Differentially Private Methods for Counts

SDSS 2022

Statistical Data Privacy Techniques for Sharing Sensitive Data

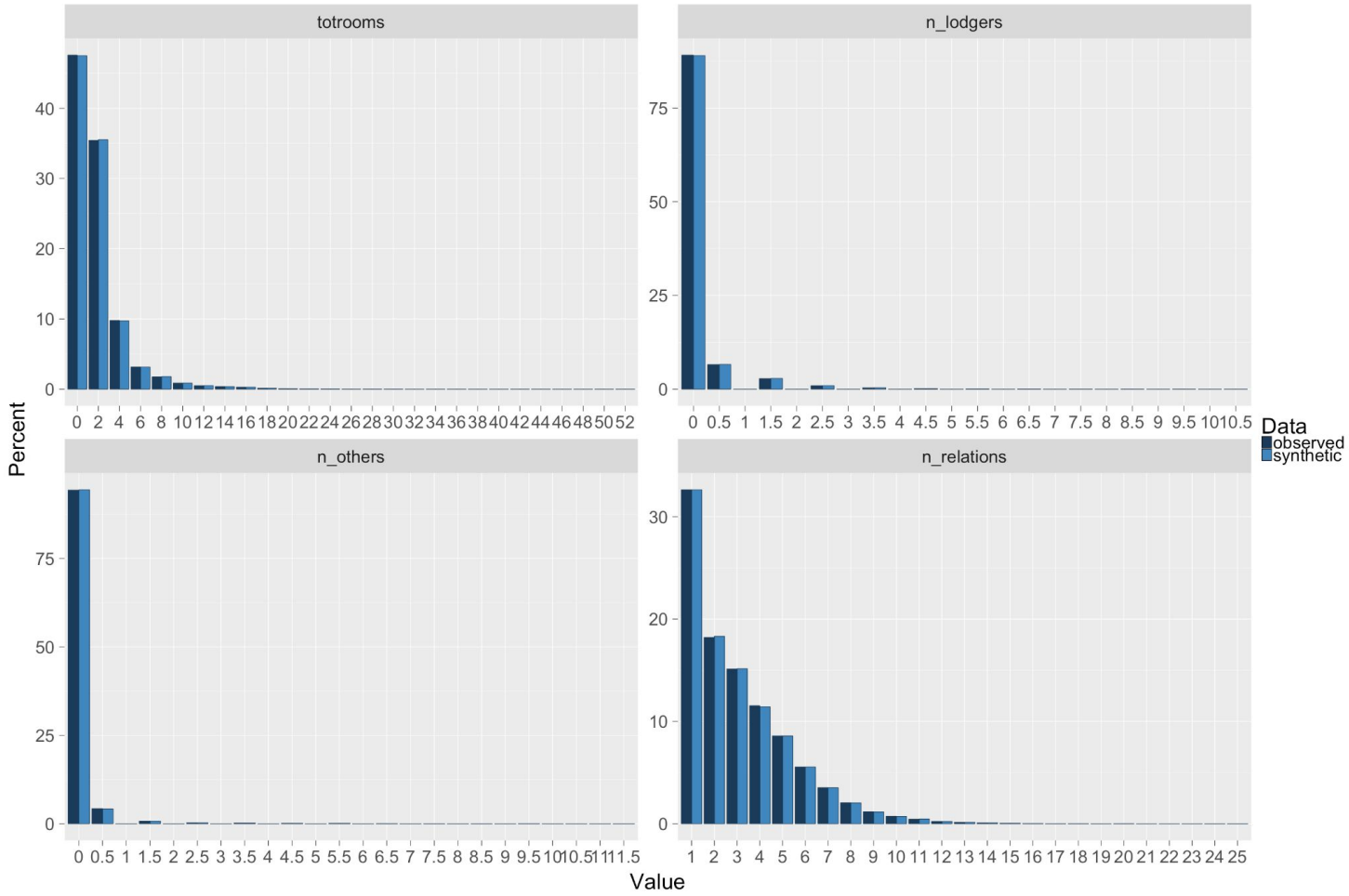
Short Course: Part 4

Differential Privacy is an Entirely New Framework

Motivation for a New Privacy Framework

- Recall from previous lessons:
  - Assumptions on population data
  - Assumptions on attacker knowledge on participation
  - Assumptions on publicly accessible variables
  - Lack of consensus on risk definitions for synthetic data

Values are Synthetic but are They Protective?



Goal is Meet a Provable Definition of Privacy

- DP is not a method, it is a set of *definitions*, e.g.:

A randomized algorithm, $\mathcal{M}$, is ϵ-Differentially Private if for all $S \subseteq Range(\mathcal{M})$ and for all X, X' such that $\delta(X, X') = 1$:

$$\frac{P(\mathcal{M}(X') \in S)}{P(\mathcal{M}(X) \in S)} \leqslant exp(\epsilon).$$

- **ε**-DP in words:
  - The relative likelihood of publishing any given statistic value is bounded given the presence or absence of any individual
- Does not require assumptions about the attacker
- Must understand space of all possible observations

Let's Try Out an Example

- Suppose we want to release a count of COVID cases

County	Sex	Race/Ethnicity	Case Count w/ person X	Case Count w/o person X
Allegheny	F	W	101	100
Allegheny	M	W	683	683
…	…	…	…	…

- With no noise added, the ratio of likelihoods is unbounded
  - W/ person X $\rightarrow$ release count = 101 w.p. 1, release count = 100 w.p. 0
  - W/o person X $\rightarrow$ release count = 100 w.p. 1, release count = 101 w.p. 0

Let's Try Out an Example

- Add random noise to the counts to bound the relative likelihood

County	Sex	Race/Ethnicity	Case Count w/ person X	Case Count w/o person X
Allegheny	F	W	101 + Lap(Δ/ε)	100 + Lap(Δ/ε)
Allegheny	M	W	683	683
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Let's Try Out an Example

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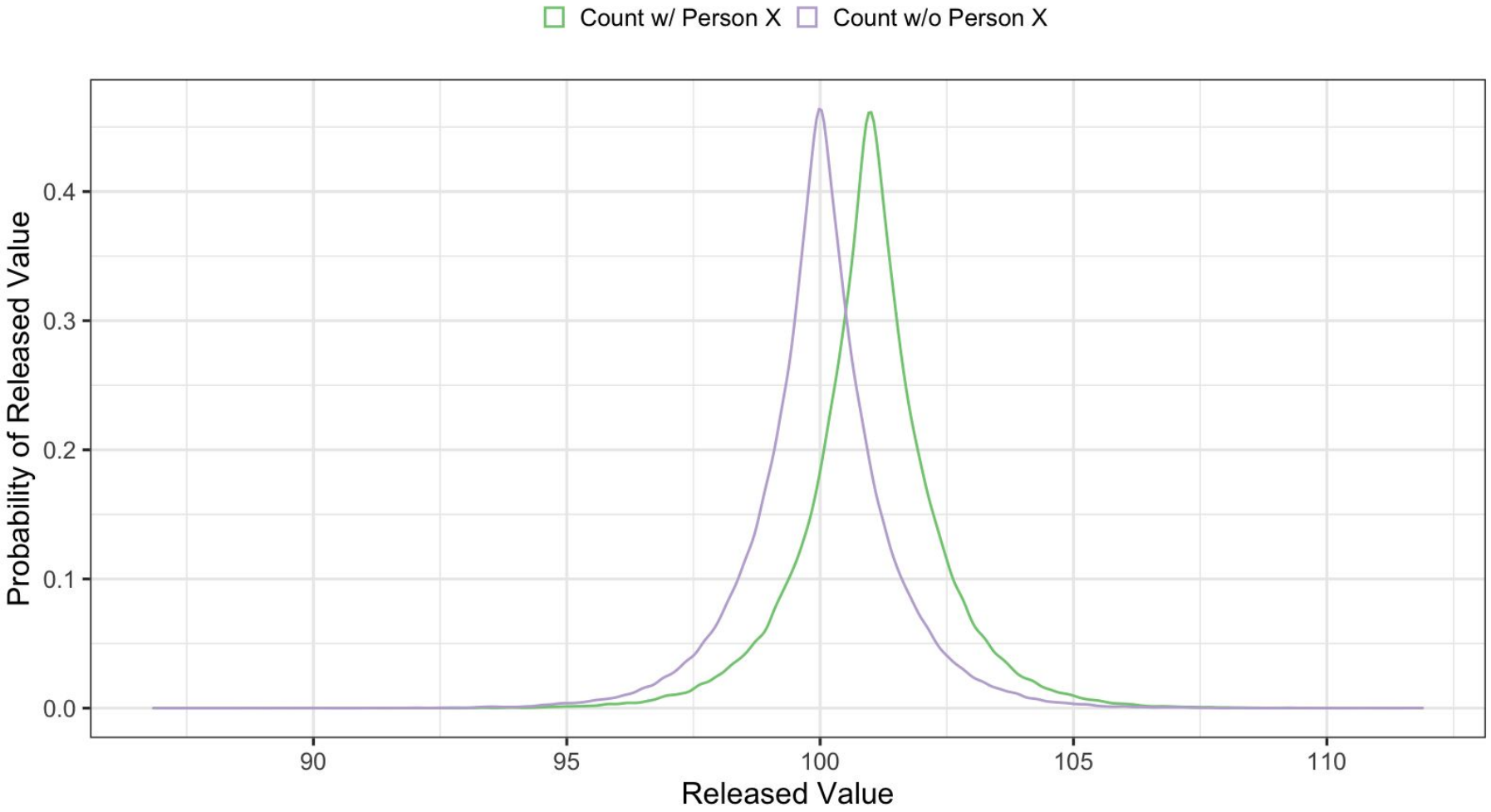
- Proven that Laplace noise meets the definition of ε-DP
  - Laplace scale parameter (variance) must: Δ / ε

Max possible difference in count due to one person.
In this case Δ = 1 (Global sensitivity)

Privacy parameter

We are Bounding the Difference Between Two Distributions of Possible Outputs



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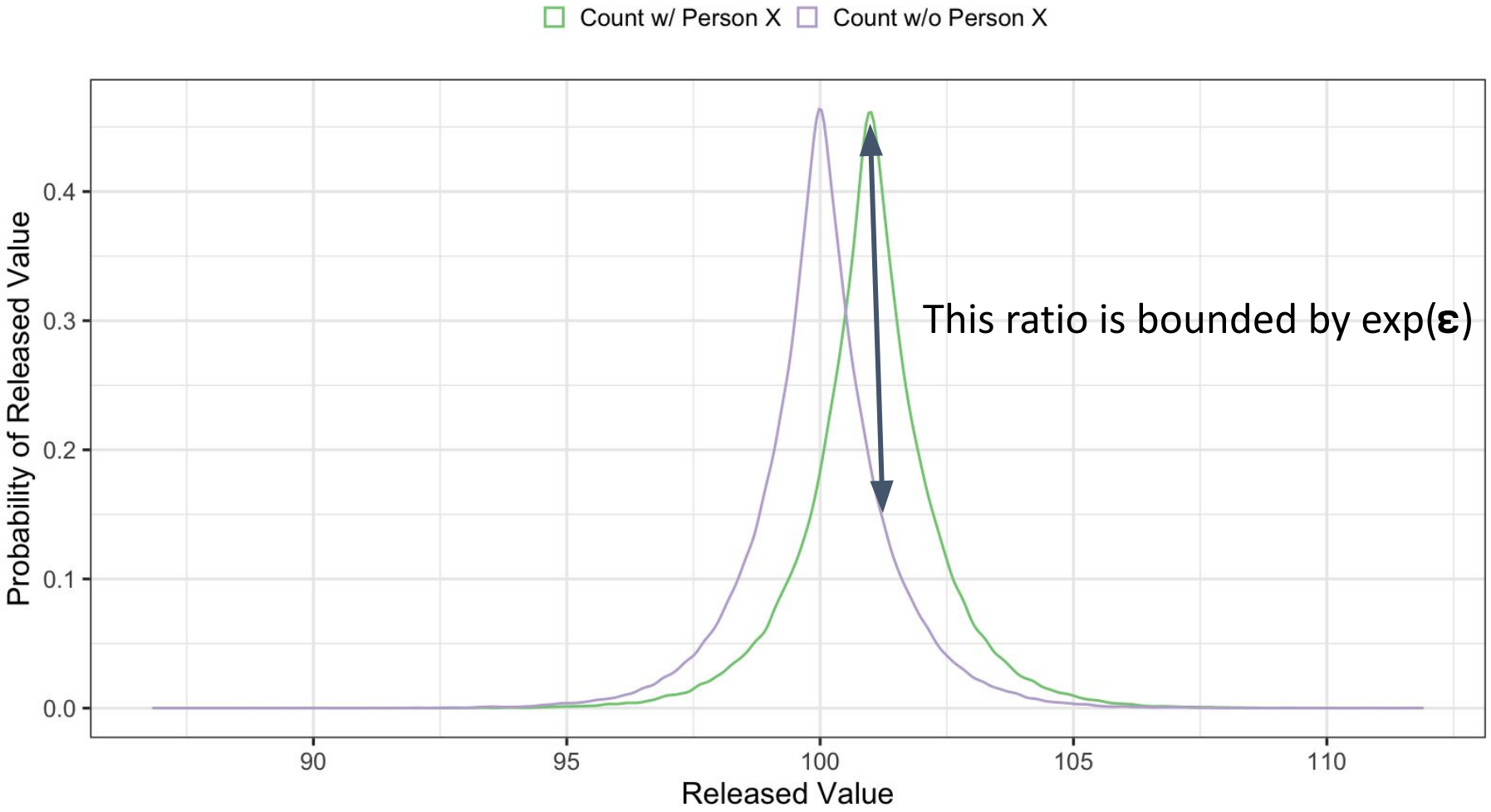
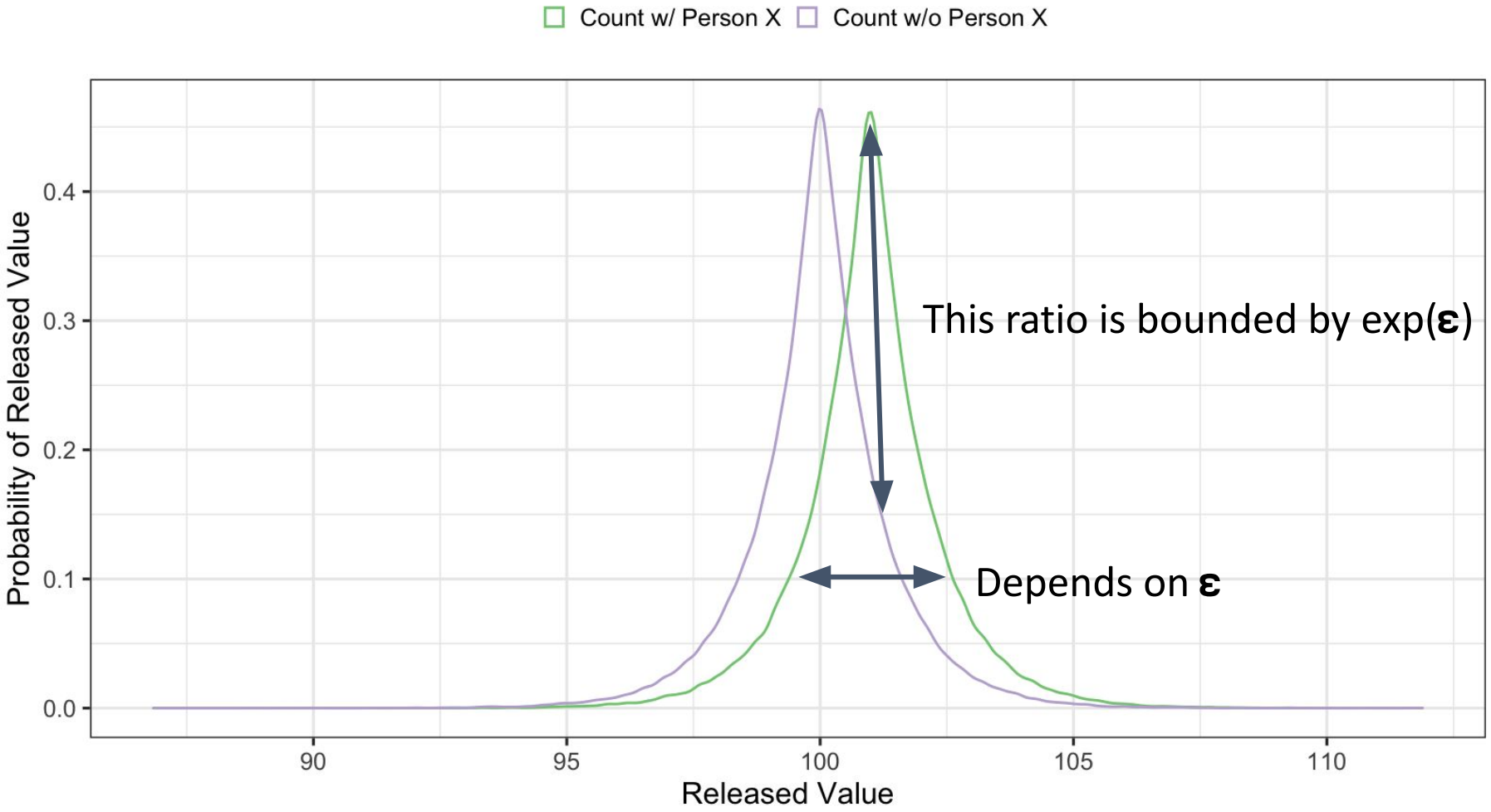


The Variance Depends on the Privacy Loss (and the Sensitivity)



We Can Release Multiple Counts

- If counts are from disjoint subsets, no need to add **ε**

County	Sex	Race/Ethnicity	Case Count
Allegheny	F	W	101 + Lap(1/**ε**)
Allegheny	M	W	683 + Lap(1/**ε**)

- If counts are based on the same individuals, add **ε**

County	Sex	Race/Ethnicity	Case Count	Death Count
Allegheny	F	W	101 + Lap(1/ε_1)	1 + Lap(1/ε_2)

Total $\varepsilon = \varepsilon_1 + \varepsilon_2$

- Leads to the idea of a “privacy budget”
- More complex composition theorems exist

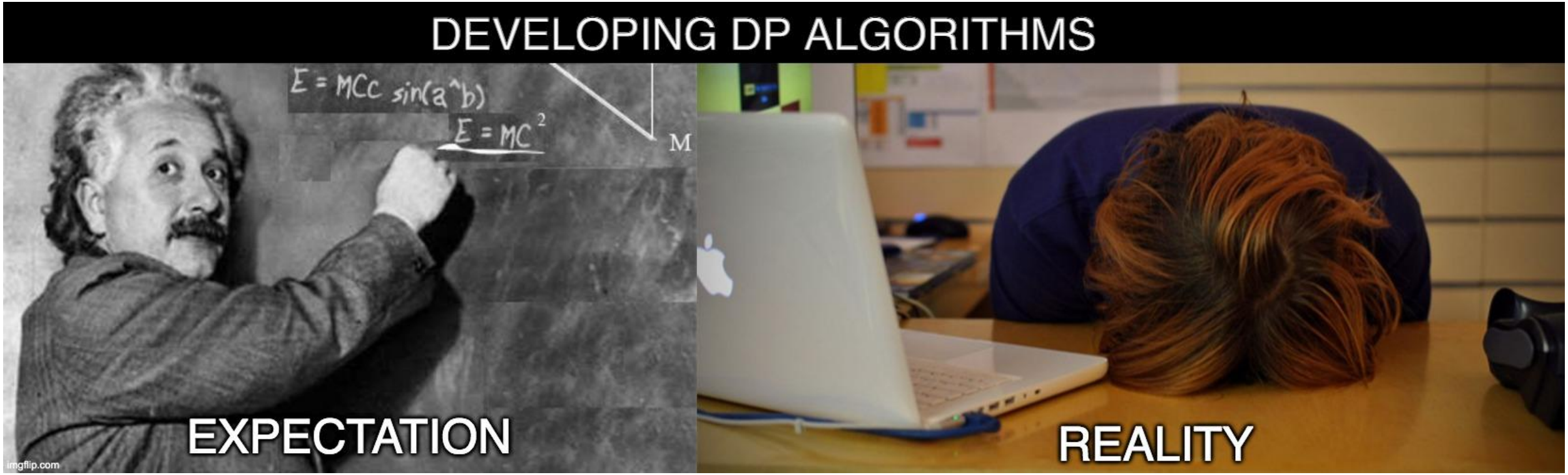
Noisy Results can be Post-Processed

- Any function of an output that satisfies **ε**-DP is also **ε**-DP
- Post-processing means applying other functions to the data
  - Rounding the counts
  - Enforcing no counts to be negative
  - Rescaling so that total counts remain fixed (invariants)
- **Important: Cannot use information from the confidential data**

Why use Differential Privacy to Protect Data?

- The privacy loss as defined is guaranteed
  - Does not require unknowable assumptions about the attacker
- The amount of loss is quantifiable
  - In many cases it can be totaled across multiple releases and tracked like a "budget"

What are Drawbacks to Differential Privacy?



What are Drawbacks to Differential Privacy?

- Meeting the definition is hard for complex statistics/microdata
  - Counts/histograms are the most developed
- The meaning of the privacy loss is less intuitive
  - Compared with measures such as risk of re-identification
- No consensus on how to set the privacy parameter

Further Reading

- Dwork, Cynthia, Frank McSherry, Kobbi Nissim, and Adam Smith. "Calibrating noise to sensitivity in private data analysis." In *Theory of cryptography conference*, pp. 265-284. Springer, Berlin, Heidelberg, 2006.
- Wasserman, Larry, and Shuheng Zhou. "A statistical framework for differential privacy." *Journal of the American Statistical Association* 105, no. 489 (2010): 375-389.
- Abowd, John M., and Lars Vilhuber. "How protective are synthetic data?." In *International Conference on Privacy in Statistical Databases*, pp. 239-246. Springer, Berlin, Heidelberg, 2008.
- Wood, Alexandra, Micah Altman, Aaron Bembenek, Mark Bun, Marco Gaboardi, James Honaker, Kobbi Nissim, David R. O'Brien, Thomas Steinke, and Salil Vadhan. "Differential privacy: A primer for a non-technical audience." *Vand. J. Ent. & Tech. L.* 21 (2018): 209.
- Snoke, Joshua, and Claire McKay Bowen. "How statisticians should grapple with privacy in a changing data landscape." *Chance* 33, no. 4 (2020): 6-13.
- Snoke, Joshua, and Claire McKay Bowen. "Differential Privacy: What Is It?." *AMSTAT news: the membership magazine of the American Statistical Association* 501 (2019): 26-28.