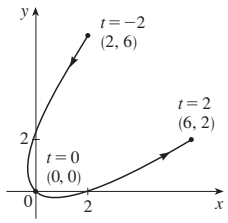
H Answers to Odd-Numbered Exercises

CHAPTER 10

EXERCISES 10.1 ■ PAGE 665

1.



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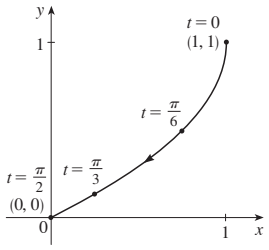
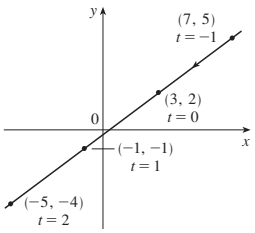
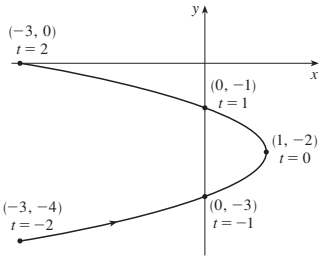


5. (a)



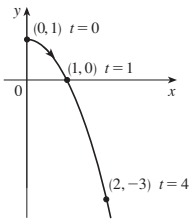
(b) $y = \frac{3}{4}x - \frac{1}{4}$

7. (a)



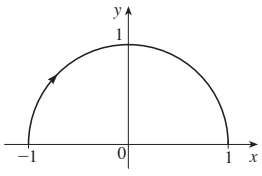
(b) $x = -(y + 2)^2 + 1$, $-4 \leqslant y \leqslant 0$

9. (a)

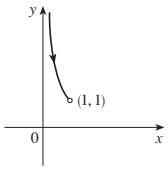


(b) $y = 1 - x^2$, $x \geqslant 0$

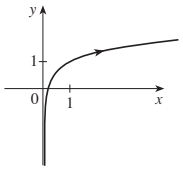
11. (a) $x^2 + y^2 = 1, y \geqslant 0$ (b)



13. (a) $y = 1/x, y > 1$ (b)

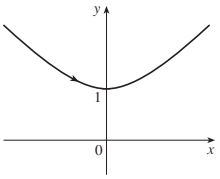


15. (a) $y = \frac{1}{2}\ln x + 1$ (b)



17. (a) $y^2 - x^2 = 1, y \geqslant 1$

(b)

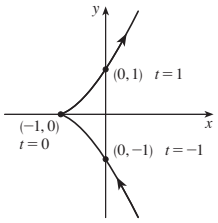


19. Moves counterclockwise along the circle $(x - 3)^2 + (y - 1)^2 = 4$ from $(3, 3)$ to $(3, -1)$

21. Moves 3 times clockwise around the ellipse $(x^2/25) + (y^2/4) = 1$, starting and ending at $(0, -2)$

23. It is contained in the rectangle described by $1 \leqslant x \leqslant 4$ and $2 \leqslant y \leqslant 3$.

25.



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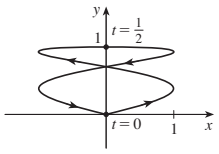

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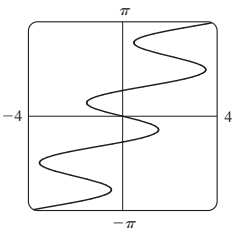


31. (b) $x = -2 + 5t, y = 7 - 8t, 0 \leqslant t \leqslant 1$

33. (a) $x = 2\cos t, y = 1 - 2\sin t, 0 \leqslant t \leqslant 2\pi$

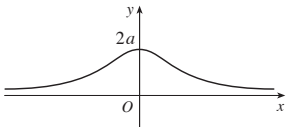
(b) $x = 2\cos t, y = 1 + 2\sin t, 0 \leqslant t \leqslant 6\pi$

(c) $x = 2\cos t, y = 1 + 2\sin t, \pi/2 \leqslant t \leqslant 3\pi/2$

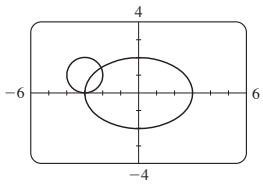
37. The curve $y = x^{2/3}$ is generated in (a). In (b), only the portion with $x \geqslant 0$ is generated, and in (c) we get only the portion with $x > 0$.

41. $x = a\cos\theta, y = b\sin\theta; (x^2/a^2) + (y^2/b^2) = 1$, ellipse

43.



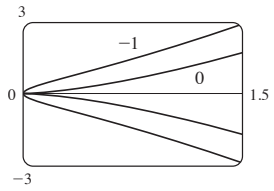
45. (a) Two points of intersection

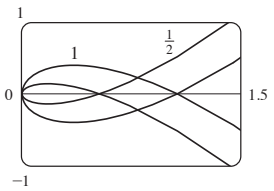


(b) One collision point at $(-3, 0)$ when $t = 3\pi/2$

(c) There are still two intersection points, but no collision point.

47. For $c = 0$, there is a cusp; for $c > 0$, there is a loop whose size increases as c increases.





49. The curves roughly follow the line $y = x$, and they start having loops when a is between 1.4 and 1.6 (more precisely, when $a > \sqrt{2}$). The loops increase in size as a increases.

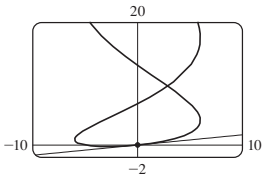
51. As n increases, the number of oscillations increases; a and b determine the width and height.

EXERCISES 10.2 ■ PAGE 675

1. $\dfrac{2t + 1}{t\cos t + \sin t}$ **3.** $y = -\frac{3}{2}x + 7$ **5.** $y = \pi x + \pi^2$

7. $y = 2x + 1$

9. $y = \frac{1}{6}x$



11. $\dfrac{2t + 1}{2t}, -\dfrac{1}{4t^3}, t < 0$ **13.** $e^{-2t}(1 - t), e^{-3t}(2t - 3),\ t > \frac{3}{2}$

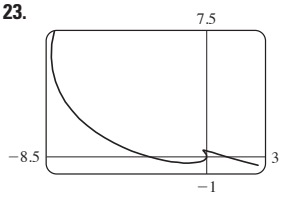
15. $-\frac{3}{2}\tan t, -\frac{3}{4}\sec^3 t,\ \pi/2 < t < 3\pi/2$

17. Horizontal at $(0, -3)$, vertical at $(\pm 2, -2)$

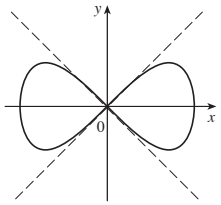
19. Horizontal at $\left(\frac{1}{2}, -1\right)$ and $\left(-\frac{1}{2}, 1\right)$, no vertical

21. $(0.6, 2); \left(5 \cdot 6^{-6/5}, e^{6^{-1/5}}\right)$

23.



25. $y = x, y = -x$



27. (a) $d\sin\theta/(r - d\cos\theta)$ **29.** $\left(\frac{16}{27}, \frac{29}{9}\right), (-2, -4)$

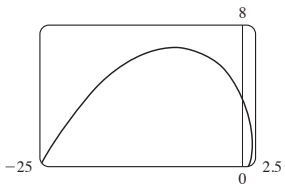
31. πab **33.** $3 - e$ **35.** $2\pi r^2 + \pi d^2$

37. $\int_0^2 \sqrt{2 + 2e^{-2t}}\, dt \approx 3.1416$

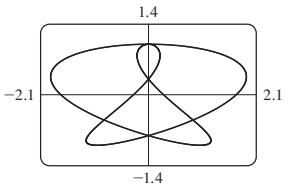
39. $\int_0^{4\pi} \sqrt{5 - 4\cos t}\, dt \approx 26.7298$ **41.** $4\sqrt{2} - 2$

43. $\frac{1}{2}\sqrt{2} + \frac{1}{2}\ln\left(1 + \sqrt{2}\right)$

45. $\sqrt{2}\,(e^{\pi} - 1)$

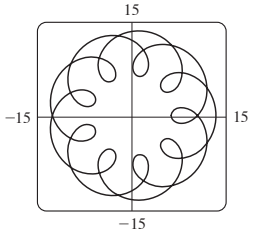


47. 16.7102



49. 612.3053 **51.** $6\sqrt{2}, \sqrt{2}$

55. (a) $t \in [0, 4\pi]$



(b) 294

57. $\int_0^{\pi/2} 2\pi t\cos t\sqrt{t^2 + 1}\, dt \approx 4.7394$

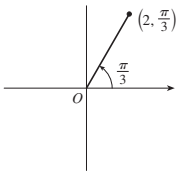
59. $\int_0^1 2\pi(t^2 + 1)e^t\sqrt{e^{2t}(t + 1)^2(t^2 + 2t + 2)}\, dt \approx 103.5999$

61. $\frac{2}{1215}\pi\left(247\sqrt{13} + 64\right)$ **63.** $\frac{6}{5}\pi a^2$

65. $\frac{24}{5}\pi\left(949\sqrt{26} + 1\right)$ **71.** $\frac{1}{4}$

EXERCISES 10.3 ■ PAGE 686

1. (a)



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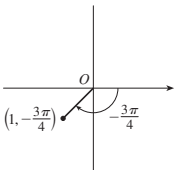
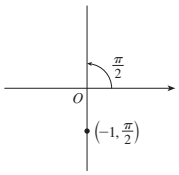


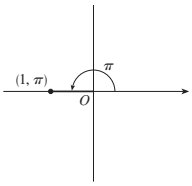
$(2, 7\pi/3), (-2, 4\pi/3)$ $(1, 5\pi/4), (-1, \pi/4)$

(c)



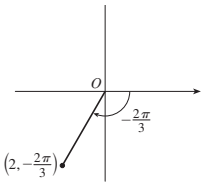
$(1, 3\pi/2), (-1, 5\pi/2)$

3. (a)



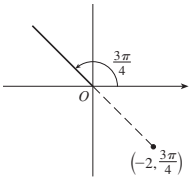
$(-1, 0)$

(b)



$(-1, -\sqrt{3})$

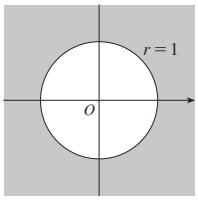
(c)



$(\sqrt{2}, -\sqrt{2})$

5. (a) (i) $(2\sqrt{2}, 7\pi/4)$ (ii) $(-2\sqrt{2}, 3\pi/4)$
(b) (i) $(2, 2\pi/3)$ (ii) $(-2, 5\pi/3)$

7.



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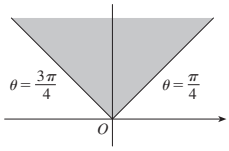

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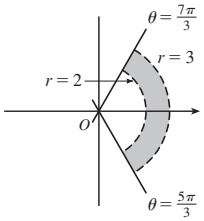
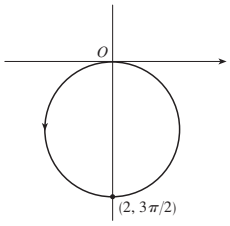


13. $2\sqrt{3}$ **15.** Circle, center O, radius $\sqrt{5}$
17. Circle, center $(1, 0)$, radius 1
19. Hyperbola, center O, foci on x-axis
21. $r = 2 \csc \theta$ **23.** $r = 1/(\sin \theta - 3 \cos \theta)$
25. $r = 2c \cos \theta$ **27.** (a) $\theta = \pi/6$ (b) $x = 3$

29.



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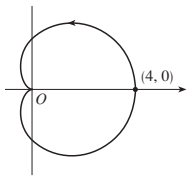

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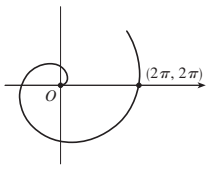

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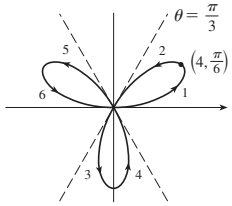

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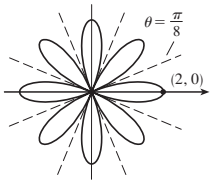

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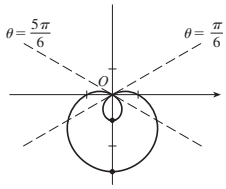

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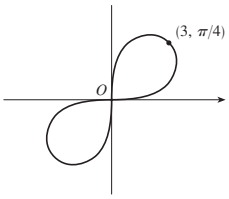

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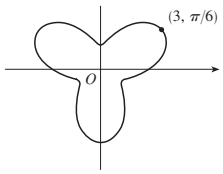

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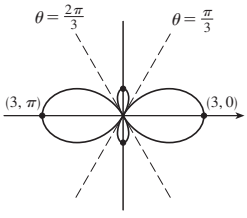

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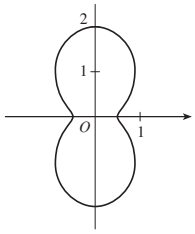

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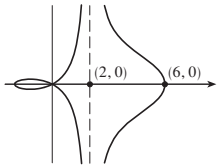

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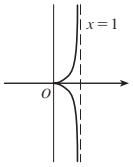


53. (a) For $c < -1$, the inner loop begins at $\theta = \sin^{-1}(-1/c)$ and ends at $\theta = \pi - \sin^{-1}(-1/c)$; for $c > 1$, it begins at $\theta = \pi + \sin^{-1}(1/c)$ and ends at $\theta = 2\pi - \sin^{-1}(1/c)$.

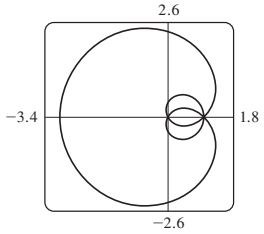
55. $\sqrt{3}$ **57.** $-\pi$ **59.** 1

61. Horizontal at $\left(3/\sqrt{2}, \pi/4\right), \left(-3/\sqrt{2}, 3\pi/4\right)$; vertical at $(3, 0), (0, \pi/2)$

63. Horizontal at $\left(\frac{3}{2}, \pi/3\right), (0, \pi)$ [the pole], and $\left(\frac{3}{2}, 5\pi/3\right)$; vertical at $(2, 0), \left(\frac{1}{2}, 2\pi/3\right), \left(\frac{1}{2}, 4\pi/3\right)$

65. Center $(b/2, a/2)$, radius $\sqrt{a^2 + b^2}/2$

67.



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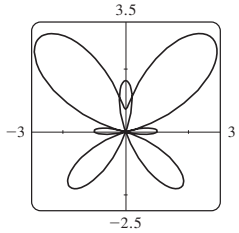

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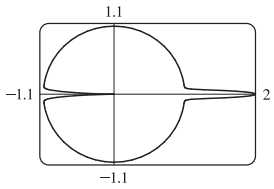


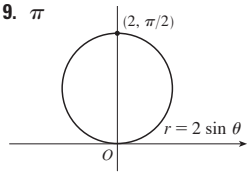
73. By counterclockwise rotation through angle $\pi/6$, $\pi/3$, or α about the origin

75. For $c = 0$, the curve is a circle. As c increases, the left side gets flatter, then has a dimple for $0.5 < c < 1$, a cusp for $c = 1$, and a loop for $c > 1$.

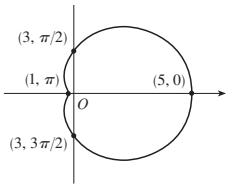
EXERCISES 10.4 ■ PAGE 692

1. $e^{-\pi/4} - e^{-\pi/2}$ **3.** $\frac{9}{2}$ **5.** π^2 **7.** $\frac{41}{4}\pi$

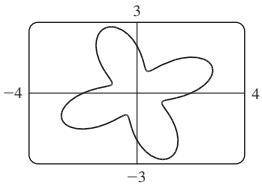
9. π



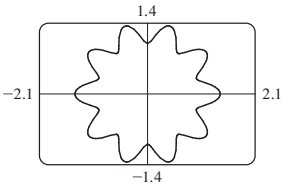
11. 11π



13. $\frac{9}{2}\pi$



15. $\frac{3}{2}\pi$



17. $\frac{4}{3}\pi$ **19.** $\frac{1}{16}\pi$ **21.** $\pi - \frac{3}{2}\sqrt{3}$ **23.** $\frac{1}{3}\pi + \frac{1}{2}\sqrt{3}$

25. $4\sqrt{3} - \frac{4}{3}\pi$ **27.** π **29.** $\frac{5}{24}\pi - \frac{1}{4}\sqrt{3}$ **31.** $\frac{1}{2}\pi - 1$

33. $1 - \frac{1}{2}\sqrt{2}$ **35.** $\frac{1}{4}\left(\pi + 3\sqrt{3}\right)$

37. $\left(\frac{3}{2}, \pi/6\right), \left(\frac{3}{2}, 5\pi/6\right)$, and the pole

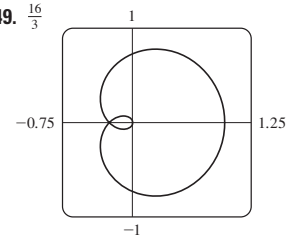
39. $(1, \theta)$ where $\theta = \pi/12, 5\pi/12, 13\pi/12, 17\pi/12$ and $(-1, \theta)$ where $\theta = 7\pi/12, 11\pi/12, 19\pi/12, 23\pi/12$

41. $\left(\frac{1}{2}\sqrt{3}, \pi/3\right), \left(\frac{1}{2}\sqrt{3}, 2\pi/3\right)$, and the pole

43. Intersection at $\theta \approx 0.89, 2.25$; area ≈ 3.46

45. 2π **47.** $\frac{8}{3}[(\pi^2 + 1)^{3/2} - 1]$

49. $\frac{16}{3}$

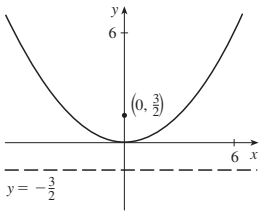


51. 2.4221 **53.** 8.0091

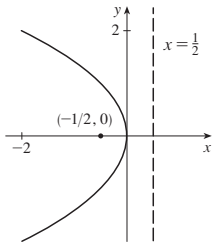
55. (b) $2\pi\left(2 - \sqrt{2}\right)$

EXERCISES 10.5 ■ PAGE 700

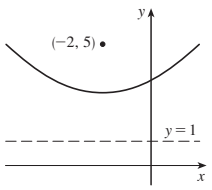
1. $(0, 0), \left(0, \frac{3}{2}\right), y = -\frac{3}{2}$



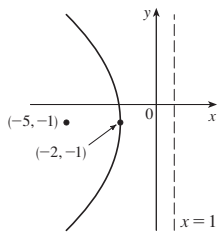
3. $(0, 0), \left(-\frac{1}{2}, 0\right), x = \frac{1}{2}$



5. $(-2, 3), (-2, 5), y = 1$

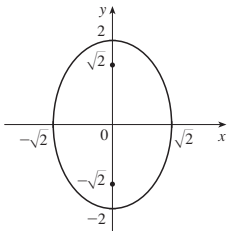


7. $(-2, -1), (-5, -1), x = 1$

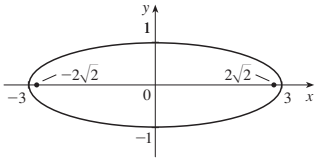


9. $x = -y^2$, focus $\left(-\frac{1}{4}, 0\right)$, directrix $x = \frac{1}{4}$

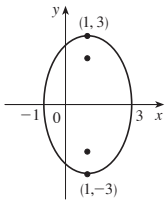
11. $(0, \pm 2), \left(0, \pm\sqrt{2}\right)$



13. $(\pm 3, 0), \left(\pm 2\sqrt{2}, 0\right)$

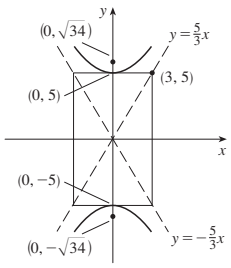


15. $(1, \pm 3), \left(1, \pm\sqrt{5}\right)$

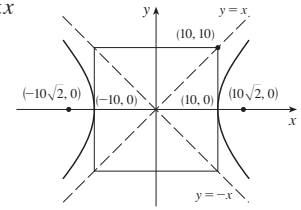


17. $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$, foci $\left(0, \pm\sqrt{5}\right)$

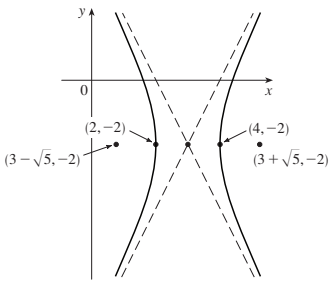
19. $(0, \pm 5); \left(0, \pm\sqrt{34}\right); y = \pm\frac{5}{3}x$



21. $(\pm 10, 0), \left(\pm 10\sqrt{2}, 0\right), y = \pm x$



23. $(4, -2), (2, -2)$; $\left(3 \pm \sqrt{5}, -2\right)$; $y + 2 = \pm 2(x - 3)$



25. Parabola, $(0, -1), \left(0, -\frac{3}{4}\right)$

27. Ellipse, $\left(\pm\sqrt{2}, 1\right), (\pm 1, 1)$

29. Hyperbola, $(0, 1), (0, -3); \left(0, -1 \pm \sqrt{5}\right)$

31. $y^2 = 4x$ **33.** $y^2 = -12(x + 1)$ **35.** $y - 3 = 2(x - 2)^2$

37. $\dfrac{x^2}{25} + \dfrac{y^2}{21} = 1$ **39.** $\dfrac{x^2}{12} + \dfrac{(y-4)^2}{16} = 1$

41. $\dfrac{(x+1)^2}{12} + \dfrac{(y-4)^2}{16} = 1$ **43.** $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

45. $\dfrac{(y-1)^2}{25} - \dfrac{(x+3)^2}{39} = 1$ **47.** $\dfrac{x^2}{9} - \dfrac{y^2}{36} = 1$

49. $\dfrac{x^2}{3{,}763{,}600} + \dfrac{y^2}{3{,}753{,}196} = 1$

51. (a) $\dfrac{121x^2}{1{,}500{,}625} - \dfrac{121y^2}{3{,}339{,}375} = 1$ (b) ≈ 248 mi

55. (a) Ellipse (b) Hyperbola (c) No curve

59. 15.9

61. $\dfrac{b^2 c}{a} + ab \ln\left(\dfrac{a}{b + c}\right)$ where $c^2 = a^2 + b^2$

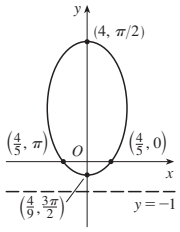
63. $(0, 4/\pi)$

EXERCISES 10.6 ■ PAGE 708

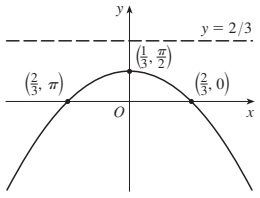
1. $r = \dfrac{4}{2 + \cos\theta}$ **3.** $r = \dfrac{6}{2 + 3\sin\theta}$

5. $r = \dfrac{8}{1 - \sin\theta}$ **7.** $r = \dfrac{4}{2 + \cos\theta}$

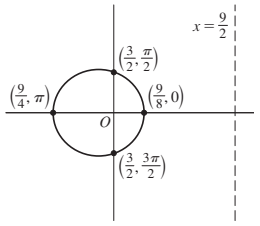
9. (a) $\frac{4}{5}$ (b) Ellipse (c) $y = -1$
(d)



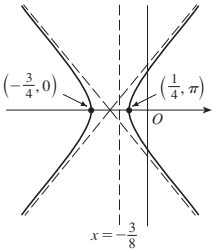
11. (a) 1 (b) Parabola (c) $y = \frac{2}{3}$
(d)



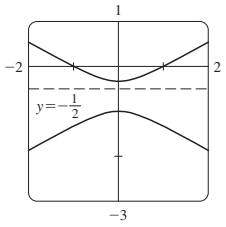
13. (a) $\frac{1}{3}$ (b) Ellipse (c) $x = \frac{9}{2}$
(d)



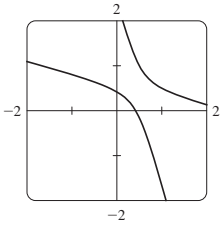
15. (a) 2 (b) Hyperbola (c) $x = -\frac{3}{8}$
(d)



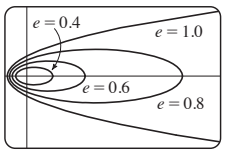
17. (a) $2, y = -\frac{1}{2}$



(b) $r = \dfrac{1}{1 - 2\sin(\theta - 3\pi/4)}$



19. The ellipse is nearly circular when e is close to 0 and becomes more elongated as $e \to 1^-$. At $e = 1$, the curve becomes a parabola.



25. $r = \dfrac{2.26 \times 10^8}{1 + 0.093\cos\theta}$

27. 35.64 AU **29.** 7.0×10^7 km **31.** 3.6×10^8 km

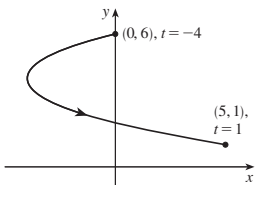
CHAPTER 10 REVIEW ■ PAGE 709

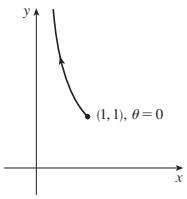
True-False Quiz

1. False **3.** False **5.** True **7.** False **9.** True

Exercises

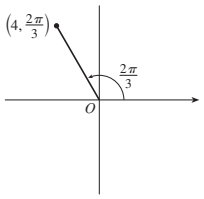
1. $x = y^2 - 8y + 12$ **3.** $y = 1/x$





5. $x = t, y = \sqrt{t}$; $x = t^4, y = t^2$; $x = \tan^2 t, y = \tan t, 0 \le t < \pi/2$

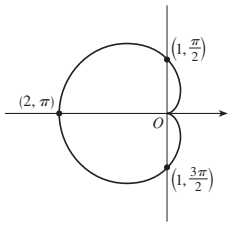
7. (a)



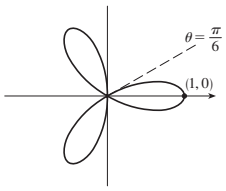
(b) $\left(3\sqrt{2}, 3\pi/4\right)$, $\left(-3\sqrt{2}, 7\pi/4\right)$

$\left(-2, 2\sqrt{3}\right)$

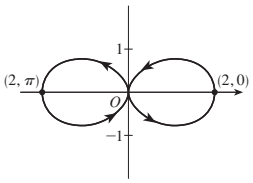
9.



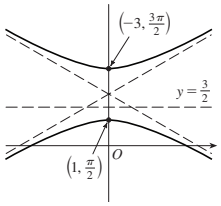
11.



13.

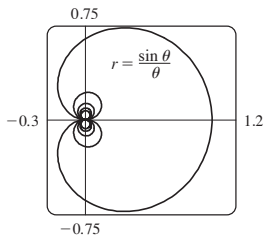


15.



17. $r = \dfrac{2}{\cos\theta + \sin\theta}$

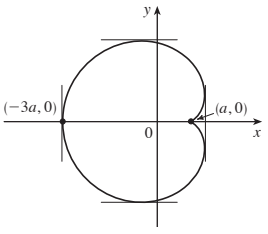
19.



21. 2 **23.** −1

25. $\dfrac{1 + \sin t}{1 + \cos t}, \dfrac{1 + \cos t + \sin t}{(1 + \cos t)^3}$ **27.** $\left(\frac{11}{8}, \frac{3}{4}\right)$

29. Vertical tangent at $\left(\frac{3}{2}a, \pm\frac{1}{2}\sqrt{3}\,a\right), (-3a, 0)$; horizontal tangent at $(a, 0), \left(-\frac{1}{2}a, \pm\frac{3}{2}\sqrt{3}a\right)$



31. 18 **33.** $(2, \pm\pi/3)$ **35.** $\frac{1}{2}(\pi - 1)$

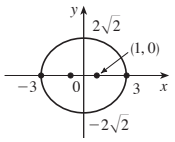
37. $2\left(5\sqrt{5} - 1\right)$

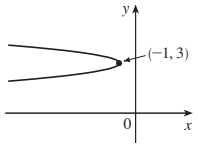
39. $\dfrac{2\sqrt{\pi^2 + 1} - \sqrt{4\pi^2 + 1}}{2\pi} + \ln\left(\dfrac{2\pi + \sqrt{4\pi^2 + 1}}{\pi + \sqrt{\pi^2 + 1}}\right)$

41. $471{,}295\pi/1024$

43. All curves have the vertical asymptote $x = 1$. For $c < -1$, the curve bulges to the right. At $c = -1$, the curve is the line $x = 1$. For $-1 < c < 0$, it bulges to the left. At $c = 0$ there is a cusp at $(0, 0)$. For $c > 0$, there is a loop.

45. $(\pm 1, 0), (\pm 3, 0)$ **47.** $\left(-\frac{25}{24}, 3\right), (-1, 3)$

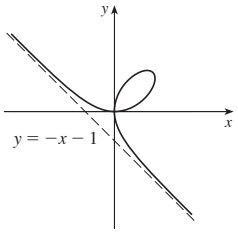




49. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$ **51.** $\dfrac{y^2}{72/5} - \dfrac{x^2}{8/5} = 1$

53. $\dfrac{x^2}{25} + \dfrac{(8y - 399)^2}{160{,}801} = 1$ **55.** $r = \dfrac{4}{3 + \cos\theta}$

57. (a) At $(0, 0)$ and $\left(\frac{3}{2}, \frac{3}{2}\right)$
(b) Horizontal tangents at $(0, 0)$ and $\left(\sqrt[3]{2}, \sqrt[3]{4}\right)$; vertical tangents at $(0, 0)$ and $\left(\sqrt[3]{4}, \sqrt[3]{2}\right)$
(d) (g) $\frac{3}{2}$



PROBLEMS PLUS ■ PAGE 712

1. $\ln(\pi/2)$ **3.** $\left[-\frac{3}{4}\sqrt{3}, \frac{3}{4}\sqrt{3}\right] \times [-1, 2]$

CHAPTER 11

EXERCISES 11.1 ■ PAGE 724

Abbreviations: C, convergent; D, divergent

1. (a) A sequence is an ordered list of numbers. It can also be defined as a function whose domain is the set of positive integers.
(b) The terms a_n approach 8 as n becomes large.
(c) The terms a_n become large as n becomes large.

3. $1, \frac{4}{5}, \frac{3}{5}, \frac{8}{17}, \frac{5}{13}$ **5.** $\frac{1}{5}, -\frac{1}{25}, \frac{1}{125}, -\frac{1}{625}, \frac{1}{3125}$ **7.** $\frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \frac{1}{720}$

9. 1, 2, 7, 32, 157 **11.** $2, \frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \frac{2}{9}$ **13.** $a_n = 1/(2n - 1)$

15. $a_n = -3\left(-\frac{2}{3}\right)^{n-1}$ **17.** $a_n = (-1)^{n+1}\dfrac{n^2}{n + 1}$

19. 0.4286, 0.4615, 0.4737, 0.4800, 0.4839, 0.4865, 0.4884, 0.4898, 0.4909, 0.4918; yes; $\frac{1}{2}$

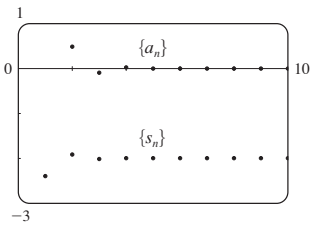
21. 0.5000, 1.2500, 0.8750, 1.0625, 0.9688, 1.0156, 0.9922, 1.0039, 0.9980, 1.0010; yes; 1

23. 1 **25.** 5 **27.** 1 **29.** 1 **31.** D **33.** 0
35. D **37.** 0 **39.** 0 **41.** 0 **43.** 0 **45.** 1
47. e^2 **49.** $\ln 2$ **51.** $\pi/2$ **53.** D **55.** D
57. 1 **59.** $\frac{1}{2}$ **61.** D **63.** 0
65. (a) 1060, 1123.60, 1191.02, 1262.48, 1338.23 (b) D
67. (a) $P_n = 1.08P_{n-1} - 300$ (b) 5734
69. $-1 < r < 1$
71. Convergent by the Monotonic Sequence Theorem; $5 \leqslant L < 8$
73. Decreasing; yes **75.** Not monotonic; no
77. Decreasing; yes
79. 2 **81.** $\frac{1}{2}(3 + \sqrt{5})$ **83.** (b) $\frac{1}{2}(1 + \sqrt{5})$
85. (a) 0 (b) 9, 11

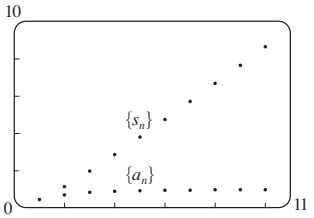
EXERCISES 11.2 ■ PAGE 735

1. (a) A sequence is an ordered list of numbers whereas a series is the *sum* of a list of numbers.
(b) A series is convergent if the sequence of partial sums is a convergent sequence. A series is divergent if it is not convergent.
3. 2
5. 1, 1.125, 1.1620, 1.1777, 1.1857, 1.1903, 1.1932, 1.1952; C
7. 0.5, 1.3284, 2.4265, 3.7598, 5.3049, 7.0443, 8.9644, 11.0540; D

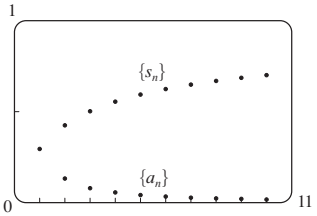
9. $-2.40000, -1.92000, -2.01600, -1.99680, -2.00064, -1.99987, -2.00003, -1.99999, -2.00000, -2.00000$; convergent, sum $= -2$



11. 0.44721, 1.15432, 1.98637, 2.88080, 3.80927, 4.75796, 5.71948, 6.68962, 7.66581, 8.64639; divergent



13. 0.29289, 0.42265, 0.50000, 0.55279, 0.59175, 0.62204, 0.64645, 0.66667, 0.68377, 0.69849; convergent, sum $= 1$



15. (a) C (b) D **17.** D **19.** $\frac{25}{3}$ **21.** 60 **23.** $\frac{1}{7}$
25. D **27.** D **29.** D **31.** $\frac{5}{2}$ **33.** D **35.** D
37. D **39.** D **41.** $e/(e - 1)$ **43.** $\frac{3}{2}$ **45.** $\frac{11}{6}$ **47.** $e - 1$
49. (b) 1 (c) 2 (d) All rational numbers with a terminating decimal representation, except 0.
51. $\frac{8}{9}$ **53.** $\frac{838}{333}$ **55.** $5063/3300$
57. $-\dfrac{1}{5} < x < \dfrac{1}{5}; \dfrac{-5x}{1 + 5x}$ **59.** $-1 < x < 5; \dfrac{3}{5 - x}$
61. $x > 2$ or $x < -2; \dfrac{x}{x - 2}$ **63.** $x < 0; \dfrac{1}{1 - e^x}$
65. 1 **67.** $a_1 = 0, a_n = \dfrac{2}{n(n + 1)}$ for $n > 1$, sum $= 1$

69. (a) 157.875 mg; $\frac{3000}{19}(1 - 0.05^n)$ (b) 157.895 mg

71. (a) $S_n = \dfrac{D(1 - c^n)}{1 - c}$ (b) 5 **73.** $\frac{1}{2}(\sqrt{3} - 1)$

77. $\dfrac{1}{n(n + 1)}$ **79.** The series is divergent.

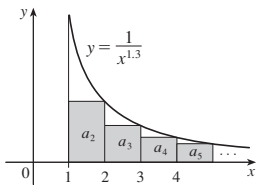
85. $\{s_n\}$ is bounded and increasing.

87. (a) $0, \frac{1}{9}, \frac{2}{9}, \frac{1}{3}, \frac{2}{3}, \frac{7}{9}, \frac{8}{9}, 1$

89. (a) $\frac{1}{2}, \frac{5}{6}, \frac{23}{24}, \frac{119}{120}$; $\dfrac{(n + 1)! - 1}{(n + 1)!}$ (c) 1

EXERCISES 11.3 ■ PAGE 744

1. C



3. D **5.** C **7.** D **9.** C **11.** C **13.** D
15. C **17.** C **19.** C **21.** D **23.** C **25.** C
27. f is neither positive nor decreasing.
29. $p > 1$ **31.** $p < -1$ **33.** $(1, \infty)$
35. (a) $\frac{9}{10}\pi^4$ (b) $\frac{1}{90}\pi^4 - \frac{17}{16}$
37. (a) 1.54977, error $\leqslant 0.1$ (b) 1.64522, error $\leqslant 0.005$
(c) 1.64522 compared to 1.64493 (d) $n > 1000$
39. 0.00145 **45.** $b < 1/e$

EXERCISES 11.4 ■ PAGE 750

1. (a) Nothing (b) C **3.** C **5.** D **7.** C **9.** D
11. C **13.** C **15.** D **17.** D **19.** D **21.** C
23. C **25.** D **27.** C **29.** C **31.** D
33. 1.249, error < 0.1 **35.** 0.0739, error $< 6.4 \times 10^{-8}$
45. Yes

EXERCISES 11.5 ■ PAGE 755

1. (a) A series whose terms are alternately positive and negative (b) $0 < b_{n+1} \leqslant b_n$ and $\lim_{n\to\infty} b_n = 0$, where $b_n = |a_n|$ (c) $|R_n| \leqslant b_{n+1}$
3. C **5.** C **7.** D **9.** C **11.** C **13.** D **15.** C
17. C **19.** D **21.** -0.5507 **23.** 5 **25.** 4
27. -0.4597 **29.** 0.0676 **31.** An underestimate
33. p is not a negative integer **35.** $\{b_n\}$ is not decreasing

EXERCISES 11.6 ■ PAGE 761

Abbreviations: AC, absolutely convergent; CC, conditionally convergent

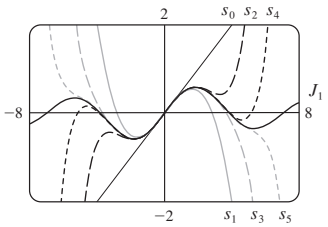
1. (a) D (b) C (c) May converge or diverge
3. AC **5.** CC **7.** AC **9.** D **11.** AC **13.** AC
15. AC **17.** CC **19.** AC **21.** AC **23.** D **25.** AC
27. AC **29.** D **31.** D **33.** AC
35. (a) and (d)
39. (a) $\frac{661}{960} \approx 0.68854$, error < 0.00521
(b) $n \geqslant 11, 0.693109$
45. (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$; $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

EXERCISES 11.7 ■ PAGE 764

1. C **3.** D **5.** C **7.** D **9.** C **11.** C
13. C **15.** C **17.** C **19.** C **21.** D **23.** D
25. C **27.** C **29.** C **31.** D
33. C **35.** D **37.** C

EXERCISES 11.8 ■ PAGE 769

1. A series of the form $\sum_{n=0}^{\infty} c_n(x - a)^n$, where x is a variable and a and the c_n's are constants
3. $1, (-1, 1)$ **5.** $1, [-1, 1)$
7. $\infty, (-\infty, \infty)$ **9.** $2, (-2, 2)$ **11.** $\frac{1}{3}, \left[-\frac{1}{3}, \frac{1}{3}\right]$
13. $4, (-4, 4]$ **15.** $1, [1, 3]$ **17.** $\frac{1}{3}, \left[-\frac{13}{3}, -\frac{11}{3}\right)$
19. $\infty, (-\infty, \infty)$ **21.** $b, (a - b, a + b)$ **23.** $0, \left\{\frac{1}{2}\right\}$
25. $\frac{1}{5}, \left[\frac{3}{5}, 1\right]$ **27.** $\infty, (-\infty, \infty)$ **29.** (a) Yes (b) No
31. k^k **33.** No
35. (a) $(-\infty, \infty)$
(b), (c)



37. $(-1, 1), f(x) = (1 + 2x)/(1 - x^2)$ **41.** 2

EXERCISES 11.9 ■ PAGE 775

1. 10 **3.** $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^n, (-1, 1)$ **5.** $\displaystyle 2\sum_{n=0}^{\infty} \frac{1}{3^{n+1}} x^n, (-3, 3)$

7. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{1}{9^{n+1}} x^{2n+1}, (-3, 3)$ **9.** $\displaystyle 1 + 2\sum_{n=1}^{\infty} x^n, (-1, 1)$

11. $\displaystyle\sum_{n=0}^{\infty} \left[(-1)^{n+1} - \frac{1}{2^{n+1}}\right] x^n, (-1, 1)$

13. (a) $\displaystyle\sum_{n=0}^{\infty} (-1)^n (n + 1) x^n, R = 1$

(b) $\displaystyle\frac{1}{2}\sum_{n=0}^{\infty} (-1)^n (n + 2)(n + 1) x^n, R = 1$

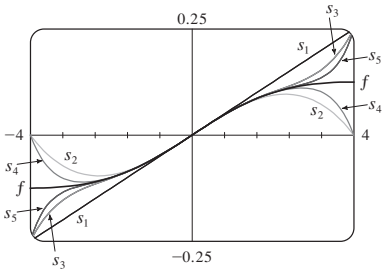
(c) $\displaystyle\frac{1}{2}\sum_{n=2}^{\infty} (-1)^n n(n - 1) x^n, R = 1$

15. $\displaystyle \ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n5^n}, R = 5$

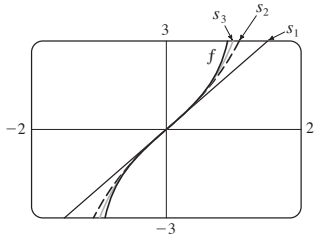
17. $\displaystyle\sum_{n=0}^{\infty} (-1)^n 4^n (n + 1) x^{n+1}, R = \frac{1}{4}$

19. $\sum_{n=0}^{\infty}(2n+1)x^n, R=1$

21. $\sum_{n=0}^{\infty}(-1)^n\frac{1}{16^{n+1}}x^{2n+1}, R=4$



23. $\sum_{n=0}^{\infty}\frac{2x^{2n+1}}{2n+1}, R=1$



25. $C+\sum_{n=0}^{\infty}\frac{t^{8n+2}}{8n+2}, R=1$

27. $C+\sum_{n=1}^{\infty}(-1)^n\frac{x^{n+3}}{n(n+3)}, R=1$

29. 0.199989 **31.** 0.000983 **33.** 0.19740

35. (b) 0.920 **39.** $[-1,1],[-1,1),(-1,1)$

EXERCISES 11.10 ■ PAGE 789

1. $b_8=f^{(8)}(5)/8!$ **3.** $\sum_{n=0}^{\infty}(n+1)x^n, R=1$

5. $\sum_{n=0}^{\infty}(n+1)x^n, R=1$

7. $\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n+1}}{(2n+1)!}x^{2n+1}, R=\infty$

9. $\sum_{n=0}^{\infty}\frac{(\ln 2)^n}{n!}x^n, R=\infty$ **11.** $\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}, R=\infty$

13. $-1-2(x-1)+3(x-1)^2+4(x-1)^3+(x-1)^4, R=\infty$

15. $\ln 2+\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n\,2^n}(x-2)^n, R=2$

17. $\sum_{n=0}^{\infty}\frac{2^n e^6}{n!}(x-3)^n, R=\infty$

19. $\sum_{n=0}^{\infty}(-1)^{n+1}\frac{1}{(2n)!}(x-\pi)^{2n}, R=\infty$

25. $1-\frac{1}{4}x-\sum_{n=2}^{\infty}\frac{3\cdot 7\cdot\cdots\cdot(4n-5)}{4^n\cdot n!}x^n, R=1$

27. $\sum_{n=0}^{\infty}(-1)^n\frac{(n+1)(n+2)}{2^{n+4}}x^n, R=2$

29. $\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n+1}}{(2n+1)!}x^{2n+1}, R=\infty$

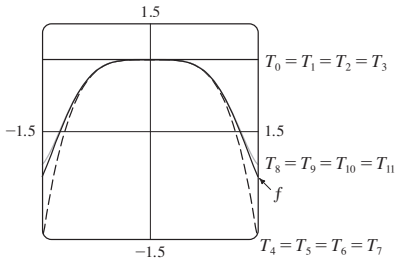
31. $\sum_{n=0}^{\infty}\frac{2^n+1}{n!}x^n, R=\infty$

33. $\sum_{n=0}^{\infty}(-1)^n\frac{1}{2^{2n}(2n)!}x^{4n+1}, R=\infty$

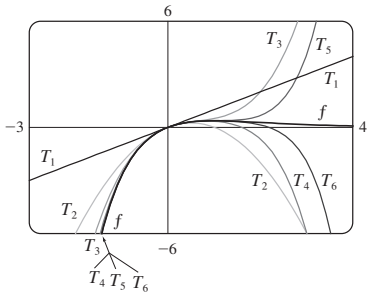
35. $\frac{1}{2}x+\sum_{n=1}^{\infty}(-1)^n\frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{n!\,2^{3n+1}}x^{2n+1}, R=2$

37. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^{2n-1}}{(2n)!}x^{2n}, R=\infty$

39. $\sum_{n=0}^{\infty}(-1)^n\frac{1}{(2n)!}x^{4n}, R=\infty$



41. $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(n-1)!}x^n, R=\infty$



43. 0.99619

45. (a) $1+\sum_{n=1}^{\infty}\frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{2^n n!}x^{2n}$

(b) $x+\sum_{n=1}^{\infty}\frac{1\cdot 3\cdot 5\cdot\cdots\cdot(2n-1)}{(2n+1)2^n n!}x^{2n+1}$

47. $C+\sum_{n=0}^{\infty}(-1)^n\frac{x^{6n+2}}{(6n+2)(2n)!}, R=\infty$

49. $C+\sum_{n=1}^{\infty}(-1)^n\frac{1}{2n\,(2n)!}x^{2n}, R=\infty$

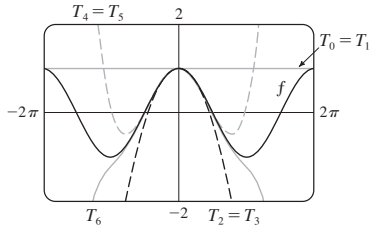
51. 0.0059 **53.** 0.40102 **55.** $\frac{1}{2}$ **57.** $\frac{1}{120}$

59. $1-\frac{3}{2}x^2+\frac{25}{24}x^4$ **61.** $1+\frac{1}{6}x^2+\frac{7}{360}x^4$ **63.** e^{-x^4}

65. $\ln\frac{8}{5}$ **67.** $1/\sqrt{2}$ **69.** e^3-1

EXERCISES 11.11 ■ PAGE 798

1. (a) $T_0(x) = 1 = T_1(x)$, $T_2(x) = 1 - \frac{1}{2}x^2 = T_3(x)$, $T_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 = T_5(x)$, $T_6(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6$

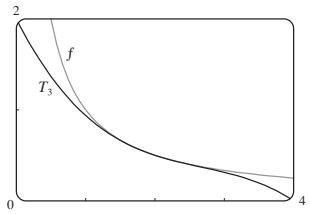


(b)

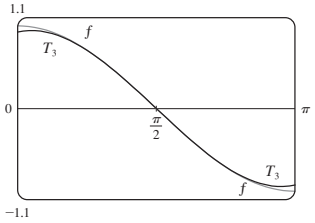
x	f	$T_0 = T_1$	$T_2 = T_3$	$T_4 = T_5$	T_6
$\frac{\pi}{4}$	0.7071	1	0.6916	0.7074	0.7071
$\frac{\pi}{2}$	0	1	−0.2337	0.0200	−0.0009
π	−1	1	−3.9348	0.1239	−1.2114

(c) As n increases, $T_n(x)$ is a good approximation to $f(x)$ on a larger and larger interval.

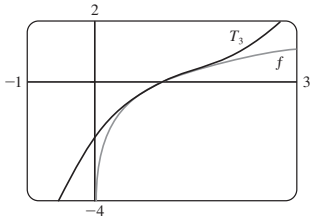
3. $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$



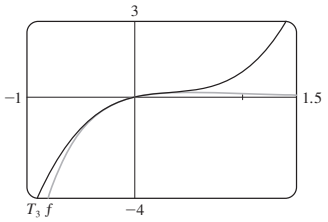
5. $-\left(x - \frac{\pi}{2}\right) + \frac{1}{6}\left(x - \frac{\pi}{2}\right)^3$



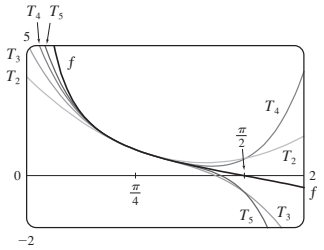
7. $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$



9. $x - 2x^2 + 2x^3$



11. $T_5(x) = 1 - 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 - \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3 + \frac{10}{3}\left(x - \frac{\pi}{4}\right)^4 - \frac{64}{15}\left(x - \frac{\pi}{4}\right)^5$



13. (a) $2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$ (b) 1.5625×10^{-5}

15. (a) $1 + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{4}{81}(x-1)^3$ (b) 0.000097

17. (a) $1 + \frac{1}{2}x^2$ (b) 0.0014

19. (a) $1 + x^2$ (b) 0.00006 **21.** (a) $x^2 - \frac{1}{6}x^4$ (b) 0.042

23. 0.17365 **25.** Four **27.** $-1.037 < x < 1.037$

29. $-0.86 < x < 0.86$ **31.** 21 m, no

37. (c) They differ by about 8×10^{-9} km.

CHAPTER 11 REVIEW ■ PAGE 802

True-False Quiz

1. False **3.** True **5.** False **7.** False **9.** False
11. True **13.** True **15.** False **17.** True
19. True **21.** True

Exercises

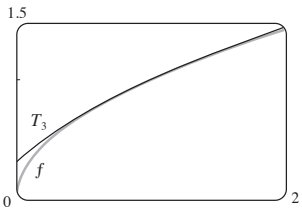
1. $\frac{1}{2}$ **3.** D **5.** 0 **7.** e^{12} **9.** 2 **11.** C **13.** C
15. D **17.** C **19.** C **21.** C **23.** CC **25.** AC
27. $\frac{1}{11}$ **29.** $\pi/4$ **31.** e^{-e} **35.** 0.9721
37. 0.18976224, error $< 6.4 \times 10^{-7}$
41. 4, $[-6, 2)$ **43.** 0.5, [2.5, 3.5)
45. $\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n\left[\frac{1}{(2n)!}\left(x - \frac{\pi}{6}\right)^{2n} + \frac{\sqrt{3}}{(2n+1)!}\left(x - \frac{\pi}{6}\right)^{2n+1}\right]$
47. $\sum_{n=0}^{\infty}(-1)^n x^{n+2}$, $R = 1$ **49.** $\ln 4 - \sum_{n=1}^{\infty}\frac{x^n}{n4^n}$, $R = 4$
51. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{8n+4}}{(2n+1)!}$, $R = \infty$

53. $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{1 \cdot 5 \cdot 9 \cdot \cdots \cdot (4n-3)}{n!\,2^{6n+1}} x^n, R = 16$

55. $C + \ln|x| + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}$

57. (a) $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$

(b)



(c) 0.000006

59. $-\frac{1}{6}$

PROBLEMS PLUS ■ PAGE 805

1. $15!/5! = 10{,}897{,}286{,}400$

3. (b) 0 if $x = 0$, $(1/x) - \cot x$ if $x \neq k\pi$, k an integer

5. (a) $s_n = 3 \cdot 4^n$, $l_n = 1/3^n$, $p_n = 4^n/3^{n-1}$ (c) $\frac{2}{5}\sqrt{3}$

9. $(-1, 1)$, $\dfrac{x^3 + 4x^2 + x}{(1-x)^4}$

11. $\ln \frac{1}{2}$ **13.** (a) $\frac{250}{101}\pi(e^{-(n-1)\pi/5} - e^{-n\pi/5})$ (b) $\frac{250}{101}\pi$

19. $\dfrac{\pi}{2\sqrt{3}} - 1$

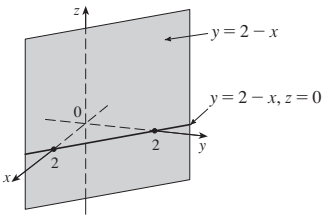
21. $-\left(\dfrac{\pi}{2} - \pi k\right)^2$ where k is a positive integer

CHAPTER 12

EXERCISES 12.1 ■ PAGE 814

1. $(4, 0, -3)$ **3.** C; A

5. A vertical plane that intersects the xy-plane in the line $y = 2 - x$, $z = 0$



7. (a) $|PQ| = 6$, $|QR| = 2\sqrt{10}$, $|RP| = 6$; isosceles triangle

9. (a) No (b) Yes

11. $(x+3)^2 + (y-2)^2 + (z-5)^2 = 16$; $(y-2)^2 + (z-5)^2 = 7, x = 0$ (a circle)

13. $(x-3)^2 + (y-8)^2 + (z-1)^2 = 30$

15. $(1, 2, -4)$, 6 **17.** $(2, 0, -6)$, $9/\sqrt{2}$

19. (b) $\frac{5}{2}, \frac{1}{2}\sqrt{94}, \frac{1}{2}\sqrt{85}$

21. (a) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 36$

(b) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 4$

(c) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 9$

23. A plane parallel to the yz-plane and 5 units in front of it

25. A half-space consisting of all points to the left of the plane $y = 8$

27. All points on or between the horizontal planes $z = 0$ and $z = 6$

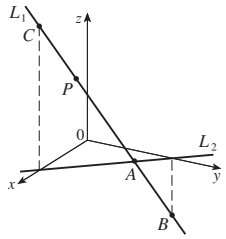
29. All points on a circle with radius 2 with center on the z-axis that is contained in the plane $z = -1$

31. All points on or inside a sphere with radius $\sqrt{3}$ and center O

33. All points on or inside a circular cylinder of radius 3 with axis the y-axis

35. $0 < x < 5$ **37.** $r^2 < x^2 + y^2 + z^2 < R^2$

39. (a) $(2, 1, 4)$ (b)



41. $14x - 6y - 10z = 9$, a plane perpendicular to AB

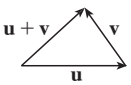
43. $2\sqrt{3} - 3$

EXERCISES 12.2 ■ PAGE 822

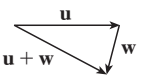
1. (a) Scalar (b) Vector (c) Vector (d) Scalar

3. $\overrightarrow{AB} = \overrightarrow{DC}, \overrightarrow{DA} = \overrightarrow{CB}, \overrightarrow{DE} = \overrightarrow{EB}, \overrightarrow{EA} = \overrightarrow{CE}$

5. (a)

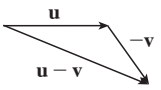


(b)

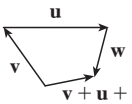


(c)

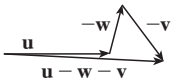
(d)



(e)

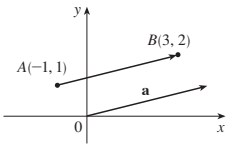


(f)

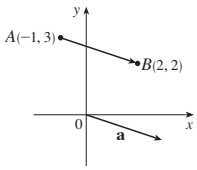


7. $\mathbf{c} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$, $\mathbf{d} = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$

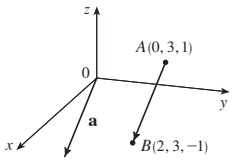
9. $\mathbf{a} = \langle 4, 1 \rangle$



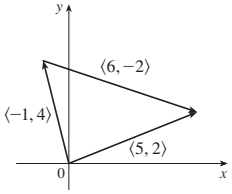
11. $\mathbf{a} = \langle 3, -1 \rangle$



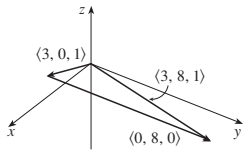
13. $\mathbf{a} = \langle 2, 0, -2 \rangle$



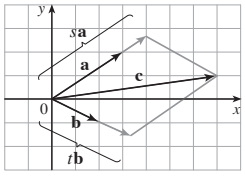
15. $\langle 5, 2 \rangle$



17. $\langle 3, 8, 1 \rangle$



19. $\langle 2, -18 \rangle, \langle 1, -42 \rangle, 13, 10$
21. $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}, -4\mathbf{i} + \mathbf{j} + 9\mathbf{k}, \sqrt{14}, \sqrt{82}$
23. $-\dfrac{3}{\sqrt{58}}\mathbf{i} + \dfrac{7}{\sqrt{58}}\mathbf{j}$ **25.** $\frac{8}{9}\mathbf{i} - \frac{1}{9}\mathbf{j} + \frac{4}{9}\mathbf{k}$ **27.** $60°$
29. $\langle 2, 2\sqrt{3} \rangle$ **31.** ≈ 45.96 ft/s, ≈ 38.57 ft/s
33. $100\sqrt{7} \approx 264.6$ N, $\approx 139.1°$
35. $\sqrt{493} \approx 22.2$ mi/h, N8°W
37. $\mathbf{T}_1 = -196\,\mathbf{i} + 3.92\,\mathbf{j}$, $\mathbf{T}_2 = 196\,\mathbf{i} + 3.92\,\mathbf{j}$
39. (a) At an angle of 43.4° from the bank, toward upstream
(b) 20.2 min
41. $\pm(\mathbf{i} + 4\,\mathbf{j})/\sqrt{17}$ **43.** **0**
45. (a), (b)



(d) $s = \frac{9}{7}, t = \frac{11}{7}$
47. A sphere with radius 1, centered at (x_0, y_0, z_0)

EXERCISES 12.3 ■ PAGE 830

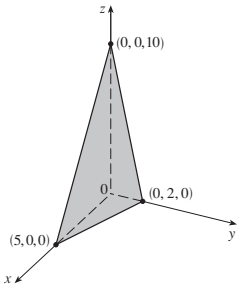
1. (b), (c), (d) are meaningful **3.** 14 **5.** 19 **7.** 1
9. -15 **11.** $\mathbf{u} \cdot \mathbf{v} = \frac{1}{2}, \mathbf{u} \cdot \mathbf{w} = -\frac{1}{2}$
15. $\cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right) \approx 63°$ **17.** $\cos^{-1}\left(\dfrac{5}{\sqrt{1015}}\right) \approx 81°$
19. $\cos^{-1}\left(\dfrac{7}{\sqrt{130}}\right) \approx 52°$ **21.** 48°, 75°, 57°
23. (a) Neither (b) Orthogonal
(c) Orthogonal (d) Parallel
25. Yes **27.** $(\mathbf{i} - \mathbf{j} - \mathbf{k})/\sqrt{3}$ $\left[\text{or } (-\mathbf{i} + \mathbf{j} + \mathbf{k})/\sqrt{3}\right]$
29. 45° **31.** 0° at (0, 0), 8.1° at (1, 1)
33. $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}$; 48°, 71°, 48°
35. $1/\sqrt{14}, -2/\sqrt{14}, -3/\sqrt{14}$; 74°, 122°, 143°
37. $1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}$; 55°, 55°, 55° **39.** $4, \left\langle -\frac{20}{13}, \frac{48}{13} \right\rangle$
41. $\frac{9}{7}, \left\langle \frac{27}{49}, \frac{54}{49}, -\frac{18}{49} \right\rangle$ **43.** $1/\sqrt{21}, \frac{2}{21}\mathbf{i} - \frac{1}{21}\mathbf{j} + \frac{4}{21}\mathbf{k}$
47. $\langle 0, 0, -2\sqrt{10} \rangle$ or any vector of the form
$\langle s, t, 3s - 2\sqrt{10} \rangle, s, t \in \mathbb{R}$
49. 144 J **51.** 2400 cos(40°) ≈ 1839 ft-lb
53. $\frac{13}{5}$ **55.** $\cos^{-1}(1/\sqrt{3}) \approx 55°$

EXERCISES 12.4 ■ PAGE 838

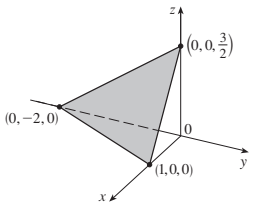
1. $16\,\mathbf{i} + 48\,\mathbf{k}$ **3.** $15\,\mathbf{i} - 3\,\mathbf{j} + 3\,\mathbf{k}$ **5.** $\frac{1}{2}\mathbf{i} - \mathbf{j} + \frac{3}{2}\mathbf{k}$
7. $(1 - t)\,\mathbf{i} + (t^3 - t^2)\,\mathbf{k}$ **9.** **0** **11.** $\mathbf{i} + \mathbf{j} + \mathbf{k}$
13. (a) Scalar (b) Meaningless (c) Vector
(d) Meaningless (e) Meaningless (f) Scalar
15. $96\sqrt{3}$; into the page **17.** $\langle -7, 10, 8 \rangle, \langle 7, -10, -8 \rangle$
19. $\left\langle -\dfrac{1}{3\sqrt{3}}, -\dfrac{1}{3\sqrt{3}}, \dfrac{5}{3\sqrt{3}} \right\rangle, \left\langle \dfrac{1}{3\sqrt{3}}, \dfrac{1}{3\sqrt{3}}, -\dfrac{5}{3\sqrt{3}} \right\rangle$
27. 16 **29.** (a) $\langle 0, 18, -9 \rangle$ (b) $\frac{9}{2}\sqrt{5}$
31. (a) $\langle 13, -14, 5 \rangle$ (b) $\frac{1}{2}\sqrt{390}$
33. 9 **35.** 16 **39.** $10.8 \sin 80° \approx 10.6$ N·m
41. ≈ 417 N **43.** 60°
45. (b) $\sqrt{97/3}$ **53.** (a) No (b) No (c) Yes

EXERCISES 12.5 ■ PAGE 848

1. (a) True (b) False (c) True (d) False (e) False
(f) True (g) False (h) True (i) True (j) False
(k) True
3. $\mathbf{r} = (2\,\mathbf{i} + 2.4\,\mathbf{j} + 3.5\,\mathbf{k}) + t(3\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k})$;
$x = 2 + 3t, y = 2.4 + 2t, z = 3.5 - t$
5. $\mathbf{r} = (\mathbf{i} + 6\mathbf{k}) + t(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$;
$x = 1 + t, y = 3t, z = 6 + t$
7. $x = 2 + 2t, y = 1 + \frac{1}{2}t, z = -3 - 4t$;
$(x - 2)/2 = 2y - 2 = (z + 3)/(-4)$
9. $x = -8 + 11t, y = 1 - 3t, z = 4$; $\dfrac{x + 8}{11} = \dfrac{y - 1}{-3}, z = 4$
11. $x = 1 + t, y = -1 + 2t, z = 1 + t$;
$x - 1 = (y + 1)/2 = z - 1$
13. Yes
15. (a) $(x - 1)/(-1) = (y + 5)/2 = (z - 6)/(-3)$
(b) $(-1, -1, 0), \left(-\frac{3}{2}, 0, -\frac{3}{2}\right), (0, -3, 3)$
17. $\mathbf{r}(t) = (2\mathbf{i} - \mathbf{j} + 4\mathbf{k}) + t(2\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}), 0 \leqslant t \leqslant 1$
19. Skew **21.** $(4, -1, -5)$ **23.** $x - 2y + 5z = 0$
25. $x + 4y + z = 4$ **27.** $5x - y - z = 7$
29. $6x + 6y + 6z = 11$ **31.** $x + y + z = 2$
33. $-13x + 17y + 7z = -42$ **35.** $33x + 10y + 4z = 190$
37. $x - 2y + 4z = -1$ **39.** $3x - 8y - z = -38$
41.



43.



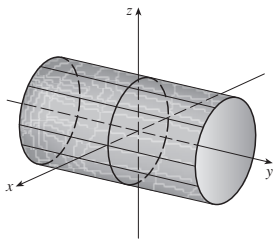
45. (2, 3, 5) **47.** (2, 3, 1) **49.** 1, 0, −1
51. Perpendicular **53.** Neither, $\cos^{-1}\left(\frac{1}{3}\right) \approx 70.5°$
55. Parallel
57. (a) $x = 1, y = -t, z = t$ (b) $\cos^{-1}\left(\dfrac{5}{3\sqrt{3}}\right) \approx 15.8°$
59. $x = 1, y - 2 = -z$ **61.** $x + 2y + z = 5$
63. $(x/a) + (y/b) + (z/c) = 1$
65. $x = 3t, y = 1 - t, z = 2 - 2t$
67. P_2 and P_3 are parallel, P_1 and P_4 are identical

69. $\sqrt{61/14}$ **71.** $\frac{18}{7}$ **73.** $5/(2\sqrt{14})$ **77.** $1/\sqrt{6}$
79. $13/\sqrt{69}$

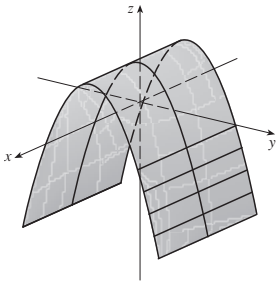
EXERCISES 12.6 ■ PAGE 856

1. (a) Parabola
(b) Parabolic cylinder with rulings parallel to the z-axis
(c) Parabolic cylinder with rulings parallel to the x-axis

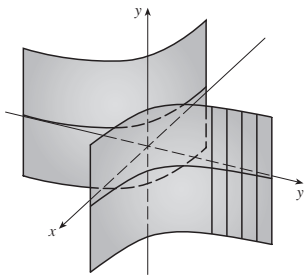
3. Circular cylinder



5. Parabolic cylinder

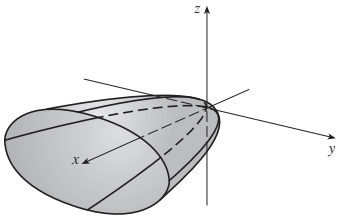


7. Hyperbolic cylinder

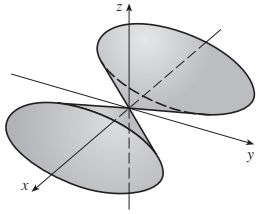


9. (a) $x = k, y^2 - z^2 = 1 - k^2$, hyperbola ($k \neq \pm 1$);
$y = k, x^2 - z^2 = 1 - k^2$, hyperbola ($k \neq \pm 1$);
$z = k, x^2 + y^2 = 1 + k^2$, circle
(b) The hyperboloid is rotated so that it has axis the y-axis
(c) The hyperboloid is shifted one unit in the negative y-direction

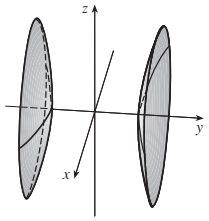
11. Elliptic paraboloid with axis the x-axis



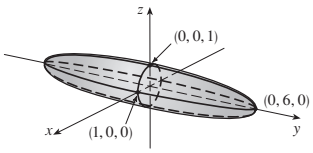
13. Elliptic cone with axis the x-axis



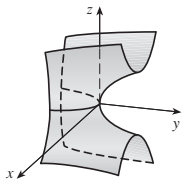
15. Hyperboloid of two sheets



17. Ellipsoid



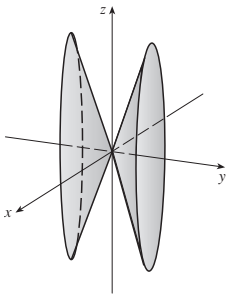
19. Hyperbolic paraboloid



21. VII **23.** II **25.** VI **27.** VIII

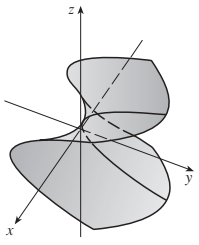
29. $y^2 = x^2 + \dfrac{z^2}{9}$

Elliptic cone with axis the y-axis



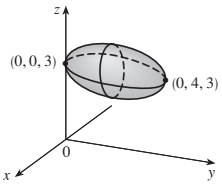
31. $y = z^2 - \dfrac{x^2}{2}$

Hyperbolic paraboloid

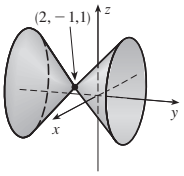


33. $x^2 + \dfrac{(y-2)^2}{4} + (z-3)^2 = 1$

Ellipsoid with center $(0, 2, 3)$



35. $(y+1)^2=(x-2)^2+(z-1)^2$
Circular cone with vertex $(2,-1,1)$ and axis parallel to the y-axis



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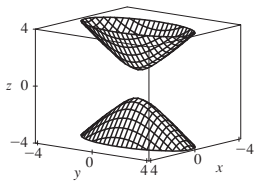

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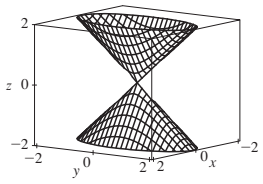

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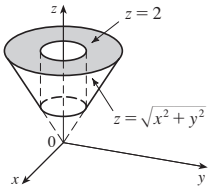
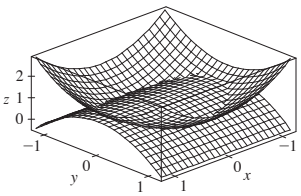


43. $y=x^2+z^2$ **45.** $-4x=y^2+z^2$, paraboloid

47. (a) $\dfrac{x^2}{(6378.137)^2}+\dfrac{y^2}{(6378.137)^2}+\dfrac{z^2}{(6356.523)^2}=1$
(b) Circle (c) Ellipse

51.



CHAPTER 12 REVIEW ■ PAGE 858

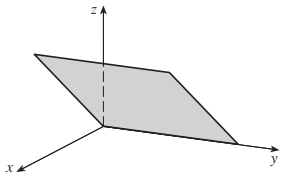
True-False Quiz

1. False **3.** False **5.** True **7.** True **9.** True
11. True **13.** True **15.** False **17.** False
19. False **21.** True

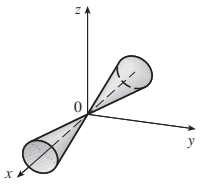
Exercises

1. (a) $(x+1)^2+(y-2)^2+(z-1)^2=69$
(b) $(y-2)^2+(z-1)^2=68,\ x=0$
(c) Center $(4,-1,-3)$, radius 5
3. $\mathbf{u}\cdot\mathbf{v}=3\sqrt{2}$; $|\mathbf{u}\times\mathbf{v}|=3\sqrt{2}$; out of the page
5. $-2,-4$ **7.** (a) 2 (b) -2 (c) -2 (d) 0
9. $\cos^{-1}\left(\frac{1}{3}\right)\approx 71°$ **11.** (a) $\langle 4,-3,4\rangle$ (b) $\sqrt{41}/2$
13. 166 N, 114 N
15. $x=4-3t,\ y=-1+2t,\ z=2+3t$
17. $x=-2+2t,\ y=2-t,\ z=4+5t$
19. $-4x+3y+z=-14$ **21.** $(1,4,4)$ **23.** Skew
25. $x+y+z=4$ **27.** $22/\sqrt{26}$

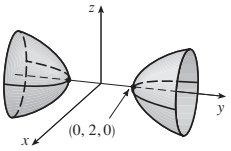
29. Plane



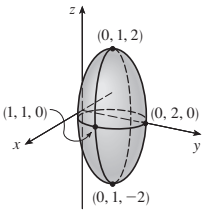
31. Cone



33. Hyperboloid of two sheets



35. Ellipsoid



37. $4x^2+y^2+z^2=16$

PROBLEMS PLUS ■ PAGE 861

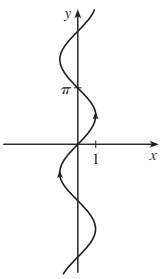
1. $\left(\sqrt{3}-\frac{3}{2}\right)$ m
3. (a) $(x+1)/(-2c)=(y-c)/(c^2-1)=(z-c)/(c^2+1)$
(b) $x^2+y^2=t^2+1,\ z=t$ (c) $4\pi/3$
5. 20

CHAPTER 13

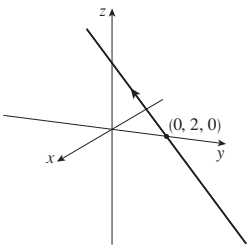
EXERCISES 13.1 ■ PAGE 869

1. $(-1,2]$ **3.** $\mathbf{i}+\mathbf{j}+\mathbf{k}$ **5.** $\langle -1,\pi/2,0\rangle$

7.



9.



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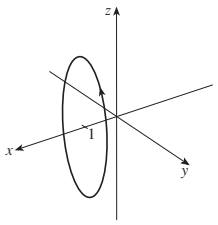

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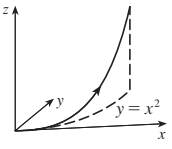

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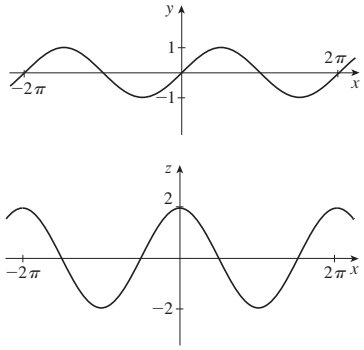
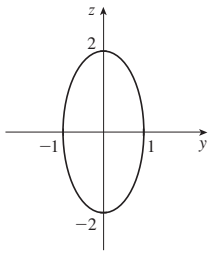
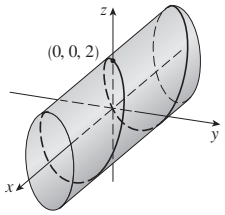




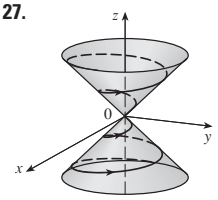


17. $\mathbf{r}(t) = \langle 2 + 4t, 2t, -2t \rangle, 0 \leqslant t \leqslant 1$;
$x = 2 + 4t, y = 2t, z = -2t, 0 \leqslant t \leqslant 1$

19. $\mathbf{r}(t) = \left\langle \frac{1}{2}t, -1 + \frac{4}{3}t, 1 - \frac{3}{4}t \right\rangle, 0 \leqslant t \leqslant 1$;
$x = \frac{1}{2}t, y = -1 + \frac{4}{3}t, z = 1 - \frac{3}{4}t, 0 \leqslant t \leqslant 1$

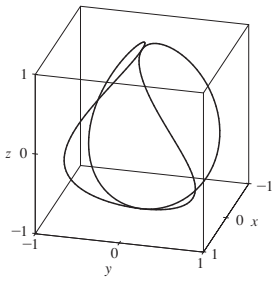
21. II **23.** V **25.** IV

27.

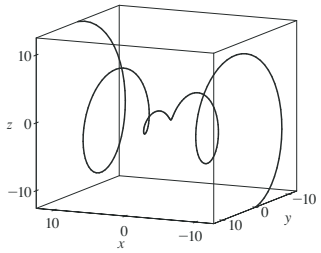


29. (0, 0, 0), (1, 0, 1)

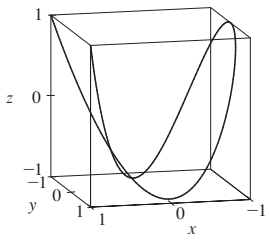
31.



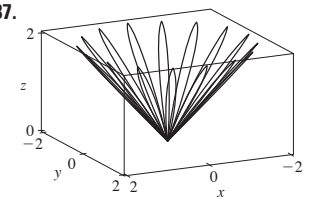
33.



35.



37.



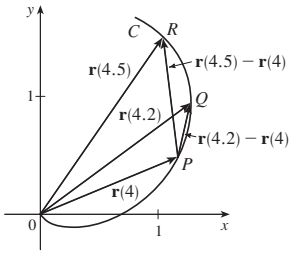
41. $\mathbf{r}(t) = t\,\mathbf{i} + \frac{1}{2}(t^2 - 1)\,\mathbf{j} + \frac{1}{2}(t^2 + 1)\,\mathbf{k}$

43. $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \cos 2t\,\mathbf{k}, 0 \leqslant t \leqslant 2\pi$

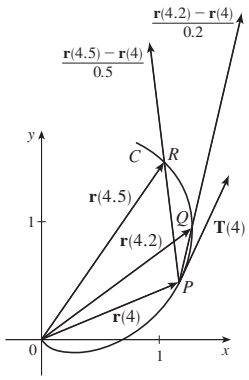
45. $x = 2\cos t, y = 2\sin t, z = 4\cos^2 t$ **47.** Yes

EXERCISES 13.2 ■ PAGE 876

1. (a)

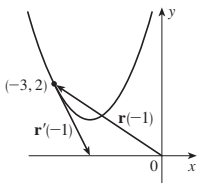


(b), (d)



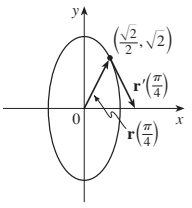
(c) $\mathbf{r}'(4) = \lim_{h \to 0} \dfrac{\mathbf{r}(4 + h) - \mathbf{r}(4)}{h}$; $\mathbf{T}(4) = \dfrac{\mathbf{r}'(4)}{|\mathbf{r}'(4)|}$

3. (a), (c)



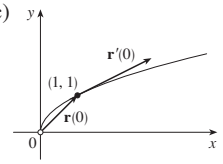
(b) $\mathbf{r}'(t) = \langle 1, 2t \rangle$

5. (a), (c)



(b) $\mathbf{r}'(t) = \cos t\,\mathbf{i} - 2\sin t\,\mathbf{j}$

7. (a), (c)



(b) $\mathbf{r}'(t) = 2e^{2t}\,\mathbf{i} + e^{t}\,\mathbf{j}$

9. $\mathbf{r}'(t) = \langle t\cos t + \sin t,\ 2t,\ \cos 2t - 2t\sin 2t\rangle$

11. $\mathbf{r}'(t) = \mathbf{i} + \left(1/\sqrt{t}\,\right)\mathbf{k}$

13. $\mathbf{r}'(t) = 2te^{t^2}\,\mathbf{i} + [3/(1+3t)]\,\mathbf{k}$ **15.** $\mathbf{r}'(t) = \mathbf{b} + 2t\mathbf{c}$

17. $\left\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right\rangle$ **19.** $\frac{3}{5}\,\mathbf{j} + \frac{4}{5}\,\mathbf{k}$

21. $\langle 1, 2t, 3t^2\rangle, \left\langle 1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14}\right\rangle, \langle 0, 2, 6t\rangle, \langle 6t^2, -6t, 2\rangle$

23. $x = 3 + t, y = 2t, z = 2 + 4t$

25. $x = 1 - t, y = t, z = 1 - t$

27. $\mathbf{r}(t) = (3 - 4t)\,\mathbf{i} + (4 + 3t)\,\mathbf{j} + (2 - 6t)\,\mathbf{k}$

29. $x = t, y = 1 - t, z = 2t$

31. $x = -\pi - t, y = \pi + t, z = -\pi t$

33. 66° **35.** $2\,\mathbf{i} - 4\,\mathbf{j} + 32\,\mathbf{k}$ **37.** $\mathbf{i} + \mathbf{j} + \mathbf{k}$

39. $\tan t\,\mathbf{i} + \frac{1}{8}(t^2+1)^4\,\mathbf{j} + \left(\frac{1}{3}t^3\ln t - \frac{1}{9}t^3\right)\mathbf{k} + \mathbf{C}$

41. $t^2\,\mathbf{i} + t^3\,\mathbf{j} + \left(\frac{2}{3}t^{3/2} - \frac{2}{3}\right)\mathbf{k}$

47. $2t\cos t + 2\sin t - 2\cos t\sin t$ **49.** 35

EXERCISES 13.3 ■ PAGE 884

1. $10\sqrt{10}$ **3.** $e - e^{-1}$ **5.** $\frac{1}{27}(13^{3/2} - 8)$ **7.** 18.6833

9. 1.2780 **11.** 42

13. $\mathbf{r}(t(s)) = \dfrac{2}{\sqrt{29}}s\,\mathbf{i} + \left(1 - \dfrac{3}{\sqrt{29}}s\right)\mathbf{j} + \left(5 + \dfrac{4}{\sqrt{29}}s\right)\mathbf{k}$

15. $(3\sin 1, 4, 3\cos 1)$

17. (a) $\left\langle 1/\sqrt{10}, \left(-3/\sqrt{10}\right)\sin t, \left(3/\sqrt{10}\right)\cos t\right\rangle$, $\langle 0, -\cos t, -\sin t\rangle$ (b) $\frac{3}{10}$

19. (a) $\dfrac{1}{e^{2t}+1}\left\langle \sqrt{2}e^{t}, e^{2t}, -1\right\rangle, \dfrac{1}{e^{2t}+1}\left\langle 1 - e^{2t}, \sqrt{2}e^{t}, \sqrt{2}e^{t}\right\rangle$

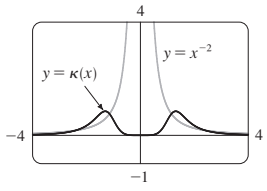
(b) $\sqrt{2}e^{2t}/(e^{2t}+1)^2$

21. $6t^2/(9t^4 + 4t^2)^{3/2}$ **23.** $\frac{4}{25}$ **25.** $\frac{1}{7}\sqrt{\frac{19}{14}}$

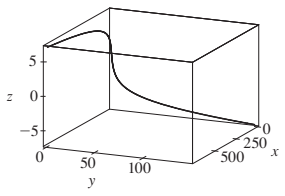
27. $12x^2/(1 + 16x^6)^{3/2}$ **29.** $e^x|x+2|/[1 + (xe^x + e^x)^2]^{3/2}$

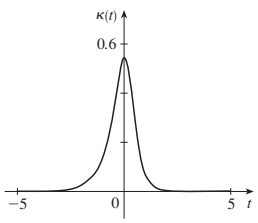
31. $\left(-\frac{1}{2}\ln 2, 1/\sqrt{2}\right)$; approaches 0 **33.** (a) P (b) 1.3, 0.7

35.



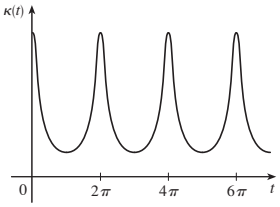
37.





39. a is $y = f(x)$, b is $y = \kappa(x)$

41. $\kappa(t) = \dfrac{6\sqrt{4\cos^2 t - 12\cos t + 13}}{(17 - 12\cos t)^{3/2}}$



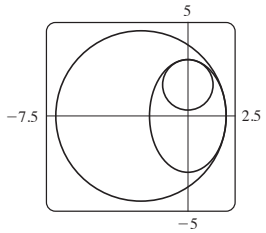
integer multiples of 2π

43. $6t^2/(4t^2 + 9t^4)^{3/2}$

45. $1/\left(\sqrt{2}e^{t}\right)$ **47.** $\left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle, \left\langle -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right\rangle, \left\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle$

49. $y = 6x + \pi, x + 6y = 6\pi$

51. $\left(x + \frac{5}{2}\right)^2 + y^2 = \frac{81}{4}, x^2 + \left(y - \frac{5}{3}\right)^2 = \frac{16}{9}$



53. $(-1, -3, 1)$

55. $2x + y + 4z = 7, 6x - 8y - z = -3$

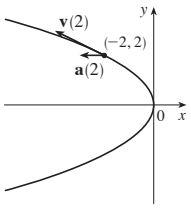
63. $2/(t^4 + 4t^2 + 1)$ **65.** 2.07×10^{10} Å ≈ 2 m

EXERCISES 13.4 ■ PAGE 894

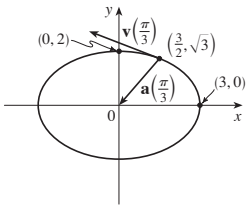
1. (a) $1.8\,\mathbf{i} - 3.8\,\mathbf{j} - 0.7\,\mathbf{k}, 2.0\,\mathbf{i} - 2.4\,\mathbf{j} - 0.6\,\mathbf{k}$, $2.8\,\mathbf{i} + 1.8\,\mathbf{j} - 0.3\,\mathbf{k}, 2.8\,\mathbf{i} + 0.8\,\mathbf{j} - 0.4\,\mathbf{k}$

(b) $2.4\,\mathbf{i} - 0.8\,\mathbf{j} - 0.5\,\mathbf{k}, 2.58$

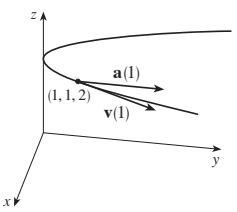
3. $\mathbf{v}(t) = \langle -t, 1\rangle$
$\mathbf{a}(t) = \langle -1, 0\rangle$
$|\,\mathbf{v}(t)\,| = \sqrt{t^2+1}$



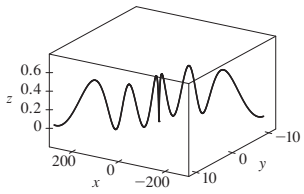
5. $\mathbf{v}(t) = -3\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j}$
$\mathbf{a}(t) = -3\cos t\,\mathbf{i} - 2\sin t\,\mathbf{j}$
$|\,\mathbf{v}(t)\,| = \sqrt{5\sin^2 t + 4}$



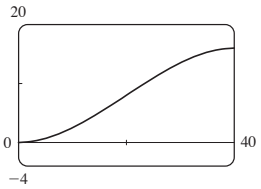
7. $\mathbf{v}(t) = \mathbf{i} + 2t\,\mathbf{j}$
$\mathbf{a}(t) = 2\,\mathbf{j}$
$|\,\mathbf{v}(t)\,| = \sqrt{1 + 4t^2}$

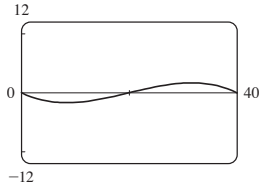


9. $\langle 2t+1, 2t-1, 3t^2\rangle$, $\langle 2, 2, 6t\rangle$, $\sqrt{9t^4+8t^2+2}$
11. $\sqrt{2}\,\mathbf{i} + e^t\,\mathbf{j} - e^{-t}\,\mathbf{k}$, $e^t\,\mathbf{j} + e^{-t}\,\mathbf{k}$, $e^t + e^{-t}$
13. $e^t[(\cos t - \sin t)\mathbf{i} + (\sin t + \cos t)\mathbf{j} + (t+1)\mathbf{k}]$,
$e^t[-2\sin t\,\mathbf{i} + 2\cos t\,\mathbf{j} + (t+2)\mathbf{k}]$, $e^t\sqrt{t^2+2t+3}$
15. $\mathbf{v}(t) = t\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{k}$, $\mathbf{r}(t) = \left(\frac{1}{2}t^2+1\right)\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k}$
17. (a) $\mathbf{r}(t) = \left(\frac{1}{3}t^3 + t\right)\mathbf{i} + (t - \sin t + 1)\,\mathbf{j} + \left(\frac{1}{4} - \frac{1}{4}\cos 2t\right)\mathbf{k}$
(b)



19. $t = 4$ **21.** $\mathbf{r}(t) = t\,\mathbf{i} - t\,\mathbf{j} + \frac{5}{2}t^2\,\mathbf{k}$, $|\mathbf{v}(t)| = \sqrt{25t^2+2}$
23. (a) ≈3535 m (b) ≈1531 m (c) 200 m/s
25. 30 m/s **27.** ≈10.2°, ≈79.8°
29. $13.0° < \theta < 36.0°$, $55.4° < \theta < 85.5°$
31. $(250, -50, 0)$; $10\sqrt{93} \approx 96.4$ ft/s
33. (a) 16 m (b) ≈23.6° upstream





35. The path is contained in a circle that lies in a plane perpendicular to $\mathbf{c}$ with center on a line through the origin in the direction of $\mathbf{c}$.
37. $6t, 6$ **39.** $0, 1$ **41.** $e^t - e^{-t}, \sqrt{2}$
43. 4.5 cm/s², 9.0 cm/s² **45.** $t = 1$

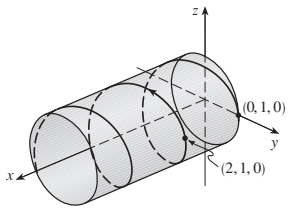
CHAPTER 13 REVIEW ■ PAGE 897

True-False Quiz

1. True **3.** False **5.** False **7.** False
9. True **11.** False **13.** True

Exercises

1. (a)



(b) $\mathbf{r}'(t) = \mathbf{i} - \pi\sin\pi t\,\mathbf{j} + \pi\cos\pi t\,\mathbf{k}$,
$\mathbf{r}''(t) = -\pi^2\cos\pi t\,\mathbf{j} - \pi^2\sin\pi t\,\mathbf{k}$
3. $\mathbf{r}(t) = 4\cos t\,\mathbf{i} + 4\sin t\,\mathbf{j} + (5 - 4\cos t)\mathbf{k}$, $0 \leqslant t \leqslant 2\pi$
5. $\frac{1}{3}\mathbf{i} - (2/\pi^2)\mathbf{j} + (2/\pi)\mathbf{k}$ **7.** 86.631 **9.** $\pi/2$
11. (a) $\langle t^2, t, 1\rangle/\sqrt{t^4+t^2+1}$
(b) $\langle t^3+2t, 1-t^4, -2t^3-t\rangle/\sqrt{t^8+5t^6+6t^4+5t^2+1}$
(c) $\sqrt{t^8+5t^6+6t^4+5t^2+1}/(t^4+t^2+1)^2$
13. $12/17^{3/2}$ **15.** $x - 2y + 2\pi = 0$
17. $\mathbf{v}(t) = (1+\ln t)\,\mathbf{i} + \mathbf{j} - e^{-t}\,\mathbf{k}$,
$|\mathbf{v}(t)| = \sqrt{2 + 2\ln t + (\ln t)^2 + e^{-2t}}$, $\mathbf{a}(t) = (1/t)\mathbf{i} + e^{-t}\mathbf{k}$
19. (a) About 3.8 ft above the ground, 60.8 ft from the athlete
(b) ≈21.4 ft (c) ≈64.2 ft from the athlete
21. (c) $-2e^{-t}\,\mathbf{v}_d + e^{-t}\,\mathbf{R}$
23. (a) $\mathbf{v} = \omega R(-\sin\omega t\,\mathbf{i} + \cos\omega t\,\mathbf{j})$ (c) $\mathbf{a} = -\omega^2\mathbf{r}$

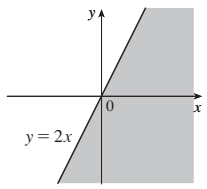
PROBLEMS PLUS ■ PAGE 900

1. (a) $90°$, $v_0^2/(2g)$
3. (a) ≈0.94 ft to the right of the table's edge, ≈15 ft/s
(b) ≈7.6° (c) ≈2.13 ft to the right of the table's edge
5. 56°
7. $\mathbf{r}(u, v) = \mathbf{c} + u\,\mathbf{a} + v\,\mathbf{b}$ where $\mathbf{a} = \langle a_1, a_2, a_3\rangle$,
$\mathbf{b} = \langle b_1, b_2, b_3\rangle$, $\mathbf{c} = \langle c_1, c_2, c_3\rangle$

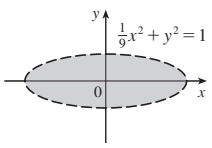
CHAPTER 14

EXERCISES 14.1 ■ PAGE 912

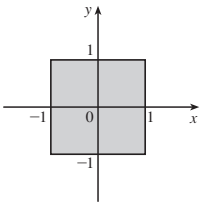
1. (a) −27; a temperature of −15°C with wind blowing at 40 km/h feels equivalent to about −27°C without wind.
(b) When the temperature is −20°C, what wind speed gives a wind chill of −30°C? 20 km/h
(c) With a wind speed of 20 km/h, what temperature gives a wind chill of −49°C? −35°C
(d) A function of wind speed that gives wind-chill values when the temperature is −5°C
(e) A function of temperature that gives wind-chill values when the wind speed is 50 km/h
3. ≈94.2; the manufacturer's yearly production is valued at $94.2 million when 120,000 labor hours are spent and $20 million in capital is invested.
5. (a) ≈20.5; the surface area of a person 70 inches tall who weighs 160 pounds is approximately 20.5 square feet.
7. (a) 25; a 40-knot wind blowing in the open sea for 15 h will create waves about 25 ft high.
(b) $f(30, t)$ is a function of t giving the wave heights produced by 30-knot winds blowing for t hours.
(c) $f(v, 30)$ is a function of v giving the wave heights produced by winds of speed v blowing for 30 hours.
9. (a) 1 (b) $\mathbb{R}^2$ (c) $[-1, 1]$
11. (a) 3 (b) $\{(x, y, z) \mid x^2 + y^2 + z^2 < 4, x \geqslant 0, y \geqslant 0, z \geqslant 0\}$, interior of a sphere of radius 2, center the origin, in the first octant
13. $\{(x, y) \mid y \leqslant 2x\}$



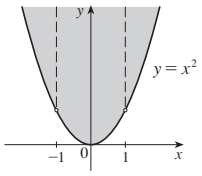
15. $\left\{(x, y) \mid \frac{1}{9}x^2 + y^2 < 1\right\}$, $(-\infty, \ln 9]$



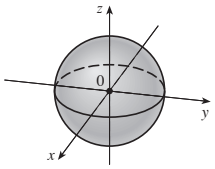
17. $\{(x, y) \mid -1 \leqslant x \leqslant 1, -1 \leqslant y \leqslant 1\}$



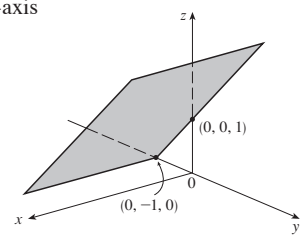
19. $\{(x, y) \mid y \geqslant x^2, x \neq \pm 1\}$



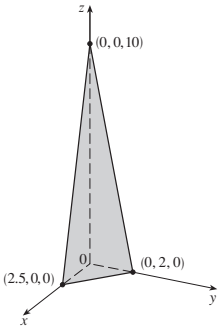
21. $\{(x, y, z) \mid x^2 + y^2 + z^2 \leqslant 1\}$



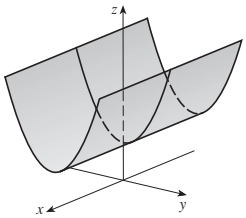
23. $z = 1 + y$, plane parallel to x-axis



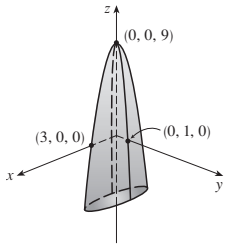
25. $4x + 5y + z = 10$, plane



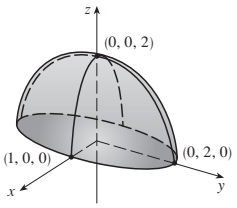
27. $z = y^2 + 1$, parabolic cylinder



29. $z = 9 - x^2 - 9y^2$, elliptic paraboloid

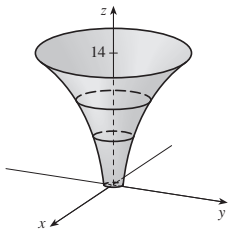


31. $z = \sqrt{4 - 4x^2 - y^2}$, top half of ellipsoid

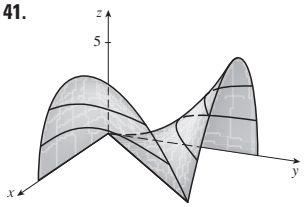


33. $\approx 56, \approx 35$ **35.** 11°C, 19.5°C **37.** Steep; nearly flat

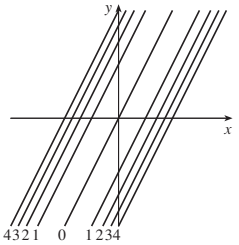
39.



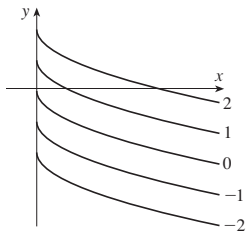
41.



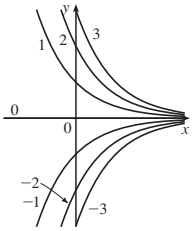
43. $(y - 2x)^2 = k$



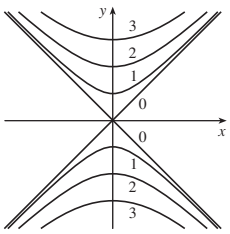
45. $y = -\sqrt{x} + k$



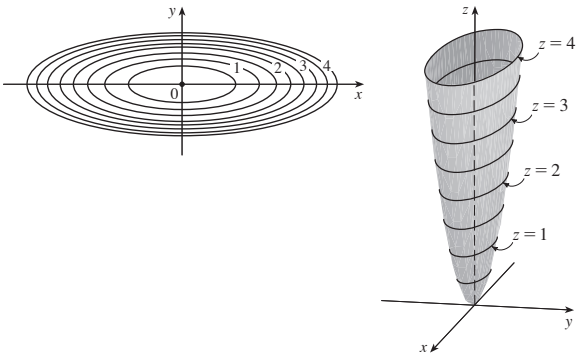
47. $y = ke^{-x}$



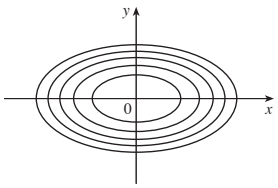
49. $y^2 - x^2 = k^2$



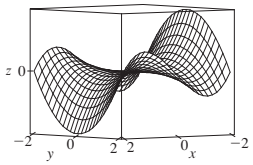
51. $x^2 + 9y^2 = k$



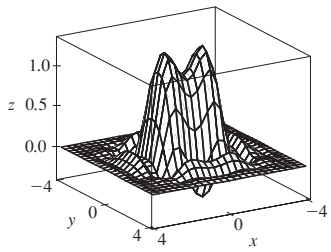
53.



55.



57.



59. (a) C (b) II **61.** (a) F (b) I

63. (a) B (b) VI **65.** Family of parallel planes

67. Family of circular cylinders with axis the x-axis ($k > 0$)

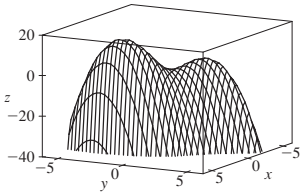
69. (a) Shift the graph of f upward 2 units

(b) Stretch the graph of f vertically by a factor of 2

(c) Reflect the graph of f about the xy-plane

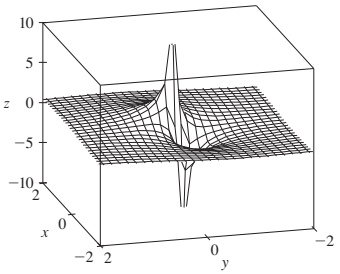
(d) Reflect the graph of f about the xy-plane and then shift it upward 2 units

71.



f appears to have a maximum value of about 15. There are two local maximum points but no local minimum point.

73.



The function values approach 0 as x, y become large; as (x, y) approaches the origin, f approaches $\pm\infty$ or 0, depending on the direction of approach.

75. If $c = 0$, the graph is a cylindrical surface. For $c > 0$, the level curves are ellipses. The graph curves upward as we leave the origin, and the steepness increases as c increases. For $c < 0$, the level curves are hyperbolas. The graph curves upward in the y-direction and downward, approaching the xy-plane, in the x-direction giving a saddle-shaped appearance near $(0, 0, 1)$.

77. $c = -2, 0, 2$ **79.** (b) $y = 0.75x + 0.01$

EXERCISES 14.2 ■ PAGE 923

1. Nothing; if f is continuous, $f(3, 1) = 6$ **3.** $-\frac{5}{2}$

5. 1 **7.** $\frac{2}{7}$ **9.** Does not exist **11.** Does not exist

13. 0 **15.** Does not exist **17.** 2

19. $\sqrt{3}$ **21.** Does not exist

23. The graph shows that the function approaches different numbers along different lines.

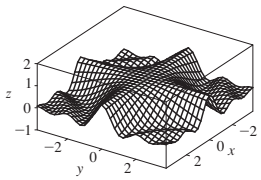
25. $h(x, y) = (2x + 3y - 6)^2 + \sqrt{2x + 3y - 6}$; $\{(x, y) \mid 2x + 3y \geqslant 6\}$

27. Along the line $y = x$ **29.** $\mathbb{R}^2$ **31.** $\{(x, y) \mid x^2 + y^2 \neq 1\}$

33. $\{(x, y) \mid x^2 + y^2 > 4\}$ **35.** $\{(x, y, z) \mid x^2 + y^2 + z^2 \leqslant 1\}$

37. $\{(x, y) \mid (x, y) \neq (0, 0)\}$ **39.** 0 **41.** -1

43.



f is continuous on $\mathbb{R}^2$

EXERCISES 14.3 ■ PAGE 935

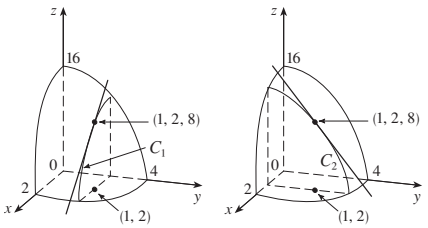
1. (a) The rate of change of temperature as longitude varies, with latitude and time fixed; the rate of change as only latitude varies; the rate of change as only time varies.

(b) Positive, negative, positive

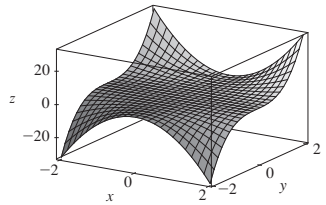
3. (a) $f_T(-15, 30) \approx 1.3$; for a temperature of -15°C and wind speed of 30 km/h, the wind-chill index rises by 1.3°C for each degree the temperature increases. $f_v(-15, 30) \approx -0.15$; for a temperature of -15°C and wind speed of 30 km/h, the wind-chill index decreases by 0.15°C for each km/h the wind speed increases.

(b) Positive, negative (c) 0

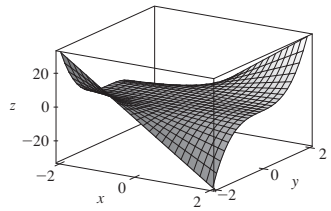
5. (a) Positive (b) Negative
7. (a) Positive (b) Negative
9. $c = f, b = f_x, a = f_y$
11. $f_x(1, 2) = -8$ = slope of C_1, $f_y(1, 2) = -4$ = slope of C_2



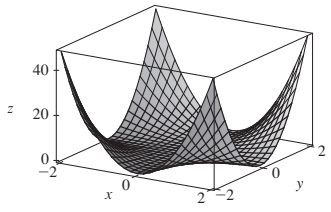
13.



$f(x, y) = x^2y^3$

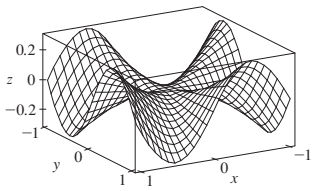


$f_x(x, y) = 2xy^3$



$f_y(x, y) = 3x^2y^2$

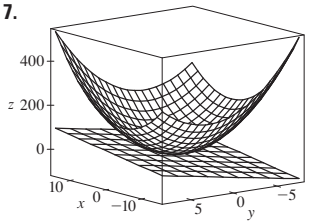
15. $f_x(x, y) = -3y$, $f_y(x, y) = 5y^4 - 3x$
17. $f_x(x, t) = -\pi e^{-t} \sin \pi x$, $f_t(x, t) = -e^{-t} \cos \pi x$
19. $\partial z/\partial x = 20(2x + 3y)^9$, $\partial z/\partial y = 30(2x + 3y)^9$
21. $f_x(x, y) = 1/y$, $f_y(x, y) = -x/y^2$
23. $f_x(x, y) = \dfrac{(ad - bc)y}{(cx + dy)^2}$, $f_y(x, y) = \dfrac{(bc - ad)x}{(cx + dy)^2}$
25. $g_u(u, v) = 10uv(u^2v - v^3)^4$, $g_v(u, v) = 5(u^2 - 3v^2)(u^2v - v^3)^4$
27. $R_p(p, q) = \dfrac{q^2}{1 + p^2q^4}$, $R_q(p, q) = \dfrac{2pq}{1 + p^2q^4}$
29. $F_x(x, y) = \cos(e^x)$, $F_y(x, y) = -\cos(e^y)$
31. $f_x = z - 10xy^3z^4$, $f_y = -15x^2y^2z^4$, $f_z = x - 20x^2y^3z^3$
33. $\partial w/\partial x = 1/(x + 2y + 3z)$, $\partial w/\partial y = 2/(x + 2y + 3z)$, $\partial w/\partial z = 3/(x + 2y + 3z)$
35. $\partial u/\partial x = y \sin^{-1}(yz)$, $\partial u/\partial y = x \sin^{-1}(yz) + xyz/\sqrt{1 - y^2z^2}$, $\partial u/\partial z = xy^2/\sqrt{1 - y^2z^2}$
37. $h_x = 2xy \cos(z/t)$, $h_y = x^2 \cos(z/t)$, $h_z = (-x^2y/t) \sin(z/t)$, $h_t = (x^2yz/t^2) \sin(z/t)$
39. $\partial u/\partial x_i = x_i/\sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$
41. $\frac{1}{5}$ **43.** $\frac{1}{4}$ **45.** $f_x(x, y) = y^2 - 3x^2y$, $f_y(x, y) = 2xy - x^3$
47. $\dfrac{\partial z}{\partial x} = -\dfrac{x}{3z}, \dfrac{\partial z}{\partial y} = -\dfrac{2y}{3z}$
49. $\dfrac{\partial z}{\partial x} = \dfrac{yz}{e^z - xy}, \dfrac{\partial z}{\partial y} = \dfrac{xz}{e^z - xy}$
51. (a) $f'(x), g'(y)$ (b) $f'(x + y), f'(x + y)$
53. $f_{xx} = 6xy^5 + 24x^2y$, $f_{xy} = 15x^2y^4 + 8x^3 = f_{yx}$, $f_{yy} = 20x^3y^3$
55. $w_{uu} = v^2/(u^2 + v^2)^{3/2}$, $w_{uv} = -uv/(u^2 + v^2)^{3/2} = w_{vu}$, $w_{vv} = u^2/(u^2 + v^2)^{3/2}$
57. $z_{xx} = -2x/(1 + x^2)^2$, $z_{xy} = 0 = z_{yx}$, $z_{yy} = -2y/(1 + y^2)^2$
63. $24xy^2 - 6y, 24x^2y - 6x$ **65.** $(2x^2y^2z^5 + 6xyz^3 + 2z)e^{xyz^2}$
67. $\theta e^{r\theta}(2 \sin \theta + \theta \cos \theta + r\theta \sin \theta)$ **69.** $4/(y + 2z)^3, 0$
71. $6yz^2$ **73.** $\approx 12.2, \approx 16.8, \approx 23.25$ **83.** R^2/R_1^2
87. $\dfrac{\partial T}{\partial P} = \dfrac{V - nb}{nR}, \dfrac{\partial P}{\partial V} = \dfrac{2n^2a}{V^3} - \dfrac{nRT}{(V - nb)^2}$
93. No **95.** $x = 1 + t, y = 2, z = 2 - 2t$ **99.** -2
101. (a)



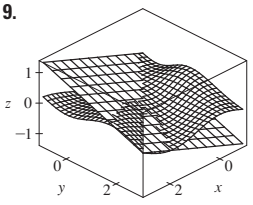
(b) $f_x(x, y) = \dfrac{x^4y + 4x^2y^3 - y^5}{(x^2 + y^2)^2}$, $f_y(x, y) = \dfrac{x^5 - 4x^3y^2 - xy^4}{(x^2 + y^2)^2}$
(c) 0, 0 (e) No, since f_{xy} and f_{yx} are not continuous.

EXERCISES 14.4 ■ PAGE 946

1. $z = -7x - 6y + 5$ **3.** $x + y - 2z = 0$
5. $x + y + z = 0$
7.



9.



11. $6x + 4y - 23$ **13.** $\frac{1}{9}x - \frac{2}{9}y + \frac{2}{3}$ **15.** $1 - \pi y$
19. 6.3 **21.** $\frac{3}{7}x + \frac{2}{7}y + \frac{6}{7}z$; 6.9914
23. $4T + H - 329$; 129°F
25. $dz = -2e^{-2x} \cos 2\pi t\, dx - 2\pi e^{-2x} \sin 2\pi t\, dt$
27. $dm = 5p^4q^3\, dp + 3p^5q^2\, dq$
29. $dR = \beta^2 \cos \gamma\, d\alpha + 2\alpha\beta \cos \gamma\, d\beta - \alpha\beta^2 \sin \gamma\, d\gamma$
31. $\Delta z = 0.9225$, $dz = 0.9$ **33.** 5.4 cm^2 **35.** 16 cm^3
37. $\approx -0.0165mg$; decrease
39. $\frac{1}{17} \approx 0.059\ \Omega$ **41.** 2.3% **43.** $\varepsilon_1 = \Delta x, \varepsilon_2 = \Delta y$

EXERCISES 14.5 ■ PAGE 954

1. $(2x + y)\cos t + (2y + x)e^t$

3. $[(x/t) - y\sin t]/\sqrt{1 + x^2 + y^2}$

5. $e^{y/z}[2t - (x/z) - (2xy/z^2)]$

7. $\partial z/\partial s = 2xy^3\cos t + 3x^2y^2\sin t$,
$\partial z/\partial t = -2sxy^3\sin t + 3sx^2y^2\cos t$

9. $\partial z/\partial s = t^2\cos\theta\cos\phi - 2st\sin\theta\sin\phi$,
$\partial z/\partial t = 2st\cos\theta\cos\phi - s^2\sin\theta\sin\phi$

11. $\dfrac{\partial z}{\partial s} = e^r\left(t\cos\theta - \dfrac{s}{\sqrt{s^2+t^2}}\sin\theta\right)$,
$\dfrac{\partial z}{\partial t} = e^r\left(s\cos\theta - \dfrac{t}{\sqrt{s^2+t^2}}\sin\theta\right)$

13. 62 **15.** 7, 2

17. $\dfrac{\partial u}{\partial r} = \dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial r} + \dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial r}$, $\dfrac{\partial u}{\partial s} = \dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial s} + \dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial s}$,
$\dfrac{\partial u}{\partial t} = \dfrac{\partial u}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial u}{\partial y}\dfrac{\partial y}{\partial t}$

19. $\dfrac{\partial w}{\partial x} = \dfrac{\partial w}{\partial r}\dfrac{\partial r}{\partial x} + \dfrac{\partial w}{\partial s}\dfrac{\partial s}{\partial x} + \dfrac{\partial w}{\partial t}\dfrac{\partial t}{\partial x}$,
$\dfrac{\partial w}{\partial y} = \dfrac{\partial w}{\partial r}\dfrac{\partial r}{\partial y} + \dfrac{\partial w}{\partial s}\dfrac{\partial s}{\partial y} + \dfrac{\partial w}{\partial t}\dfrac{\partial t}{\partial y}$

21. 1582, 3164, −700 **23.** $2\pi, -2\pi$

25. $\frac{5}{144}, -\frac{5}{96}, \frac{5}{144}$ **27.** $\dfrac{2x + y\sin x}{\cos x - 2y}$

29. $\dfrac{1 + x^4y^2 + y^2 + x^4y^4 - 2xy}{x^2 - 2xy - 2x^5y^3}$

31. $-\dfrac{x}{3z}, -\dfrac{2y}{3z}$ **33.** $\dfrac{yz}{e^z - xy}, \dfrac{xz}{e^z - xy}$

35. 2°C/s **37.** ≈ −0.33 m/s per minute

39. (a) 6 m³/s (b) 10 m²/s (c) 0 m/s

41. ≈ −0.27 L/s **43.** $-1/(12\sqrt{3})$ rad/s

45. (a) $\partial z/\partial r = (\partial z/\partial x)\cos\theta + (\partial z/\partial y)\sin\theta$,
$\partial z/\partial\theta = -(\partial z/\partial x)r\sin\theta + (\partial z/\partial y)r\cos\theta$

51. $4rs\,\partial^2z/\partial x^2 + (4r^2 + 4s^2)\partial^2z/\partial x\,\partial y + 4rs\,\partial^2z/\partial y^2 + 2\,\partial z/\partial y$

EXERCISES 14.6 ■ PAGE 967

1. ≈ −0.08 mb/km **3.** ≈ 0.778 **5.** $2 + \sqrt{3}/2$

7. (a) $\nabla f(x, y) = \langle 2\cos(2x + 3y), 3\cos(2x + 3y)\rangle$
(b) $\langle 2, 3\rangle$ (c) $\sqrt{3} - \frac{3}{2}$

9. (a) $\langle 2xyz - yz^3, x^2z - xz^3, x^2y - 3xyz^2\rangle$
(b) $\langle -3, 2, 2\rangle$ (c) $\frac{2}{5}$

11. $\dfrac{4 - 3\sqrt{3}}{10}$ **13.** $-8/\sqrt{10}$ **15.** $4/\sqrt{30}$

17. $\frac{23}{42}$ **19.** $2/5$ **21.** $\sqrt{65}, \langle 1, 8\rangle$

23. $1, \langle 0, 1\rangle$ **25.** $1, \langle 3, 6, -2\rangle$

27. (b) $\langle -12, 92\rangle$

29. All points on the line $y = x + 1$

31. (a) $-40/(3\sqrt{3})$

33. (a) $32/\sqrt{3}$ (b) $\langle 38, 6, 12\rangle$ (c) $2\sqrt{406}$

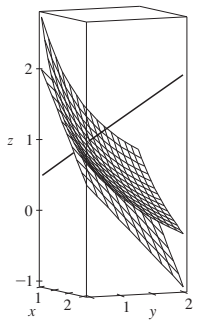
35. $\frac{327}{13}$ **39.** $\frac{774}{25}$

41. (a) $x + y + z = 11$ (b) $x - 3 = y - 3 = z - 5$

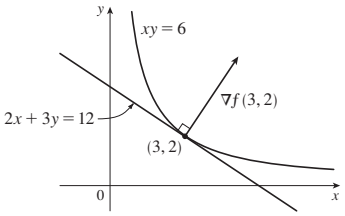
43. (a) $2x + 3y + 12z = 24$ (b) $\dfrac{x-3}{2} = \dfrac{y-2}{3} = \dfrac{z-1}{12}$

45. (a) $x + y + z = 1$ (b) $x = y = z - 1$

47.



49. $\langle 2, 3\rangle, 2x + 3y = 12$



55. No **59.** $\left(-\frac{5}{4}, -\frac{5}{4}, \frac{25}{8}\right)$

63. $x = -1 - 10t, y = 1 - 16t, z = 2 - 12t$

67. If $\mathbf{u} = \langle a, b\rangle$ and $\mathbf{v} = \langle c, d\rangle$, then $af_x + bf_y$ and $cf_x + df_y$ are known, so we solve linear equations for f_x and f_y.

EXERCISES 14.7 ■ PAGE 977

1. (a) f has a local minimum at (1, 1).
(b) f has a saddle point at (1, 1).

3. Local minimum at (1, 1), saddle point at (0, 0)

5. Minimum $f\left(\frac{1}{3}, -\frac{2}{3}\right) = -\frac{1}{3}$

7. Saddle points at (1, 1), (−1, −1)

9. Maximum $f(0, 0) = 2$, minimum $f(0, 4) = -30$,
saddle points at (2, 2), (−2, 2)

11. Minimum $f(2, 1) = -8$, saddle point at (0, 0)

13. None **15.** Minimum $f(0, 0) = 0$, saddle points at $(\pm 1, 0)$

17. Minima $f(0, 1) = f(\pi, -1) = f(2\pi, 1) = -1$,
saddle points at $(\pi/2, 0), (3\pi/2, 0)$

21. Minima $f(1, \pm 1) = 3, f(-1, \pm 1) = 3$

23. Maximum $f(\pi/3, \pi/3) = 3\sqrt{3}/2$,
minimum $f(5\pi/3, 5\pi/3) = -3\sqrt{3}/2$, saddle point at (π, π)

25. Minima $f(0, -0.794) \approx -1.191, f(\pm 1.592, 1.267) \approx -1.310$,
saddle points $(\pm 0.720, 0.259)$,
lowest points $(\pm 1.592, 1.267, -1.310)$

27. Maximum $f(0.170, -1.215) \approx 3.197$,
minima $f(-1.301, 0.549) \approx -3.145, f(1.131, 0.549) \approx -0.701$,
saddle points $(-1.301, -1.215), (0.170, 0.549), (1.131, -1.215)$,
no highest or lowest point

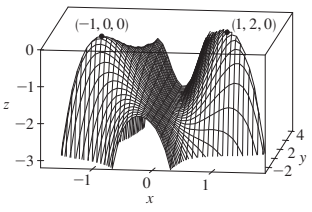
29. Maximum $f(0, \pm 2) = 4$, minimum $f(1, 0) = -1$

31. Maximum $f(\pm 1, 1) = 7$, minimum $f(0, 0) = 4$

33. Maximum $f(3, 0) = 83$, minimum $f(1, 1) = 0$

35. Maximum $f(1, 0) = 2$, minimum $f(-1, 0) = -2$

37.



39. $2/\sqrt{3}$ **41.** $(2, 1, \sqrt{5}), (2, 1, -\sqrt{5})$ **43.** $\frac{100}{3}, \frac{100}{3}, \frac{100}{3}$
45. $8r^3/(3\sqrt{3})$ **47.** $\frac{4}{3}$ **49.** Cube, edge length $c/12$
51. Square base of side 40 cm, height 20 cm **53.** $L^3/(3\sqrt{3})$

EXERCISES 14.8 ■ PAGE 987

1. $\approx 59, 30$
3. No maximum, minimum $f(1, 1) = f(-1, -1) = 2$
5. Maximum $f(0, \pm 1) = 1$, minimum $f(\pm 2, 0) = -4$
7. Maximum $f(2, 2, 1) = 9$, minimum $f(-2, -2, -1) = -9$
9. Maximum $2/\sqrt{3}$, minimum $-2/\sqrt{3}$
11. Maximum $\sqrt{3}$, minimum 1
13. Maximum $f\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) = 2$,
minimum $f\left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right) = -2$
15. Maximum $f(1, \sqrt{2}, -\sqrt{2}) = 1 + 2\sqrt{2}$,
minimum $f(1, -\sqrt{2}, \sqrt{2}) = 1 - 2\sqrt{2}$
17. Maximum $\frac{3}{2}$, minimum $\frac{1}{2}$
19. Maximum $f(3/\sqrt{2}, -3/\sqrt{2}) = 9 + 12\sqrt{2}$,
minimum $f(-2, 2) = -8$
21. Maximum $f(\pm 1/\sqrt{2}, \mp 1/(2\sqrt{2})) = e^{1/4}$,
minimum $f(\pm 1/\sqrt{2}, \pm 1/(2\sqrt{2})) = e^{-1/4}$
29–41. See Exercises 39–53 in Section 14.7.
43. Nearest $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$, farthest $(-1, -1, 2)$
45. Maximum ≈ 9.7938, minimum ≈ -5.3506
47. (a) c/n (b) When $x_1 = x_2 = \cdots = x_n$

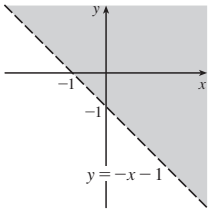
CHAPTER 14 REVIEW ■ PAGE 991

True-False Quiz

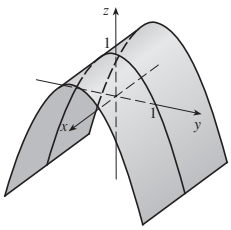
1. True **3.** False **5.** False **7.** True **9.** False
11. True

Exercises

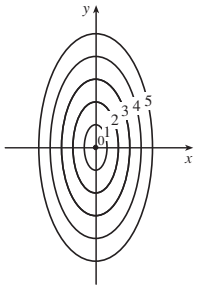
1. $\{(x, y) \mid y > -x - 1\}$



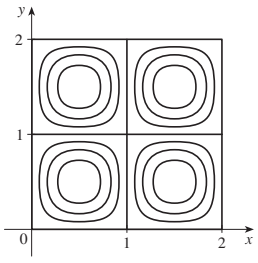
3.



5.



7.



9. $\frac{2}{3}$
11. (a) $\approx 3.5°\text{C/m}, -3.0°\text{C/m}$ (b) $\approx 0.35°\text{C/m}$ by Equation 14.6.9 (Definition 14.6.2 gives $\approx 1.1°\text{C/m}$.)
(c) -0.25
13. $f_x = 32xy(5y^3 + 2x^2y)^7, f_y = (16x^2 + 120y^2)(5y^3 + 2x^2y)^7$
15. $F_\alpha = \dfrac{2\alpha^3}{\alpha^2 + \beta^2} + 2\alpha \ln(\alpha^2 + \beta^2), F_\beta = \dfrac{2\alpha^2\beta}{\alpha^2 + \beta^2}$
17. $S_u = \arctan\left(v\sqrt{w}\right), S_v = \dfrac{u\sqrt{w}}{1 + v^2w}, S_w = \dfrac{uv}{2\sqrt{w}(1 + v^2w)}$
19. $f_{xx} = 24x, f_{xy} = -2y = f_{yx}, f_{yy} = -2x$
21. $f_{xx} = k(k-1)x^{k-2}y^lz^m, f_{xy} = klx^{k-1}y^{l-1}z^m = f_{yx}$,
$f_{xz} = kmx^{k-1}y^lz^{m-1} = f_{zx}, f_{yy} = l(l-1)x^ky^{l-2}z^m$,
$f_{yz} = lmx^ky^{l-1}z^{m-1} = f_{zy}, f_{zz} = m(m-1)x^ky^lz^{m-2}$
25. (a) $z = 8x + 4y + 1$ (b) $\dfrac{x-1}{8} = \dfrac{y+2}{4} = \dfrac{z-1}{-1}$
27. (a) $2x - 2y - 3z = 3$ (b) $\dfrac{x-2}{4} = \dfrac{y+1}{-4} = \dfrac{z-1}{-6}$
29. (a) $x + 2y + 5z = 0$
(b) $x = 2 + t, y = -1 + 2t, z = 5t$
31. $\left(2, \frac{1}{2}, -1\right), \left(-2, -\frac{1}{2}, 1\right)$
33. $60x + \frac{24}{5}y + \frac{32}{5}z - 120$; 38.656
35. $2xy^3(1 + 6p) + 3x^2y^2(pe^p + e^p) + 4z^3(p\cos p + \sin p)$
37. $-47, 108$
43. $\langle 2xe^{yz^2}, x^2z^2e^{yz^2}, 2x^2yze^{yz^2}\rangle$ **45.** $-\frac{4}{5}$
47. $\sqrt{145}/2, \langle 4, \frac{9}{2}\rangle$ **49.** $\approx \frac{5}{8}$ knot/mi
51. Minimum $f(-4, 1) = -11$
53. Maximum $f(1, 1) = 1$; saddle points $(0, 0), (0, 3), (3, 0)$
55. Maximum $f(1, 2) = 4$, minimum $f(2, 4) = -64$
57. Maximum $f(-1, 0) = 2$, minima $f(1, \pm 1) = -3$,
saddle points $(-1, \pm 1), (1, 0)$
59. Maximum $f\left(\pm\sqrt{2/3}, 1/\sqrt{3}\right) = 2/(3\sqrt{3})$,
minimum $f\left(\pm\sqrt{2/3}, -1/\sqrt{3}\right) = -2/(3\sqrt{3})$
61. Maximum 1, minimum -1
63. $\left(\pm 3^{-1/4}, 3^{-1/4}\sqrt{2}, \pm 3^{1/4}\right), \left(\pm 3^{-1/4}, -3^{-1/4}\sqrt{2}, \pm 3^{1/4}\right)$
65. $P(2 - \sqrt{3}), P(3 - \sqrt{3})/6, P(2\sqrt{3} - 3)/3$

PROBLEMS PLUS ■ PAGE 995

1. $L^2W^2, \frac{1}{4}L^2W^2$ **3.** (a) $x = w/3$, base $= w/3$ (b) Yes
7. $\sqrt{3/2}, 3/\sqrt{2}$

CHAPTER 15

EXERCISES 15.1 ■ PAGE 1005

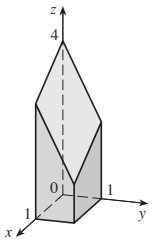
1. (a) 288 (b) 144 **3.** (a) 0.990 (b) 1.151
5. (a) 4 (b) -8 **7.** $U < V < L$
9. (a) ≈ 248 (b) ≈ 15.5 **11.** 60 **13.** 3
15. 1.141606, 1.143191, 1.143535, 1.143617, 1.143637, 1.143642

EXERCISES 15.2 ■ PAGE 1011

1. $500y^3, 3x^2$ **3.** 222 **5.** $32(e^4 - 1)$ **7.** 18
9. $\frac{21}{2}\ln 2$ **11.** $\frac{31}{30}$ **13.** π **15.** 0

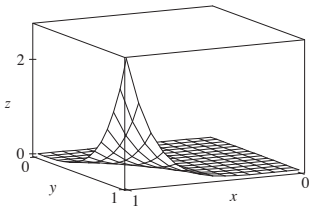
17. $9 \ln 2$ **19.** $\frac{1}{2}(\sqrt{3}-1)-\frac{1}{12}\pi$ **21.** $\frac{1}{2}e^{-6}+\frac{5}{2}$

23.



25. 51 **27.** $\frac{166}{27}$ **29.** 2 **31.** $\frac{64}{3}$

33. $21e-57$



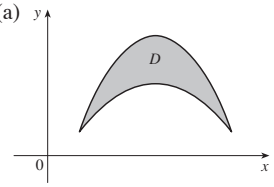
35. $\frac{5}{6}$ **37.** 0

39. Fubini's Theorem does not apply. The integrand has an infinite discontinuity at the origin.

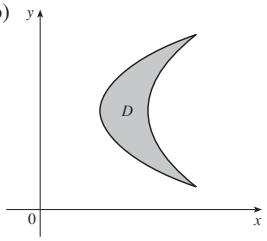
EXERCISES 15.3 ■ PAGE 1019

1. 32 **3.** $\frac{3}{10}$ **5.** $\frac{1}{3}\sin 1$ **7.** $\frac{4}{3}$ **9.** π

11. (a)



(b)



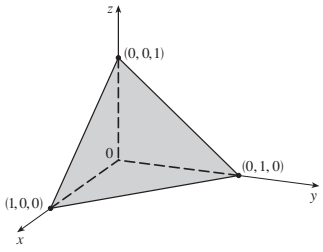
13. Type I: $D=\{(x,y)\mid 0\le x\le 1, 0\le y\le x\}$, type II: $D=\{(x,y)\mid 0\le y\le 1, y\le x\le 1\}$; $\frac{1}{3}$

15. $\int_0^1\int_{-\sqrt{x}}^{\sqrt{x}} y\,dy\,dx+\int_1^4\int_{x-2}^{\sqrt{x}} y\,dy\,dx=\int_{-1}^{2}\int_{y^2}^{y+2} y\,dx\,dy=\frac{9}{4}$

17. $\frac{1}{2}(1-\cos 1)$ **19.** $\frac{11}{3}$ **21.** 0 **23.** $\frac{17}{60}$ **25.** $\frac{31}{8}$

27. 6 **29.** $\frac{128}{15}$ **31.** $\frac{1}{3}$ **33.** 0, 1.213; 0.713 **35.** $\frac{64}{3}$

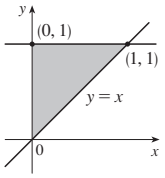
37.



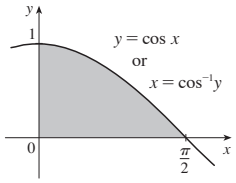
39. 13,984,735,616/14,549,535

41. $\pi/2$

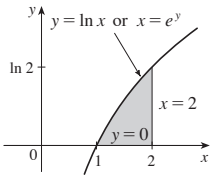
43. $\int_0^1\int_x^1 f(x,y)\,dy\,dx$



45. $\int_0^1\int_0^{\cos^{-1}y} f(x,y)\,dx\,dy$



47. $\int_0^{\ln 2}\int_{e^y}^2 f(x,y)\,dx\,dy$



49. $\frac{1}{6}(e^9-1)$ **51.** $\frac{1}{3}\ln 9$ **53.** $\frac{1}{3}(2\sqrt{2}-1)$ **55.** 1

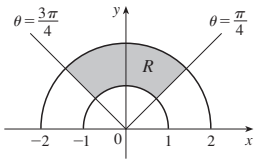
57. $(\pi/16)e^{-1/16}\le\iint_Q e^{-(x^2+y^2)^2}\,dA\le\pi/16$ **59.** $\frac{3}{4}$ **63.** 9π

65. $a^2b+\frac{3}{2}ab^2$ **67.** $\pi a^2 b$

EXERCISES 15.4 ■ PAGE 1026

1. $\int_0^{3\pi/2}\int_0^4 f(r\cos\theta, r\sin\theta)r\,dr\,d\theta$ **3.** $\int_{-1}^{1}\int_0^{(x+1)/2} f(x,y)\,dy\,dx$

5. $3\pi/4$



7. $\frac{1250}{3}$ **9.** $(\pi/4)(\cos 1-\cos 9)$

11. $(\pi/2)(1-e^{-4})$ **13.** $\frac{3}{64}\pi^2$ **15.** $\pi/12$

17. $\frac{\pi}{3}+\frac{\sqrt{3}}{2}$ **19.** $\frac{16}{3}\pi$ **21.** $\frac{4}{3}\pi$ **23.** $\frac{4}{3}\pi a^3$

25. $(2\pi/3)\left[1-\left(1/\sqrt{2}\right)\right]$ **27.** $(8\pi/3)\left(64-24\sqrt{3}\right)$

29. $\frac{1}{2}\pi(1-\cos 9)$ **31.** $2\sqrt{2}/3$ **33.** 4.5951

35. $1800\pi\ \text{ft}^3$ **37.** $2/(a+b)$ **39.** $\frac{15}{16}$

41. (a) $\sqrt{\pi}/4$ (b) $\sqrt{\pi}/2$

EXERCISES 15.5 ■ PAGE 1036

1. 285 C **3.** $42k, \left(2, \frac{85}{28}\right)$ **5.** $6, \left(\frac{3}{4}, \frac{3}{2}\right)$ **7.** $\frac{8}{15}k, \left(0, \frac{4}{7}\right)$

9. $L/4, (L/2, 16/(9\pi))$ **11.** $\left(\frac{3}{8}, 3\pi/16\right)$ **13.** $(0, 45/(14\pi))$

15. $(2a/5, 2a/5)$ if vertex is $(0,0)$ and sides are along positive axes

17. $\frac{64}{315}k, \frac{8}{105}k, \frac{88}{315}k$

19. $7ka^6/180, 7ka^6/180, 7ka^6/90$ if vertex is $(0,0)$ and sides are along positive axes

21. $\rho bh^3/3, \rho b^3h/3; b/\sqrt{3}, h/\sqrt{3}$

23. $\rho a^4\pi/16,\ \rho a^4\pi/16;\ a/2,\ a/2$

25. $m = 3\pi/64,\ (\bar{x}, \bar{y}) = \left(\dfrac{16384\sqrt{2}}{10395\pi}, 0\right),$

$I_x = \dfrac{5\pi}{384} - \dfrac{4}{105},\ I_y = \dfrac{5\pi}{384} + \dfrac{4}{105},\ I_0 = \dfrac{5\pi}{192}$

27. (a) $\frac{1}{2}$ (b) 0.375 (c) $\frac{5}{48} \approx 0.1042$

29. (b) (i) $e^{-0.2} \approx 0.8187$

(ii) $1 + e^{-1.8} - e^{-0.8} - e^{-1} \approx 0.3481$ (c) 2, 5

31. (a) ≈ 0.500 (b) ≈ 0.632

33. (a) $\iint_D k\left[1 - \frac{1}{20}\sqrt{(x - x_0)^2 + (y - y_0)^2}\right] dA$, where D is the disk with radius 10 mi centered at the center of the city

(b) $200\pi k/3 \approx 209k$, $200(\pi/2 - \frac{8}{9})k \approx 136k$, on the edge

EXERCISES 15.6 ■ PAGE 1040

1. $15\sqrt{26}$ **3.** $3\sqrt{14}$ **5.** $12\sin^{-1}(\frac{2}{3})$

7. $(\pi/6)(17\sqrt{17} - 5\sqrt{5})$ **9.** $(2\pi/3)(2\sqrt{2} - 1)$

11. $a^2(\pi - 2)$ **13.** 13.9783 **15.** (a) ≈ 1.83 (b) ≈ 1.8616

17. $\frac{45}{8}\sqrt{14} + \frac{15}{16}\ln\left[(11\sqrt{5} + 3\sqrt{70})/(3\sqrt{5} + \sqrt{70})\right]$

19. 3.3213 **23.** $(\pi/6)(101\sqrt{101} - 1)$

EXERCISES 15.7 ■ PAGE 1049

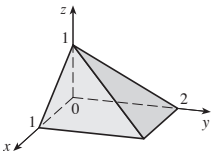
1. $\frac{27}{4}$ **3.** $\frac{16}{15}$ **5.** $\frac{5}{3}$ **7.** $-\frac{1}{3}$ **9.** $\frac{27}{2}$ **11.** $9\pi/8$

13. $\frac{65}{28}$ **15.** $\frac{1}{60}$ **17.** $16\pi/3$ **19.** $\frac{16}{3}$ **21.** $\frac{8}{15}$

23. (a) $\int_0^1 \int_0^x \int_0^{\sqrt{1-y^2}} dz\, dy\, dx$ (b) $\frac{1}{4}\pi - \frac{1}{3}$

25. 0.985

27.



29. $\int_{-2}^{2} \int_0^{4-x^2} \int_{-\sqrt{4-x^2-y}/2}^{\sqrt{4-x^2-y}/2} f(x, y, z)\, dz\, dy\, dx$

$= \int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} \int_{-\sqrt{4-x^2-y}/2}^{\sqrt{4-x^2-y}/2} f(x, y, z)\, dz\, dx\, dy$

$= \int_{-1}^{1} \int_0^{4-4z^2} \int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x, y, z)\, dx\, dy\, dz$

$= \int_0^4 \int_{-\sqrt{4-y}/2}^{\sqrt{4-y}/2} \int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x, y, z)\, dx\, dz\, dy$

$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}/2}^{\sqrt{4-x^2}/2} \int_0^{4-x^2-4z^2} f(x, y, z)\, dy\, dz\, dx$

$= \int_{-1}^{1} \int_{-\sqrt{4-4z^2}}^{\sqrt{4-4z^2}} \int_0^{4-x^2-4z^2} f(x, y, z)\, dy\, dx\, dz$

31. $\int_{-2}^{2} \int_{x^2}^{4} \int_0^{2-y/2} f(x, y, z)\, dz\, dy\, dx$

$= \int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \int_0^{2-y/2} f(x, y, z)\, dz\, dx\, dy$

$= \int_0^2 \int_0^{4-2z} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z)\, dx\, dy\, dz$

$= \int_0^4 \int_0^{2-y/2} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z)\, dx\, dz\, dy$

$= \int_{-2}^{2} \int_0^{2-x^2/2} \int_{x^2}^{4-2z} f(x, y, z)\, dy\, dz\, dx$

$= \int_0^2 \int_{-\sqrt{4-2z}}^{\sqrt{4-2z}} \int_{x^2}^{4-2z} f(x, y, z)\, dy\, dx\, dz$

33. $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z)\, dz\, dy\, dx = \int_0^1 \int_0^{y^2} \int_0^{1-y} f(x, y, z)\, dz\, dx\, dy$

$= \int_0^1 \int_0^{1-z} \int_0^{y^2} f(x, y, z)\, dx\, dy\, dz = \int_0^1 \int_0^{1-y} \int_0^{y^2} f(x, y, z)\, dx\, dz\, dy$

$= \int_0^1 \int_0^{1-\sqrt{x}} \int_{\sqrt{x}}^{1-z} f(x, y, z)\, dy\, dz\, dx = \int_0^1 \int_0^{(1-z)^2} \int_{\sqrt{x}}^{1-z} f(x, y, z)\, dy\, dx\, dz$

35. $\int_0^1 \int_y^1 \int_0^y f(x, y, z)\, dz\, dx\, dy = \int_0^1 \int_0^x \int_0^y f(x, y, z)\, dz\, dy\, dx$

$= \int_0^1 \int_z^1 \int_y^1 f(x, y, z)\, dx\, dy\, dz = \int_0^1 \int_0^y \int_y^1 f(x, y, z)\, dx\, dz\, dy$

$= \int_0^1 \int_0^x \int_z^x f(x, y, z)\, dy\, dz\, dx = \int_0^1 \int_z^1 \int_z^x f(x, y, z)\, dy\, dx\, dz$

37. 64π **39.** $\frac{79}{30}, \left(\frac{358}{553}, \frac{33}{79}, \frac{571}{553}\right)$

41. $a^5,\ (7a/12, 7a/12, 7a/12)$

43. $I_x = I_y = I_z = \frac{2}{3}kL^5$ **45.** $\frac{1}{2}\pi khb^4$

47. (a) $m = \int_{-1}^{1} \int_{x^2}^{1} \int_0^{1-y} \sqrt{x^2 + y^2}\, dz\, dy\, dx$

(b) $(\bar{x}, \bar{y}, \bar{z})$, where

$\bar{x} = (1/m) \int_{-1}^{1} \int_{x^2}^{1} \int_0^{1-y} x\sqrt{x^2 + y^2}\, dz\, dy\, dx$

$\bar{y} = (1/m) \int_{-1}^{1} \int_{x^2}^{1} \int_0^{1-y} y\sqrt{x^2 + y^2}\, dz\, dy\, dx$

$\bar{z} = (1/m) \int_{-1}^{1} \int_{x^2}^{1} \int_0^{1-y} z\sqrt{x^2 + y^2}\, dz\, dy\, dx$

(c) $\int_{-1}^{1} \int_{x^2}^{1} \int_0^{1-y} (x^2 + y^2)^{3/2}\, dz\, dy\, dx$

49. (a) $\frac{3}{32}\pi + \frac{11}{24}$

(b) $\left(\dfrac{28}{9\pi + 44}, \dfrac{30\pi + 128}{45\pi + 220}, \dfrac{45\pi + 208}{135\pi + 660}\right)$

(c) $\frac{1}{240}(68 + 15\pi)$

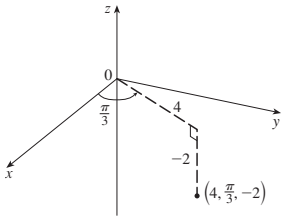
51. (a) $\frac{1}{8}$ (b) $\frac{1}{64}$ (c) $\frac{1}{5760}$ **53.** $L^3/8$

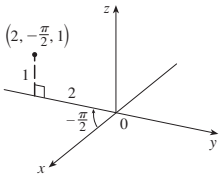
55. (a) The region bounded by the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$

(b) $4\sqrt{6}\pi/45$

EXERCISES 15.8 ■ PAGE 1055

1. (a) (b)





$(2, 2\sqrt{3}, -2)$ $(0, -2, 1)$

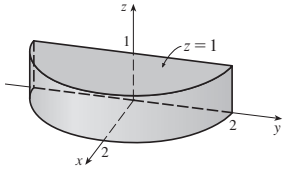
3. (a) $(\sqrt{2}, 3\pi/4, 1)$ (b) $(4, 2\pi/3, 3)$

5. Vertical half-plane through the z-axis

7. Circular paraboloid

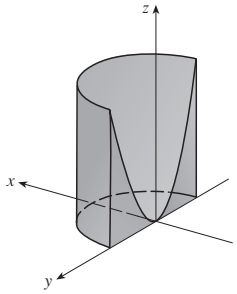
9. (a) $z^2 = 1 + r\cos\theta - r^2$ (b) $z = r^2\cos 2\theta$

11.



13. Cylindrical coordinates: $6 \leqslant r \leqslant 7,\ 0 \leqslant \theta \leqslant 2\pi,\ 0 \leqslant z \leqslant 20$

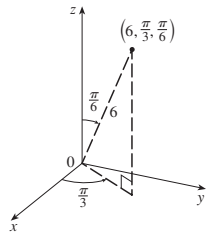
15. 4π

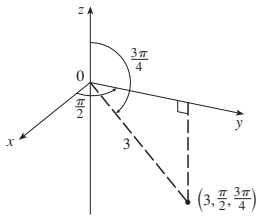


17. 384π **19.** $\frac{8}{3}\pi + \frac{128}{15}$ **21.** $2\pi/5$ **23.** $\frac{4}{3}\pi\left(\sqrt{2} - 1\right)$
25. (a) 162π (b) $(0, 0, 15)$
27. $\pi Ka^2/8, (0, 0, 2a/3)$ **29.** 0
31. (a) $\iiint_C h(P)g(P)\,dV$, where C is the cone
(b) $\approx 3.1 \times 10^{19}$ ft-lb

EXERCISES 15.9 ■ PAGE 1061

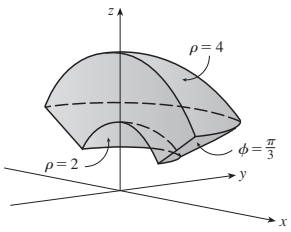
1. (a) (b)



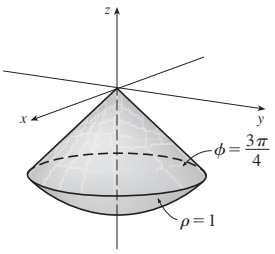


$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}, 3\sqrt{3}\right)$ $\left(0, \frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$

3. (a) $(2, 3\pi/2, \pi/2)$ (b) $(2, 3\pi/4, 3\pi/4)$
5. Half-cone **7.** Sphere, radius $\frac{1}{2}$, center $\left(0, \frac{1}{2}, 0\right)$
9. (a) $\cos^2\phi = \sin^2\phi$ (b) $\rho^2(\sin^2\phi\cos^2\theta + \cos^2\phi) = 9$
11.

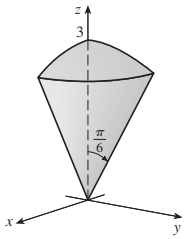


13.

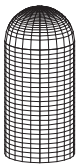


15. $0 \leq \phi \leq \pi/4, 0 \leq \rho \leq \cos\phi$

17. $(9\pi/4)\left(2 - \sqrt{3}\right)$



19. $\int_0^{\pi/2}\int_0^3\int_0^2 f(r\cos\theta, r\sin\theta, z)\,r\,dz\,dr\,d\theta$
21. $312{,}500\pi/7$ **23.** $1688\pi/15$ **25.** $\pi/8$
27. $\left(\sqrt{3} - 1\right)\pi a^3/3$ **29.** (a) 10π (b) $(0, 0, 2.1)$
31. (a) $\left(0, 0, \frac{7}{12}\right)$ (b) $11K\pi/960$
33. (a) $\left(0, 0, \frac{3}{8}a\right)$ (b) $4K\pi a^5/15$
35. $\frac{1}{3}\pi\left(2 - \sqrt{2}\right), \left(0, 0, 3/\left[8\left(2 - \sqrt{2}\right)\right]\right)$
37. $5\pi/6$ **39.** $\left(4\sqrt{2} - 5\right)/15$ **41.** $4096\pi/21$
43.



45. $136\pi/99$

EXERCISES 15.10 ■ PAGE 1071

1. 16 **3.** $\sin^2\theta - \cos^2\theta$ **5.** 0
7. The parallelogram with vertices $(0, 0), (6, 3), (12, 1), (6, -2)$
9. The region bounded by the line $y = 1$, the y-axis, and $y = \sqrt{x}$
11. $x = \frac{1}{3}(v - u), y = \frac{1}{3}(u + 2v)$ is one possible transformation, where $S = \{(u, v) \mid -1 \leq u \leq 1, 1 \leq v \leq 3\}$
13. $x = u\cos v, y = u\sin v$ is one possible transformation, where $S = \left\{(u, v) \mid 1 \leq u \leq \sqrt{2}, 0 \leq v \leq \pi/2\right\}$
15. -3 **17.** 6π **19.** $2\ln 3$
21. (a) $\frac{4}{3}\pi abc$ (b) $1.083 \times 10^{12}\ \text{km}^3$ (c) $\frac{4}{15}\pi(a^2 + b^2)abck$
23. $\frac{8}{5}\ln 8$ **25.** $\frac{3}{2}\sin 1$ **27.** $e - e^{-1}$

CHAPTER 15 REVIEW ■ PAGE 1073

True-False Quiz

1. True **3.** True **5.** True **7.** True **9.** False

Exercises

1. ≈ 64.0 **3.** $4e^2 - 4e + 3$ **5.** $\frac{1}{2}\sin 1$ **7.** $\frac{2}{3}$
9. $\int_0^{\pi}\int_2^4 f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$
11. The region inside the loop of the four-leaved rose $r = \sin 2\theta$ in the first quadrant
13. $\frac{1}{2}\sin 1$ **15.** $\frac{1}{2}e^6 - \frac{7}{2}$ **17.** $\frac{1}{4}\ln 2$ **19.** 8
21. $81\pi/5$ **23.** $\frac{81}{2}$ **25.** $\pi/96$ **27.** $\frac{64}{15}$
29. 176 **31.** $\frac{2}{3}$ **33.** $2ma^3/9$
35. (a) $\frac{1}{4}$ (b) $\left(\frac{1}{3}, \frac{8}{15}\right)$
(c) $I_x = \frac{1}{12}, I_y = \frac{1}{24}; \bar{\bar{y}} = 1/\sqrt{3}, \bar{\bar{x}} = 1/\sqrt{6}$
37. (a) $(0, 0, h/4)$ (b) $\pi a^4 h/10$
39. $\ln\left(\sqrt{2} + \sqrt{3}\right) + \sqrt{2}/3$ **41.** $\frac{486}{5}$ **43.** 0.0512

45. (a) $\frac{1}{15}$ (b) $\frac{1}{3}$ (c) $\frac{1}{45}$

47. $\int_0^1 \int_0^{1-z} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z)\, dx\, dy\, dz$ **49.** $-\ln 2$ **51.** 0

PROBLEMS PLUS ■ PAGE 1077

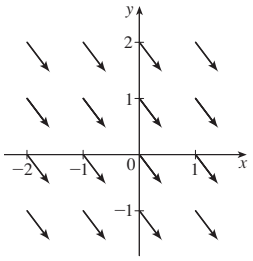
1. 30 **3.** $\frac{1}{2}\sin 1$ **7.** (b) 0.90

13. $abc\pi\left(\frac{2}{3} - \frac{8}{9\sqrt{3}}\right)$

CHAPTER 16

EXERCISES 16.1 ■ PAGE 1085

1.



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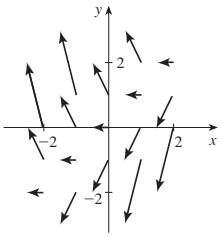

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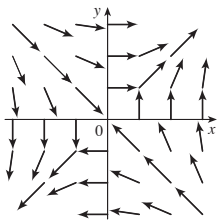

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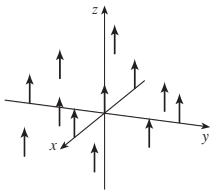

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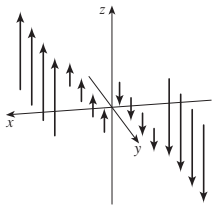
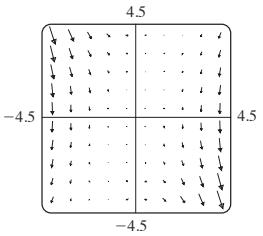


11. IV **13.** I **15.** IV **17.** III

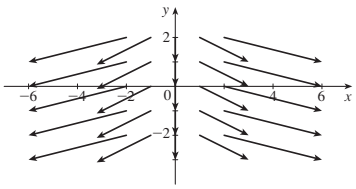
19. The line $y = 2x$



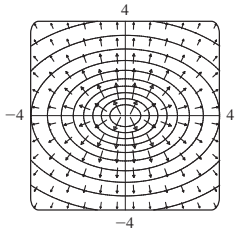
21. $\nabla f(x, y) = (xy + 1)e^{xy}\,\mathbf{i} + x^2 e^{xy}\,\mathbf{j}$

23. $\nabla f(x, y, z) = \dfrac{x}{\sqrt{x^2 + y^2 + z^2}}\,\mathbf{i} + \dfrac{y}{\sqrt{x^2 + y^2 + z^2}}\,\mathbf{j} + \dfrac{z}{\sqrt{x^2 + y^2 + z^2}}\,\mathbf{k}$

25. $\nabla f(x, y) = 2x\,\mathbf{i} - \mathbf{j}$

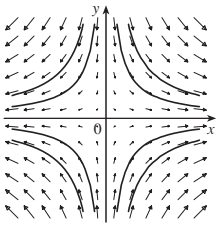


27.



29. III **31.** II **33.** (2.04, 1.03)

35. (a)



(b) $y = 1/x, x > 0$

$y = C/x$

EXERCISES 16.2 ■ PAGE 1096

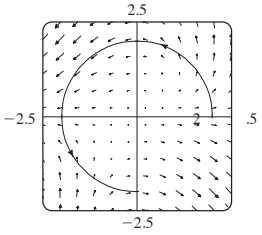
1. $\frac{1}{54}(145^{3/2} - 1)$ **3.** 1638.4 **5.** $\frac{243}{8}$ **7.** $\frac{5}{2}$

9. $\sqrt{5}\pi$ **11.** $\frac{1}{12}\sqrt{14}(e^6 - 1)$ **13.** $\frac{2}{5}(e - 1)$ **15.** $\frac{35}{3}$

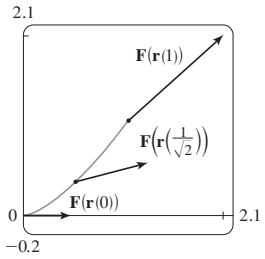
17. (a) Positive (b) Negative **19.** 45

21. $\frac{6}{5} - \cos 1 - \sin 1$ **23.** 1.9633 **25.** 15.0074

27. $3\pi + \frac{2}{3}$



29. (a) $\frac{11}{8} - 1/e$ (b)



31. $\frac{172{,}704}{5{,}632{,}705}\sqrt{2}(1 - e^{-14\pi})$ **33.** $2\pi k, (4/\pi, 0)$

35. (a) $\bar{x} = (1/m)\int_C x\rho(x, y, z)\, ds$,
$\bar{y} = (1/m)\int_C y\rho(x, y, z)\, ds$,
$\bar{z} = (1/m)\int_C z\rho(x, y, z)\, ds$, where $m = \int_C \rho(x, y, z)\, ds$
(b) $(0, 0, 3\pi)$

37. $I_x = k\left(\frac{1}{2}\pi - \frac{4}{3}\right), I_y = k\left(\frac{1}{2}\pi - \frac{2}{3}\right)$ **39.** $2\pi^2$ **41.** $\frac{7}{3}$

43. (a) $2ma\,\mathbf{i} + 6mbt\,\mathbf{j}, 0 \leq t \leq 1$ (b) $2ma^2 + \frac{9}{2}mb^2$

45. $\approx 1.67 \times 10^4$ ft-lb **47.** (b) Yes **51.** ≈ 22 J

EXERCISES 16.3 ■ PAGE 1106

1. 40 **3.** $f(x, y) = x^2 - 3xy + 2y^2 - 8y + K$

5. Not conservative **7.** $f(x, y) = ye^x + x \sin y + K$

9. $f(x, y) = x \ln y + x^2y^3 + K$

11. (b) 16 **13.** (a) $f(x, y) = \frac{1}{2}x^2y^2$ (b) 2

15. (a) $f(x, y, z) = xyz + z^2$ (b) 77

17. (a) $f(x, y, z) = ye^{xz}$ (b) 4 **19.** $4/e$

21. It doesn't matter which curve is chosen.

23. 30 **25.** No **27.** Conservative

31. (a) Yes (b) Yes (c) Yes

33. (a) No (b) Yes (c) Yes

EXERCISES 16.4 ■ PAGE 1113

1. 8π **3.** $\frac{2}{3}$ **5.** 12 **7.** $\frac{1}{3}$ **9.** -24π **11.** $-\frac{16}{3}$

13. 4π **15.** $-8e + 48e^{-1}$ **17.** $-\frac{1}{12}$ **19.** 3π **21.** (c) $\frac{9}{2}$

23. $(4a/3\pi, 4a/3\pi)$ if the region is the portion of the disk $x^2 + y^2 = a^2$ in the first quadrant

27. 0

EXERCISES 16.5 ■ PAGE 1121

1. (a) $\mathbf{0}$ (b) 3

3. (a) $ze^x\,\mathbf{i} + (xye^z - yze^x)\,\mathbf{j} - xe^z\,\mathbf{k}$ (b) $y(e^z + e^x)$

5. (a) $\mathbf{0}$ (b) $2/\sqrt{x^2 + y^2 + z^2}$

7. (a) $\langle -e^y \cos z, -e^z \cos x, -e^x \cos y\rangle$
(b) $e^x \sin y + e^y \sin z + e^z \sin x$

9. (a) Negative (b) curl $\mathbf{F} = \mathbf{0}$

11. (a) Zero (b) curl $\mathbf{F}$ points in the negative z-direction

13. $f(x, y, z) = xy^2z^3 + K$ **15.** Not conservative

17. $f(x, y, z) = xe^{yz} + K$ **19.** No

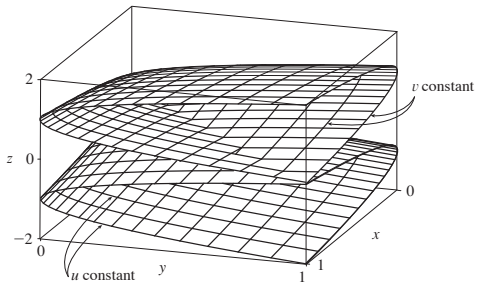
EXERCISES 16.6 ■ PAGE 1132

1. P: no; Q: yes

3. Plane through $(0, 3, 1)$ containing vectors $\langle 1, 0, 4\rangle, \langle 1, -1, 5\rangle$

5. Hyperbolic paraboloid

7.



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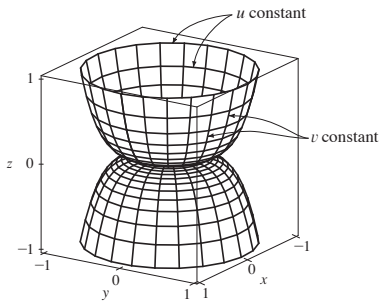

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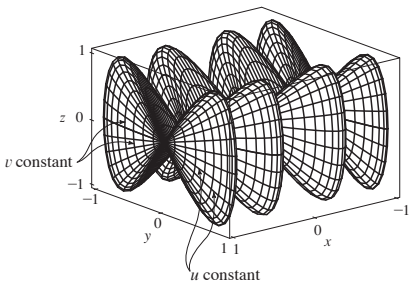


13. IV **15.** II **17.** III

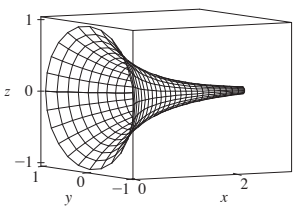
19. $x = u, y = v - u, z = -v$

21. $y = y, z = z, x = \sqrt{1 + y^2 + \frac{1}{4}z^2}$

23. $x = 2 \sin\phi \cos\theta, y = 2 \sin\phi \sin\theta,$
$z = 2 \cos\phi, 0 \leq \phi \leq \pi/4, 0 \leq \theta \leq 2\pi$
$\left[\text{or } x = x, y = y, z = \sqrt{4 - x^2 - y^2}, x^2 + y^2 \leq 2\right]$

25. $x = x, y = 4 \cos\theta, z = 4 \sin\theta, 0 \leq x \leq 5, 0 \leq \theta \leq 2\pi$

29. $x = x, y = e^{-x} \cos\theta,$
$z = e^{-x} \sin\theta, 0 \leq x \leq 3,$
$0 \leq \theta \leq 2\pi$



31. (a) Direction reverses (b) Number of coils doubles

33. $3x - y + 3z = 3$ **35.** $\frac{\sqrt{3}}{2}x - \frac{1}{2}y + z = \frac{\pi}{3}$

37. $-x + 2z = 1$ **39.** $3\sqrt{14}$ **41.** $\sqrt{14}\,\pi$

43. $\frac{4}{15}(3^{5/2} - 2^{7/2} + 1)$ **45.** $(2\pi/3)(2\sqrt{2} - 1)$

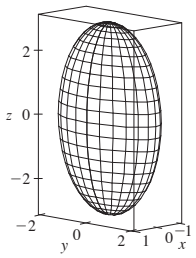
47. $\frac{1}{2}\sqrt{21} + \frac{17}{4}\left[\ln(2 + \sqrt{21}) - \ln\sqrt{17}\right]$ **49.** 4

51. $A(S) \leqslant \sqrt{3}\,\pi R^2$ **53.** 13.9783

55. (a) 24.2055 (b) 24.2476

57. $\frac{45}{8}\sqrt{14} + \frac{15}{16}\ln\left[(11\sqrt{5} + 3\sqrt{70})/(3\sqrt{5} + \sqrt{70})\right]$

59. (b)



(c) $\int_0^{2\pi}\int_0^{\pi}\sqrt{36\sin^4u\cos^2v + 9\sin^4u\sin^2v + 4\cos^2u\sin^2u}\,du\,dv$

61. 4π **63.** $2a^2(\pi - 2)$

EXERCISES 16.7 ■ PAGE 1144

1. 49.09 **3.** 900π **5.** $11\sqrt{14}$ **7.** $\frac{2}{3}(2\sqrt{2} - 1)$

9. $171\sqrt{14}$ **11.** $\sqrt{21}/3$ **13.** $364\sqrt{2}\,\pi/3$

15. $(\pi/60)(391\sqrt{17} + 1)$ **17.** 16π **19.** 12 **21.** 4

23. $\frac{713}{180}$ **25.** $-\frac{4}{3}\pi$ **27.** 0 **29.** 48 **31.** $2\pi + \frac{8}{3}$

33. 4.5822 **35.** 3.4895

37. $\iint_S \mathbf{F}\cdot d\mathbf{S} = \iint_D [P(\partial h/\partial x) - Q + R(\partial h/\partial z)]\,dA$, where D = projection of S on xz-plane

39. $(0, 0, a/2)$

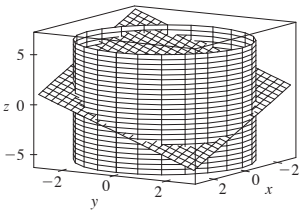
41. (a) $I_z = \iint_S (x^2 + y^2)\rho(x, y, z)\,dS$ (b) $4329\sqrt{2}\,\pi/5$

43. 0 kg/s **45.** $\frac{8}{3}\pi a^3\varepsilon_0$ **47.** 1248π

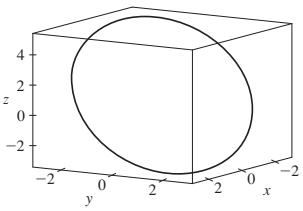
EXERCISES 16.8 ■ PAGE 1151

3. 0 **5.** 0 **7.** -1 **9.** 80π

11. (a) $81\pi/2$ (b)



(c) $x = 3\cos t$, $y = 3\sin t$, $z = 1 - 3(\cos t + \sin t)$, $0 \leqslant t \leqslant 2\pi$



17. 3

EXERCISES 16.9 ■ PAGE 1157

5. $\frac{9}{2}$ **7.** $9\pi/2$ **9.** 0 **11.** $32\pi/3$ **13.** 2π

15. $341\sqrt{2}/60 + \frac{81}{20}\arcsin(\sqrt{3}/3)$

17. $13\pi/20$ **19.** Negative at P_1, positive at P_2

21. div $\mathbf{F} > 0$ in quadrants I, II; div $\mathbf{F} < 0$ in quadrants III, IV

CHAPTER 16 REVIEW ■ PAGE 1160

True-False Quiz

1. False **3.** True **5.** False

7. False **9.** True **11.** True

Exercises

1. (a) Negative (b) Positive **3.** $6\sqrt{10}$ **5.** $\frac{4}{15}$

7. $\frac{110}{3}$ **9.** $\frac{11}{12} - 4/e$ **11.** $f(x, y) = e^y + xe^{xy}$ **13.** 0

17. -8π **25.** $\frac{1}{6}(27 - 5\sqrt{5})$ **27.** $(\pi/60)(391\sqrt{17} + 1)$

29. $-64\pi/3$ **33.** $-\frac{1}{2}$ **37.** -4 **39.** 21

CHAPTER 17

EXERCISES 17.1 ■ PAGE 1172

1. $y = c_1e^{3x} + c_2e^{-2x}$ **3.** $y = c_1\cos 4x + c_2\sin 4x$

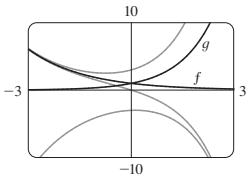
5. $y = c_1e^{2x/3} + c_2xe^{2x/3}$ **7.** $y = c_1 + c_2e^{x/2}$

9. $y = e^{2x}(c_1\cos 3x + c_2\sin 3x)$

11. $y = c_1e^{(\sqrt{3}-1)t/2} + c_2e^{-(\sqrt{3}+1)t/2}$

13. $P = e^{-t}\left[c_1\cos\left(\frac{1}{10}t\right) + c_2\sin\left(\frac{1}{10}t\right)\right]$

15.



All solutions approach either 0 or $\pm\infty$ as $x \to \pm\infty$.

17. $y = 3e^{2x} - e^{4x}$ **19.** $y = e^{-2x/3} + \frac{2}{3}xe^{-2x/3}$

21. $y = e^{3x}(2\cos x - 3\sin x)$

23. $y = \frac{1}{7}e^{4x-4} - \frac{1}{7}e^{3-3x}$ **25.** $y = 5\cos 2x + 3\sin 2x$

27. $y = 2e^{-2x} - 2xe^{-2x}$ **29.** $y = \dfrac{e - 2}{e - 1} + \dfrac{e^x}{e - 1}$

31. No solution

33. (b) $\lambda = n^2\pi^2/L^2$, n a positive integer; $y = C\sin(n\pi x/L)$

35. (a) $b - a \neq n\pi$, n any integer

(b) $b - a = n\pi$ and $\dfrac{c}{d} \neq e^{a-b}\dfrac{\cos a}{\cos b}$ unless $\cos b = 0$, then $\dfrac{c}{d} \neq e^{a-b}\dfrac{\sin a}{\sin b}$

(c) $b - a = n\pi$ and $\dfrac{c}{d} = e^{a-b}\dfrac{\cos a}{\cos b}$ unless $\cos b = 0$, then $\dfrac{c}{d} = e^{a-b}\dfrac{\sin a}{\sin b}$

EXERCISES 17.2 ■ PAGE 1179

1. $y = c_1e^{3x} + c_2e^{-x} - \frac{7}{65}\cos 2x - \frac{4}{65}\sin 2x$

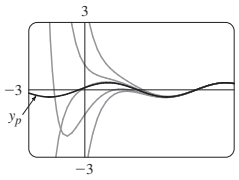
3. $y = c_1\cos 3x + c_2\sin 3x + \frac{1}{13}e^{-2x}$

5. $y = e^{2x}(c_1 \cos x + c_2 \sin x) + \frac{1}{10}e^{-x}$

7. $y = \frac{3}{2}\cos x + \frac{11}{2}\sin x + \frac{1}{2}e^{x} + x^3 - 6x$

9. $y = e^{x}\left(\frac{1}{2}x^2 - x + 2\right)$

11.



The solutions are all asymptotic to $y_p = \frac{1}{10}\cos x + \frac{3}{10}\sin x$ as $x \to \infty$. Except for y_p, all solutions approach either ∞ or $-\infty$ as $x \to -\infty$.

13. $y_p = (Ax + B)e^{x}\cos x + (Cx + D)e^{x}\sin x$

15. $y_p = Axe^{x} + B\cos x + C\sin x$

17. $y_p = xe^{-x}[(Ax^2 + Bx + C)\cos 3x + (Dx^2 + Ex + F)\sin 3x]$

19. $y = c_1\cos\left(\frac{1}{2}x\right) + c_2\sin\left(\frac{1}{2}x\right) - \frac{1}{3}\cos x$

21. $y = c_1e^{x} + c_2xe^{x} + e^{2x}$

23. $y = c_1\sin x + c_2\cos x + \sin x\ln(\sec x + \tan x) - 1$

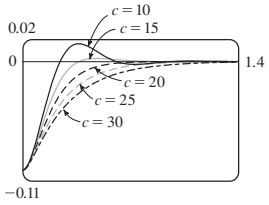
25. $y = [c_1 + \ln(1 + e^{-x})]e^{x} + [c_2 - e^{-x} + \ln(1 + e^{-x})]e^{2x}$

27. $y = e^{x}\left[c_1 + c_2x - \frac{1}{2}\ln(1 + x^2) + x\tan^{-1}x\right]$

EXERCISES 17.3 ■ PAGE 1187

1. $x = 0.35\cos\left(2\sqrt{5}\,t\right)$ **3.** $x = -\frac{1}{5}e^{-6t} + \frac{6}{5}e^{-t}$ **5.** $\frac{49}{12}$ kg

7.



13. $Q(t) = (-e^{-10t}/250)(6\cos 20t + 3\sin 20t) + \frac{3}{125}$, $I(t) = \frac{3}{5}e^{-10t}\sin 20t$

15. $Q(t) = e^{-10t}\left[\frac{3}{250}\cos 20t - \frac{3}{500}\sin 20t\right] - \frac{3}{250}\cos 10t + \frac{3}{125}\sin 10t$

EXERCISES 17.4 ■ PAGE 1192

1. $c_0\sum_{n=0}^{\infty}\frac{x^n}{n!} = c_0e^{x}$ **3.** $c_0\sum_{n=0}^{\infty}\frac{x^{3n}}{3^n n!} = c_0e^{x^3/3}$

5. $c_0\sum_{n=0}^{\infty}\frac{(-1)^n}{2^n n!}x^{2n} + c_1\sum_{n=0}^{\infty}\frac{(-2)^n n!}{(2n+1)!}x^{2n+1}$

7. $c_0 + c_1\sum_{n=1}^{\infty}\frac{x^n}{n} = c_0 - c_1\ln(1 - x)$ for $|x| < 1$

9. $\sum_{n=0}^{\infty}\frac{x^{2n}}{2^n n!} = e^{x^2/2}$

11. $x + \sum_{n=1}^{\infty}\frac{(-1)^n 2^2 5^2 \cdot \cdots \cdot (3n-1)^2}{(3n+1)!}x^{3n+1}$

CHAPTER 17 REVIEW ■ PAGE 1193

True-False Quiz

1. True **3.** True

Exercises

1. $y = c_1e^{x/2} + c_2e^{-x/2}$

3. $y = c_1\cos\left(\sqrt{3}x\right) + c_2\sin\left(\sqrt{3}x\right)$

5. $y = e^{2x}(c_1\cos x + c_2\sin x + 1)$

7. $y = c_1e^{x} + c_2xe^{x} - \frac{1}{2}\cos x - \frac{1}{2}(x + 1)\sin x$

9. $y = c_1e^{3x} + c_2e^{-2x} - \frac{1}{6} - \frac{1}{5}xe^{-2x}$

11. $y = 5 - 2e^{-6(x-1)}$ **13.** $y = (e^{4x} - e^{x})/3$

15. No solution

17. $\sum_{n=0}^{\infty}\frac{(-2)^n n!}{(2n+1)!}x^{2n+1}$

19. $Q(t) = -0.02e^{-10t}(\cos 10t + \sin 10t) + 0.03$

21. (c) $2\pi/k \approx 85$ min (d) ≈17,600 mi/h

APPENDIXES

EXERCISES G ■ PAGE A12

1. $8 - 4i$ **3.** $13 + 18i$ **5.** $12 - 7i$ **7.** $\frac{11}{13} + \frac{10}{13}i$

9. $\frac{1}{2} - \frac{1}{2}i$ **11.** $-i$ **13.** $5i$ **15.** $12 + 5i, 13$

17. $4i, 4$ **19.** $\pm\frac{3}{2}i$ **21.** $-1 \pm 2i$

23. $-\frac{1}{2} \pm (\sqrt{7}/2)i$ **25.** $3\sqrt{2}\,[\cos(3\pi/4) + i\sin(3\pi/4)]$

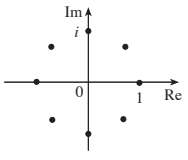
27. $5\left\{\cos\left[\tan^{-1}\left(\frac{4}{3}\right)\right] + i\sin\left[\tan^{-1}\left(\frac{4}{3}\right)\right]\right\}$

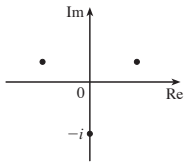
29. $4[\cos(\pi/2) + i\sin(\pi/2)]$, $\cos(-\pi/6) + i\sin(-\pi/6)$, $\frac{1}{2}[\cos(-\pi/6) + i\sin(-\pi/6)]$

31. $4\sqrt{2}\,[\cos(7\pi/12) + i\sin(7\pi/12)]$, $(2\sqrt{2})[\cos(13\pi/12) + i\sin(13\pi/12)]$, $\frac{1}{4}[\cos(\pi/6) + i\sin(\pi/6)]$

33. -1024 **35.** $-512\sqrt{3} + 512i$

37. $\pm 1, \pm i, (1/\sqrt{2})(\pm 1 \pm i)$ **39.** $\pm\left(\sqrt{3}/2\right) + \frac{1}{2}i, -i$





41. i **43.** $\frac{1}{2} + (\sqrt{3}/2)i$ **45.** $-e^2$

47. $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$, $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$

Index

RP denotes Reference Page numbers.

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DIFFERENTIATION RULES

General Formulas

1. $\frac{d}{dx}(c) = 0$
2. $\frac{d}{dx}[cf(x)] = cf'(x)$
3. $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
4. $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$
5. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$ (Product Rule)
6. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ (Quotient Rule)
7. $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$ (Chain Rule)
8. $\frac{d}{dx}(x^n) = nx^{n-1}$ (Power Rule)

Exponential and Logarithmic Functions

9. $\frac{d}{dx}(e^x) = e^x$
10. $\frac{d}{dx}(a^x) = a^x \ln a$
11. $\frac{d}{dx}\ln|x| = \frac{1}{x}$
12. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Trigonometric Functions

13. $\frac{d}{dx}(\sin x) = \cos x$
14. $\frac{d}{dx}(\cos x) = -\sin x$
15. $\frac{d}{dx}(\tan x) = \sec^2 x$
16. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
17. $\frac{d}{dx}(\sec x) = \sec x \tan x$
18. $\frac{d}{dx}(\cot x) = -\csc^2 x$

Inverse Trigonometric Functions

19. $\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1 - x^2}}$
20. $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1 - x^2}}$
21. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1 + x^2}$
22. $\frac{d}{dx}(\csc^{-1}x) = -\frac{1}{x\sqrt{x^2 - 1}}$
23. $\frac{d}{dx}(\sec^{-1}x) = \frac{1}{x\sqrt{x^2 - 1}}$
24. $\frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1 + x^2}$

Hyperbolic Functions

25. $\frac{d}{dx}(\sinh x) = \cosh x$
26. $\frac{d}{dx}(\cosh x) = \sinh x$
27. $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$
28. $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$
29. $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$
30. $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

Inverse Hyperbolic Functions

31. $\frac{d}{dx}(\sinh^{-1}x) = \frac{1}{\sqrt{1 + x^2}}$
32. $\frac{d}{dx}(\cosh^{-1}x) = \frac{1}{\sqrt{x^2 - 1}}$
33. $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1 - x^2}$
34. $\frac{d}{dx}(\operatorname{csch}^{-1}x) = -\frac{1}{|x|\sqrt{x^2 + 1}}$
35. $\frac{d}{dx}(\operatorname{sech}^{-1}x) = -\frac{1}{x\sqrt{1 - x^2}}$
36. $\frac{d}{dx}(\coth^{-1}x) = \frac{1}{1 - x^2}$

TABLE OF INTEGRALS

Basic Forms

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \dfrac{du}{u} = \ln|u| + C$
4. $\int e^u\,du = e^u + C$
5. $\int a^u\,du = \dfrac{a^u}{\ln a} + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \sec^2 u\,du = \tan u + C$
9. $\int \csc^2 u\,du = -\cot u + C$
10. $\int \sec u \tan u\,du = \sec u + C$
11. $\int \csc u \cot u\,du = -\csc u + C$
12. $\int \tan u\,du = \ln|\sec u| + C$
13. $\int \cot u\,du = \ln|\sin u| + C$
14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
15. $\int \csc u\,du = \ln|\csc u - \cot u| + C$
16. $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\dfrac{u}{a} + C, \quad a > 0$
17. $\int \dfrac{du}{a^2 + u^2} = \dfrac{1}{a}\tan^{-1}\dfrac{u}{a} + C$
18. $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{a}\sec^{-1}\dfrac{u}{a} + C$
19. $\int \dfrac{du}{a^2 - u^2} = \dfrac{1}{2a}\ln\left|\dfrac{u + a}{u - a}\right| + C$
20. $\int \dfrac{du}{u^2 - a^2} = \dfrac{1}{2a}\ln\left|\dfrac{u - a}{u + a}\right| + C$

Forms Involving $\sqrt{a^2 + u^2}$, $a > 0$

21. $\int \sqrt{a^2 + u^2}\,du = \dfrac{u}{2}\sqrt{a^2 + u^2} + \dfrac{a^2}{2}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$
22. $\int u^2\sqrt{a^2 + u^2}\,du = \dfrac{u}{8}(a^2 + 2u^2)\sqrt{a^2 + u^2} - \dfrac{a^4}{8}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$
23. $\int \dfrac{\sqrt{a^2 + u^2}}{u}\,du = \sqrt{a^2 + u^2} - a\ln\left|\dfrac{a + \sqrt{a^2 + u^2}}{u}\right| + C$
24. $\int \dfrac{\sqrt{a^2 + u^2}}{u^2}\,du = -\dfrac{\sqrt{a^2 + u^2}}{u} + \ln\left(u + \sqrt{a^2 + u^2}\right) + C$
25. $\int \dfrac{du}{\sqrt{a^2 + u^2}} = \ln\left(u + \sqrt{a^2 + u^2}\right) + C$
26. $\int \dfrac{u^2\,du}{\sqrt{a^2 + u^2}} = \dfrac{u}{2}\sqrt{a^2 + u^2} - \dfrac{a^2}{2}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$
27. $\int \dfrac{du}{u\sqrt{a^2 + u^2}} = -\dfrac{1}{a}\ln\left|\dfrac{\sqrt{a^2 + u^2} + a}{u}\right| + C$
28. $\int \dfrac{du}{u^2\sqrt{a^2 + u^2}} = -\dfrac{\sqrt{a^2 + u^2}}{a^2 u} + C$
29. $\int \dfrac{du}{(a^2 + u^2)^{3/2}} = \dfrac{u}{a^2\sqrt{a^2 + u^2}} + C$

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TABLE OF INTEGRALS

Forms Involving $\sqrt{a^2 - u^2}$, $a > 0$

30. $\displaystyle\int \sqrt{a^2 - u^2}\, du = \frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

31. $\displaystyle\int u^2\sqrt{a^2 - u^2}\, du = \frac{u}{8}(2u^2 - a^2)\sqrt{a^2 - u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C$

32. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u}\, du = \sqrt{a^2 - u^2} - a\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

33. $\displaystyle\int \frac{\sqrt{a^2 - u^2}}{u^2}\, du = -\frac{1}{u}\sqrt{a^2 - u^2} - \sin^{-1}\frac{u}{a} + C$

34. $\displaystyle\int \frac{u^2\, du}{\sqrt{a^2 - u^2}} = -\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

35. $\displaystyle\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C$

36. $\displaystyle\int \frac{du}{u^2\sqrt{a^2 - u^2}} = -\frac{1}{a^2 u}\sqrt{a^2 - u^2} + C$

37. $\displaystyle\int (a^2 - u^2)^{3/2}\, du = -\frac{u}{8}(2u^2 - 5a^2)\sqrt{a^2 - u^2} + \frac{3a^4}{8}\sin^{-1}\frac{u}{a} + C$

38. $\displaystyle\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2 - u^2}} + C$

Forms Involving $\sqrt{u^2 - a^2}$, $a > 0$

39. $\displaystyle\int \sqrt{u^2 - a^2}\, du = \frac{u}{2}\sqrt{u^2 - a^2} - \frac{a^2}{2}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$

40. $\displaystyle\int u^2\sqrt{u^2 - a^2}\, du = \frac{u}{8}(2u^2 - a^2)\sqrt{u^2 - a^2} - \frac{a^4}{8}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$

41. $\displaystyle\int \frac{\sqrt{u^2 - a^2}}{u}\, du = \sqrt{u^2 - a^2} - a\cos^{-1}\frac{a}{|u|} + C$

42. $\displaystyle\int \frac{\sqrt{u^2 - a^2}}{u^2}\, du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln\left|u + \sqrt{u^2 - a^2}\right| + C$

43. $\displaystyle\int \frac{du}{\sqrt{u^2 - a^2}} = \ln\left|u + \sqrt{u^2 - a^2}\right| + C$

44. $\displaystyle\int \frac{u^2\, du}{\sqrt{u^2 - a^2}} = \frac{u}{2}\sqrt{u^2 - a^2} + \frac{a^2}{2}\ln\left|u + \sqrt{u^2 - a^2}\right| + C$

45. $\displaystyle\int \frac{du}{u^2\sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$

46. $\displaystyle\int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2 - a^2}} + C$

TABLE OF INTEGRALS

Forms Involving $a + bu$

47. $\displaystyle\int \frac{u\,du}{a+bu} = \frac{1}{b^2}\left(a + bu - a\ln|a+bu|\right) + C$

48. $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{1}{2b^3}\left[(a+bu)^2 - 4a(a+bu) + 2a^2\ln|a+bu|\right] + C$

49. $\displaystyle\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

50. $\displaystyle\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

51. $\displaystyle\int \frac{u\,du}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b^2}\ln|a+bu| + C$

52. $\displaystyle\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

53. $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left(a + bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$

54. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu - 2a)(a+bu)^{3/2} + C$

55. $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu - 2a)\sqrt{a+bu} + C$

56. $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(8a^2 + 3b^2u^2 - 4abu)\sqrt{a+bu} + C$

57. $\displaystyle\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C, \quad \text{if } a > 0$

$\displaystyle= \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C, \qquad \text{if } a < 0$

58. $\displaystyle\int \frac{\sqrt{a+bu}}{u}\,du = 2\sqrt{a+bu} + a\int \frac{du}{u\sqrt{a+bu}}$

59. $\displaystyle\int \frac{\sqrt{a+bu}}{u^2}\,du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2}\int \frac{du}{u\sqrt{a+bu}}$

60. $\displaystyle\int u^n\sqrt{a+bu}\,du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$

61. $\displaystyle\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)}\int \frac{u^{n-1}\,du}{\sqrt{a+bu}}$

62. $\displaystyle\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{du}{u^{n-1}\sqrt{a+bu}}$

Cut here and keep for reference

TABLE OF INTEGRALS

Trigonometric Forms

63. $\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

64. $\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

65. $\int \tan^2 u\,du = \tan u - u + C$

66. $\int \cot^2 u\,du = -\cot u - u + C$

67. $\int \sin^3 u\,du = -\frac{1}{3}(2 + \sin^2 u)\cos u + C$

68. $\int \cos^3 u\,du = \frac{1}{3}(2 + \cos^2 u)\sin u + C$

69. $\int \tan^3 u\,du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$

70. $\int \cot^3 u\,du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$

71. $\int \sec^3 u\,du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$

72. $\int \csc^3 u\,du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$

73. $\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u \cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du$

74. $\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u \sin u + \frac{n-1}{n}\int \cos^{n-2} u\,du$

75. $\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\,du$

76. $\int \cot^n u\,du = \frac{-1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\,du$

77. $\int \sec^n u\,du = \frac{1}{n-1}\tan u \sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$

78. $\int \csc^n u\,du = \frac{-1}{n-1}\cot u \csc^{n-2} u + \frac{n-2}{n-1}\int \csc^{n-2} u\,du$

79. $\int \sin au \sin bu\,du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$

80. $\int \cos au \cos bu\,du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$

81. $\int \sin au \cos bu\,du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

82. $\int u \sin u\,du = \sin u - u\cos u + C$

83. $\int u \cos u\,du = \cos u + u\sin u + C$

84. $\int u^n \sin u\,du = -u^n \cos u + n\int u^{n-1}\cos u\,du$

85. $\int u^n \cos u\,du = u^n \sin u - n\int u^{n-1}\sin u\,du$

86. $\int \sin^n u \cos^m u\,du = -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m}\int \sin^{n-2} u \cos^m u\,du$

$= \frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m}\int \sin^n u \cos^{m-2} u\,du$

Inverse Trigonometric Forms

87. $\int \sin^{-1} u\,du = u\sin^{-1} u + \sqrt{1-u^2} + C$

88. $\int \cos^{-1} u\,du = u\cos^{-1} u - \sqrt{1-u^2} + C$

89. $\int \tan^{-1} u\,du = u\tan^{-1} u - \frac{1}{2}\ln(1+u^2) + C$

90. $\int u \sin^{-1} u\,du = \frac{2u^2-1}{4}\sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$

91. $\int u \cos^{-1} u\,du = \frac{2u^2-1}{4}\cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$

92. $\int u \tan^{-1} u\,du = \frac{u^2+1}{2}\tan^{-1} u - \frac{u}{2} + C$

93. $\int u^n \sin^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\sin^{-1} u - \int \frac{u^{n+1}\,du}{\sqrt{1-u^2}}\right], \quad n \neq -1$

94. $\int u^n \cos^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\cos^{-1} u + \int \frac{u^{n+1}\,du}{\sqrt{1-u^2}}\right], \quad n \neq -1$

95. $\int u^n \tan^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\tan^{-1} u - \int \frac{u^{n+1}\,du}{1+u^2}\right], \quad n \neq -1$

TABLE OF INTEGRALS

Exponential and Logarithmic Forms

96. $\int ue^{au}\,du = \frac{1}{a^2}(au-1)e^{au} + C$

97. $\int u^n e^{au}\,du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}\,du$

98. $\int e^{au}\sin bu\,du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$

99. $\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu) + C$

100. $\int \ln u\,du = u\ln u - u + C$

101. $\int u^n \ln u\,du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$

102. $\int \frac{1}{u\ln u}\,du = \ln|\ln u| + C$

Hyperbolic Forms

103. $\int \sinh u\,du = \cosh u + C$

104. $\int \cosh u\,du = \sinh u + C$

105. $\int \tanh u\,du = \ln\cosh u + C$

106. $\int \coth u\,du = \ln|\sinh u| + C$

107. $\int \operatorname{sech} u\,du = \tan^{-1}|\sinh u| + C$

108. $\int \operatorname{csch} u\,du = \ln|\tanh \tfrac{1}{2}u| + C$

109. $\int \operatorname{sech}^2 u\,du = \tanh u + C$

110. $\int \operatorname{csch}^2 u\,du = -\coth u + C$

111. $\int \operatorname{sech} u\tanh u\,du = -\operatorname{sech} u + C$

112. $\int \operatorname{csch} u\coth u\,du = -\operatorname{csch} u + C$

Forms Involving $\sqrt{2au-u^2}$, $a>0$

113. $\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2} + \frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

114. $\int u\sqrt{2au-u^2}\,du = \frac{2u^2-au-3a^2}{6}\sqrt{2au-u^2} + \frac{a^3}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

115. $\int \frac{\sqrt{2au-u^2}}{u}\,du = \sqrt{2au-u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$

116. $\int \frac{\sqrt{2au-u^2}}{u^2}\,du = -\frac{2\sqrt{2au-u^2}}{u} - \cos^{-1}\left(\frac{a-u}{a}\right) + C$

117. $\int \frac{du}{\sqrt{2au-u^2}} = \cos^{-1}\left(\frac{a-u}{a}\right) + C$

118. $\int \frac{u\,du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2} + a\cos^{-1}\left(\frac{a-u}{a}\right) + C$

119. $\int \frac{u^2\,du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2}\sqrt{2au-u^2} + \frac{3a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right) + C$

120. $\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$