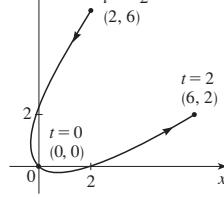


## H Answers to Odd-Numbered Exercises

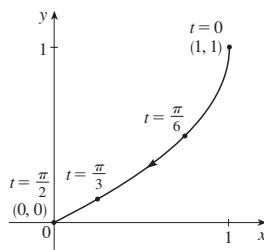
### CHAPTER 10

#### EXERCISES 10.1 ■ PAGE 665

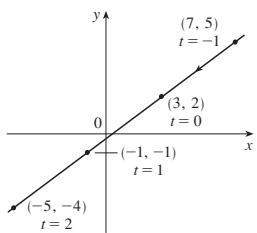
**1.**



**3.**

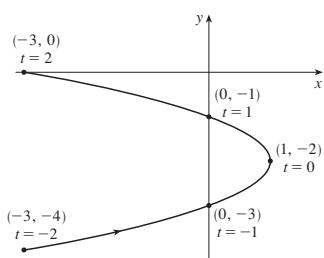


**5. (a)**



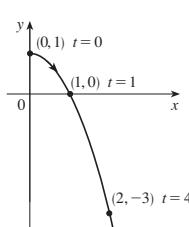
(b)  $y = \frac{3}{4}x - \frac{1}{4}$

**7. (a)**



(b)  $x = -(y + 2)^2 + 1, -4 \leq y \leq 0$

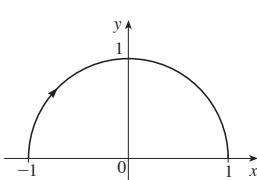
**9. (a)**



(b)  $y = 1 - x^2, x \geq 0$

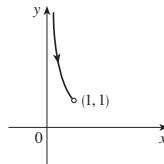
**11. (a)**  $x^2 + y^2 = 1, y \geq 0$

(b)



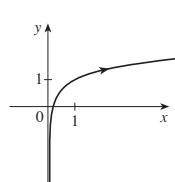
**13. (a)**  $y = 1/x, y > 1$

(b)



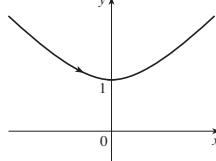
**15. (a)**  $y = \frac{1}{2}\ln x + 1$

(b)



**17. (a)**  $y^2 - x^2 = 1, y \geq 1$

(b)

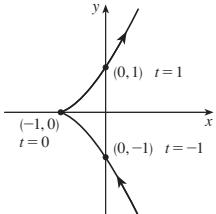


**19.** Moves counterclockwise along the circle  $(x - 3)^2 + (y - 1)^2 = 4$  from  $(3, 3)$  to  $(3, -1)$

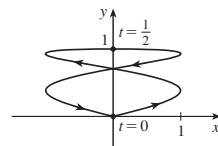
**21.** Moves 3 times clockwise around the ellipse  $(x^2/25) + (y^2/4) = 1$ , starting and ending at  $(0, -2)$

**23.** It is contained in the rectangle described by  $1 \leq x \leq 4$  and  $2 \leq y \leq 3$ .

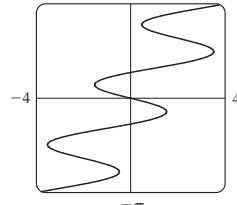
**25.**



**27.**



**29.**



**31. (b)**  $x = -2 + 5t, y = 7 - 8t, 0 \leq t \leq 1$

**33. (a)**  $x = 2 \cos t, y = 1 - 2 \sin t, 0 \leq t \leq 2\pi$

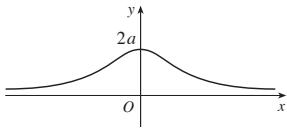
(b)  $x = 2 \cos t, y = 1 + 2 \sin t, 0 \leq t \leq 6\pi$

(c)  $x = 2 \cos t, y = 1 + 2 \sin t, \pi/2 \leq t \leq 3\pi/2$

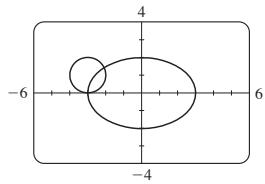
**37.** The curve  $y = x^{2/3}$  is generated in (a). In (b), only the portion with  $x \geq 0$  is generated, and in (c) we get only the portion with  $x > 0$ .

**41.**  $x = a \cos \theta, y = b \sin \theta; (x^2/a^2) + (y^2/b^2) = 1$ , ellipse

**43.**

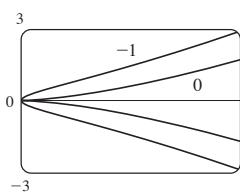


**45.** (a) Two points of intersection



(b) One collision point at  $(-3, 0)$  when  $t = 3\pi/2$

(c) There are still two intersection points, but no collision point.  
**47.** For  $c = 0$ , there is a cusp; for  $c > 0$ , there is a loop whose size increases as  $c$  increases.



**49.** The curves roughly follow the line  $y = x$ , and they start having loops when  $a$  is between 1.4 and 1.6 (more precisely, when  $a > \sqrt{2}$ ). The loops increase in size as  $a$  increases.

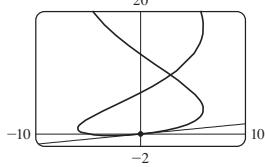
**51.** As  $n$  increases, the number of oscillations increases;  $a$  and  $b$  determine the width and height.

**EXERCISES 10.2 ■ PAGE 675**

**1.**  $\frac{2t+1}{t \cos t + \sin t}$     **3.**  $y = -\frac{3}{2}x + 7$     **5.**  $y = \pi x + \pi^2$

**7.**  $y = 2x + 1$

**9.**  $y = \frac{1}{6}x$



**11.**  $\frac{2t+1}{2t}, -\frac{1}{4t^3}, t < 0$     **13.**  $e^{-2t}(1-t), e^{-3t}(2t-3), t > \frac{3}{2}$

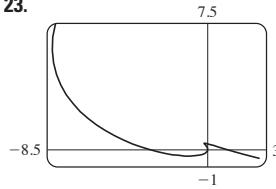
**15.**  $-\frac{3}{2} \tan t, -\frac{3}{4} \sec^3 t, \pi/2 < t < 3\pi/2$

**17.** Horizontal at  $(0, -3)$ , vertical at  $(\pm 2, -2)$

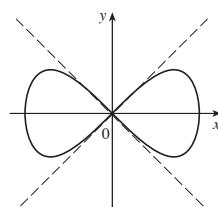
**19.** Horizontal at  $(\frac{1}{2}, -1)$  and  $(-\frac{1}{2}, 1)$ , no vertical

**21.**  $(0.6, 2); (5 \cdot 6^{-6/5}, e^{6^{-1/5}})$

**23.**



**25.**  $y = x, y = -x$



**27.** (a)  $d \sin \theta / (r - d \cos \theta)$     **29.**  $(\frac{16}{27}, \frac{29}{9}), (-2, -4)$

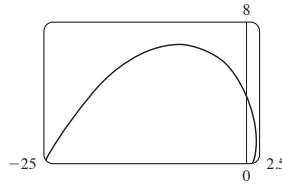
**31.**  $\pi ab$     **33.**  $3 - e$     **35.**  $2\pi r^2 + \pi d^2$

**37.**  $\int_0^2 \sqrt{2 + 2e^{-2t}} dt \approx 3.1416$

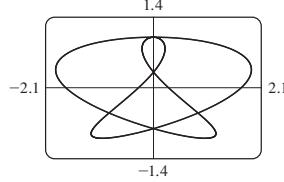
**39.**  $\int_0^{4\pi} \sqrt{5 - 4 \cos t} dt \approx 26.7298$     **41.**  $4\sqrt{2} - 2$

**43.**  $\frac{1}{2}\sqrt{2} + \frac{1}{2} \ln(1 + \sqrt{2})$

**45.**  $\sqrt{2} (e^\pi - 1)$

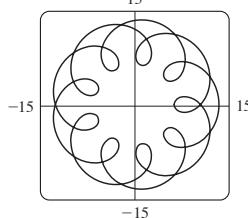


**47.** 16.7102



**49.** 612.3053    **51.**  $6\sqrt{2}, \sqrt{2}$

**55.** (a)  $t \in [0, 4\pi]$



(b) 294

**57.**  $\int_0^{\pi/2} 2\pi t \cos t \sqrt{t^2 + 1} dt \approx 4.7394$

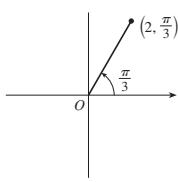
**59.**  $\int_0^1 2\pi(t^2 + 1)e^t \sqrt{e^{2t}(t+1)^2(t^2 + 2t + 2)} dt \approx 103.5999$

**61.**  $\frac{2}{1215}\pi(247\sqrt{13} + 64)$     **63.**  $\frac{6}{5}\pi a^2$

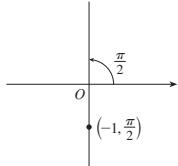
**65.**  $\frac{24}{5}\pi(949\sqrt{26} + 1)$     **71.**  $\frac{1}{4}$

**EXERCISES 10.3 ■ PAGE 686**

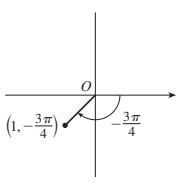
1. (a)

 $(2, 7\pi/3), (-2, 4\pi/3)$ 

(c)

 $(1, 3\pi/2), (-1, 5\pi/2)$ 

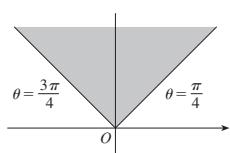
(b)

 $(1, 5\pi/4), (-1, \pi/4)$ 

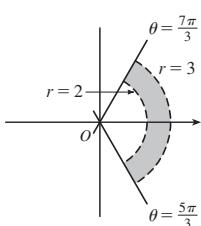
3. (a)
- A polar coordinate system with the origin labeled  $O$ . A point is plotted on the negative x-axis at an angle of  $\pi$  from the positive x-axis. The distance from the origin is 1.
- $(-1, 0)$
3. (b)
- A polar coordinate system with the origin labeled  $O$ . A point is plotted in the second quadrant at an angle of  $-\frac{2\pi}{3}$  from the positive x-axis. The distance from the origin is 2.
- $(-1, -\sqrt{3})$
3. (c)
- A polar coordinate system with the origin labeled  $O$ . A point is plotted in the fourth quadrant at an angle of  $-\frac{3\pi}{4}$  from the positive x-axis. The distance from the origin is  $\sqrt{2}$ .
- $(\sqrt{2}, -\sqrt{2})$
5. (a) (i)  $(2\sqrt{2}, 7\pi/4)$  (ii)  $(-2\sqrt{2}, 3\pi/4)$   
 (b) (i)  $(2, 2\pi/3)$  (ii)  $(-2, 5\pi/3)$
7. 

A polar coordinate system with the origin labeled  $O$ . A circle is drawn in the first quadrant, centered at the origin with a radius of 1. The entire circle is shaded gray.

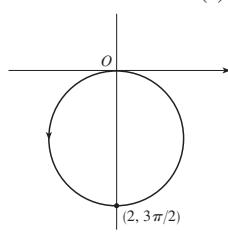
9.



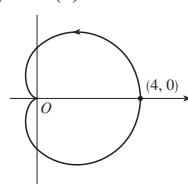
11.

13.  $2\sqrt{3}$ 15. Circle, center  $O$ , radius  $\sqrt{5}$ 17. Circle, center  $(1, 0)$ , radius 119. Hyperbola, center  $O$ , foci on  $x$ -axis21.  $r = 2 \csc \theta$ 23.  $r = 1/(\sin \theta - 3 \cos \theta)$ 25.  $r = 2c \cos \theta$ 27. (a)  $\theta = \pi/6$  (b)  $x = 3$ 

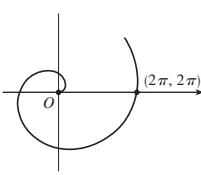
29.



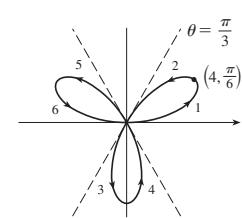
31.



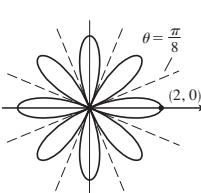
33.



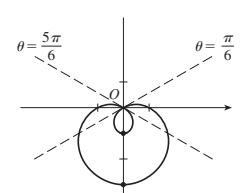
35.



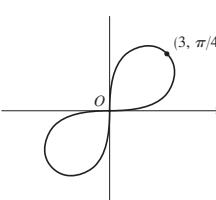
37.



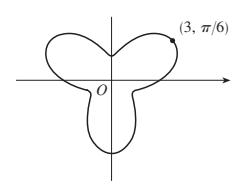
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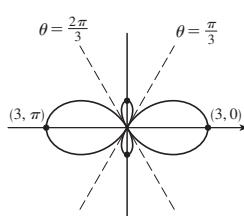
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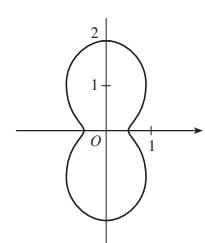
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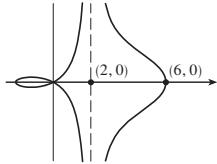
45.



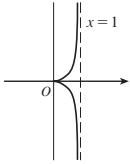
47.



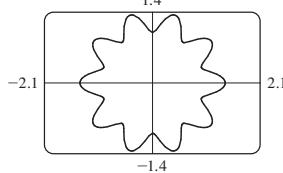
**49.**



**51.**



**15.**  $\frac{3}{2}\pi$



- 53.** (a) For  $c < -1$ , the inner loop begins at  $\theta = \sin^{-1}(-1/c)$  and ends at  $\theta = \pi - \sin^{-1}(-1/c)$ ; for  $c > 1$ , it begins at  $\theta = \pi + \sin^{-1}(1/c)$  and ends at  $\theta = 2\pi - \sin^{-1}(1/c)$ .

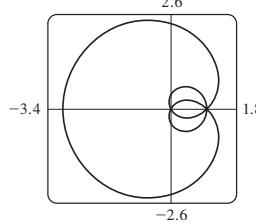
**55.**  $\sqrt{3}$     **57.**  $-\pi$     **59.** 1

- 61.** Horizontal at  $(3/\sqrt{2}, \pi/4), (-3/\sqrt{2}, 3\pi/4)$ ; vertical at  $(3, 0), (0, \pi/2)$

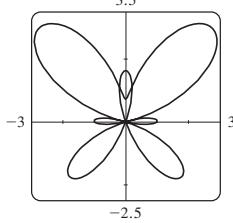
- 63.** Horizontal at  $(\frac{3}{2}, \pi/3), (0, \pi)$  [the pole], and  $(\frac{3}{2}, 5\pi/3)$ ; vertical at  $(2, 0), (\frac{1}{2}, 2\pi/3), (\frac{1}{2}, 4\pi/3)$

- 65.** Center  $(b/2, a/2)$ , radius  $\sqrt{a^2 + b^2}/2$

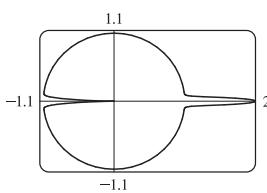
**67.**



**69.**



**71.**



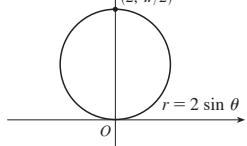
- 73.** By counterclockwise rotation through angle  $\pi/6, \pi/3$ , or  $\alpha$  about the origin

- 75.** For  $c = 0$ , the curve is a circle. As  $c$  increases, the left side gets flatter, then has a dimple for  $0.5 < c < 1$ , a cusp for  $c = 1$ , and a loop for  $c > 1$ .

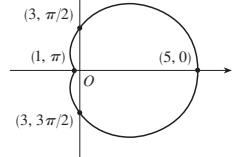
**EXERCISES 10.4 ■ PAGE 692**

**1.**  $e^{-\pi/4} - e^{-\pi/2}$     **3.**  $\frac{9}{2}$     **5.**  $\pi^2$     **7.**  $\frac{41}{4}\pi$

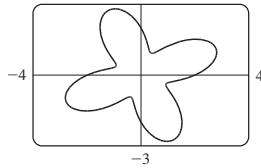
**9.**  $\pi$



**11.**  $11\pi$



**13.**  $\frac{9}{2}\pi$



**17.**  $\frac{4}{3}\pi$     **19.**  $\frac{1}{16}\pi$     **21.**  $\pi - \frac{3}{2}\sqrt{3}$     **23.**  $\frac{1}{3}\pi + \frac{1}{2}\sqrt{3}$

**25.**  $4\sqrt{3} - \frac{4}{3}\pi$     **27.**  $\pi$     **29.**  $\frac{5}{24}\pi - \frac{1}{4}\sqrt{3}$     **31.**  $\frac{1}{2}\pi - 1$

**33.**  $1 - \frac{1}{2}\sqrt{2}$     **35.**  $\frac{1}{4}(\pi + 3\sqrt{3})$

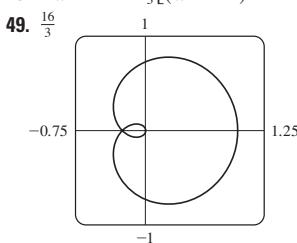
**37.**  $(\frac{3}{2}, \pi/6), (\frac{3}{2}, 5\pi/6)$ , and the pole

- 39.**  $(1, \theta)$  where  $\theta = \pi/12, 5\pi/12, 13\pi/12, 17\pi/12$  and  $(-1, \theta)$  where  $\theta = 7\pi/12, 11\pi/12, 19\pi/12, 23\pi/12$

- 41.**  $(\frac{1}{2}\sqrt{3}, \pi/3), (\frac{1}{2}\sqrt{3}, 2\pi/3)$ , and the pole

- 43.** Intersection at  $\theta \approx 0.89, 2.25$ ; area  $\approx 3.46$

**45.**  $2\pi$     **47.**  $\frac{8}{3}[(\pi^2 + 1)^{3/2} - 1]$

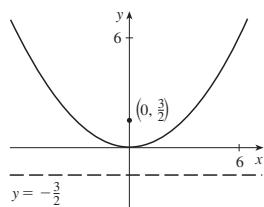


**51.** 2.4221    **53.** 8.0091

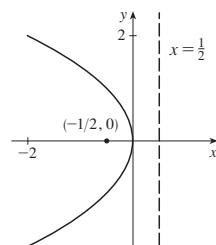
**55.** (b)  $2\pi(2 - \sqrt{2})$

**EXERCISES 10.5 ■ PAGE 700**

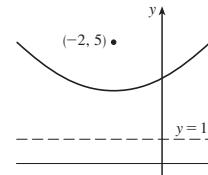
**1.**  $(0, 0), (0, \frac{3}{2}), y = -\frac{3}{2}$



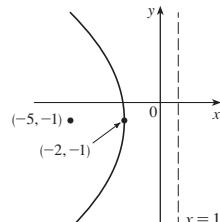
**3.**  $(0, 0), (-\frac{1}{2}, 0), x = \frac{1}{2}$



**5.**  $(-2, 3), (-2, 5), y = 1$

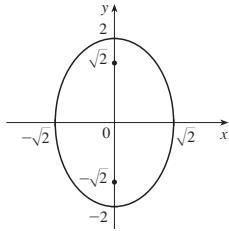


**7.**  $(-2, -1), (-5, -1), x = 1$

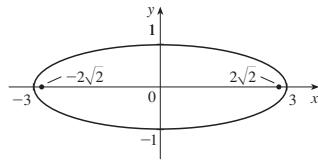


**9.**  $x = -y^2$ , focus  $(-\frac{1}{4}, 0)$ , directrix  $x = \frac{1}{4}$

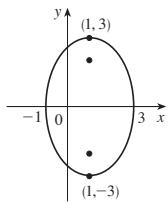
11.  $(0, \pm 2), (0, \pm \sqrt{2})$



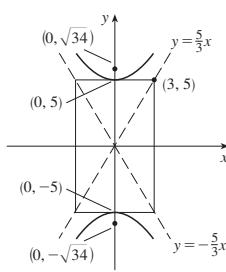
13.  $(\pm 3, 0), (\pm 2\sqrt{2}, 0)$



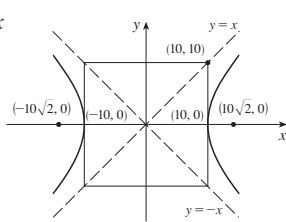
15.  $(1, \pm 3), (1, \pm \sqrt{5})$



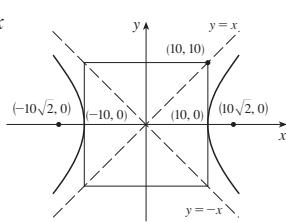
17.  $\frac{x^2}{4} + \frac{y^2}{9} = 1$ , foci  $(0, \pm\sqrt{5})$



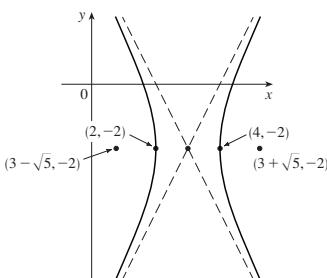
19.  $(0, \pm 5); (0, \pm\sqrt{34})$ ;  $y = \pm\frac{5}{3}x$



21.  $(\pm 10, 0), (\pm 10\sqrt{2}, 0)$ ,  $y = \pm x$



23.  $(4, -2), (2, -2);$   
 $(3 \pm \sqrt{5}, -2);$   
 $y + 2 = \pm 2(x - 3)$



25. Parabola,  $(0, -1), (0, -\frac{3}{4})$

27. Ellipse,  $(\pm\sqrt{2}, 1), (\pm 1, 1)$

29. Hyperbola,  $(0, 1), (0, -3); (0, -1 \pm \sqrt{5})$

31.  $y^2 = 4x$     33.  $y^2 = -12(x + 1)$     35.  $y - 3 = 2(x - 2)^2$

37.  $\frac{x^2}{25} + \frac{y^2}{21} = 1$     39.  $\frac{x^2}{12} + \frac{(y - 4)^2}{16} = 1$

41.  $\frac{(x + 1)^2}{12} + \frac{(y - 4)^2}{16} = 1$     43.  $\frac{x^2}{9} - \frac{y^2}{16} = 1$

45.  $\frac{(y - 1)^2}{25} - \frac{(x + 3)^2}{39} = 1$     47.  $\frac{x^2}{9} - \frac{y^2}{36} = 1$

49.  $\frac{x^2}{3,763,600} + \frac{y^2}{3,753,196} = 1$

51. (a)  $\frac{121x^2}{1,500,625} - \frac{121y^2}{3,339,375} = 1$     (b)  $\approx 248$  mi

55. (a) Ellipse    (b) Hyperbola    (c) No curve

59. 15.9

61.  $\frac{b^2c}{a} + ab \ln\left(\frac{a}{b+c}\right)$  where  $c^2 = a^2 + b^2$

63.  $(0, 4/\pi)$

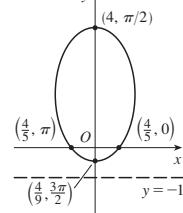
#### EXERCISES 10.6 ■ PAGE 708

1.  $r = \frac{4}{2 + \cos \theta}$     3.  $r = \frac{6}{2 + 3 \sin \theta}$

5.  $r = \frac{8}{1 - \sin \theta}$     7.  $r = \frac{4}{2 + \cos \theta}$

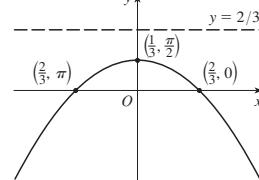
9. (a)  $\frac{4}{5}$     (b) Ellipse    (c)  $y = -1$

(d)



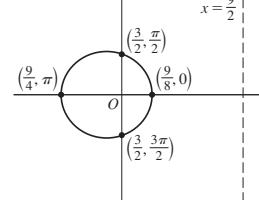
11. (a) 1    (b) Parabola    (c)  $y = \frac{2}{3}$

(d)

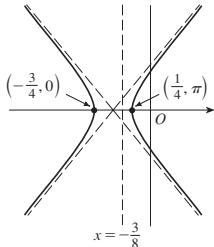


13. (a)  $\frac{1}{3}$     (b) Ellipse    (c)  $x = \frac{9}{2}$

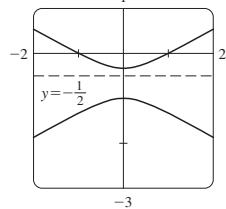
(d)



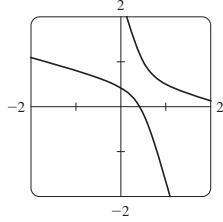
- 15.** (a) 2 (b) Hyperbola (c)  $x = -\frac{3}{8}$   
 (d)



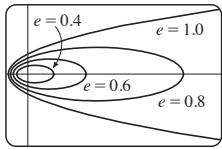
- 17.** (a)  $2, y = -\frac{1}{2}$



(b)  $r = \frac{1}{1 - 2 \sin(\theta - 3\pi/4)}$



**19.** The ellipse is nearly circular when  $e$  is close to 0 and becomes more elongated as  $e \rightarrow 1^-$ . At  $e = 1$ , the curve becomes a parabola.



**25.**  $r = \frac{2.26 \times 10^8}{1 + 0.093 \cos \theta}$

**27.** 35.64 AU    **29.**  $7.0 \times 10^7$  km

**31.**  $3.6 \times 10^8$  km

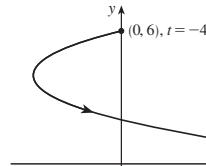
**CHAPTER 10 REVIEW ■ PAGE 709**

**True-False Quiz**

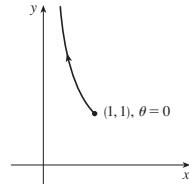
1. False    3. False    5. True    7. False    9. True

**Exercises**

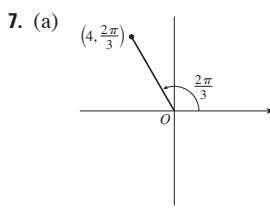
**1.**  $x = y^2 - 8y + 12$



**3.**  $y = 1/x$

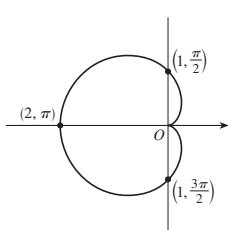


- 5.**  $x = t, y = \sqrt{t}; x = t^4, y = t^2;$   
 $x = \tan^2 t, y = \tan t, 0 \leq t < \pi/2$

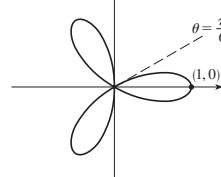


- (b)  $(3\sqrt{2}, 3\pi/4), (-3\sqrt{2}, 7\pi/4)$

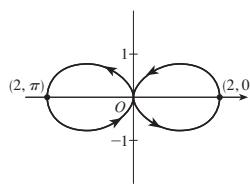
**9.**  $(-2, 2\sqrt{3})$



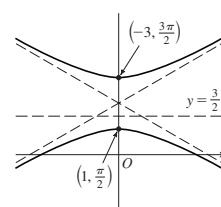
**11.**



**13.**

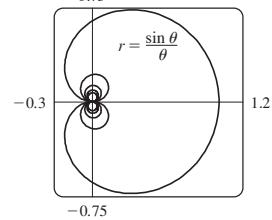


**15.**



**17.**  $r = \frac{2}{\cos \theta + \sin \theta}$

**19.**

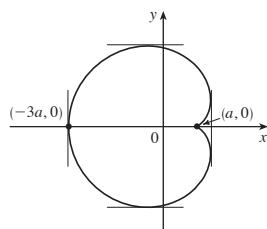


**21.** 2    **23.** -1

**25.**  $\frac{1 + \sin t}{1 + \cos t}, \frac{1 + \cos t + \sin t}{(1 + \cos t)^3}$

**27.**  $(\frac{11}{8}, \frac{3}{4})$

**29.** Vertical tangent at  $(\frac{3}{2}a, \pm \frac{1}{2}\sqrt{3}a)$ ,  $(-3a, 0)$ ;  
 horizontal tangent at  $(a, 0)$ ,  $(-\frac{1}{2}a, \pm \frac{3}{2}\sqrt{3}a)$



**31.** 18    **33.**  $(2, \pm \pi/3)$     **35.**  $\frac{1}{2}(\pi - 1)$

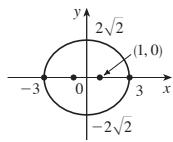
**37.**  $2(5\sqrt{5} - 1)$

**39.**  $\frac{2\sqrt{\pi^2 + 1} - \sqrt{4\pi^2 + 1}}{2\pi} + \ln\left(\frac{2\pi + \sqrt{4\pi^2 + 1}}{\pi + \sqrt{\pi^2 + 1}}\right)$

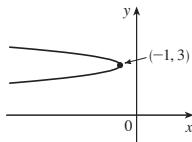
**41.**  $471,295\pi/1024$

- 43.** All curves have the vertical asymptote  $x = 1$ . For  $c < -1$ , the curve bulges to the right. At  $c = -1$ , the curve is the line  $x = 1$ . For  $-1 < c < 0$ , it bulges to the left. At  $c = 0$  there is a cusp at  $(0, 0)$ . For  $c > 0$ , there is a loop.

**45.**  $(\pm 1, 0), (\pm 3, 0)$



**47.**  $(-\frac{25}{24}, 3), (-1, 3)$



**49.**  $\frac{x^2}{25} + \frac{y^2}{9} = 1$     **51.**  $\frac{y^2}{72/5} - \frac{x^2}{8/5} = 1$

**53.**  $\frac{x^2}{25} + \frac{(8y - 399)^2}{160,801} = 1$     **55.**  $r = \frac{4}{3 + \cos \theta}$

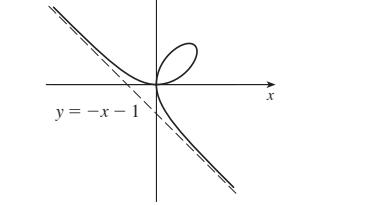
**57.** (a) At  $(0, 0)$  and  $(\frac{3}{2}, \frac{3}{2})$

(b) Horizontal tangents at  $(0, 0)$  and  $(\sqrt[3]{2}, \sqrt[3]{4})$ ;

vertical tangents at  $(0, 0)$  and  $(\sqrt[3]{4}, \sqrt[3]{2})$

(d)

(g)  $\frac{3}{2}$



### PROBLEMS PLUS ■ PAGE 712

**1.**  $\ln(\pi/2)$     **3.**  $[-\frac{3}{4}\sqrt{3}, \frac{3}{4}\sqrt{3}] \times [-1, 2]$

## CHAPTER 11

### EXERCISES 11.1 ■ PAGE 724

*Abbreviations:* C, convergent; D, divergent

- 1.** (a) A sequence is an ordered list of numbers. It can also be defined as a function whose domain is the set of positive integers.  
 (b) The terms  $a_n$  approach 8 as  $n$  becomes large.  
 (c) The terms  $a_n$  become large as  $n$  becomes large.

- 3.**  $1, \frac{4}{5}, \frac{3}{5}, \frac{8}{17}, \frac{5}{13}$     **5.**  $\frac{1}{5}, -\frac{1}{25}, \frac{1}{125}, -\frac{1}{625}, \frac{1}{3125}$     **7.**  $\frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \frac{1}{720}$   
**9.**  $1, 2, 7, 32, 157$     **11.**  $2, \frac{2}{3}, \frac{2}{5}, \frac{2}{7}, \frac{2}{9}$     **13.**  $a_n = 1/(2n - 1)$   
**15.**  $a_n = -3(-\frac{2}{3})^{n-1}$     **17.**  $a_n = (-1)^{n+1} \frac{n^2}{n+1}$   
**19.** 0.4286, 0.4615, 0.4737, 0.4800, 0.4839, 0.4865, 0.4884, 0.4898, 0.4909, 0.4918; yes;  $\frac{1}{2}$   
**21.** 0.5000, 1.2500, 0.8750, 1.0625, 0.9688, 1.0156, 0.9922, 1.0039, 0.9980, 1.0010; yes; 1  
**23.** 1    **25.** 5    **27.** 1    **29.** 1    **31.** D    **33.** 0  
**35.** D    **37.** 0    **39.** 0    **41.** 0    **43.** 0    **45.** 1  
**47.**  $e^2$     **49.**  $\ln 2$     **51.**  $\pi/2$     **53.** D    **55.** D  
**57.** 1    **59.**  $\frac{1}{2}$     **61.** D    **63.** 0  
**65.** (a) 1060, 1123.60, 1191.02, 1262.48, 1338.23    (b) D

- 67.** (a)  $P_n = 1.08P_{n-1} - 300$     (b) 5734

**69.**  $-1 < r < 1$

**71.** Convergent by the Monotonic Sequence Theorem;  $5 \leq L \leq 8$

**73.** Decreasing; yes    **75.** Not monotonic; no

**77.** Decreasing; yes

**79.** 2    **81.**  $\frac{1}{2}(3 + \sqrt{5})$     **83.** (b)  $\frac{1}{2}(1 + \sqrt{5})$

**85.** (a) 0    (b) 9, 11

### EXERCISES 11.2 ■ PAGE 735

**1.** (a) A sequence is an ordered list of numbers whereas a series is the *sum* of a list of numbers.

(b) A series is convergent if the sequence of partial sums is a convergent sequence. A series is divergent if it is not convergent.

**3.** 2

**5.** 1, 1.125, 1.1620, 1.1777, 1.1857, 1.1903, 1.1932, 1.1952; C

**7.** 0.5, 1.3284, 2.4265, 3.7598, 5.3049, 7.0443, 8.9644, 11.0540; D

**9.** -2.40000, -1.92000,

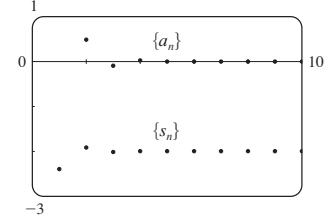
-2.01600, -1.99680,

-2.00064, -1.99987,

-2.00003, -1.99999,

-2.00000, -2.00000;

convergent, sum = -2



**11.** 0.44721, 1.15432,

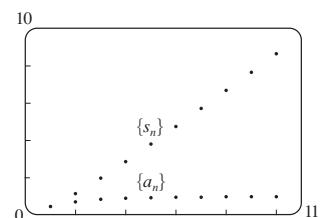
1.98637, 2.88080,

3.80927, 4.75796,

5.71948, 6.68962,

7.66581, 8.64639;

divergent



**13.** 0.29289, 0.42265,

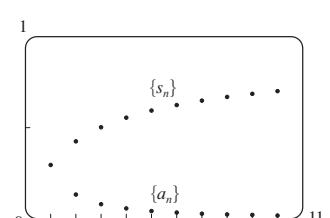
0.50000, 0.55279,

0.59175, 0.62204,

0.64645, 0.66667,

0.68377, 0.69849;

convergent, sum = 1



**15.** (a) C    (b) D    **17.** D    **19.**  $\frac{25}{3}$     **21.** 60    **23.**  $\frac{1}{7}$

**25.** D    **27.** D    **29.** D    **31.**  $\frac{5}{2}$     **33.** D    **35.** D

**37.** D    **39.** D    **41.**  $e/(e-1)$     **43.**  $\frac{3}{2}$     **45.**  $\frac{11}{6}$     **47.**  $e-1$

**49.** (b) 1    (c) 2    (d) All rational numbers with a terminating decimal representation, except 0.

**51.**  $\frac{8}{9}$     **53.**  $\frac{838}{333}$     **55.**  $5063/3300$

**57.**  $-\frac{1}{5} < x < \frac{1}{5}; \frac{-5x}{1+5x}$     **59.**  $-1 < x < 5; \frac{3}{5-x}$

**61.**  $x > 2$  or  $x < -2; \frac{x}{x-2}$     **63.**  $x < 0; \frac{1}{1-e^x}$

**65.** 1    **67.**  $a_1 = 0, a_n = \frac{2}{n(n+1)}$  for  $n > 1$ , sum = 1

**69.** (a)  $157.875 \text{ mg}$ ;  $\frac{3000}{19}(1 - 0.05^n)$  (b)  $157.895 \text{ mg}$

**71.** (a)  $S_n = \frac{D(1 - c^n)}{1 - c}$  (b) 5 **73.**  $\frac{1}{2}(\sqrt{3} - 1)$

**77.**  $\frac{1}{n(n+1)}$  **79.** The series is divergent.

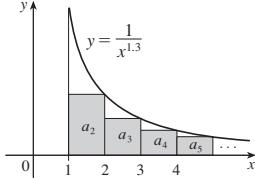
**85.**  $\{s_n\}$  is bounded and increasing.

**87.** (a)  $0, \frac{1}{9}, \frac{2}{9}, \frac{1}{3}, \frac{2}{3}, \frac{7}{9}, \frac{8}{9}, 1$

**89.** (a)  $\frac{1}{2}, \frac{5}{6}, \frac{23}{24}, \frac{119}{120}$ ;  $\frac{(n+1)! - 1}{(n+1)!}$  (c) 1

**EXERCISES 11.3 ■ PAGE 744**

**1.** C



- 3.** D **5.** C **7.** D **9.** C **11.** C **13.** D  
**15.** C **17.** C **19.** C **21.** D **23.** C **25.** C  
**27.**  $f$  is neither positive nor decreasing.  
**29.**  $p > 1$  **31.**  $p < -1$  **33.**  $(1, \infty)$   
**35.** (a)  $\frac{9}{10}\pi^4$  (b)  $\frac{1}{90}\pi^4 - \frac{17}{16}$   
**37.** (a) 1.54977, error  $\leq 0.1$  (b) 1.64522, error  $\leq 0.005$   
(c) 1.64522 compared to 1.64493 (d)  $n > 1000$   
**39.** 0.00145 **45.**  $b < 1/e$

**EXERCISES 11.4 ■ PAGE 750**

- 1.** (a) Nothing (b) C **3.** C **5.** D **7.** C **9.** D  
**11.** C **13.** C **15.** D **17.** D **19.** D **21.** C  
**23.** C **25.** D **27.** C **29.** C **31.** D  
**33.** 1.249, error  $< 0.1$  **35.** 0.0739, error  $< 6.4 \times 10^{-8}$   
**45.** Yes

**EXERCISES 11.5 ■ PAGE 755**

- 1.** (a) A series whose terms are alternately positive and negative (b)  $0 < b_{n+1} \leq b_n$  and  $\lim_{n \rightarrow \infty} b_n = 0$ , where  $b_n = |a_n|$  (c)  $|R_n| \leq b_{n+1}$   
**3.** C **5.** C **7.** D **9.** C **11.** C **13.** D **15.** C  
**17.** C **19.** D **21.**  $-0.5507$  **23.** 5 **25.** 4  
**27.**  $-0.4597$  **29.** 0.0676 **31.** An underestimate  
**33.**  $p$  is not a negative integer **35.**  $\{b_n\}$  is not decreasing

**EXERCISES 11.6 ■ PAGE 761**

*Abbreviations:* AC, absolutely convergent;  
CC, conditionally convergent

- 1.** (a) D (b) C (c) May converge or diverge  
**3.** AC **5.** CC **7.** AC **9.** D **11.** AC **13.** AC  
**15.** AC **17.** CC **19.** AC **21.** AC **23.** D **25.** AC  
**27.** AC **29.** D **31.** D **33.** AC

**35.** (a) and (d)

**39.** (a)  $\frac{661}{960} \approx 0.68854$ , error  $< 0.00521$

(b)  $n \geq 11, 0.693109$

**45.** (b)  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ ;  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

**EXERCISES 11.7 ■ PAGE 764**

- 1.** C **3.** D **5.** C **7.** D **9.** C **11.** C  
**13.** C **15.** C **17.** C **19.** C **21.** D **23.** D  
**25.** C **27.** C **29.** C **31.** D  
**33.** C **35.** D **37.** C

**EXERCISES 11.8 ■ PAGE 769**

**1.** A series of the form  $\sum_{n=0}^{\infty} c_n(x - a)^n$ , where  $x$  is a variable and  $a$  and the  $c_n$ 's are constants

**3.** 1,  $(-1, 1)$  **5.** 1,  $[-1, 1)$

**7.**  $\infty, (-\infty, \infty)$  **9.** 2,  $(-2, 2)$  **11.**  $\frac{1}{3}, [-\frac{1}{3}, \frac{1}{3}]$

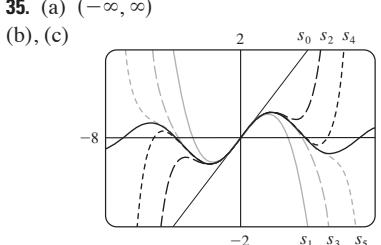
**13.** 4,  $(-4, 4]$  **15.** 1,  $[1, 3]$  **17.**  $\frac{1}{3}, [-\frac{13}{3}, -\frac{11}{3}]$

**19.**  $\infty, (-\infty, \infty)$  **21.**  $b, (a - b, a + b)$  **23.** 0,  $\{\frac{1}{2}\}$

**25.**  $\frac{1}{5}, [\frac{3}{5}, 1]$  **27.**  $\infty, (-\infty, \infty)$  **29.** (a) Yes (b) No

**31.**  $k^k$  **33.** No

**35.** (a)  $(-\infty, \infty)$



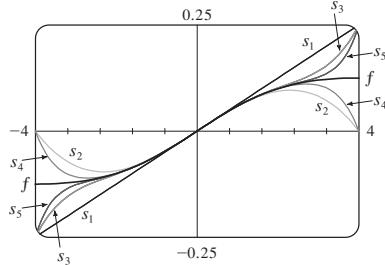
**37.**  $(-1, 1), f(x) = (1 + 2x)/(1 - x^2)$  **41.** 2

**EXERCISES 11.9 ■ PAGE 775**

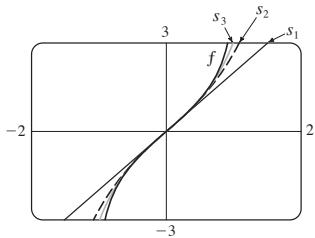
- 1.** 10 **3.**  $\sum_{n=0}^{\infty} (-1)^n x^n, (-1, 1)$  **5.**  $2 \sum_{n=0}^{\infty} \frac{1}{3^{n+1}} x^n, (-3, 3)$   
**7.**  $\sum_{n=0}^{\infty} (-1)^n \frac{1}{9^{n+1}} x^{2n+1}, (-3, 3)$  **9.**  $1 + 2 \sum_{n=1}^{\infty} x^n, (-1, 1)$   
**11.**  $\sum_{n=0}^{\infty} \left[ (-1)^{n+1} - \frac{1}{2^{n+1}} \right] x^n, (-1, 1)$   
**13.** (a)  $\sum_{n=0}^{\infty} (-1)^n (n+1)x^n, R = 1$   
(b)  $\frac{1}{2} \sum_{n=0}^{\infty} (-1)^n (n+2)(n+1)x^n, R = 1$   
(c)  $\frac{1}{2} \sum_{n=2}^{\infty} (-1)^n n(n-1)x^n, R = 1$   
**15.**  $\ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n 5^n}, R = 5$   
**17.**  $\sum_{n=0}^{\infty} (-1)^n 4^n (n+1)x^{n+1}, R = \frac{1}{4}$

19.  $\sum_{n=0}^{\infty} (2n+1)x^n, R = 1$

21.  $\sum_{n=0}^{\infty} (-1)^n \frac{1}{16^{n+1}} x^{2n+1}, R = 4$



23.  $\sum_{n=0}^{\infty} \frac{2x^{2n+1}}{2n+1}, R = 1$



25.  $C + \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2}, R = 1$

27.  $C + \sum_{n=1}^{\infty} (-1)^n \frac{x^{n+3}}{n(n+3)}, R = 1$

29. 0.199989    31. 0.000983    33. 0.19740  
35. (b) 0.920    39.  $[-1, 1], [-1, 1], (-1, 1)$

### EXERCISES 11.10 ■ PAGE 789

1.  $b_8 = f^{(8)}(5)/8!$     3.  $\sum_{n=0}^{\infty} (n+1)x^n, R = 1$

5.  $\sum_{n=0}^{\infty} (n+1)x^n, R = 1$

7.  $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} x^{2n+1}, R = \infty$

9.  $\sum_{n=0}^{\infty} \frac{(\ln 2)^n}{n!} x^n, R = \infty$     11.  $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}, R = \infty$

13.  $-1 - 2(x-1) + 3(x-1)^2 + 4(x-1)^3 + (x-1)^4, R = \infty$

15.  $\ln 2 + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n2^n} (x-2)^n, R = 2$

17.  $\sum_{n=0}^{\infty} \frac{2^n e^6}{n!} (x-3)^n, R = \infty$

19.  $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{1}{(2n)!} (x-\pi)^{2n}, R = \infty$

25.  $1 - \frac{1}{4}x - \sum_{n=2}^{\infty} \frac{3 \cdot 7 \cdots (4n-5)}{4^n \cdot n!} x^n, R = 1$

27.  $\sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2^{n+4}} x^n, R = 2$

29.  $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} x^{2n+1}, R = \infty$

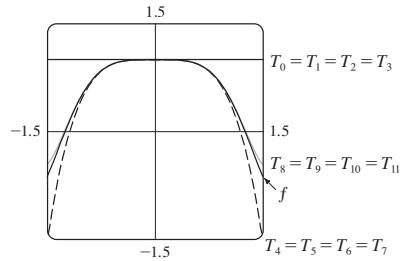
31.  $\sum_{n=0}^{\infty} \frac{2^n + 1}{n!} x^n, R = \infty$

33.  $\sum_{n=0}^{\infty} (-1)^n \frac{1}{2^{2n}(2n)!} x^{4n+1}, R = \infty$

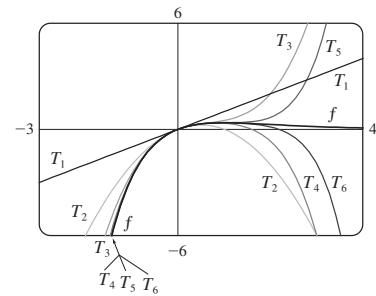
35.  $\frac{1}{2}x + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n! 2^{3n+1}} x^{2n+1}, R = 2$

37.  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^{2n-1}}{(2n)!} x^{2n}, R = \infty$

39.  $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} x^{4n}, R = \infty$



41.  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n-1)!} x^n, R = \infty$



43. 0.99619

45. (a)  $1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n n!} x^{2n}$

(b)  $x + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{(2n+1)2^n n!} x^{2n+1}$

47.  $C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+2}}{(6n+2)(2n)!}, R = \infty$

49.  $C + \sum_{n=1}^{\infty} (-1)^n \frac{1}{2n(2n)!} x^{2n}, R = \infty$

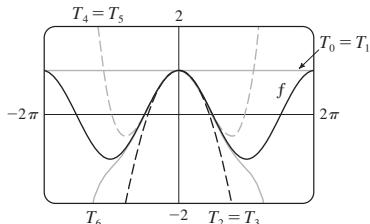
51. 0.0059    53. 0.40102    55.  $\frac{1}{2}$     57.  $\frac{1}{120}$

59.  $1 - \frac{3}{2}x^2 + \frac{25}{24}x^4$     61.  $1 + \frac{1}{6}x^2 + \frac{7}{360}x^4$     63.  $e^{-x^4}$

65.  $\ln \frac{8}{5}$     67.  $1/\sqrt{2}$     69.  $e^3 - 1$

## EXERCISES 11.11 ■ PAGE 798

1. (a)  $T_0(x) = 1 = T_1(x)$ ,  $T_2(x) = 1 - \frac{1}{2}x^2 = T_3(x)$ ,  
 $T_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 = T_5(x)$ ,  
 $T_6(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6$

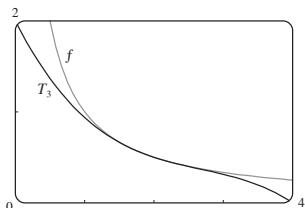


(b)

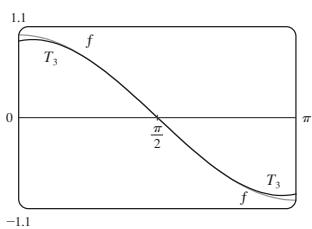
$x$	$f$	$T_0 = T_1$	$T_2 = T_3$	$T_4 = T_5$	$T_6$
$\frac{\pi}{4}$	0.7071	1	0.6916	0.7074	0.7071
$\frac{\pi}{2}$	0	1	-0.2337	0.0200	-0.0009
$\pi$	-1	1	-3.9348	0.1239	-1.2114

(c) As  $n$  increases,  $T_n(x)$  is a good approximation to  $f(x)$  on a larger and larger interval.

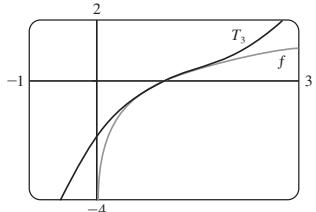
3.  $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$



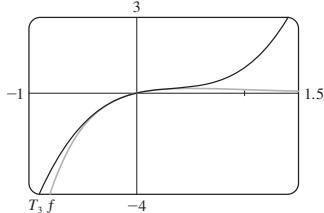
5.  $-\left(x - \frac{\pi}{2}\right) + \frac{1}{6}\left(x - \frac{\pi}{2}\right)^3$



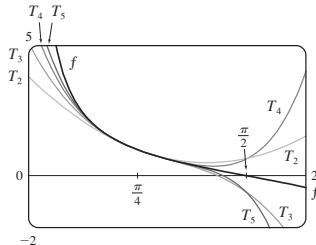
7.  $(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$



9.  $x - 2x^2 + 2x^3$



11.  $T_5(x) = 1 - 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 - \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3 + \frac{10}{3}\left(x - \frac{\pi}{4}\right)^4 - \frac{64}{15}\left(x - \frac{\pi}{4}\right)^5$



13. (a)  $2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$  (b)  $1.5625 \times 10^{-5}$

15. (a)  $1 + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{4}{81}(x-1)^3$  (b) 0.000097

17. (a)  $1 + \frac{1}{2}x^2$  (b) 0.0014

19. (a)  $1 + x^2$  (b) 0.00006 21. (a)  $x^2 - \frac{1}{6}x^4$  (b) 0.042

23. 0.17365 25. Four 27.  $-1.037 < x < 1.037$

29.  $-0.86 < x < 0.86$  31. 21 m, no

37. (c) They differ by about  $8 \times 10^{-9}$  km.

## CHAPTER 11 REVIEW ■ PAGE 802

## True-False Quiz

1. False 3. True 5. False 7. False 9. False  
 11. True 13. True 15. False 17. True  
 19. True 21. True

## Exercises

1.  $\frac{1}{2}$  3. D 5. 0 7.  $e^{12}$  9. 2 11. C 13. C

15. D 17. C 19. C 21. C 23. CC 25. AC

27.  $\frac{1}{11}$  29.  $\pi/4$  31.  $e^{-e}$  35. 0.9721

37. 0.18976224, error  $< 6.4 \times 10^{-7}$

41. 4,  $[-6, 2)$  43. 0.5,  $[2.5, 3.5)$

45.  $\frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \left[ \frac{1}{(2n)!} \left( x - \frac{\pi}{6} \right)^{2n} + \frac{\sqrt{3}}{(2n+1)!} \left( x - \frac{\pi}{6} \right)^{2n+1} \right]$

47.  $\sum_{n=0}^{\infty} (-1)^n x^{n+2}, R = 1$  49.  $\ln 4 - \sum_{n=1}^{\infty} \frac{x^n}{n 4^n}, R = 4$

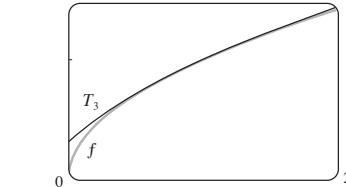
51.  $\sum_{n=0}^{\infty} (-1)^n \frac{x^{8n+4}}{(2n+1)!}, R = \infty$

53.  $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{1 \cdot 5 \cdot 9 \cdots (4n-3)}{n! 2^{6n+1}} x^n, R = 16$

55.  $C + \ln|x| + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}$

57. (a)  $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$

(b) 1.5 (c) 0.000006



59.  $-\frac{1}{6}$

### PROBLEMS PLUS ■ PAGE 805

1.  $15!/5! = 10,897,286,400$

3. (b) 0 if  $x = 0$ ,  $(1/x) - \cot x$  if  $x \neq k\pi$ ,  $k$  an integer  
5. (a)  $s_n = 3 \cdot 4^n$ ,  $l_n = 1/3^n$ ,  $p_n = 4^n/3^{n-1}$  (c)  $\frac{2}{5}\sqrt{3}$

9.  $(-1, 1), \frac{x^3 + 4x^2 + x}{(1-x)^4}$

11.  $\ln \frac{1}{2}$  13. (a)  $\frac{250}{101}\pi(e^{-(n-1)\pi/5} - e^{-n\pi/5})$  (b)  $\frac{250}{101}\pi$

19.  $\frac{\pi}{2\sqrt{3}} - 1$

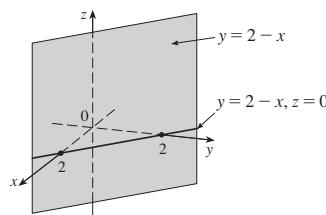
21.  $-\left(\frac{\pi}{2} - \pi k\right)^2$  where  $k$  is a positive integer

### CHAPTER 12

#### EXERCISES 12.1 ■ PAGE 814

1.  $(4, 0, -3)$  3. C; A

5. A vertical plane that intersects the  $xy$ -plane in the line  $y = 2 - x$ ,  $z = 0$



7. (a)  $|PQ| = 6$ ,  $|QR| = 2\sqrt{10}$ ,  $|RP| = 6$ ; isosceles triangle

9. (a) No (b) Yes

11.  $(x+3)^2 + (y-2)^2 + (z-5)^2 = 16$ ;  $(y-2)^2 + (z-5)^2 = 7$ ,  $x = 0$  (a circle)

13.  $(x-3)^2 + (y-8)^2 + (z-1)^2 = 30$

15.  $(1, 2, -4)$ , 6 17.  $(2, 0, -6)$ ,  $9/\sqrt{2}$

19. (b)  $\frac{5}{2}, \frac{1}{2}\sqrt{94}, \frac{1}{2}\sqrt{85}$

21. (a)  $(x-2)^2 + (y+3)^2 + (z-6)^2 = 36$

(b)  $(x-2)^2 + (y+3)^2 + (z-6)^2 = 4$

(c)  $(x-2)^2 + (y+3)^2 + (z-6)^2 = 9$

23. A plane parallel to the  $yz$ -plane and 5 units in front of it

25. A half-space consisting of all points to the left of the plane  $y = 8$

27. All points on or between the horizontal planes  $z = 0$  and  $z = 6$

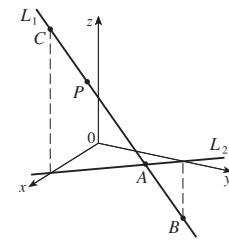
29. All points on a circle with radius 2 with center on the  $z$ -axis that is contained in the plane  $z = -1$

31. All points on or inside a sphere with radius  $\sqrt{3}$  and center  $O$

33. All points on or inside a circular cylinder of radius 3 with axis the  $y$ -axis

35.  $0 < x < 5$  37.  $r^2 < x^2 + y^2 + z^2 < R^2$

39. (a)  $(2, 1, 4)$  (b)



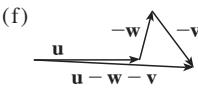
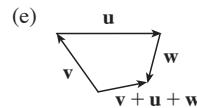
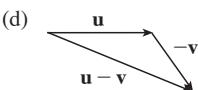
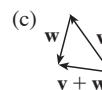
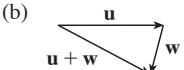
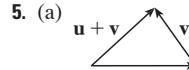
41.  $14x - 6y - 10z = 9$ , a plane perpendicular to  $AB$

43.  $2\sqrt{3} - 3$

#### EXERCISES 12.2 ■ PAGE 822

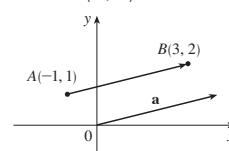
1. (a) Scalar (b) Vector (c) Vector (d) Scalar

3.  $\overrightarrow{AB} = \overrightarrow{DC}, \overrightarrow{DA} = \overrightarrow{CB}, \overrightarrow{DE} = \overrightarrow{EB}, \overrightarrow{EA} = \overrightarrow{CE}$

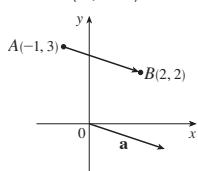


7.  $\mathbf{c} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}, \mathbf{d} = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$

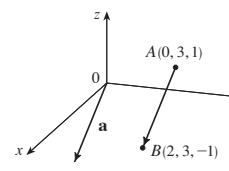
9.  $\mathbf{a} = \langle 4, 1 \rangle$



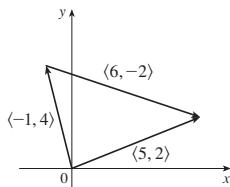
11.  $\mathbf{a} = \langle 3, -1 \rangle$



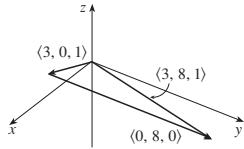
13.  $\mathbf{a} = \langle 2, 0, -2 \rangle$



15.  $\langle 5, 2 \rangle$



**17.**  $\langle 3, 8, 1 \rangle$



**19.**  $\langle 2, -18 \rangle, \langle 1, -42 \rangle, 13, 10$

**21.**  $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}, -4\mathbf{i} + \mathbf{j} + 9\mathbf{k}, \sqrt{14}, \sqrt{82}$

**23.**  $-\frac{3}{\sqrt{58}}\mathbf{i} + \frac{7}{\sqrt{58}}\mathbf{j}$     **25.**  $\frac{8}{9}\mathbf{i} - \frac{1}{9}\mathbf{j} + \frac{4}{9}\mathbf{k}$     **27.**  $60^\circ$

**29.**  $\langle 2, 2\sqrt{3} \rangle$     **31.**  $\approx 45.96 \text{ ft/s}, \approx 38.57 \text{ ft/s}$

**33.**  $100\sqrt{7} \approx 264.6 \text{ N}, \approx 139.1^\circ$

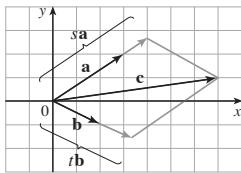
**35.**  $\sqrt{493} \approx 22.2 \text{ mi/h, } N8^\circ W$

**37.**  $\mathbf{T}_1 = -196\mathbf{i} + 3.92\mathbf{j}, \mathbf{T}_2 = 196\mathbf{i} + 3.92\mathbf{j}$

**39.** (a) At an angle of  $43.4^\circ$  from the bank, toward upstream  
(b) 20.2 min

**41.**  $\pm(\mathbf{i} + 4\mathbf{j})/\sqrt{17}$     **43.**  $\mathbf{0}$

**45.** (a), (b)



(d)  $s = \frac{9}{7}, t = \frac{11}{7}$

**47.** A sphere with radius 1, centered at  $(x_0, y_0, z_0)$

**EXERCISES 12.3 ■ PAGE 830**

**1.** (b), (c), (d) are meaningful    **3.** 14    **5.** 19    **7.** 1

**9.** -15

**11.**  $\mathbf{u} \cdot \mathbf{v} = \frac{1}{2}, \mathbf{u} \cdot \mathbf{w} = -\frac{1}{2}$

**15.**  $\cos^{-1}\left(\frac{1}{\sqrt{5}}\right) \approx 63^\circ$     **17.**  $\cos^{-1}\left(\frac{5}{\sqrt{1015}}\right) \approx 81^\circ$

**19.**  $\cos^{-1}\left(\frac{7}{\sqrt{130}}\right) \approx 52^\circ$     **21.**  $48^\circ, 75^\circ, 57^\circ$

**23.** (a) Neither    (b) Orthogonal  
(c) Orthogonal    (d) Parallel

**25.** Yes    **27.**  $(\mathbf{i} - \mathbf{j} - \mathbf{k})/\sqrt{3}$  [or  $(-\mathbf{i} + \mathbf{j} + \mathbf{k})/\sqrt{3}$ ]

**29.**  $45^\circ$     **31.**  $0^\circ$  at  $(0, 0)$ ,  $8.1^\circ$  at  $(1, 1)$

**33.**  $\frac{2}{3}, \frac{1}{3}, \frac{2}{3}; 48^\circ, 71^\circ, 48^\circ$

**35.**  $1/\sqrt{14}, -2/\sqrt{14}, -3/\sqrt{14}; 74^\circ, 122^\circ, 143^\circ$

**37.**  $1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}; 55^\circ, 55^\circ, 55^\circ$     **39.** 4,  $\left\langle -\frac{20}{13}, \frac{48}{13} \right\rangle$

**41.**  $\frac{9}{7}, \left\langle \frac{27}{49}, \frac{54}{49}, -\frac{18}{49} \right\rangle$     **43.**  $1/\sqrt{21}, \frac{2}{21}\mathbf{i} - \frac{1}{21}\mathbf{j} + \frac{4}{21}\mathbf{k}$

**47.**  $\langle 0, 0, -2\sqrt{10} \rangle$  or any vector of the form

$\langle s, t, 3s - 2\sqrt{10} \rangle, s, t \in \mathbb{R}$

**49.** 144 J    **51.**  $2400 \cos(40^\circ) \approx 1839 \text{ ft-lb}$

**53.**  $\frac{13}{5}$     **55.**  $\cos^{-1}(1/\sqrt{3}) \approx 55^\circ$

**EXERCISES 12.4 ■ PAGE 838**

**1.**  $16\mathbf{i} + 48\mathbf{k}$     **3.**  $15\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$     **5.**  $\frac{1}{2}\mathbf{i} - \mathbf{j} + \frac{3}{2}\mathbf{k}$

**7.**  $(1-t)\mathbf{i} + (t^3 - t^2)\mathbf{k}$     **9.**  $\mathbf{0}$     **11.**  $\mathbf{i} + \mathbf{j} + \mathbf{k}$

**13.** (a) Scalar    (b) Meaningless    (c) Vector  
(d) Meaningless    (e) Meaningless    (f) Scalar

**15.**  $96\sqrt{3}$ ; into the page    **17.**  $\langle -7, 10, 8 \rangle, \langle 7, -10, -8 \rangle$

**19.**  $\left\langle -\frac{1}{3\sqrt{3}}, -\frac{1}{3\sqrt{3}}, \frac{5}{3\sqrt{3}} \right\rangle, \left\langle \frac{1}{3\sqrt{3}}, \frac{1}{3\sqrt{3}}, -\frac{5}{3\sqrt{3}} \right\rangle$

**27.** 16    **29.** (a)  $\langle 0, 18, -9 \rangle$     (b)  $\frac{9}{2}\sqrt{5}$

**31.** (a)  $\langle 13, -14, 5 \rangle$     (b)  $\frac{1}{2}\sqrt{390}$

**33.** 9    **35.** 16    **39.**  $10.8 \sin 80^\circ \approx 10.6 \text{ N} \cdot \text{m}$

**41.**  $\approx 417 \text{ N}$     **43.**  $60^\circ$

**45.** (b)  $\sqrt{97/3}$     **53.** (a) No    (b) No    (c) Yes

**EXERCISES 12.5 ■ PAGE 848**

**1.** (a) True    (b) False    (c) True    (d) False    (e) False  
(f) True    (g) False    (h) True    (i) True    (j) False

(k) True

**3.**  $\mathbf{r} = (2\mathbf{i} + 2.4\mathbf{j} + 3.5\mathbf{k}) + t(3\mathbf{i} + 2\mathbf{j} - \mathbf{k});$   
 $x = 2 + 3t, y = 2.4 + 2t, z = 3.5 - t$

**5.**  $\mathbf{r} = (\mathbf{i} + 6\mathbf{k}) + t(\mathbf{i} + 3\mathbf{j} + \mathbf{k});$   
 $x = 1 + t, y = 3t, z = 6 + t$

**7.**  $x = 2 + 2t, y = 1 + \frac{1}{2}t, z = -3 - 4t;$   
 $(x-2)/2 = 2y-2 = (z+3)/(-4)$

**9.**  $x = -8 + 11t, y = 1 - 3t, z = 4; \frac{x+8}{11} = \frac{y-1}{-3}, z = 4$

**11.**  $x = 1 + t, y = -1 + 2t, z = 1 + t;$   
 $x-1 = (y+1)/2 = z-1$

**13.** Yes

**15.** (a)  $(x-1)/(-1) = (y+5)/2 = (z-6)/(-3)$   
(b)  $(-1, -1, 0), (-\frac{3}{2}, 0, -\frac{3}{2}), (0, -3, 3)$

**17.**  $\mathbf{r}(t) = (2\mathbf{i} - \mathbf{j} + 4\mathbf{k}) + t(2\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}), 0 \leq t \leq 1$

**19.** Skew    **21.**  $(4, -1, -5)$     **23.**  $x - 2y + 5z = 0$

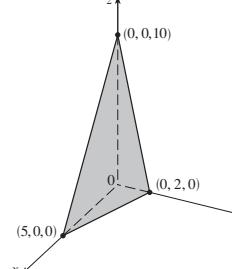
**25.**  $x + 4y + z = 4$     **27.**  $5x - y - z = 7$

**29.**  $6x + 6y + 6z = 11$     **31.**  $x + y + z = 2$

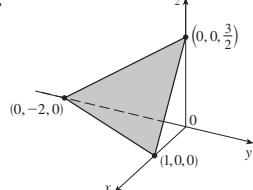
**33.**  $-13x + 17y + 7z = -42$     **35.**  $33x + 10y + 4z = 190$

**37.**  $x - 2y + 4z = -1$     **39.**  $3x - 8y - z = -38$

**41.**



**43.**



**45.** (2, 3, 5)    **47.** (2, 3, 1)    **49.** 1, 0, -1

**51.** Perpendicular    **53.** Neither,  $\cos^{-1}(\frac{1}{3}) \approx 70.5^\circ$

**55.** Parallel

**57.** (a)  $x = 1, y = -t, z = t$     (b)  $\cos^{-1}\left(\frac{5}{3\sqrt{3}}\right) \approx 15.8^\circ$

**59.**  $x = 1, y = 2 - z$     **61.**  $x + 2y + z = 5$

**63.**  $(x/a) + (y/b) + (z/c) = 1$

**65.**  $x = 3t, y = 1 - t, z = 2 - 2t$

**67.**  $P_2$  and  $P_3$  are parallel,  $P_1$  and  $P_4$  are identical

69.  $\sqrt{61/14}$

71.  $\frac{18}{7}$

73.  $5/(2\sqrt{14})$

77.  $1/\sqrt{6}$

79.  $13/\sqrt{69}$

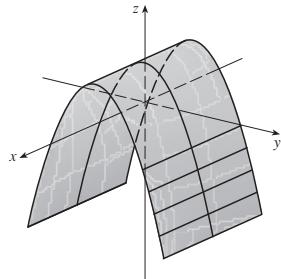
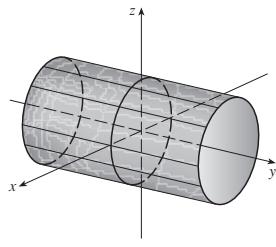
**EXERCISES 12.6 ■ PAGE 856**

1. (a) Parabola

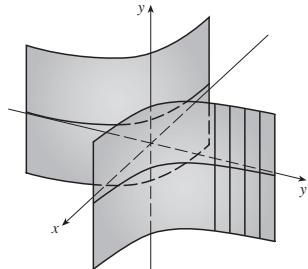
(b) Parabolic cylinder with rulings parallel to the  $z$ -axis  
(c) Parabolic cylinder with rulings parallel to the  $x$ -axis

3. Circular cylinder

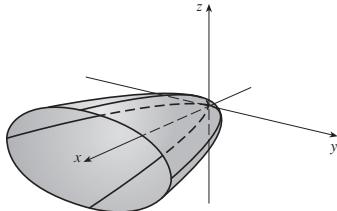
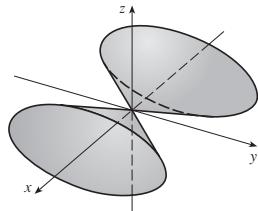
5. Parabolic cylinder



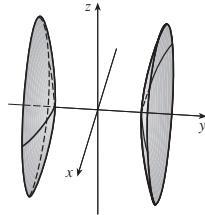
7. Hyperbolic cylinder



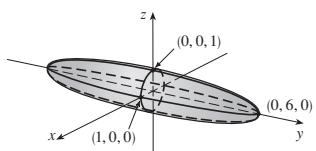
9. (a)  $x = k, y^2 - z^2 = 1 - k^2$ , hyperbola ( $k \neq \pm 1$ );  
 $y = k, x^2 - z^2 = 1 - k^2$ , hyperbola ( $k \neq \pm 1$ );  
 $z = k, x^2 + y^2 = 1 + k^2$ , circle  
(b) The hyperboloid is rotated so that it has axis the  $y$ -axis  
(c) The hyperboloid is shifted one unit in the negative  $y$ -direction

11. Elliptic paraboloid with axis the  $x$ -axis13. Elliptic cone with axis the  $x$ -axis

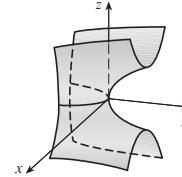
15. Hyperboloid of two sheets



17. Ellipsoid

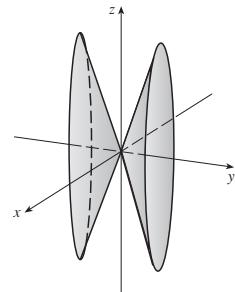


19. Hyperbolic paraboloid



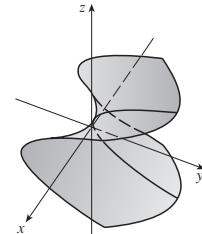
21. VII    23. II    25. VI    27. VIII

29.  $y^2 = x^2 + \frac{z^2}{9}$

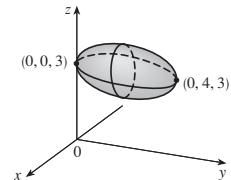
Elliptic cone with axis the  $y$ -axis

31.  $y = z^2 - \frac{x^2}{2}$

Hyperbolic paraboloid

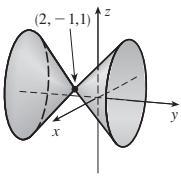


33.  $x^2 + \frac{(y-2)^2}{4} + (z-3)^2 = 1$

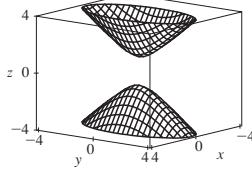
Ellipsoid with center  
 $(0, 2, 3)$ 

**35.**  $(y + 1)^2 = (x - 2)^2 + (z - 1)^2$

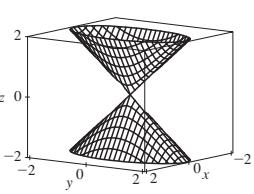
Circular cone with vertex  $(2, -1, 1)$  and axis parallel to the  $y$ -axis



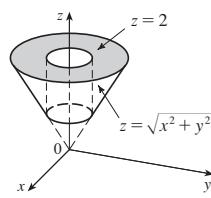
**37.**



**39.**



**41.**



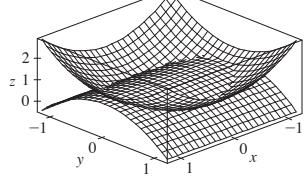
**43.**  $y = x^2 + z^2$

**45.**  $-4x = y^2 + z^2$ , paraboloid

**47.** (a)  $\frac{x^2}{(6378.137)^2} + \frac{y^2}{(6378.137)^2} + \frac{z^2}{(6356.523)^2} = 1$

(b) Circle (c) Ellipse

**51.**



### CHAPTER 12 REVIEW ■ PAGE 858

#### True-False Quiz

1. False 3. False 5. True 7. True 9. True  
 11. True 13. True 15. False 17. False  
 19. False 21. True

#### Exercises

1. (a)  $(x + 1)^2 + (y - 2)^2 + (z - 1)^2 = 69$

(b)  $(y - 2)^2 + (z - 1)^2 = 68$ ,  $x = 0$

(c) Center  $(4, -1, -3)$ , radius 5

3.  $\mathbf{u} \cdot \mathbf{v} = 3\sqrt{2}$ ;  $|\mathbf{u} \times \mathbf{v}| = 3\sqrt{2}$ ; out of the page

5.  $-2, -4$  7. (a) 2 (b) -2 (c) -2 (d) 0

9.  $\cos^{-1}\left(\frac{1}{3}\right) \approx 71^\circ$  11. (a)  $\langle 4, -3, 4 \rangle$  (b)  $\sqrt{41}/2$

13. 166 N, 114 N

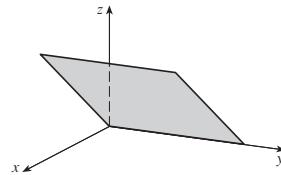
15.  $x = 4 - 3t$ ,  $y = -1 + 2t$ ,  $z = 2 + 3t$

17.  $x = -2 + 2t$ ,  $y = 2 - t$ ,  $z = 4 + 5t$

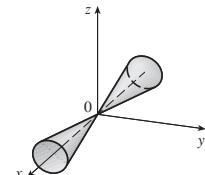
19.  $-4x + 3y + z = -14$  21.  $(1, 4, 4)$  23. Skew

25.  $x + y + z = 4$  27.  $22/\sqrt{26}$

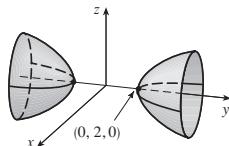
**29. Plane**



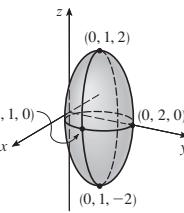
**31. Cone**



**33. Hyperboloid of two sheets**



**35. Ellipsoid**



**37.**  $4x^2 + y^2 + z^2 = 16$

### PROBLEMS PLUS ■ PAGE 861

1.  $(\sqrt{3} - \frac{3}{2})$  m

3. (a)  $(x + 1)/(-2c) = (y - c)/(c^2 - 1) = (z - c)/(c^2 + 1)$

(b)  $x^2 + y^2 = t^2 + 1$ ,  $z = t$  (c)  $4\pi/3$

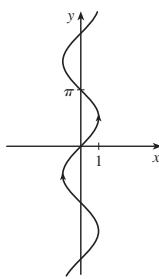
5. 20

### CHAPTER 13

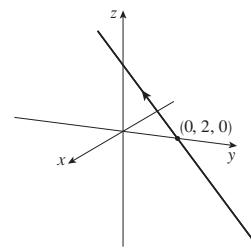
#### EXERCISES 13.1 ■ PAGE 869

1.  $(-1, 2]$  3.  $\mathbf{i} + \mathbf{j} + \mathbf{k}$  5.  $\langle -1, \pi/2, 0 \rangle$

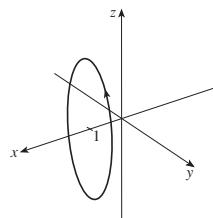
7.



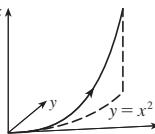
9.



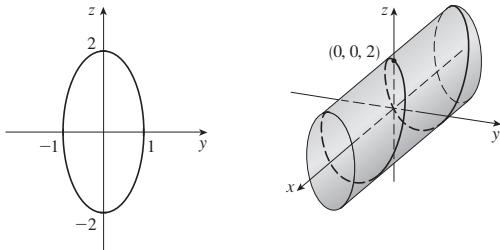
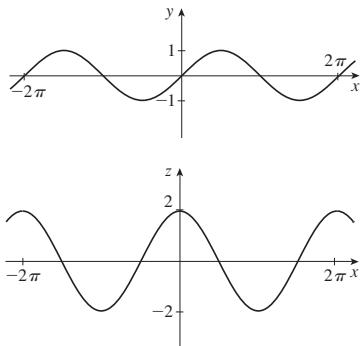
11.



13.



15.

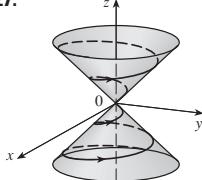


17.  $\mathbf{r}(t) = \langle 2 + 4t, 2t, -2t \rangle, 0 \leq t \leq 1;$   
 $x = 2 + 4t, y = 2t, z = -2t, 0 \leq t \leq 1$

19.  $\mathbf{r}(t) = \left\langle \frac{1}{2}t, -1 + \frac{4}{3}t, 1 - \frac{3}{4}t \right\rangle, 0 \leq t \leq 1;$   
 $x = \frac{1}{2}t, y = -1 + \frac{4}{3}t, z = 1 - \frac{3}{4}t, 0 \leq t \leq 1$

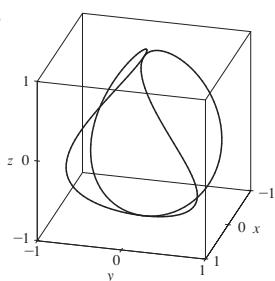
21. II    23. V    25. IV

27.

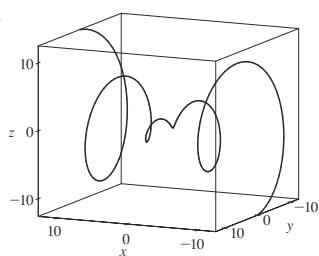


29.  $(0, 0, 0), (1, 0, 1)$

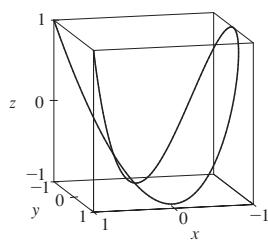
31.



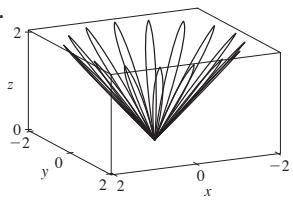
33.



35.



37.



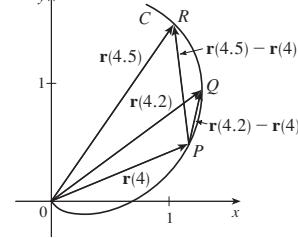
41.  $\mathbf{r}(t) = t \mathbf{i} + \frac{1}{2}(t^2 - 1) \mathbf{j} + \frac{1}{2}(t^2 + 1) \mathbf{k}$

43.  $\mathbf{r}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + \cos 2t \mathbf{k}, 0 \leq t \leq 2\pi$

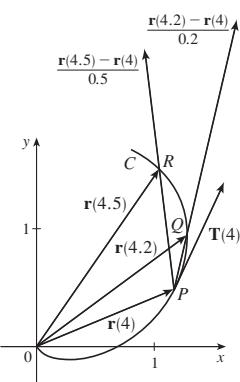
45.  $x = 2 \cos t, y = 2 \sin t, z = 4 \cos^2 t$     47. Yes

### EXERCISES 13.2 ■ PAGE 876

1. (a)

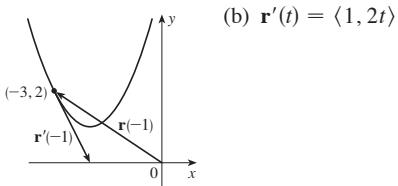


(b), (d)



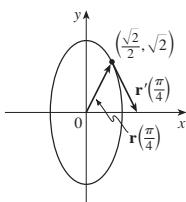
(c)  $\mathbf{r}'(4) = \lim_{h \rightarrow 0} \frac{\mathbf{r}(4+h) - \mathbf{r}(4)}{h}; \mathbf{T}(4) = \frac{\mathbf{r}'(4)}{\|\mathbf{r}'(4)\|}$

3. (a), (c)

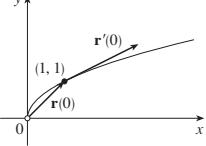


(b)  $\mathbf{r}'(t) = \langle 1, 2t \rangle$

5. (a), (c)



(b)  $\mathbf{r}'(t) = \cos t \mathbf{i} - 2 \sin t \mathbf{j}$

7. (a), (c) 

(b)  $\mathbf{r}'(t) = 2e^{2t}\mathbf{i} + e^t\mathbf{j}$

9.  $\mathbf{r}'(t) = \langle t \cos t + \sin t, 2t, \cos 2t - 2t \sin 2t \rangle$

11.  $\mathbf{r}'(t) = \mathbf{i} + (\frac{1}{\sqrt{t}})\mathbf{k}$

13.  $\mathbf{r}'(t) = 2te^{t^2}\mathbf{i} + [3/(1+3t)]\mathbf{k}$

15.  $\mathbf{r}'(t) = \mathbf{b} + 2t\mathbf{c}$

17.  $\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$

19.  $\frac{3}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}$

21.  $\langle 1, 2t, 3t^2 \rangle, \langle 1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14} \rangle, \langle 0, 2, 6t \rangle, \langle 6t^2, -6t, 2 \rangle$

23.  $x = 3 + t, y = 2t, z = 2 + 4t$

25.  $x = 1 - t, y = t, z = 1 - t$

27.  $\mathbf{r}(t) = (3 - 4t)\mathbf{i} + (4 + 3t)\mathbf{j} + (2 - 6t)\mathbf{k}$

29.  $x = t, y = 1 - t, z = 2t$

31.  $x = -\pi - t, y = \pi + t, z = -\pi t$

33.  $66^\circ$

35.  $2\mathbf{i} - 4\mathbf{j} + 32\mathbf{k}$

37.  $\mathbf{i} + \mathbf{j} + \mathbf{k}$

39.  $\tan t\mathbf{i} + \frac{1}{8}(t^2 + 1)^4\mathbf{j} + (\frac{1}{3}t^3 \ln t - \frac{1}{9}t^3)\mathbf{k} + \mathbf{C}$

41.  $t^2\mathbf{i} + t^3\mathbf{j} + (\frac{2}{3}t^{3/2} - \frac{2}{3})\mathbf{k}$

47.  $2t \cos t + 2 \sin t - 2 \cos t \sin t$

49. 35

**EXERCISES 13.3 ■ PAGE 884**

1.  $10\sqrt{10}$

3.  $e - e^{-1}$

5.  $\frac{1}{27}(13^{3/2} - 8)$

7. 18.6833

9. 1.2780

11. 42

13.  $\mathbf{r}(t(s)) = \frac{2}{\sqrt{29}}s\mathbf{i} + \left(1 - \frac{3}{\sqrt{29}}s\right)\mathbf{j} + \left(5 + \frac{4}{\sqrt{29}}s\right)\mathbf{k}$

15.  $(3 \sin 1, 4, 3 \cos 1)$

17. (a)  $\langle 1/\sqrt{10}, (-3/\sqrt{10})\sin t, (3/\sqrt{10})\cos t \rangle, \langle 0, -\cos t, -\sin t \rangle$

(b)  $\frac{3}{10}$

19. (a)  $\frac{1}{e^{2t}+1}\langle \sqrt{2}e^t, e^{2t}, -1 \rangle, \frac{1}{e^{2t}+1}\langle 1 - e^{2t}, \sqrt{2}e^t, \sqrt{2}e^t \rangle$

(b)  $\sqrt{2}e^{2t}/(e^{2t} + 1)^2$

21.  $6t^2/(9t^4 + 4t^2)^{3/2}$

23.  $\frac{4}{25}$

25.  $\frac{1}{7}\sqrt{\frac{19}{14}}$

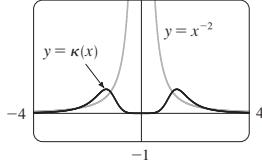
27.  $12x^2/(1 + 16x^6)^{3/2}$

29.  $e^x|x + 2|/[1 + (xe^x + e^x)^2]^{3/2}$

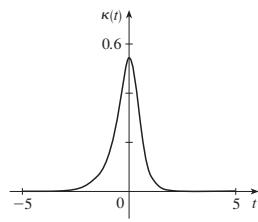
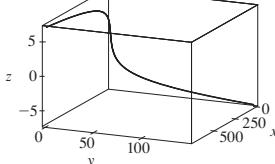
31.  $(-\frac{1}{2} \ln 2, 1/\sqrt{2})$ ; approaches 0

33. (a)  $P$

35.

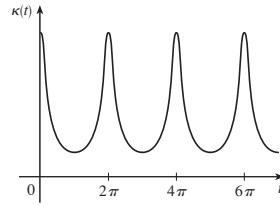


37.



39.  $a$  is  $y = f(x)$ ,  $b$  is  $y = \kappa(x)$

41.  $\kappa(t) = \frac{6\sqrt{4 \cos^2 t - 12 \cos t + 13}}{(17 - 12 \cos t)^{3/2}}$



integer multiples of  $2\pi$

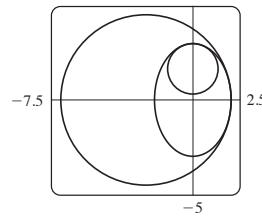
43.  $6t^2/(4t^2 + 9t^4)^{3/2}$

45.  $1/(\sqrt{2}e^t)$

47.  $\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle, \langle -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \rangle, \langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \rangle$

49.  $y = 6x + \pi, x + 6y = 6\pi$

51.  $(x + \frac{5}{2})^2 + y^2 = \frac{81}{4}, x^2 + (y - \frac{5}{3})^2 = \frac{16}{9}$



53.  $(-1, -3, 1)$

55.  $2x + y + 4z = 7, 6x - 8y - z = -3$

63.  $2/(t^4 + 4t^2 + 1)$

65.  $2.07 \times 10^{10} \text{ \AA} \approx 2 \text{ m}$

**EXERCISES 13.4 ■ PAGE 894**

1. (a)  $1.8\mathbf{i} - 3.8\mathbf{j} - 0.7\mathbf{k}, 2.0\mathbf{i} - 2.4\mathbf{j} - 0.6\mathbf{k},$

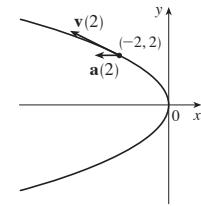
2.  $8\mathbf{i} + 1.8\mathbf{j} - 0.3\mathbf{k}, 2.8\mathbf{i} + 0.8\mathbf{j} - 0.4\mathbf{k}$

(b)  $2.4\mathbf{i} - 0.8\mathbf{j} - 0.5\mathbf{k}, 2.58$

3.  $\mathbf{v}(t) = \langle -t, 1 \rangle$

$\mathbf{a}(t) = \langle -1, 0 \rangle$

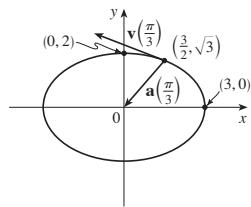
$|\mathbf{v}(t)| = \sqrt{t^2 + 1}$



5.  $\mathbf{v}(t) = -3 \sin t \mathbf{i} + 2 \cos t \mathbf{j}$

$\mathbf{a}(t) = -3 \cos t \mathbf{i} - 2 \sin t \mathbf{j}$

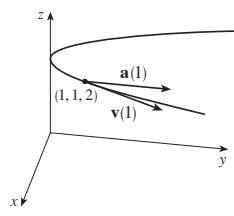
$|\mathbf{v}(t)| = \sqrt{9 \sin^2 t + 4}$



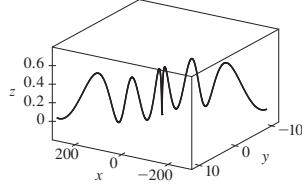
7.  $\mathbf{v}(t) = \mathbf{i} + 2t\mathbf{j}$

$\mathbf{a}(t) = 2\mathbf{j}$

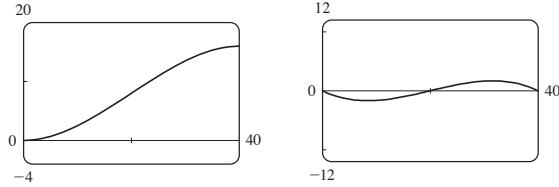
$|\mathbf{v}(t)| = \sqrt{1 + 4t^2}$



9.  $\langle 2t+1, 2t-1, 3t^2 \rangle$ ,  $\langle 2, 2, 6t \rangle$ ,  $\sqrt{9t^4 + 8t^2 + 2}$   
 11.  $\sqrt{2}\mathbf{i} + e^t\mathbf{j} - e^{-t}\mathbf{k}$ ,  $e^t\mathbf{j} + e^{-t}\mathbf{k}$ ,  $e^t + e^{-t}$   
 13.  $e^t[(\cos t - \sin t)\mathbf{i} + (\sin t + \cos t)\mathbf{j} + (t+1)\mathbf{k}]$ ,  
 $e^t[-2\sin t\mathbf{i} + 2\cos t\mathbf{j} + (t+2)\mathbf{k}]$ ,  $e^t\sqrt{t^2 + 2t + 3}$   
 15.  $\mathbf{v}(t) = t\mathbf{i} + 2t\mathbf{j} + \mathbf{k}$ ,  $\mathbf{r}(t) = (\frac{1}{3}t^2 + 1)\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$   
 17. (a)  $\mathbf{r}(t) = (\frac{1}{3}t^3 + t)\mathbf{i} + (t - \sin t + 1)\mathbf{j} + (\frac{1}{3} - \frac{1}{4}\cos 2t)\mathbf{k}$   
 (b)



19.  $t = 4$     21.  $\mathbf{r}(t) = t\mathbf{i} - t\mathbf{j} + \frac{5}{2}t^2\mathbf{k}$ ,  $|\mathbf{v}(t)| = \sqrt{25t^2 + 2}$   
 23. (a)  $\approx 3535$  m    (b)  $\approx 1531$  m    (c)  $200$  m/s  
 25.  $30$  m/s    27.  $\approx 10.2^\circ$ ,  $\approx 79.8^\circ$   
 29.  $13.0^\circ < \theta < 36.0^\circ$ ,  $55.4^\circ < \theta < 85.5^\circ$   
 31.  $(250, -50, 0)$ ;  $10\sqrt{93} \approx 96.4$  ft/s  
 33. (a)  $16$  m    (b)  $\approx 23.6^\circ$  upstream



35. The path is contained in a circle that lies in a plane perpendicular to  $\mathbf{c}$  with center on a line through the origin in the direction of  $\mathbf{c}$ .

37.  $6t, 6$     39.  $0, 1$     41.  $e^t - e^{-t}, \sqrt{2}$   
 43.  $4.5 \text{ cm/s}^2, 9.0 \text{ cm/s}^2$     45.  $t = 1$

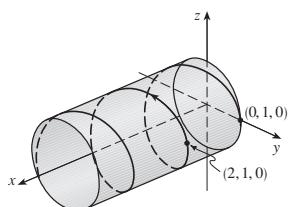
### CHAPTER 13 REVIEW ■ PAGE 897

#### True-False Quiz

1. True    3. False    5. False    7. False  
 9. True    11. False    13. True

#### Exercises

1. (a)



- (b)  $\mathbf{r}'(t) = \mathbf{i} - \pi \sin \pi t \mathbf{j} + \pi \cos \pi t \mathbf{k}$ ,  
 $\mathbf{r}''(t) = -\pi^2 \cos \pi t \mathbf{j} - \pi^2 \sin \pi t \mathbf{k}$   
 3.  $\mathbf{r}(t) = 4 \cos t \mathbf{i} + 4 \sin t \mathbf{j} + (5 - 4 \cos t) \mathbf{k}$ ,  $0 \leq t \leq 2\pi$   
 5.  $\frac{1}{3}\mathbf{i} - (2/\pi^2)\mathbf{j} + (2/\pi)\mathbf{k}$     7.  $86.631$     9.  $\pi/2$   
 11. (a)  $\langle t^2, t, 1 \rangle / \sqrt{t^4 + t^2 + 1}$   
 (b)  $\langle t^3 + 2t, 1 - t^4, -2t^3 - t \rangle / \sqrt{t^8 + 5t^6 + 6t^4 + 5t^2 + 1}$   
 (c)  $\sqrt{t^8 + 5t^6 + 6t^4 + 5t^2 + 1} / (t^4 + t^2 + 1)^2$   
 13.  $12/17^{3/2}$     15.  $x - 2y + 2\pi = 0$

17.  $\mathbf{v}(t) = (1 + \ln t)\mathbf{i} + \mathbf{j} - e^{-t}\mathbf{k}$ ,  
 $|\mathbf{v}(t)| = \sqrt{2 + 2 \ln t + (\ln t)^2 + e^{-2t}}$ ,  $\mathbf{a}(t) = (1/t)\mathbf{i} + e^{-t}\mathbf{k}$   
 19. (a) About  $3.8$  ft above the ground,  $60.8$  ft from the athlete  
 (b)  $\approx 21.4$  ft    (c)  $\approx 64.2$  ft from the athlete  
 21. (c)  $-2e^{-t}\mathbf{v}_d + e^{-t}\mathbf{R}$   
 23. (a)  $\mathbf{v} = \omega R(-\sin \omega t \mathbf{i} + \cos \omega t \mathbf{j})$     (c)  $\mathbf{a} = -\omega^2 \mathbf{r}$

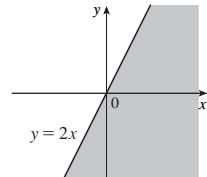
### PROBLEMS PLUS ■ PAGE 900

1. (a)  $90^\circ, v_0^2/(2g)$   
 3. (a)  $\approx 0.94$  ft to the right of the table's edge,  $\approx 15$  ft/s  
 (b)  $\approx 7.6^\circ$     (c)  $\approx 2.13$  ft to the right of the table's edge  
 5.  $56^\circ$   
 7.  $\mathbf{r}(u, v) = \mathbf{c} + u\mathbf{a} + v\mathbf{b}$  where  $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ ,  
 $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ ,  $\mathbf{c} = \langle c_1, c_2, c_3 \rangle$

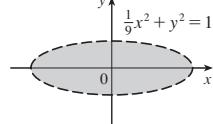
### CHAPTER 14

#### EXERCISES 14.1 ■ PAGE 912

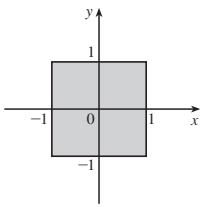
1. (a)  $-27$ ; a temperature of  $-15^\circ\text{C}$  with wind blowing at  $40$  km/h feels equivalent to about  $-27^\circ\text{C}$  without wind.  
 (b) When the temperature is  $-20^\circ\text{C}$ , what wind speed gives a wind chill of  $-30^\circ\text{C}$ ?  $20$  km/h  
 (c) With a wind speed of  $20$  km/h, what temperature gives a wind chill of  $-49^\circ\text{C}$ ?  $-35^\circ\text{C}$   
 (d) A function of wind speed that gives wind-chill values when the temperature is  $-5^\circ\text{C}$   
 (e) A function of temperature that gives wind-chill values when the wind speed is  $50$  km/h  
 3.  $\approx 94.2$ ; the manufacturer's yearly production is valued at  $\$94.2$  million when  $120,000$  labor hours are spent and  $\$20$  million in capital is invested.  
 5. (a)  $\approx 20.5$ ; the surface area of a person  $70$  inches tall who weighs  $160$  pounds is approximately  $20.5$  square feet.  
 7. (a)  $25$ ; a  $40$ -knot wind blowing in the open sea for  $15$  h will create waves about  $25$  ft high.  
 (b)  $f(30, t)$  is a function of  $t$  giving the wave heights produced by  $30$ -knot winds blowing for  $t$  hours.  
 (c)  $f(v, 30)$  is a function of  $v$  giving the wave heights produced by winds of speed  $v$  blowing for  $30$  hours.  
 9. (a)  $1$     (b)  $\mathbb{R}^2$     (c)  $[-1, 1]$   
 11. (a)  $3$     (b)  $\{(x, y, z) | x^2 + y^2 + z^2 < 4, x \geq 0, y \geq 0, z \geq 0\}$ , interior of a sphere of radius  $2$ , center the origin, in the first octant  
 13.  $\{(x, y) | y \leq 2x\}$



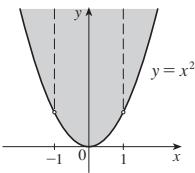
15.  $\{(x, y) | \frac{1}{9}x^2 + y^2 < 1\}, (-\infty, \ln 9]$



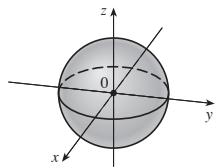
17.  $\{(x, y) \mid -1 \leq x \leq 1, -1 \leq y \leq 1\}$



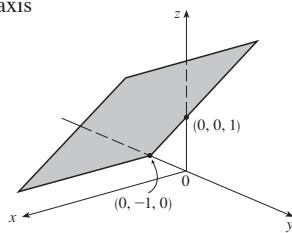
19.  $\{(x, y) \mid y \geq x^2, x \neq \pm 1\}$



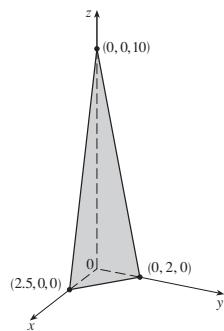
21.  $\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$



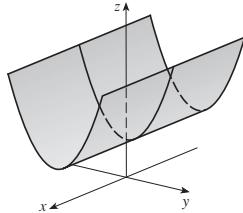
23.  $z = 1 + y$ , plane parallel to  $x$ -axis



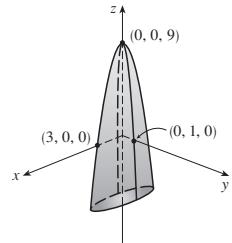
25.  $4x + 5y + z = 10$ , plane



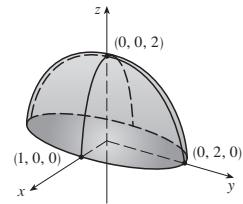
27.  $z = y^2 + 1$ , parabolic cylinder



29.  $z = 9 - x^2 - 9y^2$ , elliptic paraboloid



31.  $z = \sqrt{4 - 4x^2 - y^2}$ , top half of ellipsoid

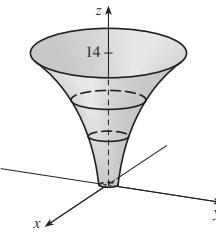


33.  $\approx 56, \approx 35$

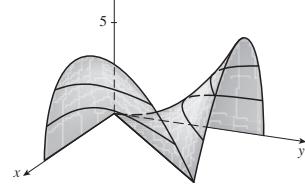
35.  $11^\circ\text{C}, 19.5^\circ\text{C}$

37. Steep; nearly flat

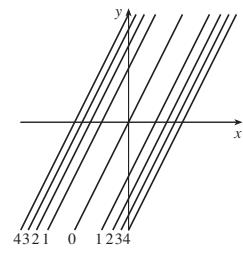
39.



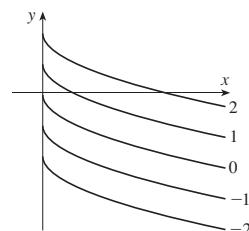
41.



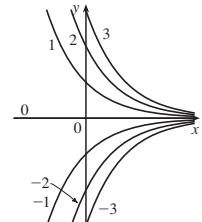
43.  $(y - 2x)^2 = k$



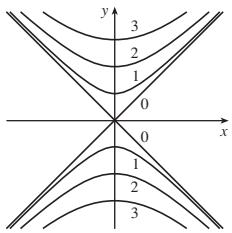
45.  $y = -\sqrt{x} + k$



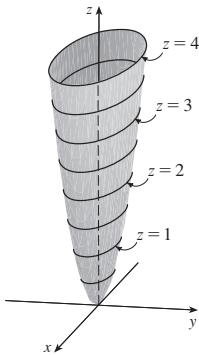
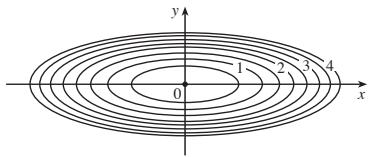
47.  $y = ke^{-x}$



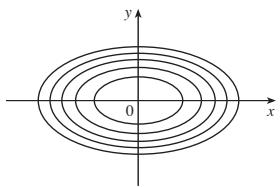
49.  $y^2 - x^2 = k^2$



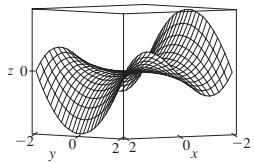
51.  $x^2 + 9y^2 = k$



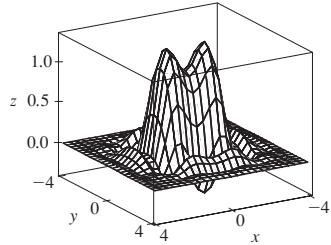
53.



55.



57.

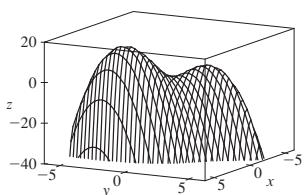


59. (a) C (b) II

63. (a) B (b) VI

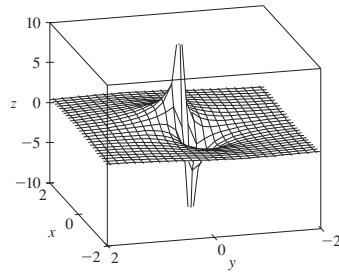
67. Family of circular cylinders with axis the  $x$ -axis ( $k > 0$ )69. (a) Shift the graph of  $f$  upward 2 units(b) Stretch the graph of  $f$  vertically by a factor of 2(c) Reflect the graph of  $f$  about the  $xy$ -plane(d) Reflect the graph of  $f$  about the  $xy$ -plane and then shift it upward 2 units

71.



$f$  appears to have a maximum value of about 15. There are two local maximum points but no local minimum point.

73.



The function values approach 0 as  $x, y$  become large; as  $(x, y)$  approaches the origin,  $f$  approaches  $\pm\infty$  or 0, depending on the direction of approach.

75. If  $c = 0$ , the graph is a cylindrical surface. For  $c > 0$ , the level curves are ellipses. The graph curves upward as we leave the origin, and the steepness increases as  $c$  increases. For  $c < 0$ , the level curves are hyperbolas. The graph curves upward in the  $y$ -direction and downward, approaching the  $xy$ -plane, in the  $x$ -direction giving a saddle-shaped appearance near  $(0, 0, 1)$ .

77.  $c = -2, 0, 2$ 79. (b)  $y = 0.75x + 0.01$ 

## EXERCISES 14.2 ■ PAGE 923

1. Nothing; if  $f$  is continuous,  $f(3, 1) = 6$ 3.  $-\frac{5}{2}$ 

5. 1

7.  $\frac{2}{7}$ 

9. Does not exist

11. Does not exist

13. 0

15. Does not exist

17. 2

19.  $\sqrt{3}$ 

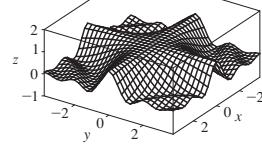
21. Does not exist

23. The graph shows that the function approaches different numbers along different lines.

25.  $h(x, y) = (2x + 3y - 6)^2 + \sqrt{2x + 3y - 6};$ { $(x, y) \mid 2x + 3y \geq 6$ }27. Along the line  $y = x$ 29.  $\mathbb{R}^2$ 31.  $\{(x, y) \mid x^2 + y^2 \neq 1\}$ 33.  $\{(x, y) \mid x^2 + y^2 > 4\}$ 35.  $\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1\}$ 37.  $\{(x, y) \mid (x, y) \neq (0, 0)\}$ 

39. 0 41. -1

43.

 $f$  is continuous on  $\mathbb{R}^2$ 

## EXERCISES 14.3 ■ PAGE 935

1. (a) The rate of change of temperature as longitude varies, with latitude and time fixed; the rate of change as only latitude varies; the rate of change as only time varies.

(b) Positive, negative, positive

3. (a)  $f_T(-15, 30) \approx 1.3$ ; for a temperature of  $-15^\circ\text{C}$  and wind speed of  $30 \text{ km/h}$ , the wind-chill index rises by  $1.3^\circ\text{C}$  for each degree the temperature increases.  $f_v(-15, 30) \approx -0.15$ ; for a temperature of  $-15^\circ\text{C}$  and wind speed of  $30 \text{ km/h}$ , the wind-chill index decreases by  $0.15^\circ\text{C}$  for each  $\text{km/h}$  the wind speed increases.

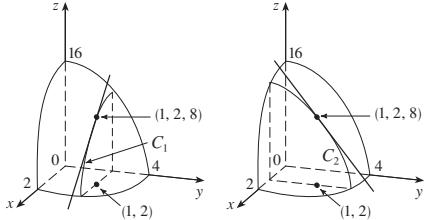
(b) Positive, negative (c) 0

5. (a) Positive (b) Negative

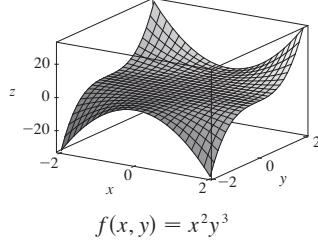
7. (a) Positive (b) Negative

9.  $c = f$ ,  $b = f_x$ ,  $a = f_y$

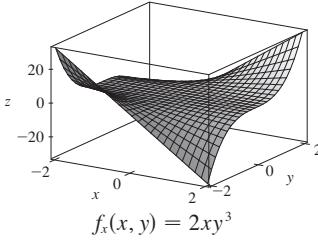
11.  $f_x(1, 2) = -8$  = slope of  $C_1$ ,  $f_y(1, 2) = -4$  = slope of  $C_2$



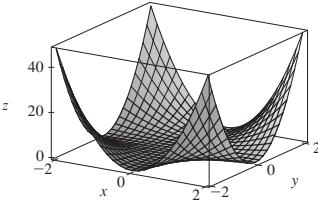
13.



$$f(x, y) = x^2y^3$$



$$f(x, y) = 2xy^3$$



$$f(y, x) = 3x^2y^2$$

15.  $f_x(x, y) = -3y$ ,  $f_y(x, y) = 5y^4 - 3x$

17.  $f_x(x, t) = -\pi e^{-t} \sin \pi x$ ,  $f_t(x, t) = -e^{-t} \cos \pi x$

19.  $\partial z / \partial x = 20(2x + 3y)^9$ ,  $\partial z / \partial y = 30(2x + 3y)^9$

21.  $f_x(x, y) = 1/y$ ,  $f_y(x, y) = -x/y^2$

23.  $f_x(x, y) = \frac{(ad - bc)y}{(cx + dy)^2}$ ,  $f_y(x, y) = \frac{(bc - ad)x}{(cx + dy)^2}$

25.  $g_u(u, v) = 10uv(u^2v - v^3)^4$ ,  $g_v(u, v) = 5(u^2 - 3v^2)(u^2v - v^3)^4$

27.  $R_p(p, q) = \frac{q^2}{1 + p^2q^4}$ ,  $R_q(p, q) = \frac{2pq}{1 + p^2q^4}$

29.  $F_x(x, y) = \cos(e^x)$ ,  $F_y(x, y) = -\cos(e^y)$

31.  $f_z = z - 10xy^3z^4$ ,  $f_y = -15x^2y^2z^4$ ,  $f_z = x - 20x^2y^3z^3$

33.  $\partial w / \partial x = 1/(x + 2y + 3z)$ ,  $\partial w / \partial y = 2/(x + 2y + 3z)$ ,  $\partial w / \partial z = 3/(x + 2y + 3z)$

35.  $\frac{\partial u}{\partial x} = y \sin^{-1}(yz)$ ,  $\frac{\partial u}{\partial y} = x \sin^{-1}(yz) + xyz / \sqrt{1 - y^2z^2}$ ,  $\frac{\partial u}{\partial z} = xy^2 / \sqrt{1 - y^2z^2}$

37.  $h_x = 2xy \cos(z/t)$ ,  $h_y = x^2 \cos(z/t)$ ,  $h_z = (-x^2y/t) \sin(z/t)$ ,  $h_t = (x^2yz/t^2) \sin(z/t)$

39.  $\partial u / \partial x_i = x_i / \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$

41.  $\frac{1}{5}$     43.  $\frac{1}{4}$     45.  $f_x(x, y) = y^2 - 3x^2y$ ,  $f_y(x, y) = 2xy - x^3$

47.  $\frac{\partial z}{\partial x} = -\frac{x}{3z}$ ,  $\frac{\partial z}{\partial y} = -\frac{2y}{3z}$

49.  $\frac{\partial z}{\partial x} = \frac{yz}{e^z - xy}$ ,  $\frac{\partial z}{\partial y} = \frac{xz}{e^z - xy}$

51. (a)  $f'(x)$ ,  $g'(y)$     (b)  $f'(x+y)$ ,  $f'(x+y)$

53.  $f_{xx} = 6xy^5 + 24x^2y$ ,  $f_{xy} = 15x^2y^4 + 8x^3 = f_{yx}$ ,  $f_{yy} = 20x^3y^3$

55.  $w_{uu} = v^2 / (u^2 + v^2)^{3/2}$ ,  $w_{uv} = -uv / (u^2 + v^2)^{3/2} = w_{vu}$ ,  $w_{vv} = u^2 / (u^2 + v^2)^{3/2}$

57.  $z_{xx} = -2x / (1 + x^2)^2$ ,  $z_{xy} = 0 = z_{yx}$ ,  $z_{yy} = -2y / (1 + y^2)^2$

63.  $24xy^2 - 6y$ ,  $24x^2y - 6x$     65.  $(2x^2y^2z^5 + 6xyz^3 + 2z)e^{xyz^2}$

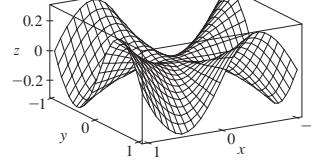
67.  $\theta e^{r\theta} (2 \sin \theta + \theta \cos \theta + r\theta \sin \theta)$     69.  $4/(y+2z)^3$ , 0

71.  $6yz^2$     73.  $\approx 12.2$ ,  $\approx 16.8$ ,  $\approx 23.25$     83.  $R^2/R_1^2$

87.  $\frac{\partial T}{\partial P} = \frac{V - nb}{nR}$ ,  $\frac{\partial P}{\partial V} = \frac{2n^2a}{V^3} - \frac{nRT}{(V - nb)^2}$

93. No    95.  $x = 1 + t$ ,  $y = 2$ ,  $z = 2 - 2t$     99. -2

101. (a)



(b)  $f_x(x, y) = \frac{x^4y + 4x^2y^3 - y^5}{(x^2 + y^2)^2}$ ,  $f_y(x, y) = \frac{x^5 - 4x^3y^2 - xy^4}{(x^2 + y^2)^2}$

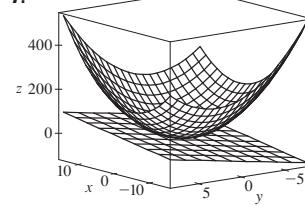
(c) 0, 0    (e) No, since  $f_{xy}$  and  $f_{yx}$  are not continuous.

#### EXERCISES 14.4 ■ PAGE 946

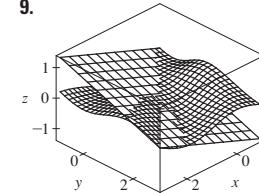
1.  $z = -7x - 6y + 5$     3.  $x + y - 2z = 0$

5.  $x + y + z = 0$

7.



9.



11.  $6x + 4y - 23$     13.  $\frac{1}{9}x - \frac{2}{9}y + \frac{2}{3}$     15.  $1 - \pi y$

19. 6.3    21.  $\frac{3}{7}x + \frac{2}{7}y + \frac{6}{7}z$ ; 6.9914

23.  $4T + H - 329$ ; 129°F

25.  $dz = -2e^{-2x} \cos 2\pi t dx - 2\pi e^{-2x} \sin 2\pi t dt$

27.  $dm = 5p^4q^3 dp + 3p^5q^2 dq$

29.  $dR = \beta^2 \cos \gamma d\alpha + 2\alpha\beta \cos \gamma d\beta - \alpha\beta^2 \sin \gamma d\gamma$

31.  $\Delta z = 0.9225$ ,  $dz = 0.9$     33.  $5.4 \text{ cm}^2$     35.  $16 \text{ cm}^3$

37.  $\approx -0.0165mg$ ; decrease

39.  $\frac{1}{17} \approx 0.059 \Omega$     41. 2.3%    43.  $\varepsilon_1 = \Delta x$ ,  $\varepsilon_2 = \Delta y$

**EXERCISES 14.5 ■ PAGE 954**

1.  $(2x + y) \cos t + (2y + x)e^t$

3.  $[(x/t) - y \sin t]/\sqrt{1 + x^2 + y^2}$

5.  $e^{y/z}[2t - (x/z) - (2xy/z^2)]$

7.  $\frac{\partial z}{\partial s} = 2xy^3 \cos t + 3x^2y^2 \sin t$ ,  
 $\frac{\partial z}{\partial t} = -2sxy^3 \sin t + 3sx^2y^2 \cos t$

9.  $\frac{\partial z}{\partial s} = t^2 \cos \theta \cos \phi - 2st \sin \theta \sin \phi$ ,  
 $\frac{\partial z}{\partial t} = 2st \cos \theta \cos \phi - s^2 \sin \theta \sin \phi$

11.  $\frac{\partial z}{\partial s} = e^r \left( t \cos \theta - \frac{s}{\sqrt{s^2 + t^2}} \sin \theta \right)$ ,  
 $\frac{\partial z}{\partial t} = e^r \left( s \cos \theta - \frac{t}{\sqrt{s^2 + t^2}} \sin \theta \right)$

13. 62    15. 7, 2

17.  $\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial r}, \frac{\partial u}{\partial s} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial s}$ ,  
 $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial t}$

19.  $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial w}{\partial s} \frac{\partial s}{\partial x} + \frac{\partial w}{\partial t} \frac{\partial t}{\partial x}$ ,  
 $\frac{\partial w}{\partial y} = \frac{\partial w}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial w}{\partial s} \frac{\partial s}{\partial y} + \frac{\partial w}{\partial t} \frac{\partial t}{\partial y}$

21. 1582, 3164, -700    23.  $2\pi, -2\pi$

25.  $\frac{5}{144}, -\frac{5}{96}, \frac{5}{144}$     27.  $\frac{2x + y \sin x}{\cos x - 2y}$

29.  $\frac{1 + x^4y^2 + y^2 + x^4y^4 - 2xy}{x^2 - 2xy - 2x^5y^3}$

31.  $-\frac{x}{3z}, -\frac{2y}{3z}$     33.  $\frac{yz}{e^z - xy}, \frac{xz}{e^z - xy}$

35.  $2^\circ\text{C}/\text{s}$     37.  $\approx -0.33 \text{ m/s per minute}$

39. (a)  $6 \text{ m}^3/\text{s}$     (b)  $10 \text{ m}^3/\text{s}$     (c)  $0 \text{ m/s}$

41.  $\approx -0.27 \text{ L/s}$     43.  $-1/(12\sqrt{3}) \text{ rad/s}$

45. (a)  $\frac{\partial z}{\partial r} = (\frac{\partial z}{\partial x}) \cos \theta + (\frac{\partial z}{\partial y}) \sin \theta$ ,

$\frac{\partial z}{\partial \theta} = -(\frac{\partial z}{\partial x})r \sin \theta + (\frac{\partial z}{\partial y})r \cos \theta$

51.  $4rs \frac{\partial^2 z}{\partial x^2} + (4r^2 + 4s^2) \frac{\partial^2 z}{\partial x \partial y} + 4rs \frac{\partial^2 z}{\partial y^2} + 2 \frac{\partial z}{\partial y}$

**EXERCISES 14.6 ■ PAGE 967**

1.  $\approx -0.08 \text{ mb/km}$     3.  $\approx 0.778$     5.  $2 + \sqrt{3}/2$

7. (a)  $\nabla f(x, y) = \langle 2 \cos(2x + 3y), 3 \cos(2x + 3y) \rangle$

(b)  $\langle 2, 3 \rangle$     (c)  $\sqrt{3} - \frac{3}{2}$

9. (a)  $\langle 2xyz - yz^3, x^2z - xz^3, x^2y - 3xyz^2 \rangle$

(b)  $\langle -3, 2, 2 \rangle$     (c)  $\frac{2}{5}$

11.  $\frac{4 - 3\sqrt{3}}{10}$     13.  $-8/\sqrt{10}$     15.  $4/\sqrt{30}$

17.  $\frac{23}{42}$     19.  $2/5$     21.  $\sqrt{65}, \langle 1, 8 \rangle$

23. 1,  $\langle 0, 1 \rangle$     25. 1,  $\langle 3, 6, -2 \rangle$

27. (b)  $\langle -12, 92 \rangle$

29. All points on the line  $y = x + 1$ 

31. (a)  $-40/(3\sqrt{3})$

33. (a)  $32/\sqrt{3}$     (b)  $\langle 38, 6, 12 \rangle$     (c)  $2\sqrt{406}$

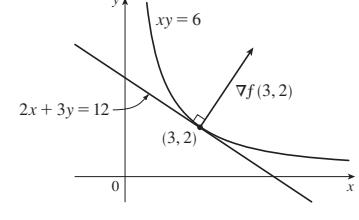
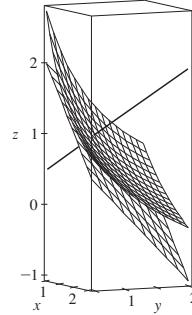
35.  $\frac{327}{13}$     39.  $\frac{774}{25}$

41. (a)  $x + y + z = 11$     (b)  $x - 3 = y - 3 = z - 5$

43. (a)  $2x + 3y + 12z = 24$     (b)  $\frac{x - 3}{2} = \frac{y - 2}{3} = \frac{z - 1}{12}$

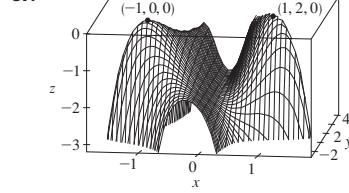
45. (a)  $x + y + z = 1$     (b)  $x = y = z - 1$

49.  $\langle 2, 3 \rangle, 2x + 3y = 12$



55. No    59.  $\left(-\frac{5}{4}, -\frac{5}{4}, \frac{25}{8}\right)$

63.  $x = -1 - 10t, y = 1 - 16t, z = 2 - 12t$

67. If  $\mathbf{u} = \langle a, b \rangle$  and  $\mathbf{v} = \langle c, d \rangle$ , then  $af_x + bf_y$  and  $cf_x + df_y$  are known, so we solve linear equations for  $f_x$  and  $f_y$ .**EXERCISES 14.7 ■ PAGE 977**1. (a)  $f$  has a local minimum at  $(1, 1)$ .(b)  $f$  has a saddle point at  $(1, 1)$ .3. Local minimum at  $(1, 1)$ , saddle point at  $(0, 0)$ 5. Minimum  $f\left(\frac{1}{3}, -\frac{2}{3}\right) = -\frac{1}{3}$ 7. Saddle points at  $(1, 1), (-1, -1)$ 9. Maximum  $f(0, 0) = 2$ , minimum  $f(0, 4) = -30$ , saddle points at  $(2, 2), (-2, 2)$ 11. Minimum  $f(2, 1) = -8$ , saddle point at  $(0, 0)$ 13. None    15. Minimum  $f(0, 0) = 0$ , saddle points at  $(\pm 1, 0)$ 17. Minima  $f(0, 1) = f(\pi, -1) = f(2\pi, 1) = -1$ , saddle points at  $(\pi/2, 0), (3\pi/2, 0)$ 21. Minima  $f(1, \pm 1) = 3, f(-1, \pm 1) = 3$ 23. Maximum  $f(\pi/3, \pi/3) = 3\sqrt{3}/2$ , minimum  $f(5\pi/3, 5\pi/3) = -3\sqrt{3}/2$ , saddle point at  $(\pi, \pi)$ 25. Minima  $f(0, -0.794) \approx -1.191, f(\pm 1.592, 1.267) \approx -1.310$ , saddle points  $(\pm 0.720, 0.259)$ , lowest points  $(\pm 1.592, 1.267, -1.310)$ 27. Maximum  $f(0.170, -1.215) \approx 3.197$ ,minima  $f(-1.301, 0.549) \approx -3.145, f(1.131, 0.549) \approx -0.701$ , saddle points  $(-1.301, -1.215), (0.170, 0.549), (1.131, -1.215)$ , no highest or lowest point29. Maximum  $f(0, \pm 2) = 4$ , minimum  $f(1, 0) = -1$ 31. Maximum  $f(\pm 1, 1) = 7$ , minimum  $f(0, 0) = 4$ 33. Maximum  $f(3, 0) = 83$ , minimum  $f(1, 1) = 0$ 35. Maximum  $f(1, 0) = 2$ , minimum  $f(-1, 0) = -2$ 

39.  $2/\sqrt{3}$     41.  $(2, 1, \sqrt{5}), (2, 1, -\sqrt{5})$     43.  $\frac{100}{3}, \frac{100}{3}, \frac{100}{3}$   
 45.  $8r^3/(3\sqrt{3})$     47.  $\frac{4}{3}$     49. Cube, edge length  $c/12$   
 51. Square base of side 40 cm, height 20 cm    53.  $L^3/(3\sqrt{3})$

**EXERCISES 14.8 ■ PAGE 987**

1.  $\approx 59, 30$   
 3. No maximum, minimum  $f(1, 1) = f(-1, -1) = 2$   
 5. Maximum  $f(0, \pm 1) = 1$ , minimum  $f(\pm 2, 0) = -4$   
 7. Maximum  $f(2, 2, 1) = 9$ , minimum  $f(-2, -2, -1) = -9$   
 9. Maximum  $2/\sqrt{3}$ , minimum  $-2/\sqrt{3}$   
 11. Maximum  $\sqrt{3}$ , minimum 1  
 13. Maximum  $f(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) = 2$ ,  
 minimum  $f(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}) = -2$   
 15. Maximum  $f(1, \sqrt{2}, -\sqrt{2}) = 1 + 2\sqrt{2}$ ,  
 minimum  $f(1, -\sqrt{2}, \sqrt{2}) = 1 - 2\sqrt{2}$   
 17. Maximum  $\frac{3}{2}$ , minimum  $\frac{1}{2}$   
 19. Maximum  $f(3/\sqrt{2}, -3/\sqrt{2}) = 9 + 12\sqrt{2}$ ,  
 minimum  $f(-2, 2) = -8$   
 21. Maximum  $f(\pm 1/\sqrt{2}, \mp 1/(2\sqrt{2})) = e^{1/4}$ ,  
 minimum  $f(\pm 1/\sqrt{2}, \pm 1/(2\sqrt{2})) = e^{-1/4}$   
**29–41.** See Exercises 39–53 in Section 14.7.  
 43. Nearest  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ , farthest  $(-1, -1, 2)$   
 45. Maximum  $\approx 9.7938$ , minimum  $\approx -5.3506$   
 47. (a)  $c/n$     (b) When  $x_1 = x_2 = \dots = x_n$

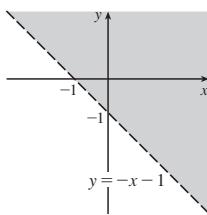
**CHAPTER 14 REVIEW ■ PAGE 991**

**True-False Quiz**

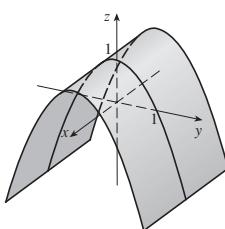
1. True    3. False    5. False    7. True    9. False  
 11. True

**Exercises**

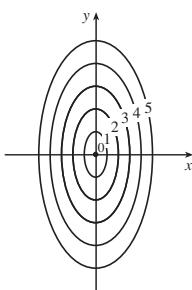
1.  $\{(x, y) | y > -x - 1\}$



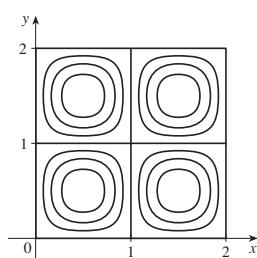
3.



5.



7.



9.  $\frac{2}{3}$   
 11. (a)  $\approx 3.5^\circ\text{C}/\text{m}$ ,  $-3.0^\circ\text{C}/\text{m}$     (b)  $\approx 0.35^\circ\text{C}/\text{m}$  by Equation 14.6.9 (Definition 14.6.2 gives  $\approx 1.1^\circ\text{C}/\text{m}$ ).  
 (c)  $-0.25$   
 13.  $f_x = 32xy(5y^3 + 2x^2y)^7, f_y = (16x^2 + 120y^2)(5y^3 + 2x^2y)^7$   
 15.  $F_\alpha = \frac{2\alpha^3}{\alpha^2 + \beta^2} + 2\alpha \ln(\alpha^2 + \beta^2), F_\beta = \frac{2\alpha^2\beta}{\alpha^2 + \beta^2}$   
 17.  $S_u = \arctan(v\sqrt{w}), S_v = \frac{u\sqrt{w}}{1 + v^2w}, S_w = \frac{uv}{2\sqrt{w}(1 + v^2w)}$   
 19.  $f_{xx} = 24x, f_{xy} = -2y = f_{yx}, f_{yy} = -2x$   
 21.  $f_{xx} = k(k-1)x^{k-2}y^l z^m, f_{xy} = klx^{k-1}y^{l-1}z^m = f_{yx},$   
 $f_{xz} = kmx^{k-1}y^l z^{m-1} = f_{zx}, f_{yy} = l(l-1)x^k y^{l-2} z^m,$   
 $f_{yz} = lmx^k y^{l-1} z^{m-1} = f_{zy}, f_{zz} = m(m-1)x^k y^l z^{m-2}$   
 25. (a)  $z = 8x + 4y + 1$     (b)  $\frac{x-1}{8} = \frac{y+2}{4} = \frac{z-1}{-1}$   
 27. (a)  $2x - 2y - 3z = 3$     (b)  $\frac{x-2}{4} = \frac{y+1}{-4} = \frac{z-1}{-6}$   
 29. (a)  $x + 2y + 5z = 0$   
 (b)  $x = 2 + t, y = -1 + 2t, z = 5t$   
 31.  $(2, \frac{1}{2}, -1), (-2, -\frac{1}{2}, 1)$   
 33.  $60x + \frac{24}{5}y + \frac{32}{5}z = 120; 38.656$   
 35.  $2xy^3(1 + 6p) + 3x^2y^2(pe^p + e^p) + 4z^3(p \cos p + \sin p)$   
 37.  $-47, 108$   
 43.  $\langle 2xe^{yz^2}, x^2z^2e^{yz^2}, 2x^2yze^{yz^2} \rangle$     45.  $-\frac{4}{5}$   
 47.  $\sqrt{145}/2, \langle 4, \frac{9}{2} \rangle$     49.  $\approx \frac{5}{8}$  knot/mi  
 51. Minimum  $f(-4, 1) = -11$   
 53. Maximum  $f(1, 1) = 1$ ; saddle points  $(0, 0), (0, 3), (3, 0)$   
 55. Maximum  $f(1, 2) = 4$ , minimum  $f(2, 4) = -64$   
 57. Maximum  $f(-1, 0) = 2$ , minima  $f(1, \pm 1) = -3$ , saddle points  $(-1, \pm 1), (1, 0)$   
 59. Maximum  $f(\pm\sqrt{2/3}, 1/\sqrt{3}) = 2/(3\sqrt{3})$ , minimum  $f(\pm\sqrt{2/3}, -1/\sqrt{3}) = -2/(3\sqrt{3})$   
 61. Maximum 1, minimum  $-1$   
 63.  $(\pm 3^{-1/4}, 3^{-1/4}\sqrt{2}, \pm 3^{1/4}), (\pm 3^{-1/4}, -3^{-1/4}\sqrt{2}, \pm 3^{1/4})$   
 65.  $P(2 - \sqrt{3}), P(3 - \sqrt{3})/6, P(2\sqrt{3} - 3)/3$

**PROBLEMS PLUS ■ PAGE 995**

1.  $L^2W^2, \frac{1}{4}L^2W^2$     3. (a)  $x = w/3$ , base  $= w/3$     (b) Yes  
 7.  $\sqrt{3}/2, 3/\sqrt{2}$

**CHAPTER 15**

**EXERCISES 15.1 ■ PAGE 1005**

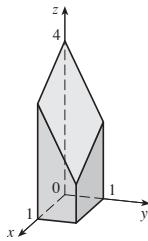
1. (a) 288    (b) 144    3. (a) 0.990    (b) 1.151  
 5. (a) 4    (b) -8    7.  $U < V < L$   
 9. (a)  $\approx 248$     (b)  $\approx 15.5$     11. 60    13. 3  
 15. 1.141606, 1.143191, 1.143535, 1.143617, 1.143637, 1.143642

**EXERCISES 15.2 ■ PAGE 1011**

1.  $500y^3, 3x^2$     3. 222    5.  $32(e^4 - 1)$     7. 18  
 9.  $\frac{21}{2} \ln 2$     11.  $\frac{31}{30}$     13.  $\pi$     15. 0

17.  $9 \ln 2$     19.  $\frac{1}{2}(\sqrt{3} - 1) - \frac{1}{12}\pi$     21.  $\frac{1}{2}e^{-6} + \frac{5}{2}$

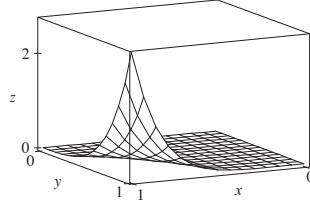
23.



25. 51    27.  $\frac{166}{27}$

33.  $21e - 57$

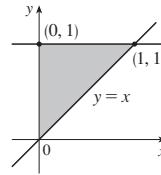
29. 2    31.  $\frac{64}{3}$



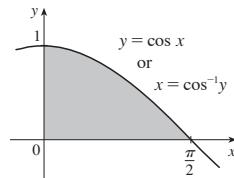
35.  $\frac{5}{6}$     37. 0

39. Fubini's Theorem does not apply. The integrand has an infinite discontinuity at the origin.

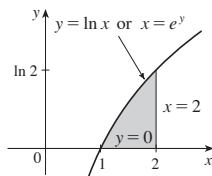
43.  $\int_0^1 \int_x^1 f(x, y) dy dx$



45.  $\int_0^1 \int_0^{\cos^{-1}y} f(x, y) dx dy$



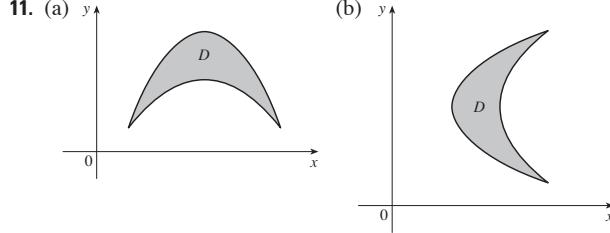
47.  $\int_0^{\ln 2} \int_{e^y}^2 f(x, y) dx dy$



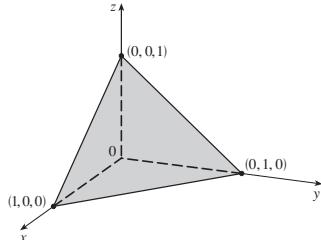
49.  $\frac{1}{6}(e^9 - 1)$     51.  $\frac{1}{3} \ln 9$     53.  $\frac{1}{2}(2\sqrt{2} - 1)$     55. 1  
57.  $(\pi/16)e^{-1/16} \leq \iint_D e^{-(x^2+y^2)^2} dA \leq \pi/16$     59.  $\frac{3}{4}$     63.  $9\pi$   
65.  $a^2b + \frac{3}{2}ab^2$     67.  $\pi a^2 b$

## EXERCISES 15.3 ■ PAGE 1019

1. 32    3.  $\frac{3}{10}$     5.  $\frac{1}{3} \sin 1$     7.  $\frac{4}{3}$     9.  $\pi$



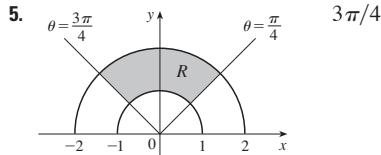
13. Type I:  $D = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq x\}$ ,  
type II:  $D = \{(x, y) \mid 0 \leq y \leq 1, y \leq x \leq 1\}$ ;  $\frac{1}{3}$   
15.  $\int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} y dy dx + \int_1^4 \int_{x-2}^{\sqrt{x}} y dy dx = \int_{-1}^2 \int_{y^2}^{y+2} y dx dy = \frac{9}{4}$   
17.  $\frac{1}{2}(1 - \cos 1)$     19.  $\frac{11}{3}$     21. 0    23.  $\frac{17}{60}$     25.  $\frac{31}{8}$   
27. 6    29.  $\frac{128}{15}$     31.  $\frac{1}{3}$     33. 0, 1.213; 0.713    35.  $\frac{64}{3}$   
37.



39.  $13,984,735,616/14,549,535$   
41.  $\pi/2$

## EXERCISES 15.4 ■ PAGE 1026

1.  $\int_0^{3\pi/2} \int_0^4 f(r \cos \theta, r \sin \theta) r dr d\theta$     3.  $\int_{-1}^1 \int_0^{(x+1)/2} f(x, y) dy dx$



7.  $\frac{1250}{3}$     9.  $(\pi/4)(\cos 1 - \cos 9)$   
11.  $(\pi/2)(1 - e^{-4})$     13.  $\frac{3}{64}\pi^2$     15.  $\pi/12$   
17.  $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$     19.  $\frac{16}{3}\pi$     21.  $\frac{4}{3}\pi$     23.  $\frac{4}{3}\pi a^3$   
25.  $(2\pi/3)[1 - (1/\sqrt{2})]$     27.  $(8\pi/3)(64 - 24\sqrt{3})$   
29.  $\frac{1}{2}\pi(1 - \cos 9)$     31.  $2\sqrt{2}/3$     33. 4.5951  
35.  $1800\pi \text{ ft}^3$     37.  $2/(a+b)$     39.  $\frac{15}{16}$   
41. (a)  $\sqrt{\pi}/4$     (b)  $\sqrt{\pi}/2$

## EXERCISES 15.5 ■ PAGE 1036

1. 285 C    3.  $42k, (2, \frac{85}{28})$     5.  $6, (\frac{3}{4}, \frac{3}{2})$     7.  $\frac{8}{15}k, (0, \frac{4}{7})$   
9.  $L/4, (L/2, 16/(9\pi))$     11.  $(\frac{3}{8}, 3\pi/16)$     13.  $(0, 45/(14\pi))$   
15.  $(2a/5, 2a/5)$  if vertex is  $(0, 0)$  and sides are along positive axes  
17.  $\frac{64}{315}k, \frac{8}{105}k, \frac{88}{315}k$   
19.  $7ka^6/180, 7ka^6/180, 7ka^6/90$  if vertex is  $(0, 0)$  and sides are along positive axes  
21.  $\rho bh^3/3, \rho b^3h/3; b/\sqrt{3}, h/\sqrt{3}$

23.  $\rho a^4 \pi / 16, \rho a^4 \pi / 16; a/2, a/2$

25.  $m = 3\pi/64, (\bar{x}, \bar{y}) = \left( \frac{16384\sqrt{2}}{10395\pi}, 0 \right)$ ,

$$I_x = \frac{5\pi}{384} - \frac{4}{105}, I_y = \frac{5\pi}{384} + \frac{4}{105}, I_0 = \frac{5\pi}{192}$$

27. (a)  $\frac{1}{2}$  (b) 0.375 (c)  $\frac{5}{48} \approx 0.1042$

29. (b) (i)  $e^{-0.2} \approx 0.8187$   
 (ii)  $1 + e^{-1.8} - e^{-0.8} - e^{-1} \approx 0.3481$  (c) 2, 5

31. (a)  $\approx 0.500$  (b)  $\approx 0.632$

33. (a)  $\iint_D k \left[ 1 - \frac{1}{20} \sqrt{(x - x_0)^2 + (y - y_0)^2} \right] dA$ , where  $D$  is the disk with radius 10 mi centered at the center of the city  
 (b)  $200\pi k/3 \approx 209k, 200(\pi/2 - \frac{8}{9})k \approx 136k$ , on the edge

**EXERCISES 15.6 ■ PAGE 1040**

1.  $15\sqrt{26}$  3.  $3\sqrt{14}$  5.  $12 \sin^{-1}\left(\frac{2}{3}\right)$

7.  $(\pi/6)(17\sqrt{17} - 5\sqrt{5})$  9.  $(2\pi/3)(2\sqrt{2} - 1)$

11.  $a^2(\pi - 2)$  13. 13.9783 15. (a)  $\approx 1.83$  (b)  $\approx 1.8616$

17.  $\frac{45}{8}\sqrt{14} + \frac{15}{16}\ln[(11\sqrt{5} + 3\sqrt{70})/(3\sqrt{5} + \sqrt{70})]$

19. 3.3213 23.  $(\pi/6)(101\sqrt{101} - 1)$

**EXERCISES 15.7 ■ PAGE 1049**

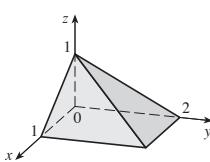
1.  $\frac{27}{4}$  3.  $\frac{16}{15}$  5.  $\frac{5}{3}$  7.  $-\frac{1}{3}$  9.  $\frac{27}{2}$  11.  $9\pi/8$

13.  $\frac{65}{28}$  15.  $\frac{1}{60}$  17.  $16\pi/3$  19.  $\frac{16}{3}$  21.  $\frac{8}{15}$

23. (a)  $\int_0^1 \int_0^x \int_0^{\sqrt{1-y^2}} dz dy dx$  (b)  $\frac{1}{4}\pi - \frac{1}{3}$

25. 0.985

27.



29.  $\int_{-2}^2 \int_0^{4-x^2} \int_{-\sqrt{4-x^2-y^2}/2}^{\sqrt{4-x^2-y^2}/2} f(x, y, z) dz dy dx$

$= \int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} \int_{-\sqrt{4-x^2-y^2}/2}^{\sqrt{4-x^2-y^2}/2} f(x, y, z) dz dy dx$

$= \int_{-1}^1 \int_0^{4-4z^2} \int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x, y, z) dx dy dz$

$= \int_0^4 \int_{-\sqrt{4-y^2}/2}^{\sqrt{4-y^2}/2} \int_{-\sqrt{4-y^2-4z^2}}^{\sqrt{4-y^2-4z^2}} f(x, y, z) dx dz dy$

$= \int_{-2}^2 \int_{-\sqrt{4-x^2}/2}^{\sqrt{4-x^2}/2} \int_0^{4-x^2-4z^2} f(x, y, z) dy dz dx$

$= \int_{-1}^1 \int_{-\sqrt{4-4z^2}}^{\sqrt{4-4z^2}} \int_0^{4-x^2-4z^2} f(x, y, z) dy dx dz$

31.  $\int_{-2}^2 \int_x^4 \int_{x^2}^{2-y/2} f(x, y, z) dz dy dx$

$= \int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \int_0^{2-y/2} f(x, y, z) dz dx dy$

$= \int_0^2 \int_0^{4-2z} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z) dx dy dz$

$= \int_0^4 \int_0^{2-y/2} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z) dx dz dy$

$= \int_{-2}^2 \int_0^{2-x^2/2} \int_{x^2}^{4-2z} f(x, y, z) dy dz dx$

$= \int_0^2 \int_{-\sqrt{4-2z}}^{\sqrt{4-2z}} \int_{x^2}^{4-2z} f(x, y, z) dy dx dz$

33.  $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z) dz dy dx = \int_0^1 \int_0^x \int_0^{1-y} f(x, y, z) dz dx dy$   
 $= \int_0^1 \int_0^{1-z} \int_0^x f(x, y, z) dx dy dz = \int_0^1 \int_0^{1-y} \int_0^x f(x, y, z) dx dz dy$   
 $= \int_0^1 \int_0^{1-\sqrt{x}} \int_{\sqrt{x}}^{1-z} f(x, y, z) dy dz dx = \int_0^1 \int_0^{(1-z)^2} \int_{\sqrt{x}}^{1-z} f(x, y, z) dy dx dz$

35.  $\int_0^1 \int_y^x \int_0^y f(x, y, z) dz dx dy = \int_0^1 \int_0^x \int_0^y f(x, y, z) dz dy dx$   
 $= \int_0^1 \int_z^1 \int_0^1 f(x, y, z) dx dy dz = \int_0^1 \int_0^y \int_z^1 f(x, y, z) dx dz dy$   
 $= \int_0^1 \int_0^x \int_z^x f(x, y, z) dy dz dx = \int_0^1 \int_z^x \int_0^x f(x, y, z) dy dx dz$

37.  $64\pi$  39.  $\frac{79}{30}, \left( \frac{358}{553}, \frac{33}{79}, \frac{571}{553} \right)$

41.  $a^5, (7a/12, 7a/12, 7a/12)$

43.  $I_x = I_y = I_z = \frac{2}{3}kL^5$  45.  $\frac{1}{2}\pi kha^4$

47. (a)  $m = \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} \sqrt{x^2 + y^2} dz dy dx$

(b)  $(\bar{x}, \bar{y}, \bar{z})$ , where

$$\bar{x} = (1/m) \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} x \sqrt{x^2 + y^2} dz dy dx$$

$$\bar{y} = (1/m) \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} y \sqrt{x^2 + y^2} dz dy dx$$

$$\bar{z} = (1/m) \int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} z \sqrt{x^2 + y^2} dz dy dx$$

(c)  $\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} (x^2 + y^2)^{3/2} dz dy dx$

49. (a)  $\frac{3}{32}\pi + \frac{11}{24}$

(b)  $\left( \frac{28}{9\pi + 44}, \frac{30\pi + 128}{45\pi + 220}, \frac{45\pi + 208}{135\pi + 660} \right)$

(c)  $\frac{1}{240}(68 + 15\pi)$

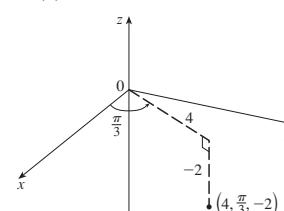
51. (a)  $\frac{1}{8}$  (b)  $\frac{1}{64}$  (c)  $\frac{1}{5760}$  53.  $L^3/8$

55. (a) The region bounded by the ellipsoid  $x^2 + 2y^2 + 3z^2 = 1$

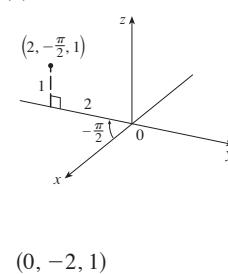
(b)  $4\sqrt{6}\pi/45$

**EXERCISES 15.8 ■ PAGE 1055**

1. (a)



(b)



(2,  $2\sqrt{3}$ , -2)

(0, -2, 1)

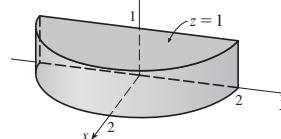
3. (a)  $(\sqrt{2}, 3\pi/4, 1)$  (b)  $(4, 2\pi/3, 3)$

5. Vertical half-plane through the  $z$ -axis

7. Circular paraboloid

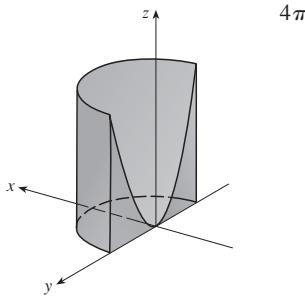
9. (a)  $z^2 = 1 + r \cos \theta - r^2$  (b)  $z = r^2 \cos 2\theta$

11.



13. Cylindrical coordinates:  $6 \leq r \leq 7, 0 \leq \theta \leq 2\pi, 0 \leq z \leq 20$

15.

 $4\pi$ 

17.  $384\pi$

19.  $\frac{8}{3}\pi + \frac{128}{15}$

21.  $2\pi/5$

23.  $\frac{4}{3}\pi(\sqrt{2} - 1)$

25. (a)  $162\pi$  (b)  $(0, 0, 15)$

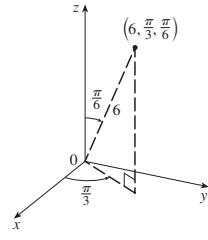
27.  $\pi K a^2/8, (0, 0, 2a/3)$

29. 0

31. (a)  $\iiint_C h(P)g(P) dV$ , where  $C$  is the cone(b)  $\approx 3.1 \times 10^{19}$  ft-lb

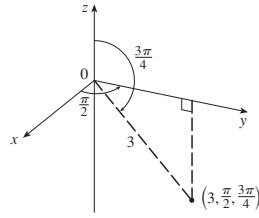
## EXERCISES 15.9 ■ PAGE 1061

1. (a)



$$\left(\frac{3}{2}, \frac{3\sqrt{3}}{2}, 3\sqrt{3}\right)$$

(b)



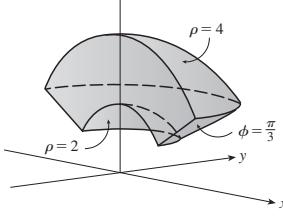
$$\left(0, \frac{3\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2}\right)$$

3. (a)  $(2, 3\pi/2, \pi/2)$  (b)  $(2, 3\pi/4, 3\pi/4)$

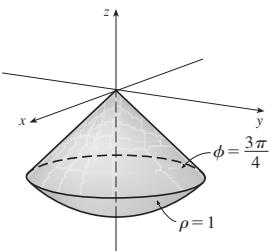
5. Half-cone 7. Sphere, radius  $\frac{1}{2}$ , center  $(0, \frac{1}{2}, 0)$ 

9. (a)  $\cos^2\phi = \sin^2\phi$  (b)  $\rho^2(\sin^2\phi \cos^2\theta + \cos^2\phi) = 9$

11.

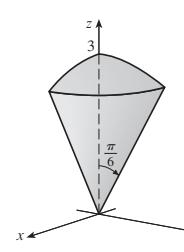


13.



15.  $0 \leq \phi \leq \pi/4, 0 \leq \rho \leq \cos \phi$

17.



$(9\pi/4)(2 - \sqrt{3})$

19.  $\int_0^{\pi/2} \int_0^3 \int_0^2 f(r \cos \theta, r \sin \theta, z) r dz dr d\theta$

21.  $312,500\pi/7$  23.  $1688\pi/15$  25.  $\pi/8$

27.  $(\sqrt{3} - 1)\pi a^3/3$  29. (a)  $10\pi$  (b)  $(0, 0, 2.1)$

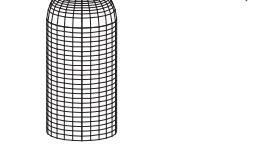
31. (a)  $(0, 0, \frac{7}{12})$  (b)  $11K\pi/960$

33. (a)  $(0, 0, \frac{3}{8}a)$  (b)  $4K\pi a^5/15$

35.  $\frac{1}{3}\pi(2 - \sqrt{2}), (0, 0, 3/[8(2 - \sqrt{2})])$

37.  $5\pi/6$  39.  $(4\sqrt{2} - 5)/15$  41.  $4096\pi/21$

43.



## EXERCISES 15.10 ■ PAGE 1071

1. 16 3.  $\sin^2\theta - \cos^2\theta$  5. 0

7. The parallelogram with vertices  $(0, 0), (6, 3), (12, 1), (6, -2)$ 9. The region bounded by the line  $y = 1$ , the  $y$ -axis, and  $y = \sqrt{x}$ 11.  $x = \frac{1}{3}(v - u), y = \frac{1}{3}(u + 2v)$  is one possible transformation, where  $S = \{(u, v) \mid -1 \leq u \leq 1, 1 \leq v \leq 3\}$ 13.  $x = u \cos v, y = u \sin v$  is one possible transformation, where  $S = \{(u, v) \mid 1 \leq u \leq \sqrt{2}, 0 \leq v \leq \pi/2\}$ 

15. -3 17.  $6\pi$  19.  $2 \ln 3$

21. (a)  $\frac{4}{3}\pi abc$  (b)  $1.083 \times 10^{12} \text{ km}^3$  (c)  $\frac{4}{15}\pi(a^2 + b^2)abck$

23.  $\frac{8}{5} \ln 8$  25.  $\frac{3}{2} \sin 1$  27.  $e - e^{-1}$

## CHAPTER 15 REVIEW ■ PAGE 1073

## True-False Quiz

1. True 3. True 5. True 7. True 9. False

## Exercises

1.  $\approx 64.0$  3.  $4e^2 - 4e + 3$  5.  $\frac{1}{2} \sin 1$  7.  $\frac{2}{3}$

9.  $\int_0^\pi \int_2^4 f(r \cos \theta, r \sin \theta) r dr d\theta$

11. The region inside the loop of the four-leaved rose  $r = \sin 2\theta$  in the first quadrant

13.  $\frac{1}{2} \sin 1$  15.  $\frac{1}{2}e^6 - \frac{7}{2}$  17.  $\frac{1}{4} \ln 2$  19. 8

21.  $81\pi/5$  23.  $\frac{81}{2}$  25.  $\pi/96$  27.  $\frac{64}{15}$

29. 176 31.  $\frac{2}{3}$  33.  $2ma^3/9$

35. (a)  $\frac{1}{4}$  (b)  $(\frac{1}{3}, \frac{8}{15})$

(c)  $I_x = \frac{1}{12}, I_y = \frac{1}{24}; \bar{x} = 1/\sqrt{3}, \bar{y} = 1/\sqrt{6}$

37. (a)  $(0, 0, h/4)$  (b)  $\pi a^4 h/10$

39.  $\ln(\sqrt{2} + \sqrt{3}) + \sqrt{2}/3$  41.  $\frac{486}{5}$  43. 0.0512

45. (a)  $\frac{1}{15}$  (b)  $\frac{1}{3}$  (c)  $\frac{1}{45}$

47.  $\int_0^1 \int_0^{1-z} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z) dx dy dz$  49.  $-\ln 2$  51. 0

**PROBLEMS PLUS ■ PAGE 1077**

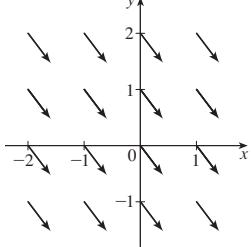
1. 30 3.  $\frac{1}{2} \sin 1$  7. (b) 0.90

13.  $abc\pi \left( \frac{2}{3} - \frac{8}{9\sqrt{3}} \right)$

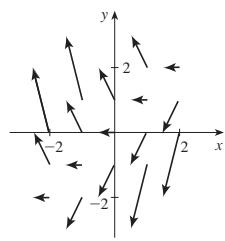
**CHAPTER 16**

**EXERCISES 16.1 ■ PAGE 1085**

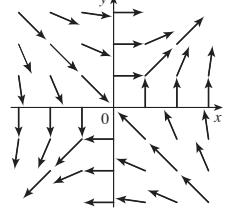
1.



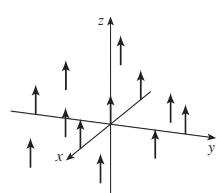
3.



5.

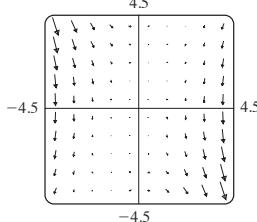


7.



11. IV 13. I 15. IV 17. III

19.

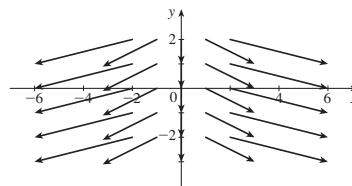


The line  $y = 2x$

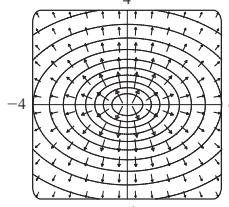
21.  $\nabla f(x, y) = (xy + 1)e^{xy} \mathbf{i} + x^2 e^{xy} \mathbf{j}$

23.  $\nabla f(x, y, z) = \frac{x}{\sqrt{x^2 + y^2 + z^2}} \mathbf{i} + \frac{y}{\sqrt{x^2 + y^2 + z^2}} \mathbf{j} + \frac{z}{\sqrt{x^2 + y^2 + z^2}} \mathbf{k}$

25.  $\nabla f(x, y) = 2x \mathbf{i} - \mathbf{j}$

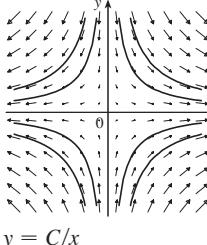


27.



29. III 31. II 33. (2.04, 1.03)

35. (a)



(b)  $y = 1/x, x > 0$

**EXERCISES 16.2 ■ PAGE 1096**

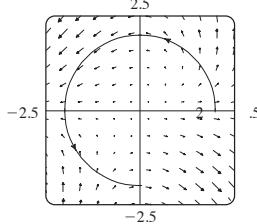
1.  $\frac{1}{54}(145^{3/2} - 1)$  3. 1638.4 5.  $\frac{243}{8}$  7.  $\frac{5}{2}$

9.  $\sqrt{5}\pi$  11.  $\frac{1}{12}\sqrt{14}(e^6 - 1)$  13.  $\frac{2}{5}(e - 1)$  15.  $\frac{35}{3}$

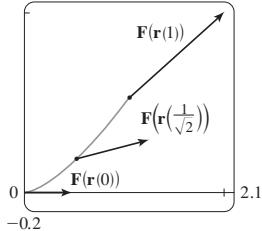
17. (a) Positive (b) Negative 19. 45

21.  $\frac{6}{5} - \cos 1 - \sin 1$  23. 1.9633 25. 15.0074

27.  $3\pi + \frac{2}{3}$



29. (a)  $\frac{11}{8} - 1/e$  (b) 2.1



31.  $\frac{172,704}{5,632,705} \sqrt{2}(1 - e^{-14\pi})$

33.  $2\pi k, (4/\pi, 0)$

35. (a)  $\bar{x} = (1/m) \int_C x\rho(x, y, z) ds$ ,

$\bar{y} = (1/m) \int_C y\rho(x, y, z) ds$ ,

$\bar{z} = (1/m) \int_C z\rho(x, y, z) ds$ , where  $m = \int_C \rho(x, y, z) ds$

(b)  $(0, 0, 3\pi)$

37.  $I_x = k(\frac{1}{2}\pi - \frac{4}{3})$ ,  $I_y = k(\frac{1}{2}\pi - \frac{2}{3})$

39.  $2\pi r^2$

41.  $\frac{7}{3}$

43. (a)  $2ma\mathbf{i} + 6mbt\mathbf{j}, 0 \leq t \leq 1$

(b)  $2ma^2 + \frac{9}{2}mb^2$

45.  $\approx 1.67 \times 10^4$  ft-lb

47. (b) Yes

51.  $\approx 22$  J

### EXERCISES 16.3 ■ PAGE 1106

1. 40    3.  $f(x, y) = x^2 - 3xy + 2y^2 - 8y + K$

5. Not conservative    7.  $f(x, y) = ye^x + x \sin y + K$

9.  $f(x, y) = x \ln y + x^2y^3 + K$

11. (b) 16    13. (a)  $f(x, y) = \frac{1}{2}x^2y^2$     (b) 2

15. (a)  $f(x, y, z) = xyz + z^2$     (b) 77

17. (a)  $f(x, y, z) = ye^{xz}$     (b) 4    19. 4/e

21. It doesn't matter which curve is chosen.

23. 30    25. No    27. Conservative

31. (a) Yes    (b) Yes    (c) Yes

33. (a) No    (b) Yes    (c) Yes

### EXERCISES 16.4 ■ PAGE 1113

1.  $8\pi$     3.  $\frac{2}{3}$     5. 12    7.  $\frac{1}{3}$     9.  $-24\pi$     11.  $-\frac{16}{3}$

13.  $4\pi$     15.  $-8e + 48e^{-1}$     17.  $-\frac{1}{12}$     19.  $3\pi$     21. (c)  $\frac{9}{2}$

23.  $(4a/3\pi, 4a/3\pi)$  if the region is the portion of the disk  $x^2 + y^2 = a^2$  in the first quadrant

27. 0

### EXERCISES 16.5 ■ PAGE 1121

1. (a)  $\mathbf{0}$     (b) 3

3. (a)  $ze^x\mathbf{i} + (xye^z - yze^x)\mathbf{j} - xe^z\mathbf{k}$     (b)  $y(e^z + e^x)$

5. (a)  $\mathbf{0}$     (b)  $2/\sqrt{x^2 + y^2 + z^2}$

7. (a)  $\langle -e^y \cos z, -e^z \cos x, -e^x \cos y \rangle$

(b)  $e^x \sin y + e^y \sin z + e^z \sin x$

9. (a) Negative    (b)  $\operatorname{curl} \mathbf{F} = \mathbf{0}$

11. (a) Zero    (b)  $\operatorname{curl} \mathbf{F}$  points in the negative  $z$ -direction

13.  $f(x, y, z) = xy^2z^3 + K$     15. Not conservative

17.  $f(x, y, z) = xe^{yz} + K$     19. No

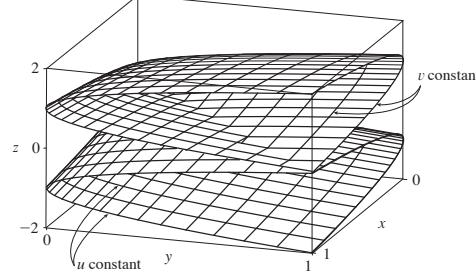
### EXERCISES 16.6 ■ PAGE 1132

1.  $P$ : no;  $Q$ : yes

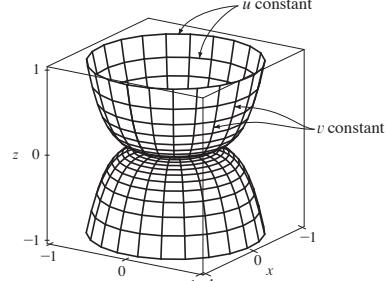
3. Plane through  $(0, 3, 1)$  containing vectors  $\langle 1, 0, 4 \rangle, \langle 1, -1, 5 \rangle$

5. Hyperbolic paraboloid

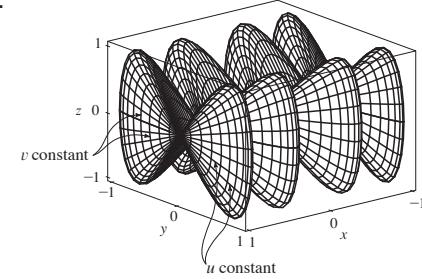
7.



9.



11.



13. IV    15. II    17. III

19.  $x = u, y = v - u, z = -v$

21.  $y = z, z = x, x = \sqrt{1 + y^2 + \frac{1}{4}z^2}$

23.  $x = 2 \sin \phi \cos \theta, y = 2 \sin \phi \sin \theta,$

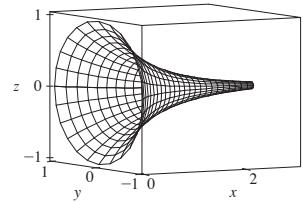
$z = 2 \cos \phi, 0 \leq \phi \leq \pi/4, 0 \leq \theta \leq 2\pi$   
[or  $x = x, y = y, z = \sqrt{4 - x^2 - y^2}, x^2 + y^2 \leq 2$ ]

25.  $x = x, y = 4 \cos \theta, z = 4 \sin \theta, 0 \leq x \leq 5, 0 \leq \theta \leq 2\pi$

29.  $x = x, y = e^{-x} \cos \theta,$

$z = e^{-x} \sin \theta, 0 \leq x \leq 3,$

$0 \leq \theta \leq 2\pi$



31. (a) Direction reverses    (b) Number of coils doubles

33.  $3x - y + 3z = 3$     35.  $\frac{\sqrt{3}}{2}x - \frac{1}{2}y + z = \frac{\pi}{3}$

37.  $-x + 2z = 1$     39.  $3\sqrt{14}$     41.  $\sqrt{14}\pi$

43.  $\frac{4}{15}(3^{5/2} - 2^{7/2} + 1)$     45.  $(2\pi/3)(2\sqrt{2} - 1)$

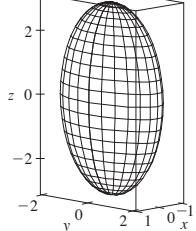
47.  $\frac{1}{2}\sqrt{21} + \frac{17}{4}[\ln(2 + \sqrt{21}) - \ln\sqrt{17}]$     49. 4

51.  $A(S) \leq \sqrt{3}\pi R^2$     53. 13.9783

55. (a) 24.2055    (b) 24.2476

57.  $\frac{45}{8}\sqrt{14} + \frac{15}{16}\ln[(11\sqrt{5} + 3\sqrt{70})/(3\sqrt{5} + \sqrt{70})]$

59. (b)



(c)  $\int_0^{2\pi} \int_0^\pi \sqrt{36 \sin^4 u \cos^2 v + 9 \sin^4 u \sin^2 v + 4 \cos^2 u \sin^2 u} \, du \, dv$

61.  $4\pi$     63.  $2a^2(\pi - 2)$

**EXERCISES 16.7 ■ PAGE 1144**

1. 49.09    3.  $900\pi$     5.  $11\sqrt{14}$     7.  $\frac{2}{3}(2\sqrt{2} - 1)$

9.  $171\sqrt{14}$     11.  $\sqrt{21}/3$     13.  $364\sqrt{2}\pi/3$

15.  $(\pi/60)(391\sqrt{17} + 1)$     17.  $16\pi$     19. 12    21. 4

23.  $\frac{713}{180}$     25.  $-\frac{4}{3}\pi$     27. 0    29. 48    31.  $2\pi + \frac{8}{3}$

33. 4.5822    35. 3.4895

37.  $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D [P(\partial h/\partial x) - Q + R(\partial h/\partial z)] \, dA$ ,

where  $D$  = projection of  $S$  on  $xz$ -plane

39.  $(0, 0, a/2)$

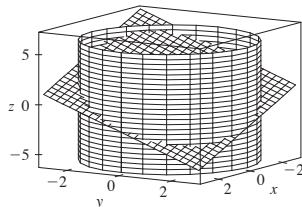
41. (a)  $I_z = \iint_S (x^2 + y^2)\rho(x, y, z) \, dS$     (b)  $4329\sqrt{2}\pi/5$

43. 0 kg/s    45.  $\frac{8}{3}\pi a^3 \varepsilon_0$     47.  $1248\pi$

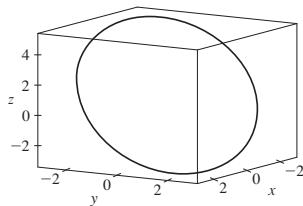
**EXERCISES 16.8 ■ PAGE 1151**

3. 0    5. 0    7. -1    9.  $80\pi$

11. (a)  $81\pi/2$     (b)



(c)  $x = 3 \cos t, y = 3 \sin t, z = 1 - 3(\cos t + \sin t), 0 \leq t \leq 2\pi$



17. 3

**EXERCISES 16.9 ■ PAGE 1157**

5.  $\frac{9}{2}$     7.  $9\pi/2$     9. 0    11.  $32\pi/3$     13.  $2\pi$

15.  $341\sqrt{2}/60 + \frac{81}{20} \arcsin(\sqrt{3}/3)$

17.  $13\pi/20$     19. Negative at  $P_1$ , positive at  $P_2$

21.  $\operatorname{div} \mathbf{F} > 0$  in quadrants I, II;  $\operatorname{div} \mathbf{F} < 0$  in quadrants III, IV

**CHAPTER 16 REVIEW ■ PAGE 1160**

**True-False Quiz**

1. False    3. True    5. False

7. False    9. True    11. True

**Exercises**

1. (a) Negative    (b) Positive    3.  $6\sqrt{10}$     5.  $\frac{4}{15}$

7.  $\frac{110}{3}$     9.  $\frac{11}{12} - 4/e$     11.  $f(x, y) = e^y + xe^{-xy}$     13. 0

17.  $-8\pi$     25.  $\frac{1}{6}(27 - 5\sqrt{5})$     27.  $(\pi/60)(391\sqrt{17} + 1)$

29.  $-64\pi/3$     33.  $-\frac{1}{2}$     37. -4    39. 21

**CHAPTER 17**

**EXERCISES 17.1 ■ PAGE 1172**

1.  $y = c_1 e^{3x} + c_2 e^{-2x}$     3.  $y = c_1 \cos 4x + c_2 \sin 4x$

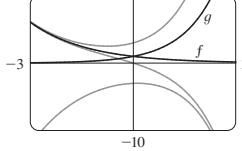
5.  $y = c_1 e^{2x/3} + c_2 x e^{2x/3}$     7.  $y = c_1 + c_2 e^{x/2}$

9.  $y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$

11.  $y = c_1 e^{(\sqrt{3}-1)t/2} + c_2 e^{-(\sqrt{3}+1)t/2}$

13.  $P = e^{-t}[c_1 \cos(\frac{1}{10}t) + c_2 \sin(\frac{1}{10}t)]$

15. All solutions approach either 0 or  $\pm\infty$  as  $x \rightarrow \pm\infty$ .



17.  $y = 3e^{2x} - e^{4x}$     19.  $y = e^{-2x/3} + \frac{2}{3}xe^{-2x/3}$

21.  $y = e^{3x}(2 \cos x - 3 \sin x)$

23.  $y = \frac{1}{7}e^{4x-4} - \frac{1}{7}e^{3-3x}$     25.  $y = 5 \cos 2x + 3 \sin 2x$

27.  $y = 2e^{-2x} - 2xe^{-2x}$     29.  $y = \frac{e-2}{e-1} + \frac{e^x}{e-1}$

31. No solution

33. (b)  $\lambda = n^2\pi^2/L^2$ ,  $n$  a positive integer;  $y = C \sin(n\pi x/L)$

35. (a)  $b - a \neq n\pi$ ,  $n$  any integer

(b)  $b - a = n\pi$  and  $\frac{c}{d} \neq e^{a-b} \frac{\cos a}{\cos b}$  unless  $\cos b = 0$ , then

$$\frac{c}{d} \neq e^{a-b} \frac{\sin a}{\sin b}$$

(c)  $b - a = n\pi$  and  $\frac{c}{d} = e^{a-b} \frac{\cos a}{\cos b}$  unless  $\cos b = 0$ , then

$$\frac{c}{d} = e^{a-b} \frac{\sin a}{\sin b}$$

**EXERCISES 17.2 ■ PAGE 1179**

1.  $y = c_1 e^{3x} + c_2 e^{-x} - \frac{7}{65} \cos 2x - \frac{4}{65} \sin 2x$

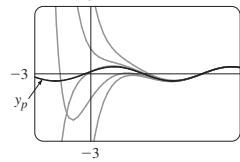
3.  $y = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{13}e^{-2x}$

5.  $y = e^{2x}(c_1 \cos x + c_2 \sin x) + \frac{1}{10}e^{-x}$

7.  $y = \frac{3}{2} \cos x + \frac{11}{2} \sin x + \frac{1}{2}e^x + x^3 - 6x$

9.  $y = e^x(\frac{1}{2}x^2 - x + 2)$

11.



The solutions are all asymptotic to  $y_p = \frac{1}{10} \cos x + \frac{3}{10} \sin x$  as  $x \rightarrow \infty$ . Except for  $y_p$ , all solutions approach either  $\infty$  or  $-\infty$  as  $x \rightarrow -\infty$ .

13.  $y_p = (Ax + B)e^x \cos x + (Cx + D)e^x \sin x$

15.  $y_p = Axe^x + B \cos x + C \sin x$

17.  $y_p = xe^{-x}[(Ax^2 + Bx + C) \cos 3x + (Dx^2 + Ex + F) \sin 3x]$

19.  $y = c_1 \cos(\frac{1}{2}x) + c_2 \sin(\frac{1}{2}x) - \frac{1}{3} \cos x$

21.  $y = c_1 e^x + c_2 x e^x + e^{2x}$

23.  $y = c_1 \sin x + c_2 \cos x + \sin x \ln(\sec x + \tan x) - 1$

25.  $y = [c_1 + \ln(1 + e^{-x})]e^x + [c_2 - e^{-x} + \ln(1 + e^{-x})]e^{2x}$

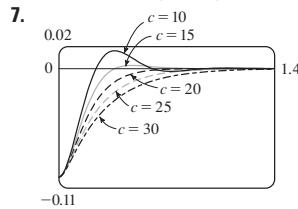
27.  $y = e^x[c_1 + c_2 x - \frac{1}{2} \ln(1 + x^2) + x \tan^{-1} x]$

**EXERCISES 17.3 ■ PAGE 1187**

1.  $x = 0.35 \cos(2\sqrt{5}t)$

3.  $x = -\frac{1}{5}e^{-6t} + \frac{6}{5}e^{-t}$

5.  $\frac{49}{12} \text{ kg}$



13.  $Q(t) = (-e^{-10t}/250)(6 \cos 20t + 3 \sin 20t) + \frac{3}{125},$   
 $I(t) = \frac{3}{5}e^{-10t} \sin 20t$

15.  $Q(t) = e^{-10t}[\frac{3}{250} \cos 20t - \frac{3}{500} \sin 20t] - \frac{3}{250} \cos 10t + \frac{3}{125} \sin 10t$

**EXERCISES 17.4 ■ PAGE 1192**

1.  $c_0 \sum_{n=0}^{\infty} \frac{x^n}{n!} = c_0 e^x$

3.  $c_0 \sum_{n=0}^{\infty} \frac{x^{3n}}{3^n n!} = c_0 e^{x^3/3}$

5.  $c_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} x^{2n} + c_1 \sum_{n=0}^{\infty} \frac{(-2)^n n!}{(2n+1)!} x^{2n+1}$

7.  $c_0 + c_1 \sum_{n=1}^{\infty} \frac{x^n}{n} = c_0 - c_1 \ln(1-x)$  for  $|x| < 1$

9.  $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!} = e^{x^2/2}$

11.  $x + \sum_{n=1}^{\infty} \frac{(-1)^n 2^2 5^2 \cdots (3n-1)^2}{(3n+1)!} x^{3n+1}$

**CHAPTER 17 REVIEW ■ PAGE 1193****True-False Quiz**

1. True    3. True

**Exercises**

1.  $y = c_1 e^{x/2} + c_2 e^{-x/2}$

3.  $y = c_1 \cos(\sqrt{3}x) + c_2 \sin(\sqrt{3}x)$

5.  $y = e^{2x}(c_1 \cos x + c_2 \sin x + 1)$

7.  $y = c_1 e^x + c_2 x e^x - \frac{1}{2} \cos x - \frac{1}{2}(x+1) \sin x$

9.  $y = c_1 e^{3x} + c_2 e^{-2x} - \frac{1}{6} - \frac{1}{5}x e^{-2x}$

11.  $y = 5 - 2e^{-6(x-1)}$

13.  $y = (e^{4x} - e^x)/3$

15. No solution

17.  $\sum_{n=0}^{\infty} \frac{(-2)^n n!}{(2n+1)!} x^{2n+1}$

19.  $Q(t) = -0.02e^{-10t}(\cos 10t + \sin 10t) + 0.03$

21. (c)  $2\pi/k \approx 85 \text{ min}$  (d)  $\approx 17,600 \text{ mi/h}$

**APPENDICES****EXERCISES G ■ PAGE A12**

1.  $8 - 4i$

3.  $13 + 18i$

5.  $12 - 7i$

7.  $\frac{11}{13} + \frac{10}{13}i$

9.  $\frac{1}{2} - \frac{1}{2}i$

11.  $-i$

13.  $5i$

15.  $12 + 5i, 13$

17.  $4i, 4$

19.  $\pm \frac{3}{2}i$

21.  $-1 \pm 2i$

23.  $-\frac{1}{2} \pm (\sqrt{7}/2)i$

25.  $3\sqrt{2} [\cos(3\pi/4) + i \sin(3\pi/4)]$

27.  $5\{\cos[\tan^{-1}(\frac{4}{3})] + i \sin[\tan^{-1}(\frac{4}{3})]\}$

29.  $4[\cos(\pi/2) + i \sin(\pi/2)], \cos(-\pi/6) + i \sin(-\pi/6),$   
 $\frac{1}{2}[\cos(-\pi/6) + i \sin(-\pi/6)]$

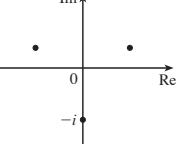
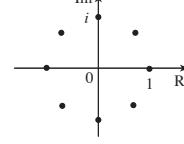
31.  $4\sqrt{2} [\cos(7\pi/12) + i \sin(7\pi/12)],$   
 $(2\sqrt{2})[\cos(13\pi/12) + i \sin(13\pi/12)], \frac{1}{4}[\cos(\pi/6) + i \sin(\pi/6)]$

33.  $-1024$

35.  $-512\sqrt{3} + 512i$

37.  $\pm 1, \pm i, (1/\sqrt{2})(\pm 1 \pm i)$

39.  $\pm(\sqrt{3}/2) + \frac{1}{2}i, -i$



41.  $i$

43.  $\frac{1}{2} + (\sqrt{3}/2)i$

45.  $-e^2$

47.  $\cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta,$

$\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta$



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**DIFFERENTIATION RULES****General Formulas**

1.  $\frac{d}{dx}(c) = 0$

3.  $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

5.  $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$  (Product Rule)

7.  $\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$  (Chain Rule)

2.  $\frac{d}{dx}[cf(x)] = cf'(x)$

4.  $\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

6.  $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$  (Quotient Rule)

8.  $\frac{d}{dx}(x^n) = nx^{n-1}$  (Power Rule)

**Exponential and Logarithmic Functions**

9.  $\frac{d}{dx}(e^x) = e^x$

11.  $\frac{d}{dx}\ln|x| = \frac{1}{x}$

10.  $\frac{d}{dx}(a^x) = a^x \ln a$

12.  $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

**Trigonometric Functions**

13.  $\frac{d}{dx}(\sin x) = \cos x$

16.  $\frac{d}{dx}(\csc x) = -\csc x \cot x$

14.  $\frac{d}{dx}(\cos x) = -\sin x$

17.  $\frac{d}{dx}(\sec x) = \sec x \tan x$

15.  $\frac{d}{dx}(\tan x) = \sec^2 x$

18.  $\frac{d}{dx}(\cot x) = -\csc^2 x$

**Inverse Trigonometric Functions**

19.  $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

22.  $\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

20.  $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

23.  $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

21.  $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

24.  $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

**Hyperbolic Functions**

25.  $\frac{d}{dx}(\sinh x) = \cosh x$

28.  $\frac{d}{dx}(\csch x) = -\csch x \coth x$

26.  $\frac{d}{dx}(\cosh x) = \sinh x$

29.  $\frac{d}{dx}(\sech x) = -\sech x \tanh x$

27.  $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

30.  $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

**Inverse Hyperbolic Functions**

31.  $\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$

34.  $\frac{d}{dx}(\csch^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$

32.  $\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$

35.  $\frac{d}{dx}(\sech^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$

33.  $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

36.  $\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$

**TABLE OF INTEGRALS****Basic Forms**

1.  $\int u \, dv = uv - \int v \, du$
2.  $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3.  $\int \frac{du}{u} = \ln |u| + C$
4.  $\int e^u \, du = e^u + C$
5.  $\int a^u \, du = \frac{a^u}{\ln a} + C$
6.  $\int \sin u \, du = -\cos u + C$
7.  $\int \cos u \, du = \sin u + C$
8.  $\int \sec^2 u \, du = \tan u + C$
9.  $\int \csc^2 u \, du = -\cot u + C$
10.  $\int \sec u \tan u \, du = \sec u + C$
11.  $\int \csc u \cot u \, du = -\csc u + C$
12.  $\int \tan u \, du = \ln |\sec u| + C$
13.  $\int \cot u \, du = \ln |\sin u| + C$
14.  $\int \sec u \, du = \ln |\sec u + \tan u| + C$
15.  $\int \csc u \, du = \ln |\csc u - \cot u| + C$
16.  $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C, \quad a > 0$
17.  $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$
18.  $\int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$
19.  $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$
20.  $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$

**Forms Involving  $\sqrt{a^2 + u^2}$ ,  $a > 0$** 

21.  $\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
22.  $\int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$
23.  $\int \frac{\sqrt{a^2 + u^2}}{u} \, du = \sqrt{a^2 + u^2} - a \ln \left| \frac{a + \sqrt{a^2 + u^2}}{u} \right| + C$
24.  $\int \frac{\sqrt{a^2 + u^2}}{u^2} \, du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$
25.  $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$
26.  $\int \frac{u^2 \, du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$
27.  $\int \frac{du}{u \sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$
28.  $\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$
29.  $\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$

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**TABLE OF INTEGRALS****Forms Involving  $\sqrt{a^2 - u^2}$ ,  $a > 0$** 

30.  $\int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$

31.  $\int u^2 \sqrt{a^2 - u^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$

32.  $\int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$

33.  $\int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2 - u^2} - \sin^{-1} \frac{u}{a} + C$

34.  $\int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$

35.  $\int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$

36.  $\int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$

37.  $\int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$

38.  $\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$

**Forms Involving  $\sqrt{u^2 - a^2}$ ,  $a > 0$** 

39.  $\int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$

40.  $\int u^2 \sqrt{u^2 - a^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln |u + \sqrt{u^2 - a^2}| + C$

41.  $\int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{|u|} + C$

42.  $\int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln |u + \sqrt{u^2 - a^2}| + C$

43.  $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln |u + \sqrt{u^2 - a^2}| + C$

44.  $\int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 - a^2}| + C$

45.  $\int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$

46.  $\int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$

**TABLE OF INTEGRALS****Forms Involving  $a + bu$** 

47.  $\int \frac{u \, du}{a + bu} = \frac{1}{b^2} (a + bu - a \ln |a + bu|) + C$

48.  $\int \frac{u^2 \, du}{a + bu} = \frac{1}{2b^3} [(a + bu)^2 - 4a(a + bu) + 2a^2 \ln |a + bu|] + C$

49.  $\int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{u}{a + bu} \right| + C$

50.  $\int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$

51.  $\int \frac{u \, du}{(a + bu)^2} = \frac{a}{b^2(a + bu)} + \frac{1}{b^2} \ln |a + bu| + C$

52.  $\int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$

53.  $\int \frac{u^2 \, du}{(a + bu)^2} = \frac{1}{b^3} \left( a + bu - \frac{a^2}{a + bu} - 2a \ln |a + bu| \right) + C$

54.  $\int u \sqrt{a + bu} \, du = \frac{2}{15b^2} (3bu - 2a)(a + bu)^{3/2} + C$

55.  $\int \frac{u \, du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a)\sqrt{a + bu} + C$

56.  $\int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2}{15b^3} (8a^2 + 3b^2u^2 - 4abu)\sqrt{a + bu} + C$

57. 
$$\begin{aligned} \int \frac{du}{u\sqrt{a + bu}} &= \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, \quad \text{if } a > 0 \\ &= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a + bu}{-a}} + C, \quad \text{if } a < 0 \end{aligned}$$

58.  $\int \frac{\sqrt{a + bu}}{u} \, du = 2\sqrt{a + bu} + a \int \frac{du}{u\sqrt{a + bu}}$

59.  $\int \frac{\sqrt{a + bu}}{u^2} \, du = -\frac{\sqrt{a + bu}}{u} + \frac{b}{2} \int \frac{du}{u\sqrt{a + bu}}$

60.  $\int u^n \sqrt{a + bu} \, du = \frac{2}{b(2n + 3)} \left[ u^n (a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} \, du \right]$

61.  $\int \frac{u^n \, du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n + 1)} - \frac{2na}{b(2n + 1)} \int \frac{u^{n-1} \, du}{\sqrt{a + bu}}$

62.  $\int \frac{du}{u^n \sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n - 1)u^{n-1}} - \frac{b(2n - 3)}{2a(n - 1)} \int \frac{du}{u^{n-1}\sqrt{a + bu}}$

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**TABLE OF INTEGRALS****Trigonometric Forms**

63.  $\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

64.  $\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

65.  $\int \tan^2 u \, du = \tan u - u + C$

66.  $\int \cot^2 u \, du = -\cot u - u + C$

67.  $\int \sin^3 u \, du = -\frac{1}{3}(2 + \sin^2 u) \cos u + C$

68.  $\int \cos^3 u \, du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$

69.  $\int \tan^3 u \, du = \frac{1}{2} \tan^2 u + \ln |\cos u| + C$

70.  $\int \cot^3 u \, du = -\frac{1}{2} \cot^2 u - \ln |\sin u| + C$

71.  $\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$

72.  $\int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$

73.  $\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du$

74.  $\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$

75.  $\int \tan^n u \, du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \, du$

76.  $\int \cot^n u \, du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du$

77.  $\int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$

78.  $\int \csc^n u \, du = \frac{-1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du$

79.  $\int \sin au \sin bu \, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$

80.  $\int \cos au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$

81.  $\int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$

82.  $\int u \sin u \, du = \sin u - u \cos u + C$

83.  $\int u \cos u \, du = \cos u + u \sin u + C$

84.  $\int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$

85.  $\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$

86. 
$$\begin{aligned} \int \sin^n u \cos^m u \, du &= -\frac{\sin^{n-1} u \cos^{m+1} u}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} u \cos^m u \, du \\ &= \frac{\sin^{n+1} u \cos^{m-1} u}{n+m} + \frac{m-1}{n+m} \int \sin^n u \cos^{m-2} u \, du \end{aligned}$$

**Inverse Trigonometric Forms**

87.  $\int \sin^{-1} u \, du = u \sin^{-1} u + \sqrt{1-u^2} + C$

92.  $\int u \tan^{-1} u \, du = \frac{u^2+1}{2} \tan^{-1} u - \frac{u}{2} + C$

88.  $\int \cos^{-1} u \, du = u \cos^{-1} u - \sqrt{1-u^2} + C$

93.  $\int u^n \sin^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \sin^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$

89.  $\int \tan^{-1} u \, du = u \tan^{-1} u - \frac{1}{2} \ln(1+u^2) + C$

94.  $\int u^n \cos^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \cos^{-1} u + \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], \quad n \neq -1$

90.  $\int u \sin^{-1} u \, du = \frac{2u^2-1}{4} \sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$

95.  $\int u^n \tan^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \tan^{-1} u - \int \frac{u^{n+1} du}{1+u^2} \right], \quad n \neq -1$

**TABLE OF INTEGRALS****Exponential and Logarithmic Forms**

**96.**  $\int ue^{au} du = \frac{1}{a^2} (au - 1)e^{au} + C$

**97.**  $\int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$

**98.**  $\int e^{au} \sin bu du = \frac{e^{au}}{a^2 + b^2} (a \sin bu - b \cos bu) + C$

**99.**  $\int e^{au} \cos bu du = \frac{e^{au}}{a^2 + b^2} (a \cos bu + b \sin bu) + C$

**100.**  $\int \ln u du = u \ln u - u + C$

**101.**  $\int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$

**102.**  $\int \frac{1}{u \ln u} du = \ln |\ln u| + C$

**Hyperbolic Forms**

**103.**  $\int \sinh u du = \cosh u + C$

**104.**  $\int \cosh u du = \sinh u + C$

**105.**  $\int \tanh u du = \ln \cosh u + C$

**106.**  $\int \coth u du = \ln |\sinh u| + C$

**107.**  $\int \operatorname{sech} u du = \tan^{-1} |\sinh u| + C$

**108.**  $\int \operatorname{csch} u du = \ln |\tanh \frac{1}{2} u| + C$

**109.**  $\int \operatorname{sech}^2 u du = \tanh u + C$

**110.**  $\int \operatorname{csch}^2 u du = -\coth u + C$

**111.**  $\int \operatorname{sech} u \tanh u du = -\operatorname{sech} u + C$

**112.**  $\int \operatorname{csch} u \coth u du = -\operatorname{csch} u + C$

**Forms Involving  $\sqrt{2au - u^2}$ ,  $a > 0$** 

**113.**  $\int \sqrt{2au - u^2} du = \frac{u - a}{2} \sqrt{2au - u^2} + \frac{a^2}{2} \cos^{-1}\left(\frac{a - u}{a}\right) + C$

**114.**  $\int u \sqrt{2au - u^2} du = \frac{2u^2 - au - 3a^2}{6} \sqrt{2au - u^2} + \frac{a^3}{2} \cos^{-1}\left(\frac{a - u}{a}\right) + C$

**115.**  $\int \frac{\sqrt{2au - u^2}}{u} du = \sqrt{2au - u^2} + a \cos^{-1}\left(\frac{a - u}{a}\right) + C$

**116.**  $\int \frac{\sqrt{2au - u^2}}{u^2} du = -\frac{2\sqrt{2au - u^2}}{u} - \cos^{-1}\left(\frac{a - u}{a}\right) + C$

**117.**  $\int \frac{du}{\sqrt{2au - u^2}} = \cos^{-1}\left(\frac{a - u}{a}\right) + C$

**118.**  $\int \frac{u du}{\sqrt{2au - u^2}} = -\sqrt{2au - u^2} + a \cos^{-1}\left(\frac{a - u}{a}\right) + C$

**119.**  $\int \frac{u^2 du}{\sqrt{2au - u^2}} = -\frac{(u + 3a)}{2} \sqrt{2au - u^2} + \frac{3a^2}{2} \cos^{-1}\left(\frac{a - u}{a}\right) + C$

**120.**  $\int \frac{du}{u \sqrt{2au - u^2}} = -\frac{\sqrt{2au - u^2}}{au} + C$