stuff about the MRI?

set-up
boundary conditions
parameter range
open questions, etc
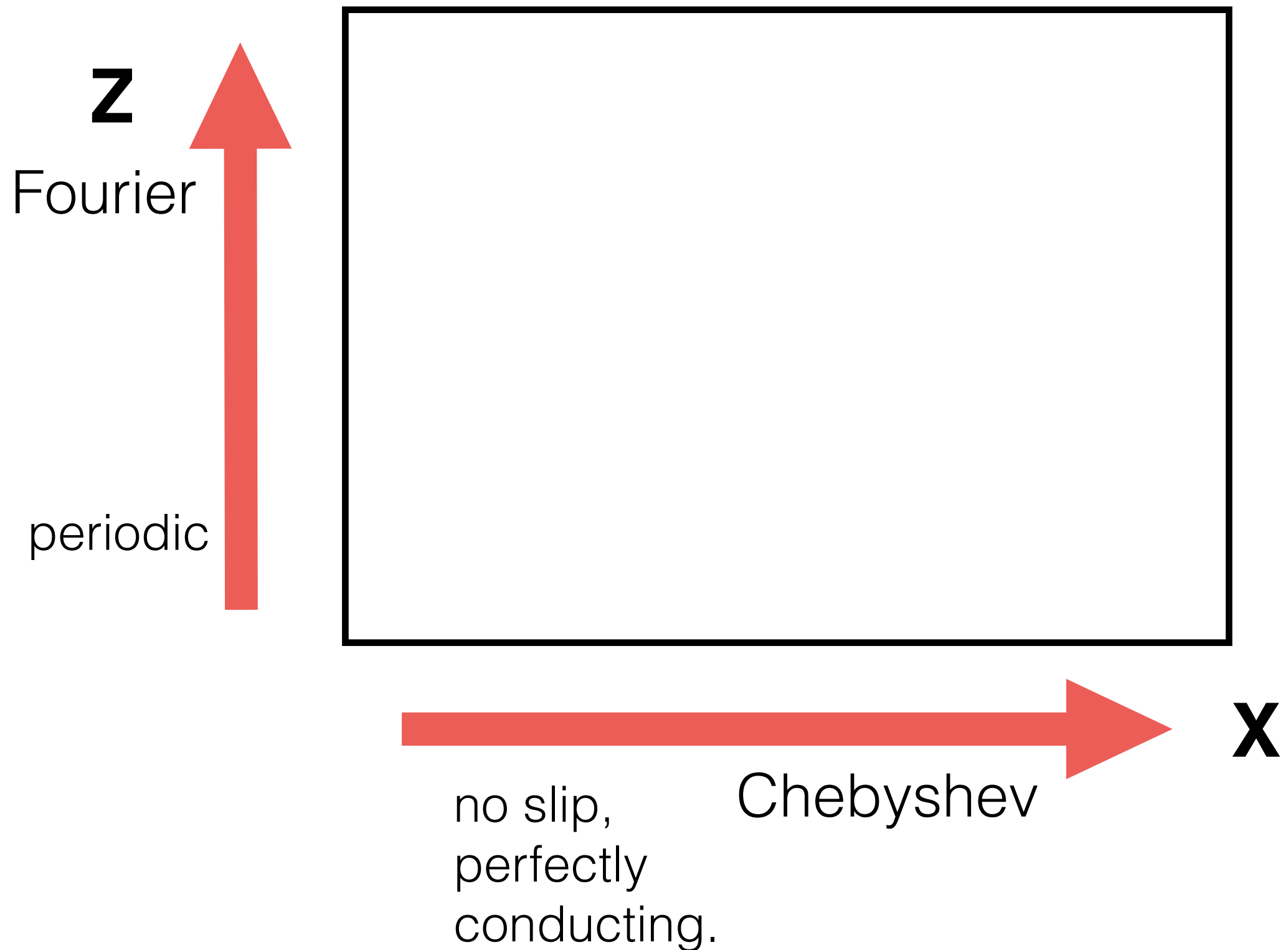
We solve the non-ideal MRI equations.

momentum

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla P - \nabla \Phi + \frac{1}{\rho} \left(\mathbf{J} \times \mathbf{B} \right) - 2\Omega \times \mathbf{u} - \Omega \times \left(\Omega \times \mathbf{r} \right) + \nu \nabla^2 \mathbf{u}$$

induction

$$\partial_t \mathbf{B} = \nabla \times \left(\mathbf{u} \times \mathbf{B} \right) + \eta \nabla^2 \mathbf{B}$$

constraints

momentum

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla P - \nabla \Phi + \frac{1}{\rho} \left(\mathbf{J} \times \mathbf{B} \right) + \nu \nabla^2 \mathbf{u} - 2\boldsymbol{\Omega} \times \mathbf{u} - \boldsymbol{\Omega} \times \left(\boldsymbol{\Omega} \times \mathbf{r} \right)$$
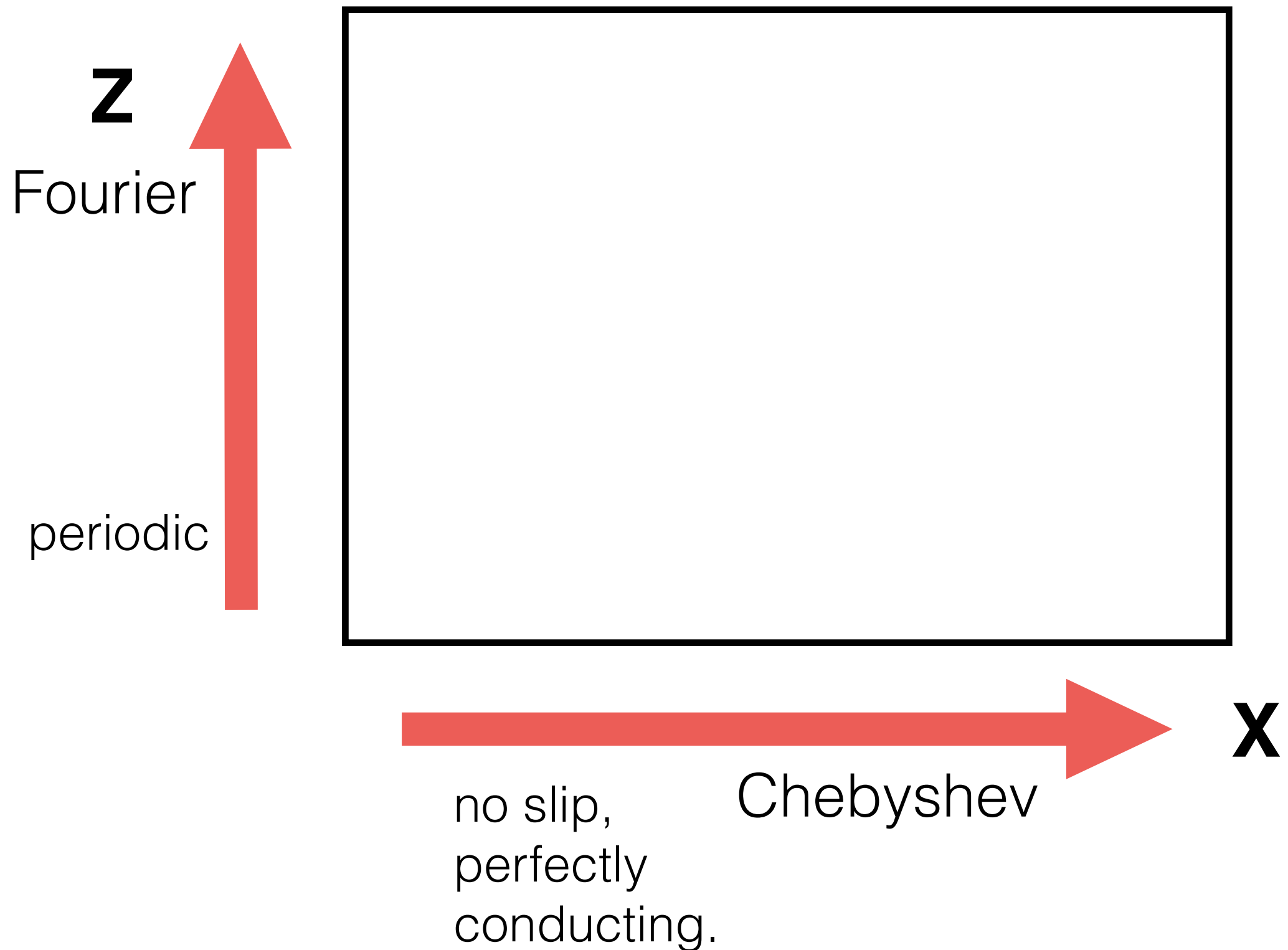
induction

$$\partial_t \mathbf{B} = \nabla \times \left(\mathbf{u} \times \mathbf{B} \right) + \eta \nabla^2 \mathbf{B}$$

Dedalus is a general-purpose spectral code.

Z

Fourier

periodic

X

Chebyshev

no slip,
perfectly
conducting.

We use experimentally relevant
boundary conditions.

Z

Fourier

periodic

X

no slip,
perfectly
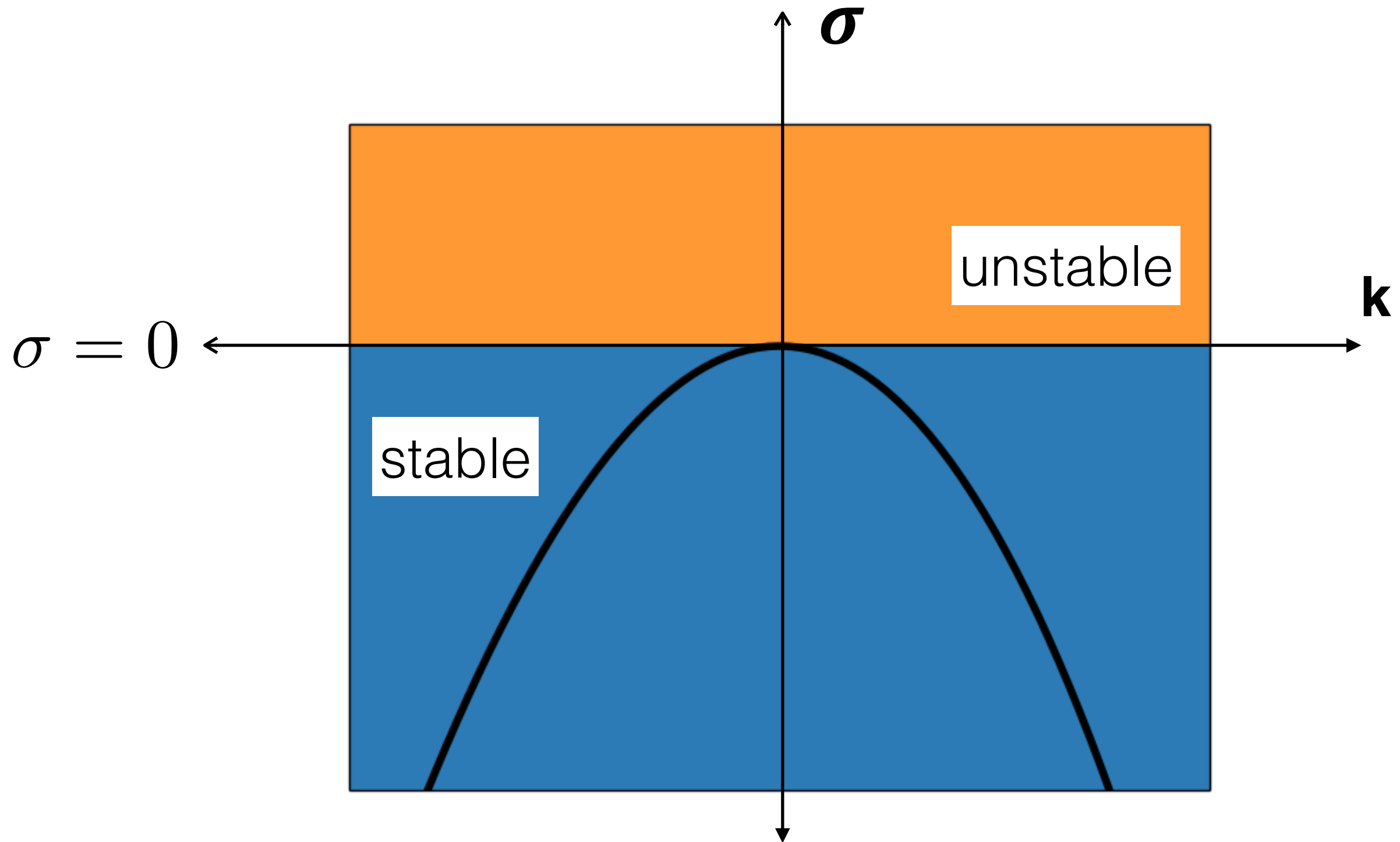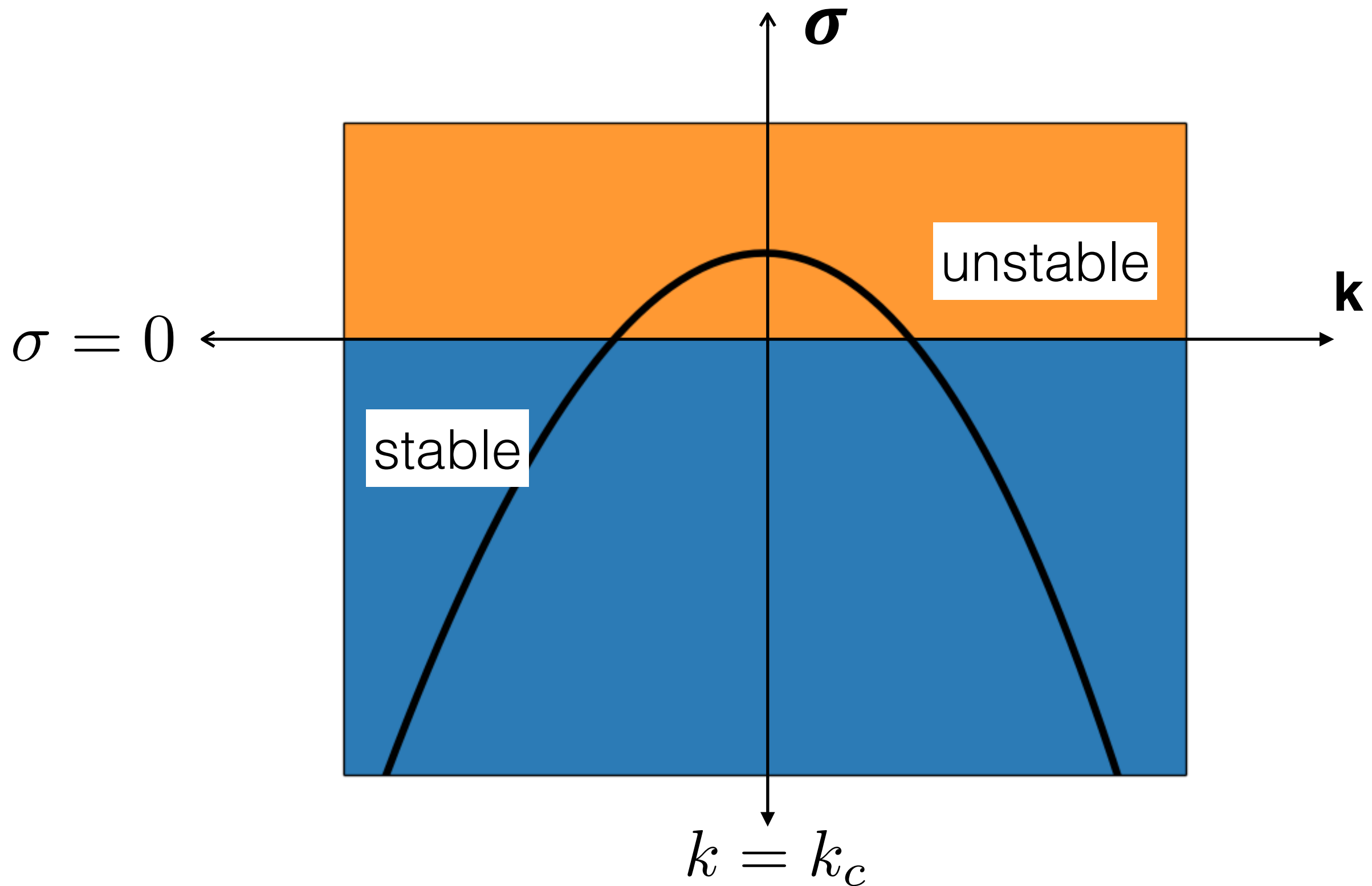conducting.

Chebyshev

Weakly nonlinear analysis explores behavior at the margin of instability.
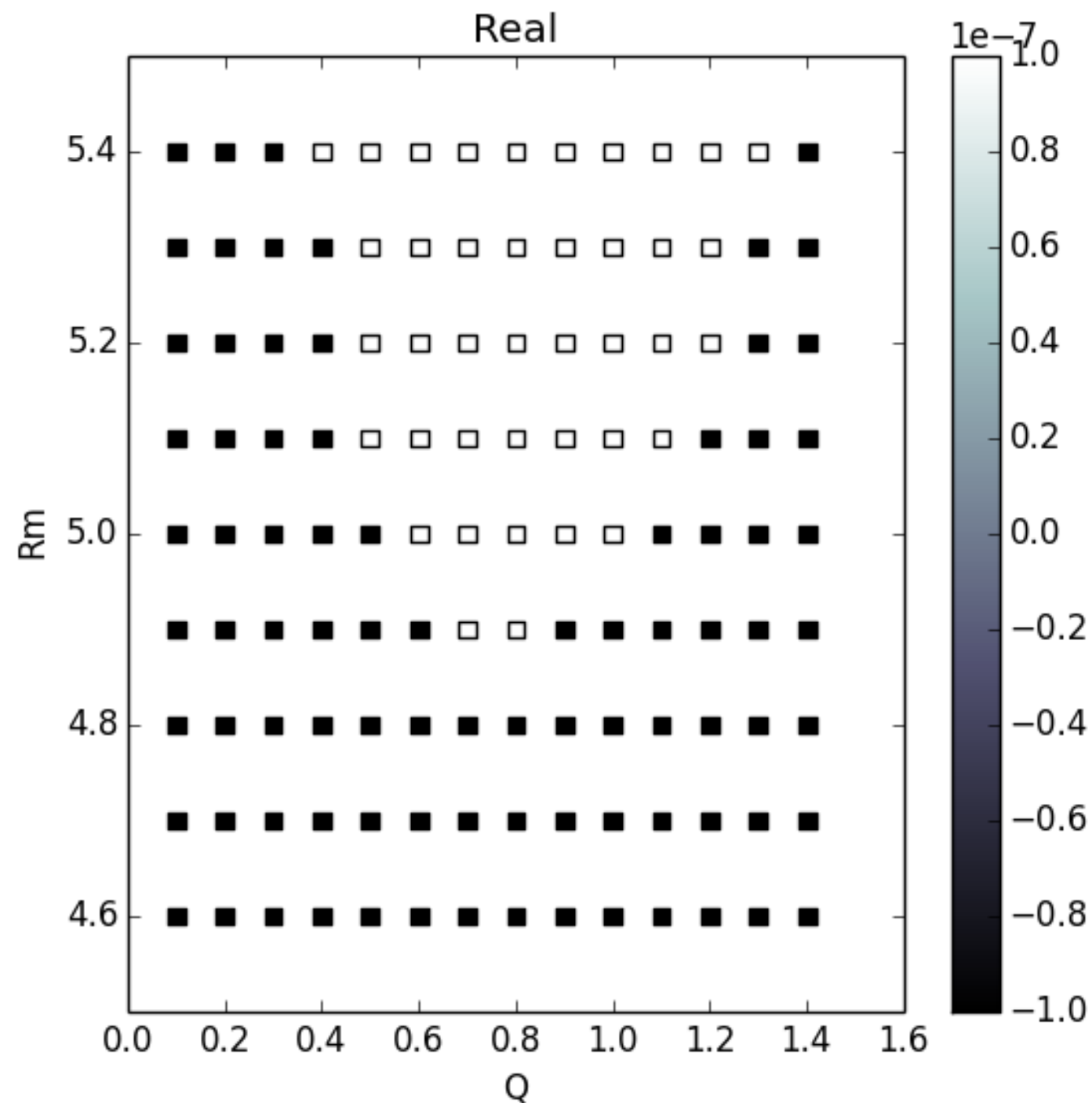
Weakly nonlinear analysis explores
behavior at the margin of instability.
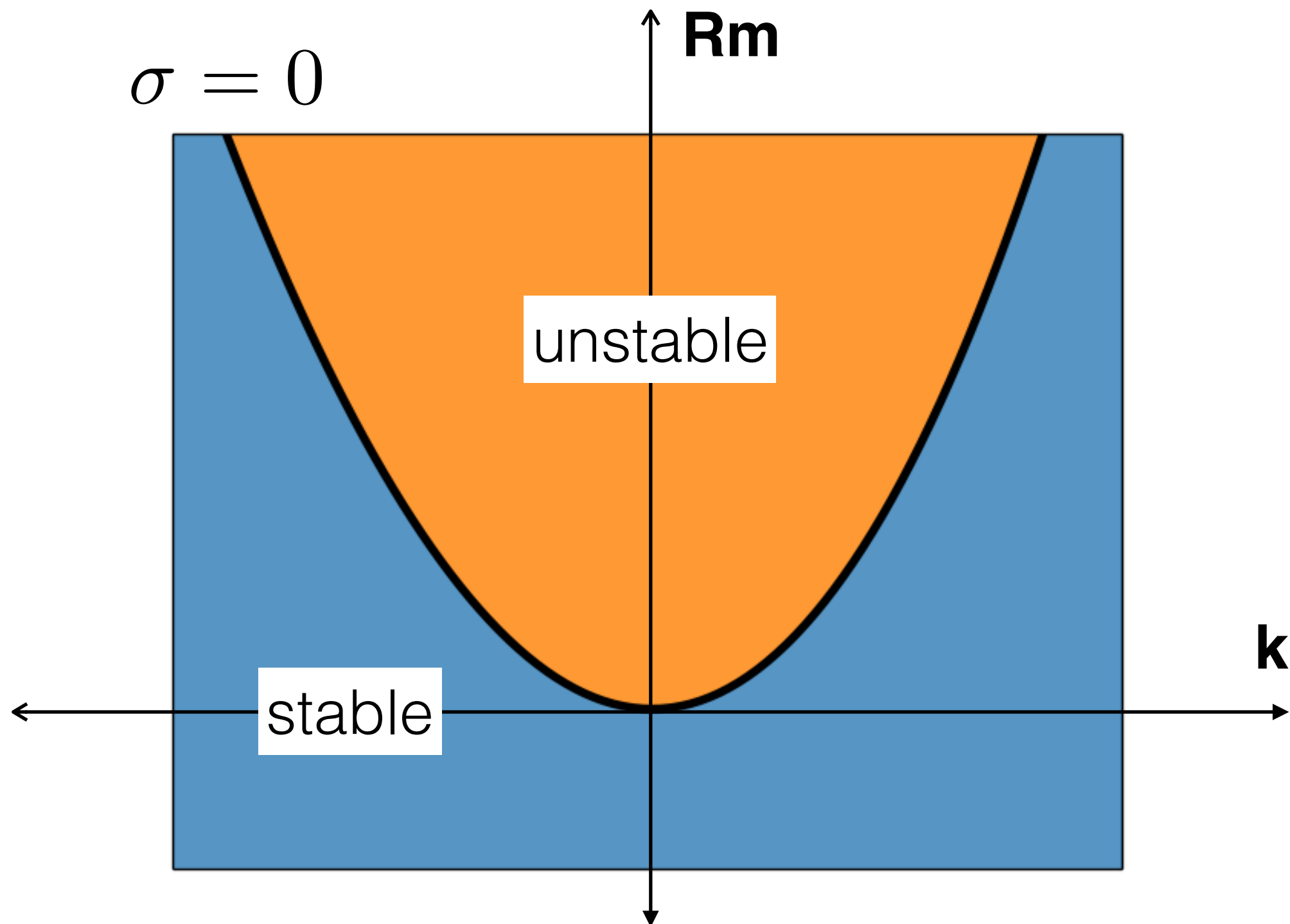
Weakly nonlinear analysis explores
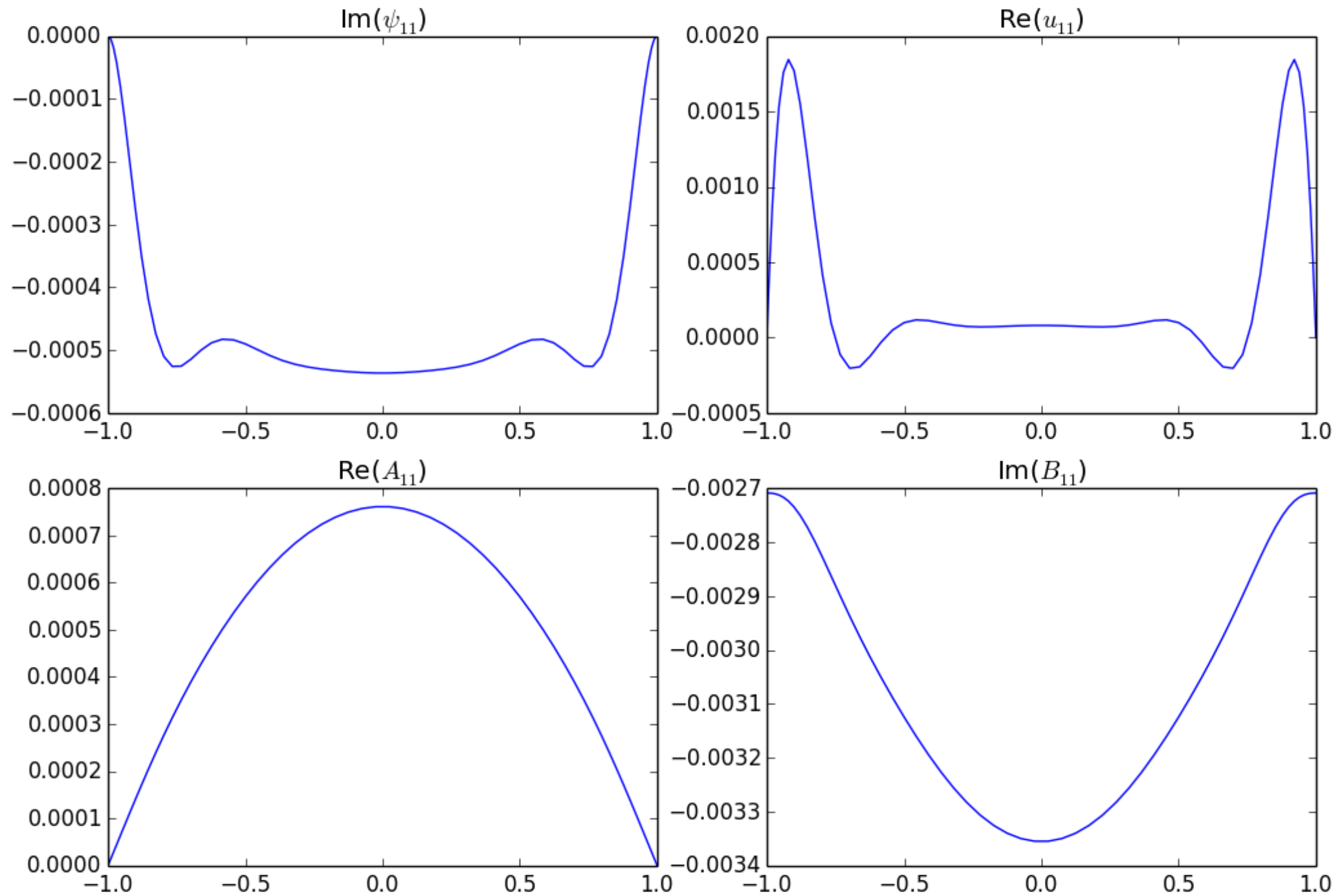behavior at the margin of instability.

Weakly nonlinear analysis explores
behavior at the margin of instability.

Weakly nonlinear analysis explores
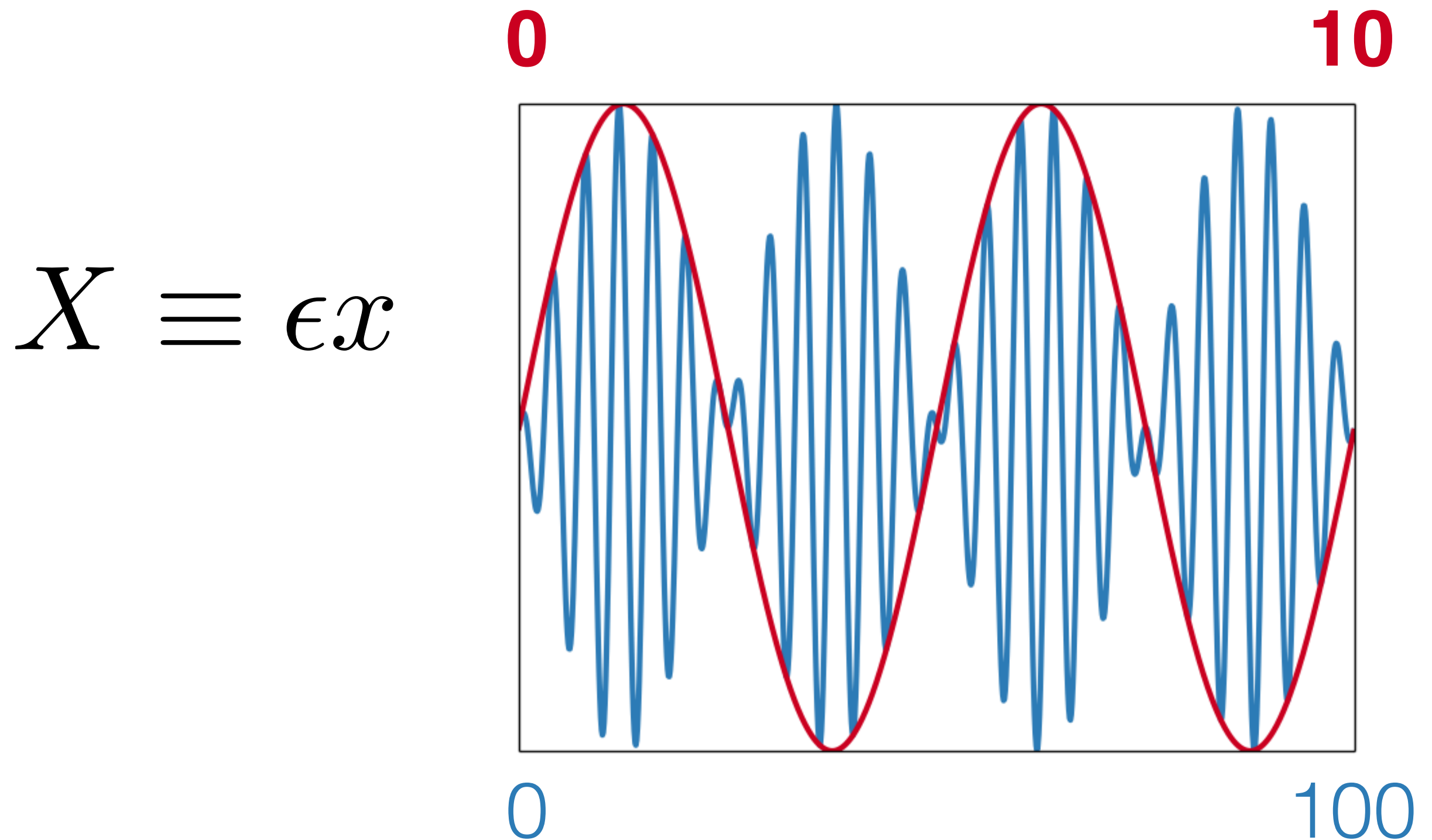behavior at the margin of instability.
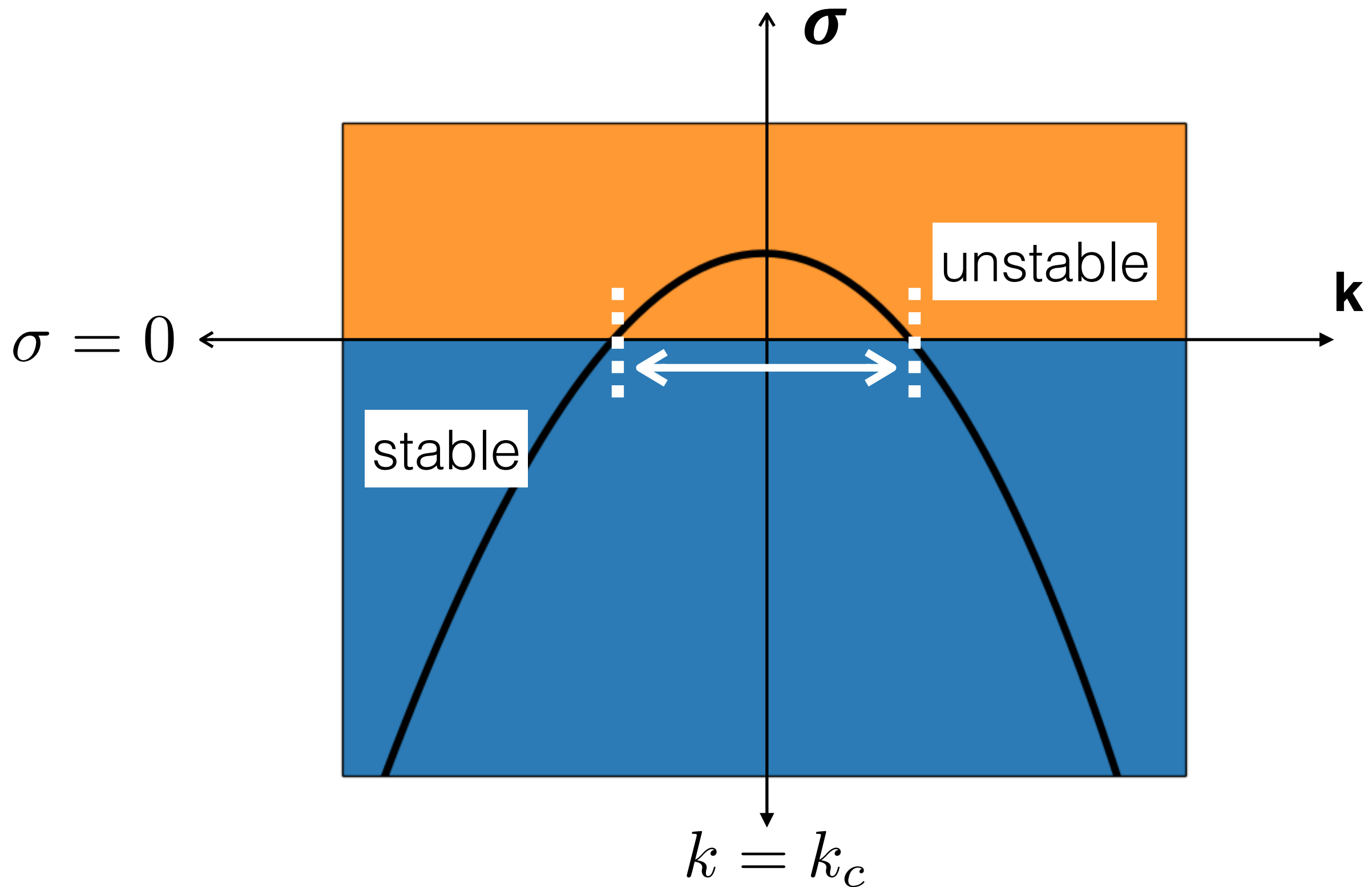
Identify the most unstable mode of the linear MRI.

Tune this mode just over the threshold of instability.

Multiscale analysis tracks the evolution of fast
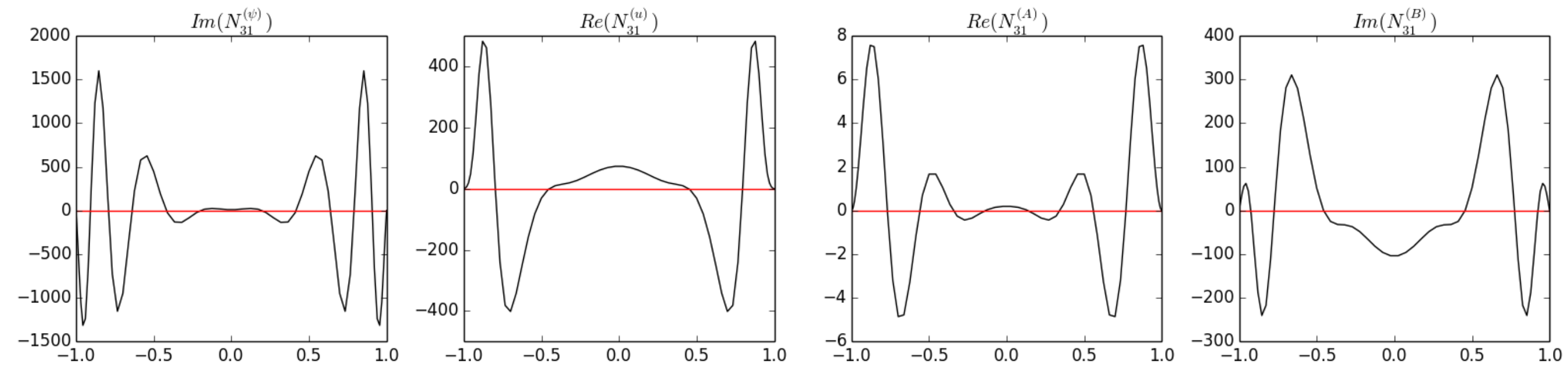and slow variables.

$$X \equiv \epsilon x$$

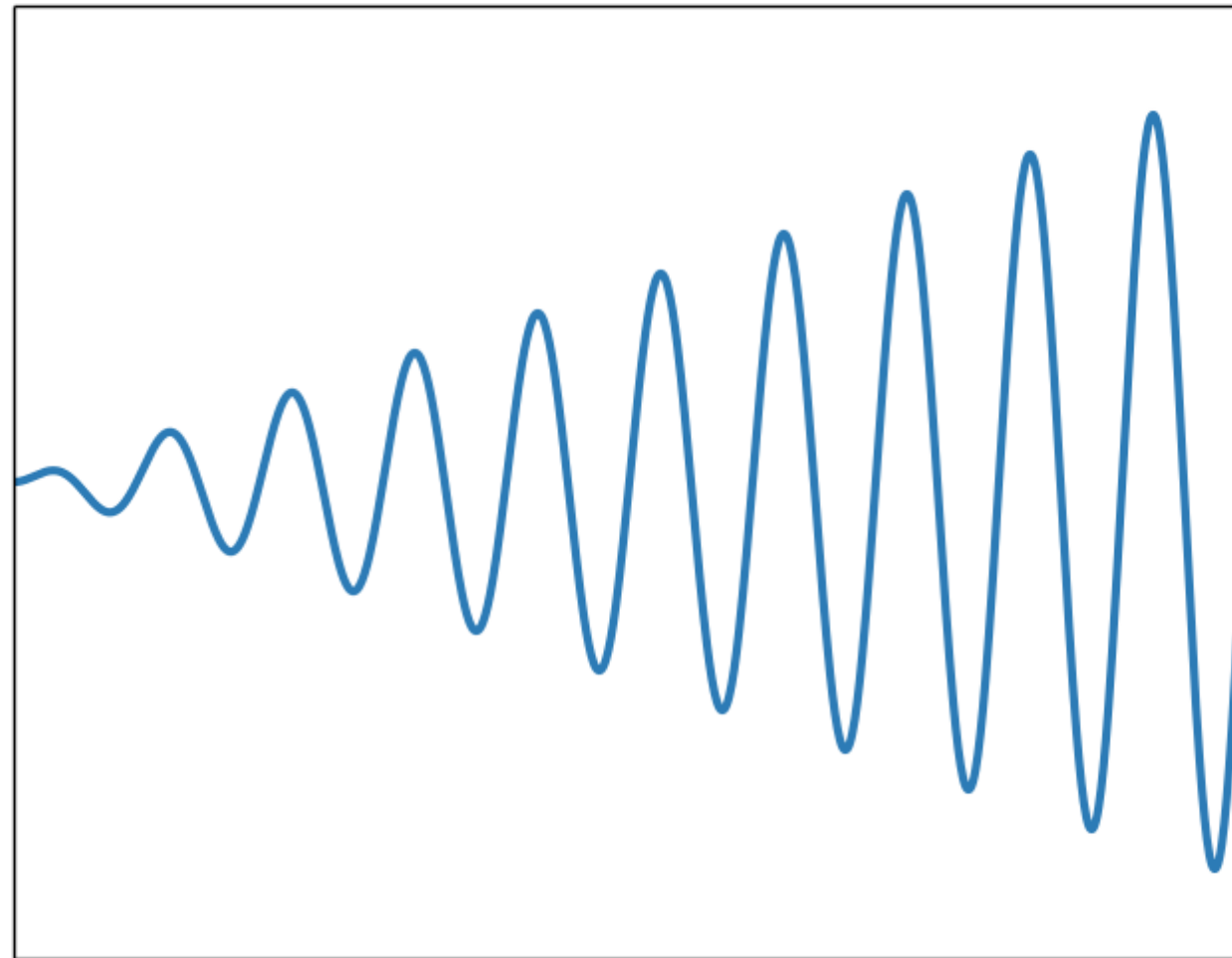Equations are solved in a
matrix formulation.

The fluid quantities are expanded
in a perturbation series.

$$\mathbf{V} = \epsilon \mathbf{V_1} + \epsilon^2 \mathbf{V_2} + \epsilon^3 \mathbf{V_3} + ...$$
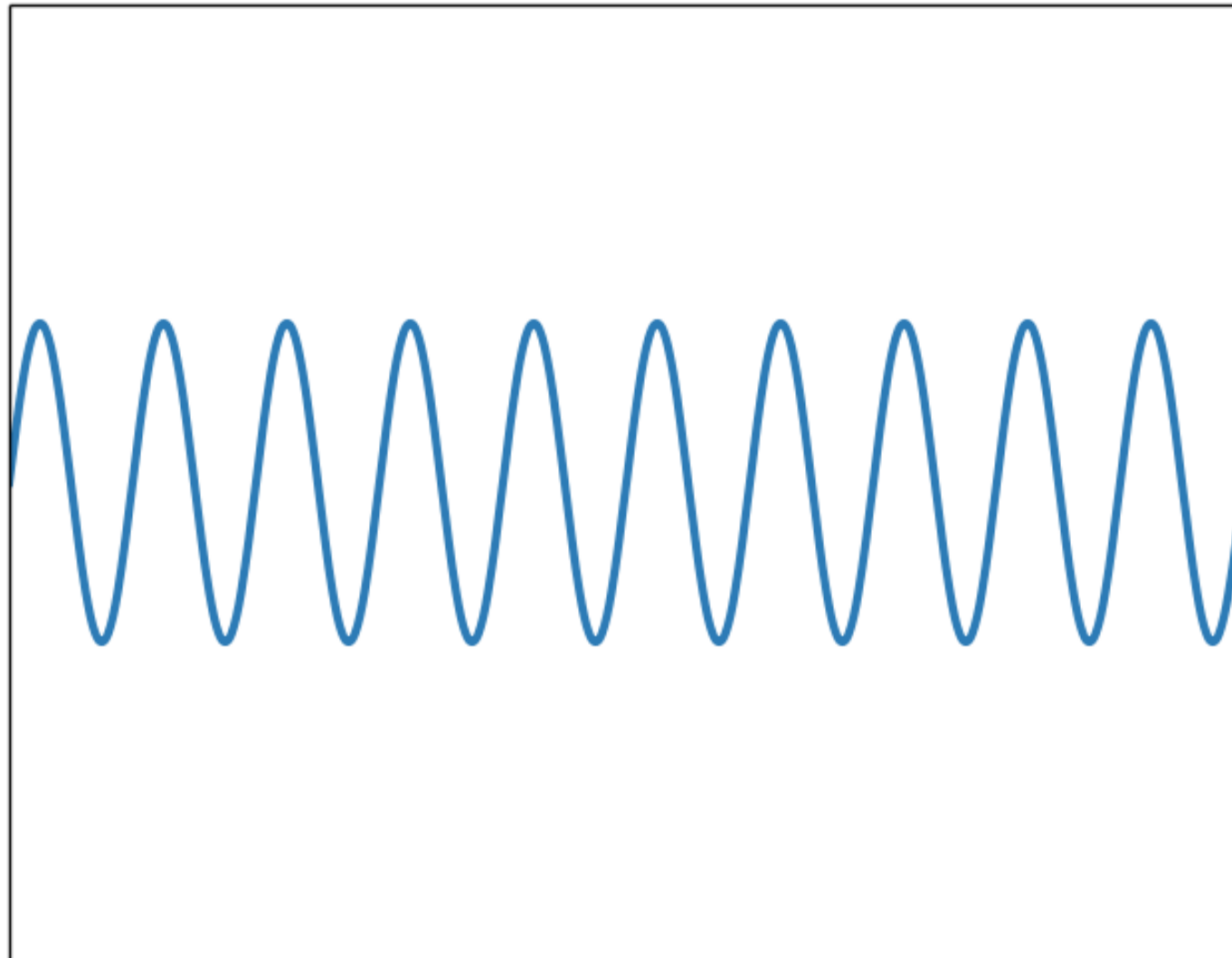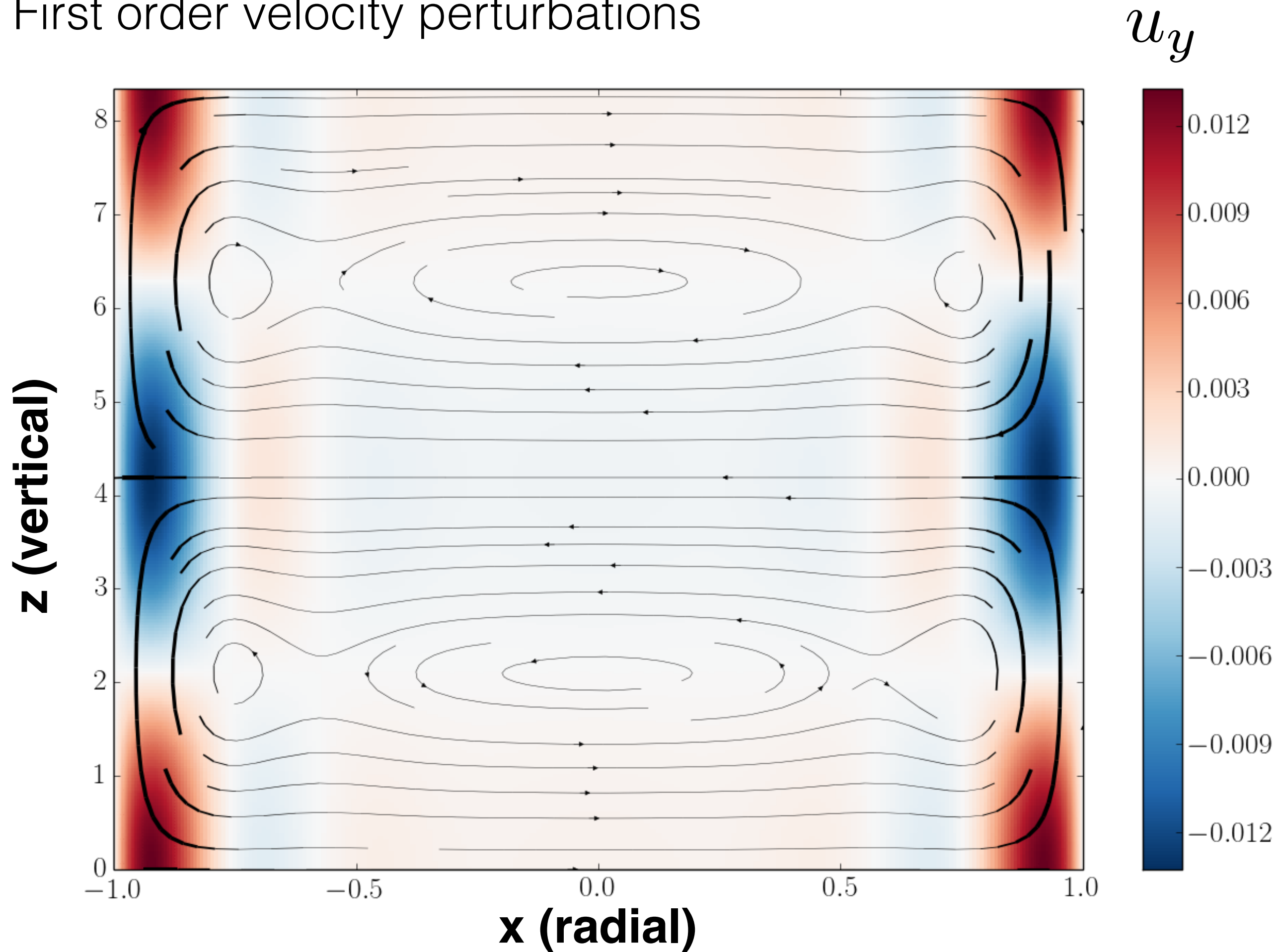
something about boundary layers?

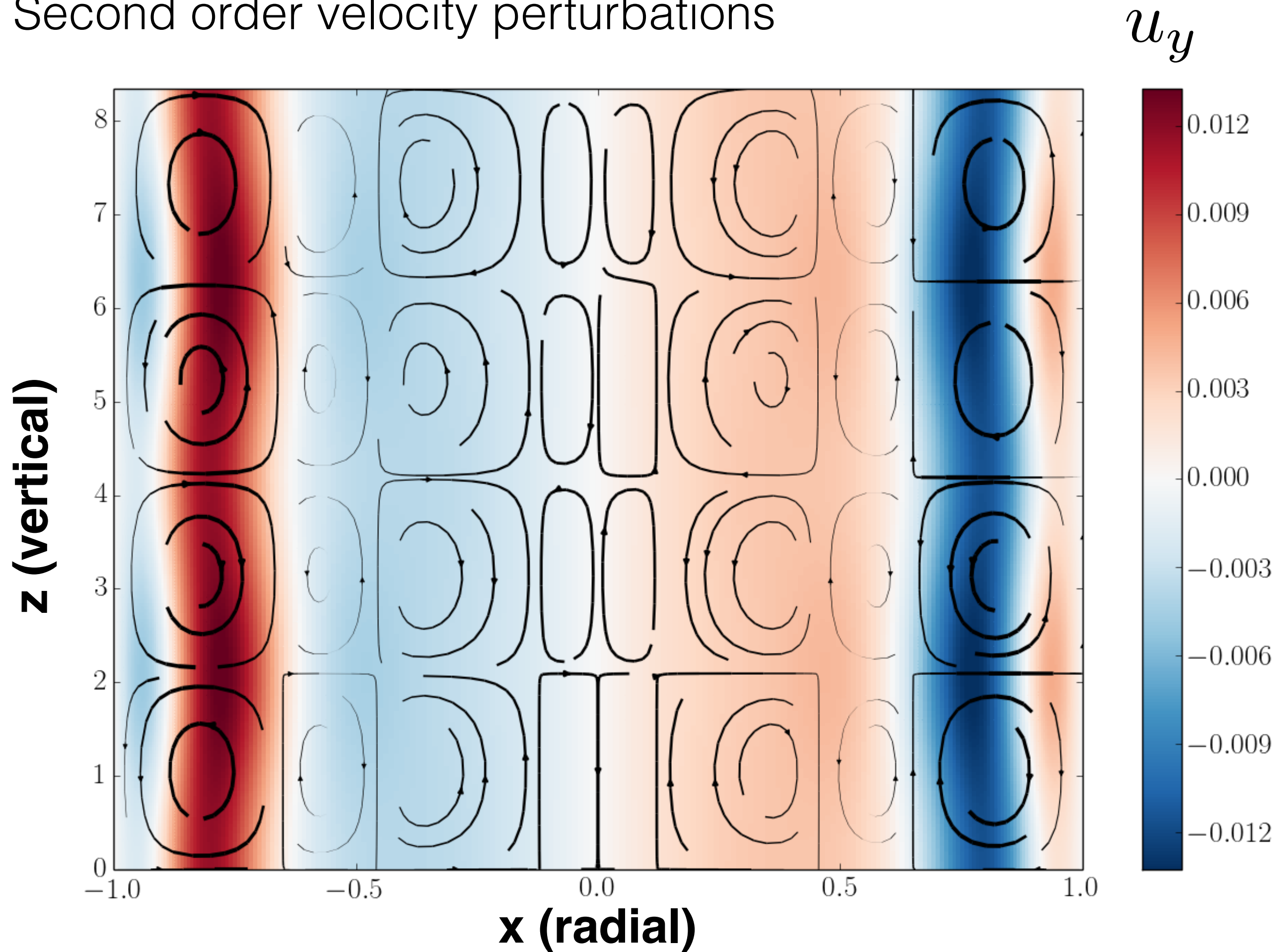The removal of secular terms yields solvability criteria.

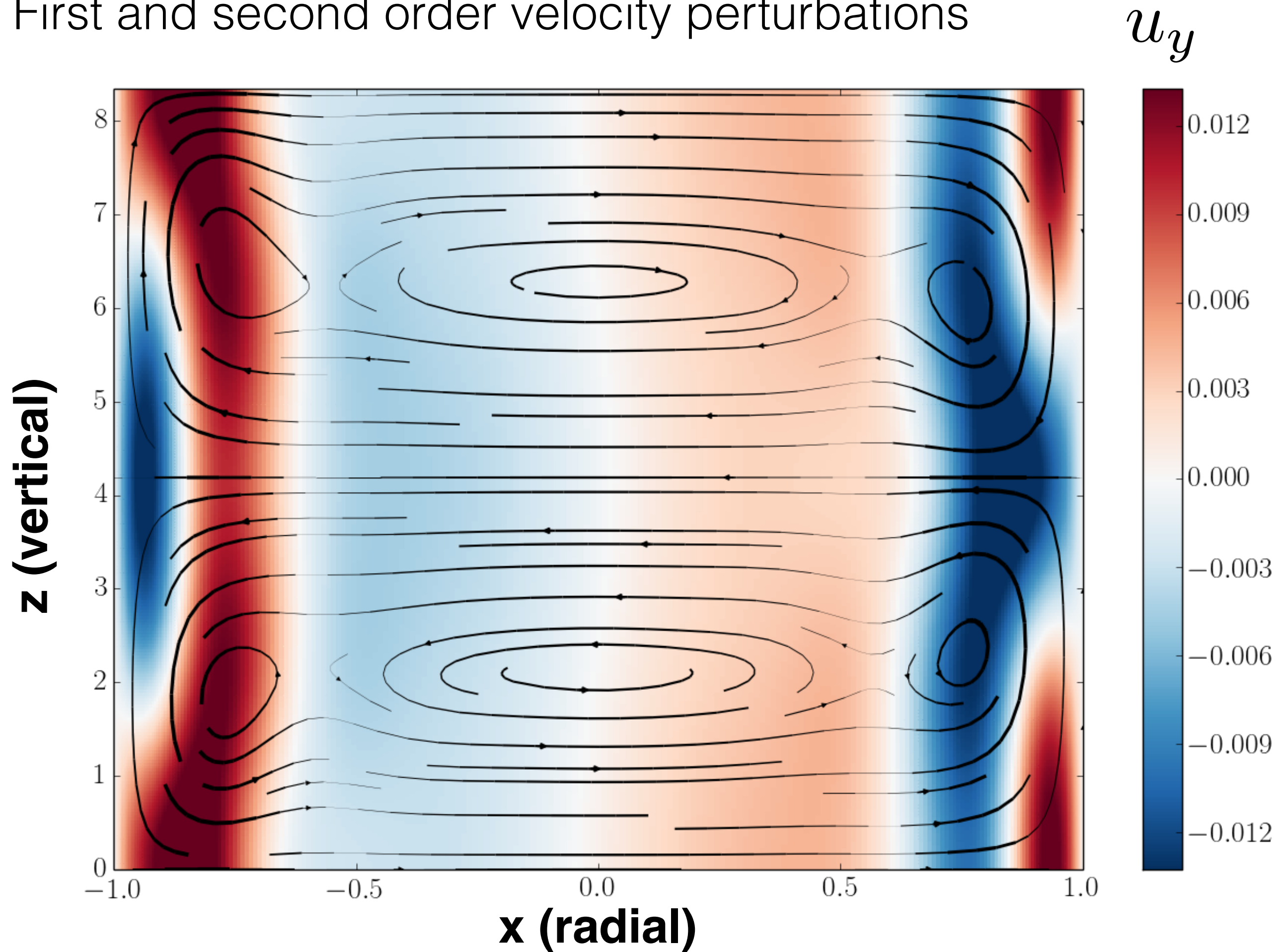# The removal of secular terms yields solvability criteria.
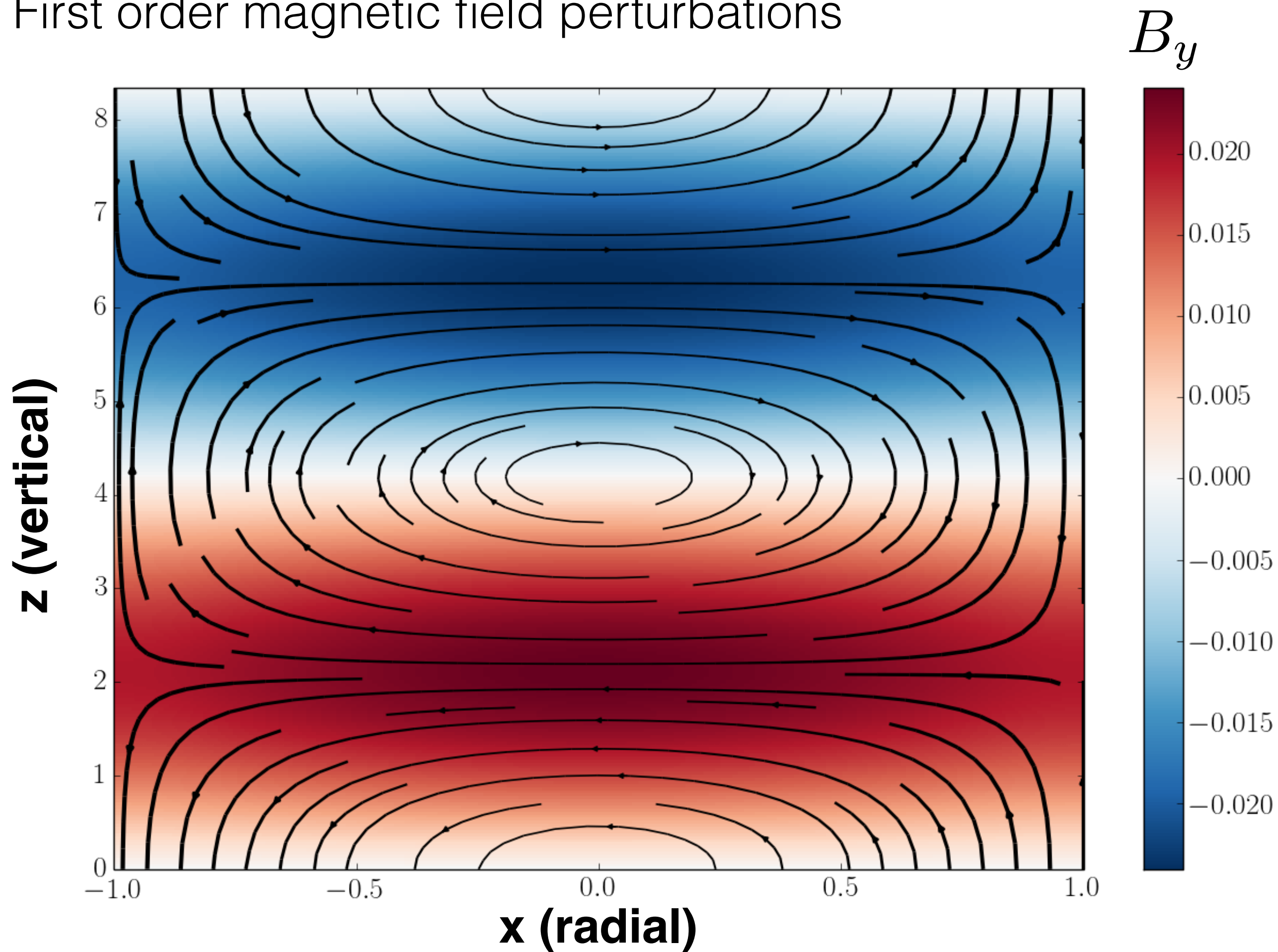
First order velocity perturbations

Second order velocity perturbations
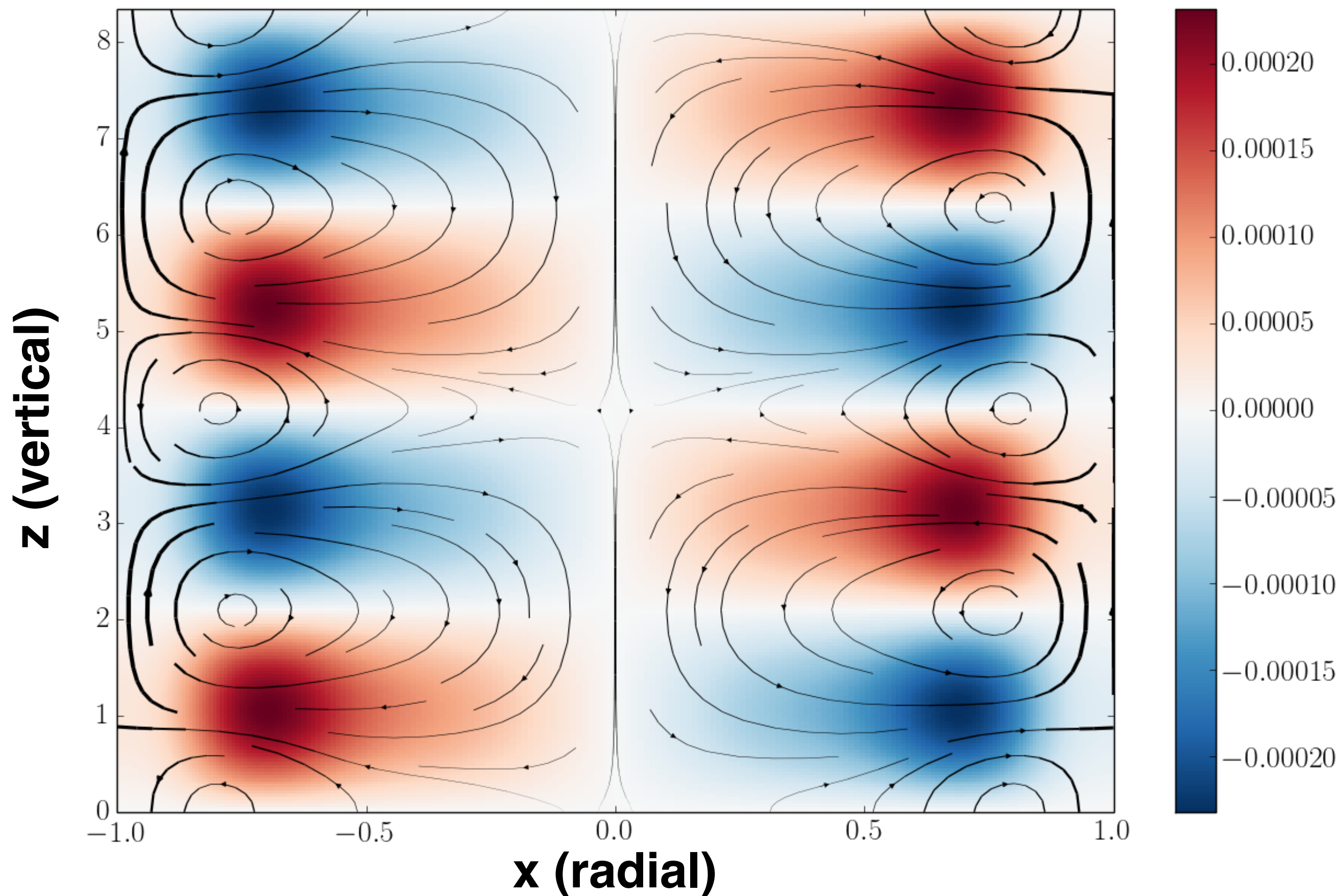
First and second order velocity perturbations

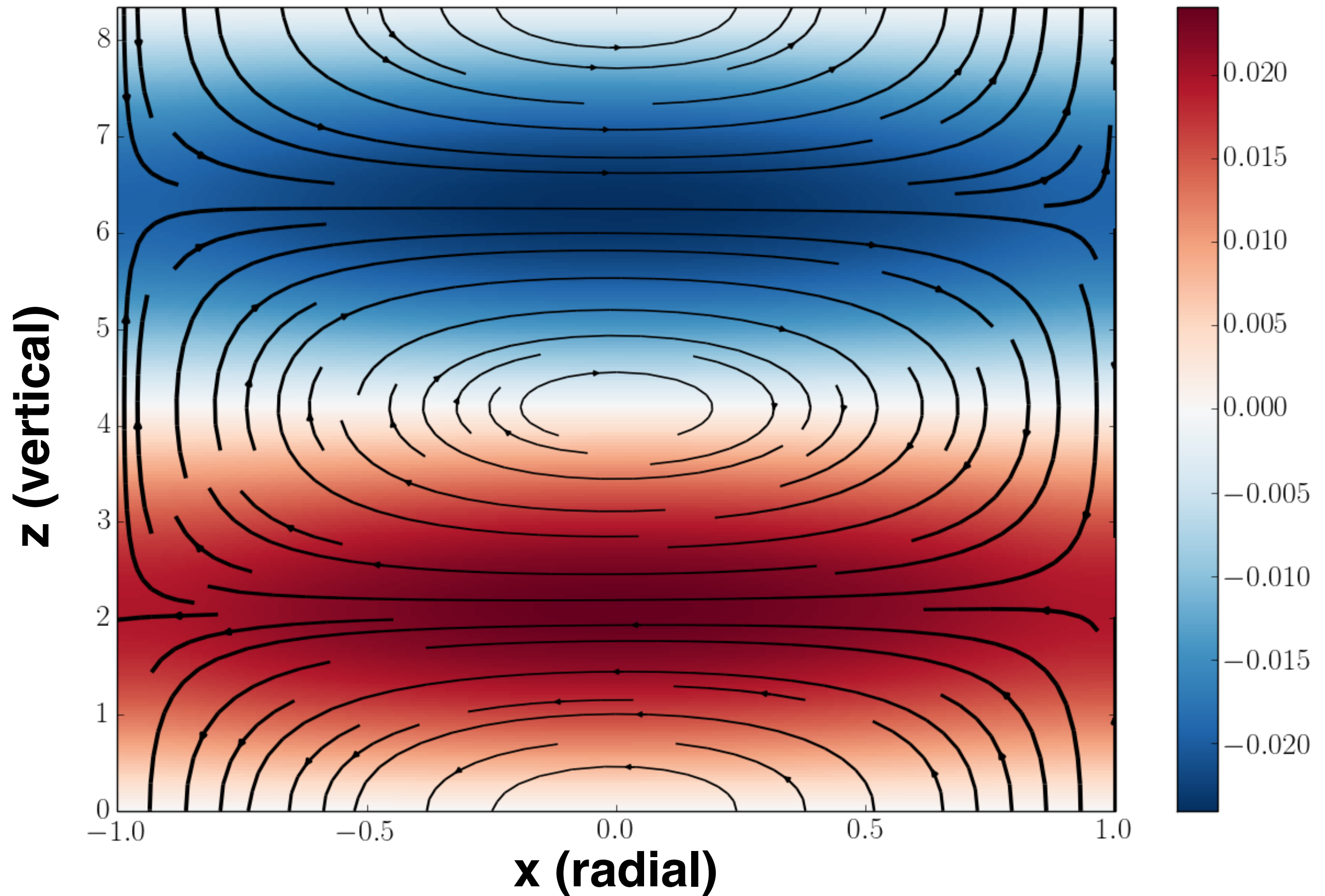First order magnetic field perturbations

Second order magnetic field perturbations

two OOM smaller!

B_y

z (vertical)

x (radial)

First and second order magnetic field perturbations

Future work:

non-thin gap approximation
helical MRI
explore parameter space
comparison to experiment