$N_0(A)_x$

$= J(A_1, \Psi_2) - J(A_2, \Psi_1) - \bar{J}(A_1, \Psi_1)$

$N_1(A)_x$

$= \partial_x A_1 \partial_z \Psi_2 - \partial_x A_2 \partial_z \Psi_1 - \partial_z A_1 \partial_x \Psi_1$

$+ \partial_z A_1 \partial_x \Psi_1 + \partial_z A_1 \partial_x \Psi_1 + \partial_x A_1 \partial_z \Psi_1$

$\Psi_2 = \alpha^2 \Psi_{22} e^{i2Q_z} + |\alpha|^2 \Psi_{20} - \partial_3 \alpha \Psi_{21} e^{iQ_z} - \partial_3 \beta \Psi_{20} + c.c.$

N_{3_1} is the portion of N_3 resonant w/ e^{iQ_z}

$\Psi_1 = \alpha \Psi_{11} e^{iQ_z} + \alpha^* \Psi_{11}^* e^{-iQ_z} + \beta \Psi_{11}$

$\partial_x \Psi_1 = iQ\alpha \Psi_{11} e^{iQ_z} - iQ\alpha^* \Psi_{11}^* e^{-iQ_z}$

$-\partial_z A_1 \partial_x \Psi_2 = (-iQ A_{11} e^{iQ_z} + iQ A_{11}^* e^{-iQ_z}).$

$\partial_x (\Psi_{22} e^{i2Q_z} + \Psi_{20} - \Psi_{21} e^{iQ_z}$

$+ \Psi_{22}^* e^{-i2Q_z} + \Psi_{20}^* - \Psi_{21}^* e^{-iQ_z})$

$= -iQ A_{11} e^{iQ_z} \cdot \partial_x \Psi_{22} e^{i2Q_z} \quad + iQ A_{11}^* e^{-iQ_z} \cdot \partial_x \Psi_{22} e^{i2Q_z}$

$-iQ A_{11} e^{iQ_z} \cdot \partial_x \Psi_{20} \quad + iQ A_{11}^* e^{-iQ_z} \cdot \partial_x \Psi_{20}$

$+iQ A_{11} e^{iQ_z} \cdot \partial_x \Psi_{21} e^{iQ_z} \quad - iQ A_{11}^* e^{-iQ_z} \cdot \partial_x \Psi_{21} e^{iQ_z}$

$-iQ A_{11} e^{iQ_z} \cdot \partial_x \Psi_{22}^* e^{-i2Q_z} \quad + iQ A_{11}^* e^{-iQ_z} \cdot \partial_x \Psi_{22}^* e^{-i2Q_z}$

$-iQ A_{11} e^{iQ_z} \cdot \partial_x \Psi_{20}^* \quad + iQ A_{11}^* e^{-iQ_z} \cdot \partial_x \Psi_{20}^*$

$+iQ A_{11} e^{iQ_z} \cdot \partial_x \Psi_{21}^* e^{-iQ_z} \quad - iQ A_{11}^* e^{-iQ_z} \cdot \partial_x \Psi_{21}^* e^{-iQ_z}$

$= -iQ A_{11} \cdot \partial_x \Psi_{22} e^{i3Q_z} \quad + iQ A_{11}^* \cdot \partial_x \Psi_{22} e^{iQ_z} \quad - iQ A_{11} \cdot \partial_x \Psi_{20} e^{iQ_z}$

$+ iQ A_{11}^* \cdot \partial_x \Psi_{20} e^{-iQ_z} \quad + iQ A_{11} \cdot \partial_x \Psi_{21} e^{i2Q_z} \quad - iQ A_{11}^* \cdot \partial_x \Psi_{21}$

$- iQ A_{11} \cdot \partial_x \Psi_{22}^* e^{-iQ_z} \quad + iQ A_{11}^* \cdot \partial_x \Psi_{22}^* e^{-i3Q_z} \quad - iQ A_{11} \cdot \partial_x \Psi_{20}^* e^{iQ_z}$

$+ iQ A_{11}^* \cdot \partial_x \Psi_{20}^* e^{-iQ_z} \quad + iQ A_{11} \cdot \partial_x \Psi_{21}^* \quad - iQ A_{11}^* \cdot \partial_x \Psi_{21}^* e^{-i2Q_z}$

$+\partial_z A_z \partial_x \Psi_1$ is the next term.

$$\partial_z V_2 = i2Q V_{22} e^{i2Qz} - iQ V_{21} e^{iQz}$$
$$- i2Q V_{22}^* e^{-i2Qz} + iQ V_{21}^* e^{-iQz}$$

$$-\partial_z A_z \partial_x \Psi_1 = (- i2Q A_{22} e^{i2Qz} + iQ A_{21} e^{iQz}$$
$$+ i2Q A_{22}^* e^{-i2Qz} - iQ A_{21}^* e^{-iQz}).$$
$$\partial_x (\Psi_{11} e^{iQz} + \Psi_{11}^* e^{-iQz})$$

$$= - i2Q A_{22} e^{i2Qz} \cdot \partial_x \Psi_{11} e^{iQz} - i2Q A_{22} e^{i2Qz} \cdot \partial_x \Psi_{11}^* e^{-iQz}$$
$$+ iQ A_{21} e^{iQz} \cdot \partial_x \Psi_{11} e^{iQz} + iQ A_{21} e^{iQz} \cdot \partial_x \Psi_{11}^* e^{-iQz}$$
$$+ i2Q A_{22}^* e^{-i2Qz} \cdot \partial_x \Psi_{11} e^{iQz} + i2Q A_{22}^* e^{-i2Qz} \cdot \partial_x \Psi_{11}^* e^{-iQz}$$
$$- iQ A_{21}^* e^{-iQz} \cdot \partial_x \Psi_{11} e^{iQz} - iQ A_{21}^* e^{-iQz} \cdot \partial_x \Psi_{11}^* e^{-iQz}$$

$$= - i2Q A_{22} \cdot \partial_x \Psi_{11} e^{i3Qz} \quad \boxed{- i2Q A_{22} \cdot \partial_x \Psi_{11}^* e^{iQz}}$$
$$+ iQ A_{21} \cdot \partial_x \Psi_{11} e^{i2Qz} + iQ A_{21} \cdot \partial_x \Psi_{11}^*$$
$$+ i2Q A_{22}^* \cdot \partial_x \Psi_{11} e^{-iQz} + i2Q A_{22}^* \cdot \partial_x \Psi_{11}^* e^{-i3Qz}$$
$$- iQ A_{21}^* \cdot \partial_x \Psi_{11} \qquad - iQ A_{21}^* \cdot \partial_x \Psi_{11}^* e^{-i2Qz}$$

$$= \partial_z (A_{11} e^{iQz} \cdot A_{11}^* e^{-iQz}) \cdot (i2Q\Psi_{22} e^{i2Qz} - iQ\Psi_{21} e^{iQz}$$
$$- i2Q\Psi_{22}^* e^{-i2Qz} + iQ\Psi_{21}^* e^{-iQz})$$

$$= i2Q \partial_x A_{11} \cdot \Psi_{22} e^{i3Qz} \qquad + i2Q \partial_x A_{11}^* \cdot \Psi_{22} e^{iQz}$$
$$- iQ \partial_x A_{11} \cdot \Psi_{21} e^{i2Qz} \qquad - iQ \partial_x A_{11}^* \cdot \Psi_{22}$$
$$- i2Q \partial_x A_{11} \cdot \Psi_{22}^* e^{-iQz} \qquad - i2Q \partial_x A_{11}^* \cdot \Psi_{21}^* e^{-i3Qz}$$
$$+ iQ \partial_x A_{11} \cdot \Psi_{21}^* \qquad + iQ \partial_x A_{11}^* \cdot \Psi_{21}^* e^{-i2Qz}$$

$$\partial_x A_2 \partial_z \Psi_1$$
$$= \partial_x (i2Q A_{22} e^{i2Qz} - iQ A_{21} e^{iQz} - i2Q A_{22}^* e^{-i2Qz} + iQ A_{21}^* e^{-iQz})$$
$$\cdot (iQ\Psi_{11} e^{iQz} - iQ\Psi_{11}^* e^{-iQz})$$

$$+ 2Q^2 \partial_x A_{22} \cdot \Psi_{11} e^{i3Qz} \qquad + 2Q^2 \partial_x A_{22} \cdot \Psi_{11}^* e^{iQz}$$
$$- Q^2 \partial_x A_{21} \cdot \Psi_{11} e^{i2Qz} \qquad - Q^2 \partial_x A_{21} \cdot \Psi_{11}^*$$
$$+ 2Q^2 \partial_x A_{22}^* \cdot \Psi_{11} e^{-iQz} \qquad - 2Q^2 \partial_x A_{22}^* \cdot \Psi_{11}^* e^{-i3Qz}$$
$$- Q^2 \partial_x A_{21}^* \cdot \Psi_{11} \qquad + Q^2 \partial_x A_{21}^* \cdot \Psi_{11}^* e^{-i2Qz}$$

$$= \partial_x (A_{22} e^{i2Qz} + A_{20} - A_{21} e^{iQz} + A_{22}^* e^{-i2Qz} + A_{20}^* - A_{21}^* e^{-iQz})$$
$$\cdot (iQ\Psi_{11} e^{iQz} - iQ\Psi_{11}^* e^{-iQz})$$

$$= iQ \partial_x A_{22} \cdot \Psi_{11} e^{i3Qz} \qquad + iQ \partial_x A_{20} \cdot \Psi_{11} e^{iQz} \qquad - iQ \partial_x A_{21} \cdot \Psi_{11} e^{i2Qz}$$
$$+ iQ \partial_x A_{22}^* \cdot \Psi_{11} e^{-iQz} \qquad + iQ \partial_x A_{20}^* \cdot \Psi_{11} e^{iQz} \qquad - iQ \partial_x A_{21}^* \cdot \Psi_{11}$$
$$- iQ \partial_x A_{22} \cdot \Psi_{11}^* e^{iQz} \qquad - iQ \partial_x A_{20} \cdot \Psi_{11}^* e^{-iQz} \qquad + iQ \partial_x A_{21} \cdot \Psi_{11}^*$$
$$- iQ \partial_x A_{22}^* \cdot \Psi_{11}^* e^{-i3Qz} \qquad - iQ \partial_x A_{20}^* \cdot \Psi_{11}^* e^{-iQz} \qquad + iQ \partial_x A_{21}^* \cdot \Psi_{11}^* e^{-i2Qz}$$