Notation and target parameter

- Observe the jump process $Z(t) = (N^a(t), A(t), N^\ell(t), L(t), N^y(t), N^d(t))$ and contain the information in the filtration $(\mathcal{F}_t)_{t \geq 0}$.
- $A(t) \in \{0, 1\}$ and $L(t) \in \mathcal{L} \subseteq \mathbb{R}^d$ with $\mathcal{L}$ finite.
- In the time interval $[0, \tau_{\text{end}}]$ there are at most $K - 1$ changes of treatment and

covariates in total for a single individual.

- Thus, $\mathcal{D}_n = \mathcal{F}_{T_{(K)}} = \left(T_{(K)}, \Delta_{(K)}, T_{(K-1)}, \Delta_{(K-1)}, A\left(T_{(K-1)}\right), L\left(T_{(K-1)}\right), \ldots, A(0), L(0)\right) \sim P \in \mathcal{M}$, where $\mathcal{F}_{T_{(k)}} = \left(T_{(k)}, \Delta_{(k)}, A\left(T_{(k)}\right), L\left(T_{(k)}\right), \ldots, A(0), L(0)\right)$.
- The counting processes N^a, N^ℓ, N^y, N^d, and N^c have with probability 1 no jump times in common ($\Rightarrow$ orthogonal martingales).
- Let $\lambda_k^x\left(t, \mathcal{F}_{T_{(k-1)}}\right)$ be the cause-specific hazard of the k'th event, $\pi_k\left(t, \mathcal{F}_{T_{(k-1)}}\right)$ be the probability of treatment given $\Delta_{(k)} = a$, $T_{(k)} = t$, and $\mathcal{F}_{T_{(k-1)}}$, amd $\mu_k\left(t, \cdot, \mathcal{F}_{T_{(k-1)}}\right)$ be the distribution of covariate given $\Delta_{(k)} = a$, $T_{(k)} = t$, and $\mathcal{F}_{T_{(k-1)}}$.

⚠ Conditional distributions

We assume that the conditional distributions $P\left(T_{(k)} \in \cdot \mid \mathcal{F}_{T_{(k-1)}}\right) \ll m$ P-a.s., where m is the Lebesgue measure on $\mathbb{R}_+$.

- Our overall goal is to estimate the interventional cumulative incidence function at time τ,

$$\Psi_\tau^g(P) = \mathbb{E}_P\left[\tilde{N}_\tau^y\right],$$

had the treatment protocol g been followed (adhering to treatment).

Identifiability (no censoring)

We can also give a martingale argument in the lines of Ryalen (2024). Let $\Lambda_t^a(A) = \pi_t(\{a\} \mid \mathcal{F}_{t-})\Lambda_t^a$ be the compensator of $N_t(A)$ - the corresponding random measure of the treatment process, i.e.,

$$N_t(A) = \sum_{j=1}^{K} \mathbb{1}\left\{T_{(j)} \leq t, \Delta_{(j)} = a, A\left(T_{(j)}\right) \in A\right\} \tag{1}$$

Identifiability

- **Consistency**: $\tilde{N}_t^y \mathbb{1}\{T^a > t\} = N_t^y \mathbb{1}\{T^a > t\}$, where $T^a = \inf\{t \geq 0 \mid A(t) = 0\}$.
- **Exchangeability**: The P-$\mathcal{F}_t$ compensator for N^a $\Lambda_t^a(\cdot)$ is also the P-$\mathcal{F}_t \vee \sigma\left(\tilde{N}^y\right)$ compensator and

$$\tilde{N}_t^y \perp A(0) \mid L(0)$$

- **Positivity**: The martingale $W(t) = \prod_{k=1}^{N_t} \left(\frac{\mathbb{1}\left\{A\left(T_{(k)}\right)=1\right\}}{\pi_k\left(T_{(k)}, \mathcal{F}_{T_{(k-1)}}\right)}\right)^{\mathbb{1}\{\Delta_{(k)}=a\}} \frac{\mathbb{1}\{A(0)=1\}}{\pi_0(L(0))}$ is uniformly integrable.

Then the estimand of interest is identifiable by

$$\Psi_t^g(P) = \mathbb{E}_P\left[\tilde{N}_t^y\right] = \mathbb{E}_P[N_t^y W(t)].$$

Proof: First note that by (2.7.7) of Andersen et al. (1993), we have $\tilde{\Lambda} = (0, \Lambda^a, \Lambda^{\ell 1}, \ldots, \Lambda^{\ell d}, \Lambda^y, \Lambda^d)$ and $\Lambda(t) = (\Lambda^a(t)\pi(0 \mid \mathcal{F}_{t-}), \Lambda^a(t)\pi(1 \mid \mathcal{F}_{t-}), \Lambda^{\ell 1}(t), \ldots, \Lambda^{\ell d}(t), \Lambda^y(t), \Lambda^d(t))$, we can simply write

$$W(t) = \frac{\mathbb{1}\{A(0) = 1\}}{\pi_0(L(0))} \frac{\prod_{s \in (0,t]} \prod_h \left(d\tilde{\Lambda}_h(s)\right)^{\Delta N_h(s)} \left(1 - d\tilde{\Lambda}_\bullet(s)\right)^{1 - \Delta N_\bullet(s)}}{\prod_{s \in (0,t]} \prod_h (d\Lambda_h(s))^{\Delta N_h(s)} (1 - d\Lambda_\bullet(s))^{1 - \Delta N_\bullet(s)}}$$

Then (2.7.8) of Andersen et al. (1993) simply states that W is the unique solution of the equation

$$W(t) = \frac{\mathbb{1}\{A(0)=1\}}{\pi_0(L(0))} + \int_0^t W(s-)\left(\frac{1}{\pi_s(1 \mid \mathcal{F}_{s-})} - 1\right) dM_s^a(\{1\}) - \int_0^t W(s-) dM_s^a(\{0\})$$

Under consistency (and regularity conditions for the integrands),

$$\begin{aligned}
\mathbb{E}_P[W(t)N_t^y] &= \mathbb{E}_P\left[\tilde{N}_t^y W(t)\right] \\
&= \mathbb{E}_P\left[\tilde{N}_t^y \frac{\mathbb{1}\{A(0)=1\}}{\pi_0(L(0))}\right] + \mathbb{E}_P\left[\int_0^t \tilde{N}_t^y W(s-)\left(\frac{1}{\pi_s(1 \mid \mathcal{F}_{s-})} - 1\right) dM_s^a(\{1\})\right] - \mathbb{E}_P\left[\int_0^t \tilde{N}_t^y W(s-) dM_s^a(\{0\})\right] \\
&= \mathbb{E}_P\left[\tilde{N}_t^y \frac{\mathbb{1}\{A(0)=1\}}{\pi_0(L(0))}\right] + 0 - 0 \\
&= \mathbb{E}_P\left[\tilde{N}_t^y\right].
\end{aligned}$$

In the third equality, we use the fact that the integrands under exchangeability are predictable, and hence the integrals are zero mean martingales. In the last equality, we simply use the baseline exchangeability condition. □

Two main recursive identification formulas:

Identification formulas

Let $\bar{Q}^g_{K,\tau} = \mathbb{1}\left\{T_{(K)} \leq \tau, \Delta_{(K)} = y\right\}$ and define inductively,

$$\begin{aligned}
\bar{Q}^g_{k-1,\tau} = \mathbb{E}_P\Big[&\mathbb{1}\left\{T_{(k)} \leq \tau, \Delta_{(k)} = \ell\right)\bar{Q}^g_{k,\tau}\left(A\left(T_{(k-1)}\right), L\left(T_{(k)}\right), T_{(k)}, \Delta_{(k)}, \mathcal{F}_{T_{(k-1)}}\right) \\
&+\mathbb{1}\left\{T_{(k)} \leq \tau, \Delta_{(k)} = a\right)\bar{Q}^g_{k,\tau}\left(1, L\left(T_{(k-1)}\right), T_{(k)}, \Delta_{(k)}, \mathcal{F}_{T_{(k-1)}}\right) \\
&+\mathbb{1}\left\{T_{(k)} \leq \tau, \Delta_{(k)} = y\right) \,\Big|\, \mathcal{F}_{T_{(k-1)}}\Big] \\
= p_{ka}&\left(t \mid \mathcal{F}_{T_{(k-1)}}\right) + p_{k\ell}\left(t \mid \mathcal{F}_{T_{(k-1)}}\right) + p_{ky}\left(t \mid \mathcal{F}_{T_{(k-1)}}\right)
\end{aligned}$$

for $k = K-1, ..., 1$ with

$$p_{ka}\left(t \mid \mathcal{F}_{T_{(k-1)}}\right) = \int_{T_{(k-1)}}^t S_k\left(s- \mid \mathcal{F}_{T_{(k-1)}}\right)\bar{Q}^g_{k+1,\tau}\left(1, L\left(T_{(k-1)}\right), s, a, \mathcal{F}_{T_{(k-1)}}\right)\Lambda_k^a\left(\mathrm{d}s \mid \mathcal{F}_{T_{(k-1)}}\right)$$

and

$$\begin{aligned}
p_{k\ell}\left(t \mid \mathcal{F}_{T_{(k-1)}}\right) = \int_{T_{(k-1)}}^t & S_k\left(s- \mid \mathcal{F}_{T_{(k-1)}}\right) \\
&\left(\mathbb{E}_P\left[\bar{Q}^g_{k+1,\tau}\left(A\left(T_{(k-1)}\right), L\left(T_{(k)}\right), T_{(k)}, \Delta_{(k)}, \mathcal{F}_{T_{(k-1)}}\right) \mid T_{(k)} = s, \Delta_{(k)} = \ell, \mathcal{F}_{T_{(k-1)}}\right]\right)\Lambda_k^\ell\left(\mathrm{d}s \mid \mathcal{F}_{T_{(k-1)}}\right)
\end{aligned}$$

and

$$p_{ky}\left(t \mid T_{(k-1)}\right) = \int_{\mathcal{F}_{T_{(k-1)}}}^t S_k\left(s- \mid \mathcal{F}_{T_{(k-1)}}\right)\Lambda_k^y\left(\mathrm{d}s \mid \mathcal{F}_{T_{(k-1)}}\right)$$

Then, $\Psi^g_\tau(P) = \mathbb{E}_P\left[\bar{Q}^g_{0,\tau}(1, L(0))\right]$.

- Notably the iterative representation using the p-functions is computationally intensive, and the resulting integral is high dimensional. We may therefore want to use Bayesian/Monte Carlo methods for the high-dimensional integral, but this is not the focus of this work.

Censoring

- We assume there exists a censoring time $C > 0$ with $N^c(t) = \mathbb{1}\{C \leq t\}$, so that we fully observe the process $(Z(t \wedge C))_t$ and $(N^c(t \wedge T^e))_t$, where T^e denotes the terminal event time. Let $\left(\bar{T}_{(k)}, \bar{\Delta}_{(k)}, A(\bar{T}_k), L(\bar{T}_k)\right)$ be the observed event times and S^c the censoring survival function.
- Let $N^{r,x}$ be a random measures corresponding to the jump times of Z and N^c for the component x for the full data filtration $\mathcal{F}_t^{\text{full}} = \mathcal{F}_t \vee \sigma(N^c(s) \mid s \leq t)$, e.g., Equation 1.

ICE-IPCW

ICE-IPCW

Assume that the intensity processes of $N^{r,x}, x = a, \ell, d, y$ with respect to the filtration $\mathcal{F}_t$ are also the intensities with respect to the filtration $\mathcal{F}_t^{\text{full}}$. Additionally, assume also that the intensity process of $N^c(t)$ with respect to the filtration $\mathcal{G}_t$ is also the intensity process with respect to the filtration $\mathcal{F}_t^{\text{full}}$. Then, we have that

$$
\begin{aligned}
\bar{Q}^g_{k-1,\tau} = \mathbb{E}_P \Bigg[& \frac{\mathbb{1}\left\{\bar{T}_{(k)} \leq \tau, \bar{\Delta}_{(k)} = \ell\right\}}{S^c_k\left(\bar{T}_{(k-1)} - \mid \mathcal{F}^{\text{obs}}_{T_{(k-1)}}\right)} \bar{Q}^g_{k,\tau}\left(A(\bar{T}_{k-1}), L(\bar{T}_k), \bar{T}_{(k)}, \bar{\Delta}_{(k)}, \mathcal{F}^{\text{obs}}_{T_{(k-1)}}\right) \\
& + \frac{\mathbb{1}\left\{\bar{T}_{(k)} \leq \tau, \bar{\Delta}_{(k)} = a\right\}}{S^c_k\left(\bar{T}_{(k-1)} - \mid \mathcal{F}^{\text{obs}}_{T_{(k-1)}}\right)} \bar{Q}^g_{k,\tau}\left(1, L(\bar{T}_{k-1}), \bar{T}_{(k)}, \bar{\Delta}_{(k)}, \mathcal{F}^{\text{obs}}_{T_{(k-1)}}\right) \\
& + \frac{\mathbb{1}\left\{\bar{T}_{(k)} \leq \tau, \bar{\Delta}_{(k)} = y\right\}}{S^c_k\left(\bar{T}_{(k-1)} - \mid \mathcal{F}^{\text{obs}}_{T_{(k-1)}}\right)} \,\Bigg|\, \mathcal{F}^{\text{obs}}_{T_{(k-1)}} \Bigg]
\end{aligned}
\tag{2}
$$

for $k = K - 1, \ldots, 1$. Then,

$$\Psi_\tau(Q) = \mathbb{E}_P\left[\bar{Q}^g_{0,\tau}(1, L(0))\right].$$

The representation is nice,

1. it is useful for (locally) efficient inference.
2. we do not use reweighting with the treatment propensity scores $\Rightarrow$ increased stability/robustness.

EIF/Debiased ML

- Want to do efficient estimation of our target functional $\Psi^g_\tau(P)$.

Efficient influence function

Let

$$
\begin{aligned}
\mathrm{MG}_k &= \int_{T_{(k-1)}}^{\tau \wedge T_{(k)}} \left(\bar{Q}^g_{k-1,\tau}\left(\tau \mid \mathcal{F}_{T_{(k-1)}}\right) - \bar{Q}^g_{k-1,\tau}\left(u \mid \mathcal{F}_{T_{(k-1)}}\right)\right) \frac{1}{S^c\left(u- \mid \mathcal{F}_{T_{(k-1)}}\right) S\left(u \mid \mathcal{F}_{T_{(k-1)}}\right)} M^c_k(\mathrm{d}u), \\
M^c_k(t) &= \mathbb{1}\left\{T_{(k-1)} < t \leq T_{(k)}\right\}\left(N^c(t) - \Lambda^c\left(t \mid \mathcal{F}_{T_{(k-1)}}\right)\right), \\
\bar{Z}^a_{k,\tau}(s) &= \frac{\mathbb{1}\left\{T_{(k)} \leq s, \Delta_{(k)} = \ell\right\}}{S^c\left(T_{(k)} - \mid \mathcal{F}_{T_{(k-1)}}\right)} \bar{Q}^g_{k,\tau}\left(A\left(T_{(k-1)}\right), L\left(T_{(k)}\right), T_{(k)}, \Delta_{(k)}, \mathcal{F}_{T_{(k-1)}}\right) \\
&\quad + \frac{\mathbb{1}\left\{T_{(k)} \leq s, \Delta_{(k)} = a\right\}}{S^c\left(T_{(k)} - \mid \mathcal{F}_{T_{(k-1)}}\right)} \bar{Q}^g_{k,\tau}\left(1, L\left(T_{(k-1)}\right), T_{(k)}, \Delta_{(k)}, \mathcal{F}_{T_{(k-1)}}\right) \\
&\quad + \frac{\mathbb{1}\left\{T_{(k)} \leq s, \Delta_{(k)} = y\right\}}{S^c\left(T_{(k)} - \mid \mathcal{F}_{T_{(k-1)}}\right)}, s < \tau
\end{aligned}
$$

and $\bar{Q}^g_{k-1,\tau}\left(s \mid \mathcal{F}_{T_{(k-1)}}\right) = \mathbb{E}_P\left[\bar{Z}^a_{k,\tau}(s) \mid \mathcal{F}_{T_{(k-1)}}\right], s < \tau$. The efficient influence function is given by

$$
\begin{aligned}
\varphi^*_\tau(P) &= \frac{\mathbb{1}\{A(0) = 1\}}{\pi_0(L(0))} \sum_{k=1}^{K} \prod_{j=1}^{k-1} \left(\frac{\mathbb{1}\left\{A\left(T_{(j)}\right) = 1\right\}}{\pi_j\left(T_{(j)}, \mathcal{F}_{T_{(j-1)}}\right)}\right)^{\mathbb{1}\{\Delta_{(j)} = a\}} \\
&\qquad \times \frac{1}{\prod_{j=1}^{k-1} S^c\left(T_{(j)} - \mid \mathcal{F}_{T_{(j-1)}}\right)} \mathbb{1}\left\{\Delta_{(k-1)} \in \{\ell, a\}, T_{(k-1)} < \tau\right\}\left(\bar{Z}^a_{k,\tau} - \bar{Q}^g_{k-1,\tau} + \mathrm{MG}_k\right) \\
&\quad + \bar{Q}^g_{0,\tau} - \Psi_\tau(P),
\end{aligned}
$$

Estimating the martingale term is evidently computationally difficult. A naive strategy is to apply the ICE-IPCW procedure for a sequence of time points. However, we can use the given ICE-IPCW estimator from the previous step $\hat{\nu}_k$ to provide a new computationally feasible estimator for $\bar{Q}^g_{k+1,\tau}(s)$ used in MG_k. First obtain the estimates $\tilde{\nu}_{k,\tau}$ by regressing $\bar{Z}^a_{k+1,\tau}\left(\widehat{S}^c, \hat{\nu}_{k+1,\tau}\right)$ on $\left(T_{(k)}, \Delta_{(k)}, A\left(T_{(k)}\right), \mathcal{F}_{T_{(k-1)}}\right)$. Now determine estimates on a fine grid $0 < s_1 < \ldots < s_{k\left(T_{(k-1)}\right)} < h_{\max}$,

$$
\begin{aligned}
\hat{\nu}^*_{k,\tau}\left(s \mid \mathcal{F}_{T_{(k)}}\right) = \int_0^{s - T_{(k)}} &\prod_{s \in \left(0, u - T_{(k)}\right)} \left(1 - \sum_{x = a, l, d, y} \hat{\Lambda}^x_{k+1}\left(ds \mid \mathcal{F}_{T_{(k)}}\right)\right)\left[\hat{\Lambda}^y_{k+1}\left(du \mid \mathcal{F}_{T_{(k)}}\right)\right. \\
&+ \tilde{\nu}_{k+1,\tau}\left(u + T_{(k)}, a, 1, \mathcal{F}_{T_{(k)}}\right)\hat{\Lambda}^a_{k+1}\left(du \mid \mathcal{F}_{T_{(k)}}\right) \\
&\left.+ \tilde{\nu}_{k+1,\tau}\left(u + T_{(k)}, \ell, A\left(T_{(k)}\right), \mathcal{F}_{T_{(k)}}\right)\hat{\Lambda}^\ell_{k+1}\left(du \mid \mathcal{F}_{T_{(k)}}\right)\right]
\end{aligned}
$$

for $s = T_{(k-1)} + s_i$. Then, we can estimate the EIF by

$$
\begin{aligned}
\varphi^*\left(\hat{P}^*_n\right) = &\frac{\mathbb{1}\{A(0) = 1\}}{\hat{\pi}_0(L(0))} \sum_{k=1}^{K} \prod_{j=1}^{k-1} \left(\frac{\mathbb{1}\left\{A\left(T_{(j)}\right) = 1\right\}}{\hat{\pi}\left(T_{(j)}, A\left(T_{(j)}\right), \mathcal{F}_{T_{(j-1)}}\right)}\right)^{\mathbb{1}\{\Delta_{(j)} = a\}} \frac{1}{\prod_{j=1}^{k-1} \hat{S}^c\left(T_{(j)} - \mid \mathcal{F}_{T_{(j-1)}}\right)} \mathbb{1}\left\{\Delta_{(k-1)} \in \{\ell, a\}, T_{(k-1)} < \tau\right\} \\
&\times \left(\bar{Z}^a_{k,\tau}\left(\hat{S}^c, \hat{\nu}_{k,\tau}\right) - \hat{\nu}_{k-1,\tau}\left(\mathcal{F}_{T_{(k-1)}}\right) + \widehat{\mathrm{MC}}_k\left(\hat{\Lambda}^c_k, \hat{\nu}^*_{k,\tau}\right)\right) \\
&+ \hat{\nu}_{0,\tau}(1, L(0)) - \mathbb{P}_n \hat{\nu}_{0,\tau}(1, \cdot)
\end{aligned}
$$

To perform double/debiased machine learning, we need to find the remainder term $R_2\left(\tilde{P}, P, P_0\right)$ and show that it has the typical product structure.

We cannot use the one in Rytgaard et al. (2022), because 1. theirs is without competing risks and 2. they facilitate estimation in a different way. This is because we end up estimating $\bar{Q}^g_{k+1,\tau}(\tau)$ in two different ways.

Remainder term

The remainder term $R_2\left(\tilde{P}, P, P_0\right) = \Psi_\tau(P) - \Psi_\tau(P_0) + \mathbb{E}_{P_0}\left[\varphi^*\left(\tilde{P}, P\right)\right]$ is given by

$$R_2\left(\tilde{P}, P, P_0\right) = \sum_{k=1}^{K} \int \mathbb{1}\{t_1 < \dots < t_k < \tau\} \prod_{j=0}^{k-2} \left(\frac{\pi_{0,j}\left(t_k, f_{j-1}^{\mathbf{1}}\right)}{\pi_j\left(t_k, f_{j-1}^{\mathbf{1}}\right)}\right)^{\mathbb{1}\{d_j = a\}}$$
$$\frac{\prod_{j=1}^{k-1} S_0^c\left(t_j - \mid f_{j-1}^{\mathbf{1}}\right)}{\prod_{j=1}^{k-1} S^c\left(t_j - \mid f_{j-1}^{\mathbf{1}}\right)} \mathbb{1}\{d_1 \in \{\ell, a\}, \dots, d_{k-1} \in \{\ell, a\}\} z_k(f_k^{\mathbf{1}}) P_{\mathcal{F}_{T_k}}(df_k),$$

Here $\tilde{P}, P$ means that we plug in two different estimators as described earlier ($\nu_{k,\tau}$ and $\nu_{k,\tau}^*$), and

$$\begin{aligned} z_k\left(\mathcal{F}_{T_{(k)}}\right) = &\left(\left(\frac{\pi_{k-1,0}\left(T_{(k)}, \mathcal{F}_{T_{(k-1)}}\right)}{\pi_{k-1}\left(T_{(k)}, \mathcal{F}_{T_{(k-1)}}\right)}\right)^{\mathbb{1}\left\{\Delta_{(k)} = a\right\}} - 1\right)\left(\bar{Q}_{k-1,\tau}^g\left(\mathcal{F}_{T_{(k-1)}}\right) - \nu_{k-1,\tau}\left(\mathcal{F}_{T_{(k-1)}}\right)\right) \\ &+ \left(\frac{\pi_{k-1,0}\left(T_{(k)}, \mathcal{F}_{T_{(k-1)}}\right)}{\pi_{k-1}\left(T_{(k)}, \mathcal{F}_{T_{(k-1)}}\right)}\right)^{\mathbb{1}\left\{\Delta_{(k)} = a\right\}} \\ &\quad \times \int_{T_{(k-1)}}^{\tau} \left(\frac{S_0^c\left(u \mid \mathcal{F}_{T_{(k-1)}}\right)}{S^c\left(u \mid \mathcal{F}_{T_{(k-1)}}\right)} - 1\right)\left(\nu_{k-1,\tau}^*\left(du \mid \mathcal{F}_{T_{(k-1)}}\right) - \bar{Q}_{k-1,\tau}^g\left(du \mid \mathcal{F}_{T_{(k-1)}}\right)\right) \\ &+ \left(\frac{\pi_{k-1,0}\left(T_{(k)}, \mathcal{F}_{T_{(k-1)}}\right)}{\pi_{k-1}\left(T_{(k)}, \mathcal{F}_{T_{(k-1)}}\right)}\right)^{\mathbb{1}\left\{\Delta_{(k)} = a\right\}} \int_{T_{(k-1)}}^{\tau} V_k\left(u, \mathcal{F}_{T_{(k-1)}}\right) \nu_{k-1,\tau}^*\left(du \mid \mathcal{F}_{T_{(k-1)}}\right), \end{aligned}$$

and $V_k\left(u, \mathcal{F}_{T_{(k)}}\right) = \int_{T_{(k-1)}}^{u} \left(\frac{S_0\left(s \mid \mathcal{F}_{T_{(k-1)}}\right)}{S\left(s \mid \mathcal{F}_{T_{(k-1)}}\right)} - 1\right) \frac{S_0^c\left(s- \mid \mathcal{F}_{T_{(k-1)}}\right)}{S^c\left(s- \mid \mathcal{F}_{T_{(k-1)}}\right)} \left(\Lambda_{k,0}^c\left(ds \mid \mathcal{F}_{T_{(k-1)}}\right) - \Lambda^c\left(ds \mid \mathcal{F}_{T_{(k-1)}}\right)\right)$. Here $f_j^{\mathbf{1}}$ simply means that we insert 1 into every place where we have $a_i, i = 1, \dots, j$ in f_j.

Simulation + data application

???

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