Wireless Communications

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2.1.1 High SNR Performance

The poor performance of wireless communications systems at high SNRs when compared to wired communicaitons systems is due to deep fades.

Comparison of Wired and wireless communications	
Wired Channel	Wireless Channel
$y = x + n$	$y = hx + n$
$BER = Q(\sqrt{SNR})$	$BER = \frac{1}{2}(1 - \sqrt{\frac{SNR}{2+SNR}})$

This can be seen why from the following analysis.

Mathematical Simplifications

Let us show why by making some mathematical simplifications for the BER expression under high SNR.

$$BER = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2 + SNR}}\right) \tag{2.1}$$

$$= \frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + \frac{2}{SNR}}}\right). \tag{2.2}$$

For high SNR we can see that $\frac{2}{SNR}$ is a small value and we know that

$$\frac{1}{\sqrt{1+x_{small}}} = 1 - \frac{x_{small}}{2}. \tag{2.3}$$

Therefore we can arrive at the approximate equation for BER of a wireless channel with a very high SNR.

$$\approx \frac{1}{2}\left(1 - \left(1 - \frac{1}{2}\frac{2}{SNR}\right)\right) \tag{2.4}$$

$$\approx \frac{1}{2}\frac{1}{SNR}. \tag{2.5}$$

Example 1

Compute the bit error rate a wireless communication system at $SNR_{dB} = 20dB$.

$$20dB = 10\log_{10} SNR \tag{2.6}$$

$$\log_{10} SNR = 2 \tag{2.7}$$

$$SNR = 10^2 = 10 \tag{2.8}$$

$$BER = \frac{1}{2SNR} = \frac{1}{2*100} = 50*10^{-4}. \tag{2.9}$$

Note that the bit error rate achieved in a wired communication system at this same SNR is only $7.8*10^{-4}$.

Example 2

Compute the SNR in dB of a wireless communication system for a $BER = 10^{-6}$

$$10^{-6} = \frac{1}{2SNR} \tag{2.10}$$

$$SNR = \frac{1}{2*10^{-6}} = \frac{10^{-6}}{2} \tag{2.11}$$

$$\implies SNR_{dB} = 10\log_{10}\left(\frac{10^{-6}}{2}\right) \tag{2.12}$$

$$= 60\text{dB} - 3\text{dB} = 57\text{dB}. \tag{2.13}$$

Note that in a Wired Communication system, the SNR required for this same BER is 13.6 dB. This is because of the desrtuctive interference that causes deep fades and is the results of multipath propogation.

Comparison Of High SNR Approximate Formula

Table 2.1: Comparison of high SNR approximate BER formula for Wired and Wireless Systems

Comparison of high SNR approximate BER formula for Wired and Wireless Systems	
Wireless System	$BER = \frac{1}{2}\left(1 - \sqrt{\frac{SNR}{2+SNR}}\right) \approx \frac{1}{2SNR}$
Wired System	$BER = Q\left(\sqrt{SNR}\right) \approx \exp^{-\frac{SNR}{2}}$

2.1.2 Probability of a Deep Fade Event

Taking a look again at the wireless communications system channel model $y = hx + n$. Let us call h the fading coefficient and n the noise. Remember that we said $\|h\|^2 P$ is the desired power at the receiver and that σ_n^2 is the noise power at the receiver. If we look at the performance of the channel when the noise power is greater than the desired power.

$$\|h\|^2 P = a^2 P < \sigma_n^2 \tag{2.14}$$

$$= a^2 < \frac{\sigma_n^2}{P} \tag{2.15}$$

$$\text{Since, } \frac{P}{\sigma_n^2} = SNR \tag{2.16}$$

$$a < \frac{1}{\sqrt{SNR}} \tag{2.17}$$

So when $a < \frac{1}{\sqrt{SNR}}$ we say we have a deep fade event as we can see that this will create very poor channel performance.

Note, this is a little missleading since we are using the terms SNR like it is the receive SNR as one would typically expect when rather here it is strictly the transmit SNR so please make sure to keep that in mind for the next paragraph and the prior paragraph. It is the tx SNR since we are separating the fading coefficient from the SNR power meaning that it is not receive SNR. This is possibly even incorrect to just

forget this section "Probability of a deep fade event"

Since we also know that the probability distribution of a is $f_A(a) = 2a\exp^{-a^2}$, we can just calculate the probability of a deep fade event by $P(a < \frac{1}{\sqrt{SNR}})$. This is just the integral of the pdf of a from 0 to a $= \int_0^{\frac{1}{\sqrt{SNR}}} f_A(a)da$ and if we approximate $\exp^{-a^2} \approx 1$ then we can say that $P(a < \frac{1}{\sqrt{SNR}}) = \frac{1}{SNR}$. Noting also that $BER = \frac{1}{2SNR}$ then we can see that $BER \sim$ The Probabililty of a deep fade event. This shows that the poor performance of a wireless system at high SNR is due to deep fade events that arise from the multipath environment that allows for destructive interference.

Adding diversity is one way to reduce the probability of a deep fade event.

2.2 Multiple Antenna Systems

Below in Figure 2.1 we can see what a multiple antenna system coutld look like. The system in the image below is what is known as a single output multiple input (MISO) system as the recevier has multiple inputs and the transmitter has a single output. From a very basic point of view, this improves high SNR performance of the wireless system as it is much less statistically likely for all channels to experience a deep fade event simultaneously.

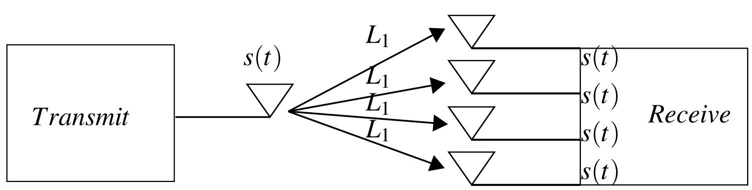

Figure 2.1: Multiple Antenna System

Using the system in the figure as an example, If we have a single transmit antenna and L receive antenna we can say that the system has L^{th} order diversity.

2.2.1 MISO System Model

Lets now form the system model for the current example system.

Table 2.2: Comparison Of SISO and MISO Systems

$y = hx + n$	SISO Wireless System
$y_1 = h_1 x + n_1$ $y_2 = h_2 x + n_2$ $\vdots$ $y_L = h_L x + n_L$	MISO Wireless System

An inspection of the MISO Wireless System makes it evident that in fact it is easiest to think of the quantities in vector form and represnt the equations as a set of vector equations.

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} h_1 \\ \vdots \\ h_n \end{bmatrix} \begin{bmatrix} x \\ \vdots \\ x \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_n \end{bmatrix} \quad = \quad \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} h_1 \\ \vdots \\ h_n \end{bmatrix} x + \begin{bmatrix} n_1 \\ \vdots \\ n_n \end{bmatrix} \tag{2.18}$$

Which is best written in matrix form as $\overline{y} = \overline{h}x + \overline{n}$.

Analysis of Receive Antenna Diversity System

We will see that the expected value of the noise at each receive antenna is σ_n^2. This is the power of the noise or in other words it is the noise variance.

$$\overline{y} = \overline{h}x + \overline{n} \tag{2.19}$$

$$E\left\{\left|n_i(k)^2\right|\right\} = \sigma_n^2 \tag{2.20}$$

signal Detection $y_1, y_2, \cdots, y_L$ are the singals received at the L receive antennas. We can combine these receive signals in the following manner $w_1^* y_1 + w_2^* y_2 + \cdots + w_L^* y_L$ then we are combining them in a weighted manner which we will call beamforming. We will refer to $\overline{W}$ as the beamformer or beamforming matrix (when in matrix form).

$$\overline{W} = \begin{bmatrix} W_1 \\ \vdots \\ W_n \end{bmatrix} \tag{2.21}$$

$$\begin{bmatrix} W_1^* & \cdots & W_n^* \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \overline{W}^H \overline{y} \tag{2.22}$$

Since $\overline{W}^H \overline{y}$ and $\overline{y} = \overline{h}x + \overline{n}$, our beamformer output is $\overline{W}^H \left(\overline{h}x + \overline{n}\right)$. Expanding that we can see that we have $\overline{W}^H \overline{h}x + \overline{W}^H \overline{n}$. We will call $\overline{W}^H \overline{h}x$ the signal component and $\overline{W}^H \overline{n}$ the noise component. Therefore we have

$$SNR = \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{\left|\overline{W}^H h\right|^2 P}{E\left\{\left|\overline{W}^H \overline{n}\right|^2\right\}}. \tag{2.23}$$

Part IV

appendix info

Appendix here

Part V

Post Appendix

COntent here [1]

Bibliography

[1] fredric harris. *Multirate Signal Processing*. Prentice Hall PTR, 2004.