year - Worldometer, accessed September 26, 2025,  
<https://www.worldometers.info/world-population/world-population-by-year/>
52. World Population (1950-2025) - Macrotrends, accessed September 26, 2025,  
<https://www.macrotrends.net/global-metrics/countries/wld/world/population>
53. Population - the United Nations, accessed September 26, 2025,  
<https://www.un.org/en/global-issues/population>
54. Human population projections - Wikipedia, accessed September 26, 2025,  
[https://en.wikipedia.org/wiki/Human\\_population\\_projections](https://en.wikipedia.org/wiki/Human_population_projections)
55. www.pewresearch.org, accessed September 26, 2025,

- <https://www.pewresearch.org/short-reads/2025/07/09/5-facts-about-how-the-worlds-population-is-expected-to-change-by-2100/#:~:text=The%20population%20more%20than%20tripled.the%20end%20of%20the%20century>.
56. The Road to 10 Billion: World Population Projections 2024, accessed September 26, 2025,  
<https://populationmatters.org/news/2024/07/the-road-to-10-billion-world-population-projections-2024/>
57. Global population could peak below 9 billion in 2050s - Earth4All, accessed September 26, 2025,  
<https://earth4all.life/news/press-release-global-population-could-peak-below-9-billion-in-2050s/>
58. Mean, Median and Mode | Introduction to Statistics - JMP, accessed September 26, 2025,  
<https://www.jmp.com/en/statistics-knowledge-portal/measures-of-central-tendency-and-variability/mean-median-and-mode>
59. Mean, Mode and Median - Measures of Central Tendency - When to use with Different Types of Variable and Skewed Distributions | Laerd Statistics, accessed September 26, 2025,  
<https://statistics.laerd.com/statistical-guides/measures-central-tendency-mean-mode-median.php>
60. Mean vs. Median: Knowing the Difference - DataCamp, accessed September 26, 2025, <https://www.datacamp.com/tutorial/mean-vs-median>
61. (PDF) A Novel Approach for Outlier Detection and Robust Sensory Data Model Learning, accessed September 26, 2025,  
[https://www.researchgate.net/publication/338938071\\_A\\_Novel\\_Approach\\_for\\_Outlier\\_Detection\\_and\\_Robust\\_Sensory\\_Data\\_Model\\_Learning](https://www.researchgate.net/publication/338938071_A_Novel_Approach_for_Outlier_Detection_and_Robust_Sensory_Data_Model_Learning)
62. A Survey of Outlier Detection Techniques in IoT: Review and Classification - MDPI, accessed September 26, 2025, <https://www.mdpi.com/2224-2708/11/1/4>
63. SPC Explained: 2023 Guide - Capvidia, accessed September 26, 2025,  
<https://www.capvidia.com/blog/spc-guide>
64. Statistical process control - Wikipedia, accessed September 26, 2025,  
[https://en.wikipedia.org/wiki/Statistical\\_process\\_control](https://en.wikipedia.org/wiki/Statistical_process_control)
65. Statistical Process Control (SPC) - MoreSteam, accessed September 26, 2025,  
<https://www.moresteam.com/toolbox/statistical-process-control>
66. An Introduction To Statistical Process Control | SPC - OEEsystems, accessed September 26, 2025,  
<https://www.oeesystems.com/knowledge/introduction-to-statistical-process-control/>
67. Median/Individual Measurements Control Charting and Analysis for Family Processes, accessed September 26, 2025,  
<https://www.nwasoft.com/resources/information-center/white-paper/medianindividual-measurements-control-charting-and-analysis>
68. Handling Machine Learning Categorical Data with Python Tutorial - DataCamp, accessed September 26, 2025,  
<https://www.datacamp.com/tutorial/categorical-data>

69. Feature Engineering from Multi-Valued Categorical Data - dotData, accessed September 26, 2025,

<https://dotdata.com/blog/feature-engineering-from-categorical-data/>

70. Predictive Maintenance with Machine Learning: A Complete Guide | SPD Technology, accessed September 26, 2025,

<https://spd.tech/machine-learning/predictive-maintenance/>