The nonlinear equations are:

$$\partial_t \mathbf{u} - 2\Omega_0 \hat{\mathbf{z}} \times \mathbf{u} - q\Omega_0 u_x \hat{\mathbf{y}} - C\mathbf{b} \cdot \nabla \mathbf{b} - CB_0 \partial_z \mathbf{b} = -\nabla\varpi + \tfrac{1}{R}\nabla^2 \mathbf{u}$$

$$\partial_t \mathbf{b} - \mathbf{b} \cdot \nabla \mathbf{u} + q\Omega_0 b_x \hat{\mathbf{y}} - B_0 \partial_z \mathbf{u} = \tfrac{1}{R_m}\nabla^2 \mathbf{b}$$

Plus incompressibility and the solenoidal magnetic field constraint:

$$\nabla \cdot \mathbf{u} = 0 \text{ and } \nabla \cdot \mathbf{b} = 0$$

```
In [1]: from sympy import *
```

```
In [2]: R, Rm, kx, ky, kz, omega, q, B0, Co = symbols("R R_m k_x k_y k_z Omega_0 q B_0 C", real=True)
        s = symbols("s",)
```

The linearized equations are as follows:

$$\left(s + \tfrac{1}{R}\left(k_x^2 + k_z^2\right)\right)u_x - 2\Omega_0 u_y - iCB_0 k_z b_x + ik_x \varpi$$

$$\Omega_0(2-q)u_x + \left(s + \tfrac{1}{R}\left(k_x^2 + k_z^2\right)\right)u_y - iCB_0 k_z b_y$$

$$\left(s + \tfrac{1}{R}\left(k_x^2 + k_z^2\right)\right)u_z - iCB_0 k_z b_z + ik_z \varpi$$

$$-iB_0 k_z u_x + \left(s + \tfrac{1}{R_m}\left(k_x^2 + k_z^2\right)\right)b_x$$

$$-iB_0 k_z u_y + \Omega_0 q b_x + \left(s + \tfrac{1}{R_m}\left(k_x^2 + k_z^2\right)\right)b_y$$

$$-iB_0 k_z u_z + \left(s + \tfrac{1}{R_m}\left(k_x^2 + k_z^2\right)\right)b_z$$

Along with the two constraints:

$$ik_x u_x + ik_z u_z = 0$$

$$ik_x b_x + ik_z b_z = 0$$

But we ignore the second constraint for now.

The state vector is: $\mathbf{x} = \left[u_x, u_y, u_z, b_x, b_y, b_z, \varpi\right]^T$

```
In [175]: M = Matrix([[s+(1/R)*(kx**2+kz**2), -2*omega, 0, -I*Co*B0*kz, 0, 0, I*kx],
                      [omega*(2-q), s+(1/R)*(kx**2+kz**2), 0, 0, -I*Co*B0*kz, 0, 0],
                      [0, 0, s+(1/R)*(kx**2+kz**2), 0, 0, -I*Co*B0*kz, I*kz],
                      [-I*B0*kz, 0, 0, s+(1/Rm)*(kx**2+kz**2), 0, 0, 0],
                      [0, -I*B0*kz, 0, q*omega, s + (1/Rm)*(kx**2+kz**2), 0, 0],
                      [0, 0, -I*B0*kz, 0, 0, s+(1/Rm)*(kx**2 + kz**2), 0],
                      [I*kx, 0, I*kz, 0, 0, 0, 0]])
          M
```

```
Out[175]: [s + (k_x**2 + k_z**2)/R,              -2*Omega_0,                    0,              -I*B_0*C*k_z,
          0,                    0, I*k_x]
          [      Omega_0*(-q + 2), s + (k_x**2 + k_z**2)/R,                    0,                         0,
          I*B_0*C*k_z,                    0,     0]                                                          -
          [                    0,                       0, s + (k_x**2 + k_z**2)/R,                         0,
          0,            -I*B_0*C*k_z, I*k_z]
          [             -I*B_0*k_z,                       0,                    0, s + (k_x**2 + k_z**2)/R_m,
          0,                    0,     0]
          [                    0,              -I*B_0*k_z,                    0,                   Omega_0*q, s + (k_x**2 +
          k_z**2)/R_m,                       0,     0]
          [                    0,                       0,           -I*B_0*k_z,                         0,
          0, s + (k_x**2 + k_z**2)/R_m,     0]
          [                  I*k_x,                       0,                I*k_z,                         0,
          0,                       0,     0]
```

$$\begin{pmatrix}
s+\frac{k_x^2+k_z^2}{R} & -2\Omega_0 & 0 & -iB_0 C k_z & 0 & 0 & ik_x \\
\Omega_0(-q+2) & s+\frac{k_x^2+k_z^2}{R} & 0 & 0 & -iB_0 C k_z & 0 & 0 \\
0 & 0 & s+\frac{k_x^2+k_z^2}{R} & 0 & 0 & -iB_0 C k_z & ik_z \\
-iB_0 k_z & 0 & 0 & s+\frac{k_x^2+k_z^2}{R_m} & 0 & 0 & 0 \\
0 & -iB_0 k_z & 0 & \Omega_0 q & s+\frac{k_x^2+k_z^2}{R_m} & 0 & 0 \\
0 & 0 & -iB_0 k_z & 0 & 0 & s+\frac{k_x^2+k_z^2}{R_m} & 0 \\
ik_x & 0 & ik_z & 0 & 0 & 0 & 0
\end{pmatrix}$$

```
In [176]: dd = M.det()
```

```
In [178]: dd
```

Out[178]:
```
B_0**4*C**2*k_x**2*k_z**4*s + B_0**4*C**2*k_z**6*s + B_0**4*C**2*k_x**4*k_z**4/R_m + 2*B_0**4*C**2*k_x**2*k_z**6/R_m +
B_0**4*C**2*k_z**8/R_m - 2*B_0**2*C*Omega_0**2*k_z**4*q*s - 2*B_0**2*C*Omega_0**2*k_x**2*k_z**4*q/R_m -
2*B_0**2*C*Omega_0**2*k_z**6*q/R_m + 2*B_0**2*C*k_x**2*k_z**2*s**3 + 2*B_0**2*C*k_z**4*s**3 +
4*B_0**2*C*k_x**4*k_z**2*s**2/R_m + 8*B_0**2*C*k_x**2*k_z**4*s**2/R_m + 4*B_0**2*C*k_z**6*s**2/R_m +
2*B_0**2*C*k_x**6*k_z**2*s/R_m**2 + 6*B_0**2*C*k_x**4*k_z**4*s/R_m**2 + 6*B_0**2*C*k_x**2*k_z**6*s/R_m**2 +
2*B_0**2*C*k_z**8*s/R_m**2 + 2*B_0**2*C*k_x**4*k_z**2*s**2/R + 4*B_0**2*C*k_x**2*k_z**4*s**2/R + 2*B_0**2*C*k_z**6*s**2/R +
4*B_0**2*C*k_x**6*k_z**2*s/(R*R_m) + 12*B_0**2*C*k_x**4*k_z**4*s/(R*R_m) + 12*B_0**2*C*k_x**2*k_z**6*s/(R*R_m) +
4*B_0**2*C*k_z**8*s/(R*R_m) + 2*B_0**2*C*k_x**8*k_z**2/(R*R_m**2) + 8*B_0**2*C*k_x**6*k_z**4/(R*R_m**2) +
12*B_0**2*C*k_x**4*k_z**6/(R*R_m**2) + 8*B_0**2*C*k_x**2*k_z**8/(R*R_m**2) + 2*B_0**2*C*k_z**10/(R*R_m**2) -
2*Omega_0**2*k_z**2*q*s**3 + 4*Omega_0**2*k_z**2*s**3 - 6*Omega_0**2*k_x**2*k_z**2*q*s**2/R_m +
12*Omega_0**2*k_x**2*k_z**2*s**2/R_m - 6*Omega_0**2*k_z**4*q*s**2/R_m + 12*Omega_0**2*k_z**4*s**2/R_m -
6*Omega_0**2*k_x**4*k_z**2*q*s/R_m**2 + 12*Omega_0**2*k_x**4*k_z**2*s/R_m**2 - 12*Omega_0**2*k_x**2*k_z**4*q*s/R_m**2 +
24*Omega_0**2*k_x**2*k_z**4*s/R_m**2 - 6*Omega_0**2*k_z**6*q*s/R_m**2 + 12*Omega_0**2*k_z**6*s/R_m**2 -
2*Omega_0**2*k_x**6*k_z**2*q/R_m**3 + 4*Omega_0**2*k_x**6*k_z**2/R_m**3 - 6*Omega_0**2*k_x**4*k_z**4*q/R_m**3 +
12*Omega_0**2*k_x**4*k_z**4/R_m**3 - 6*Omega_0**2*k_x**2*k_z**6*q/R_m**3 + 12*Omega_0**2*k_x**2*k_z**6/R_m**3 -
2*Omega_0**2*k_z**8*q/R_m**3 + 4*Omega_0**2*k_z**8/R_m**3 + k_x**2*s**5 + k_z**2*s**5 + 3*k_x**4*s**4/R_m +
6*k_x**2*k_z**2*s**4/R_m + 3*k_z**4*s**4/R_m + 3*k_x**6*s**3/R_m**2 + 9*k_x**4*k_z**2*s**3/R_m**2 +
9*k_x**2*k_z**4*s**3/R_m**2 + 3*k_z**6*s**3/R_m**2 + k_x**8*s**2/R_m**3 + 4*k_x**6*k_z**2*s**2/R_m**3 +
6*k_x**4*k_z**4*s**2/R_m**3 + 4*k_x**2*k_z**6*s**2/R_m**3 + k_z**8*s**2/R_m**3 + 2*k_x**4*s**4/R + 4*k_x**2*k_z**2*s**4/R +
2*k_z**4*s**4/R + 6*k_x**6*s**3/(R*R_m) + 18*k_x**4*k_z**2*s**3/(R*R_m) + 18*k_x**2*k_z**4*s**3/(R*R_m) +
6*k_z**6*s**3/(R*R_m) + 6*k_x**8*s**2/(R*R_m**2) + 24*k_x**6*k_z**2*s**2/(R*R_m**2) + 36*k_x**4*k_z**4*s**2/(R*R_m**2) +
24*k_x**2*k_z**6*s**2/(R*R_m**2) + 6*k_z**8*s**2/(R*R_m**2) + 2*k_x**10*s/(R*R_m**3) + 10*k_x**8*k_z**2*s/(R*R_m**3) +
20*k_x**6*k_z**4*s/(R*R_m**3) + 20*k_x**4*k_z**6*s/(R*R_m**3) + 10*k_x**2*k_z**8*s/(R*R_m**3) + 2*k_z**10*s/(R*R_m**3) +
k_x**6*s**3/R**2 + 3*k_x**4*k_z**2*s**3/R**2 + 3*k_x**2*k_z**4*s**3/R**2 + k_z**6*s**3/R**2 + 3*k_x**8*s**2/(R**2*R_m) +
12*k_x**6*k_z**2*s**2/(R**2*R_m) + 18*k_x**4*k_z**4*s**2/(R**2*R_m) + 12*k_x**2*k_z**6*s**2/(R**2*R_m) +
3*k_z**8*s**2/(R**2*R_m) + 3*k_x**10*s/(R**2*R_m**2) + 15*k_x**8*k_z**2*s/(R**2*R_m**2) + 30*k_x**6*k_z**4*s/(R**2*R_m**2) +
30*k_x**4*k_z**6*s/(R**2*R_m**2) + 15*k_x**2*k_z**8*s/(R**2*R_m**2) + 3*k_z**10*s/(R**2*R_m**2) + k_x**12/(R**2*R_m**3) +
6*k_x**10*k_z**2/(R**2*R_m**3) + 15*k_x**8*k_z**4/(R**2*R_m**3) + 20*k_x**6*k_z**6/(R**2*R_m**3) +
15*k_x**4*k_z**8/(R**2*R_m**3) + 6*k_x**2*k_z**10/(R**2*R_m**3) + k_z**12/(R**2*R_m**3)
```

$$B_0^4 C^2 k_x^2 k_z^4 s + B_0^4 C^2 k_z^6 s + \frac{B_0^4 C^2 k_x^4 k_z^4}{R_m} + 2\,\frac{B_0^4 C^2 k_x^2 k_z^6}{R_m} + \frac{B_0^4 C^2 k_z^8}{R_m} - 2 B_0^2 C \Omega_0^2 k_z^4 q s - 2\,\frac{B_0^2 C \Omega_0^2 k_x^2 k_z^4 q}{R_m} - 2\,\frac{B_0^2 C \Omega_0^2 k_z^6 q}{R_m} + 2 B_0^2 C k_x^2 k_z^2 s^3 + 2 B_0^2 C k_z^4 s^3 + 4\,\frac{B_0^2 C k_x^4 k_z^2 s^2}{R_m} + 8\,\frac{B_0^2 C k_x^2 k_z^4 s^2}{R_m}$$

$$+\,4\,\frac{B_0^2 C k_z^6 s^2}{R_m} + 2\,\frac{B_0^2 C k_x^6 k_z^2 s}{R_m^2} + 6\,\frac{B_0^2 C k_x^4 k_z^4 s}{R_m^2} + 6\,\frac{B_0^2 C k_x^2 k_z^6 s}{R_m^2} + 2\,\frac{B_0^2 C k_z^8 s}{R_m^2} + 2\,\frac{B_0^2 C k_x^4 k_z^2 s^2}{R} + 4\,\frac{B_0^2 C k_x^2 k_z^4 s^2}{R} + 2\,\frac{B_0^2 C k_z^6 s^2}{R} + 4\,\frac{B_0^2 C k_x^6 k_z^2 s}{R R_m} + 12\,\frac{B_0^2 C k_x^4 k_z^4 s}{R R_m} + 12\,\frac{B_0^2 C k_x^2 k_z^6 s}{R R_m} + 4\,\frac{B_0^2 C k_z^8 s}{R R_m} + 2\,\frac{B_0^2 Ck}{R R}$$

$$+\,12\,\frac{B_0^2 C k_x^4 k_z^6}{R R_m^2} + 8\,\frac{B_0^2 C k_x^2 k_z^8}{R R_m^2} + 2\,\frac{B_0^2 C k_z^{10}}{R R_m^2} - 2\Omega_0^2 k_z^2 q s^3 + 4\Omega_0^2 k_z^2 s^3 - 6\,\frac{\Omega_0^2 k_x^2 k_z^2 q s^2}{R_m} + 12\,\frac{\Omega_0^2 k_x^2 k_z^2 s^2}{R_m} - 6\,\frac{\Omega_0^2 k_z^4 q s^2}{R_m} + 12\,\frac{\Omega_0^2 k_z^4 s^2}{R_m} - 6\,\frac{\Omega_0^2 k_x^4 k_z^2 q s}{R_m^2} + 12\,\frac{\Omega_0^2 k_x^4 k_z^2 s}{R_m^2} - 12\,\frac{\Omega_0^2 k_x^2 k_z^4 q s}{R_m^2} + 24\,\frac{\Omega_0^2}{}$$

$$+\,12\,\frac{\Omega_0^2 k_z^6 s}{R_m^2} - 2\,\frac{\Omega_0^2 k_x^6 k_z^2 q}{R_m^3} + 4\,\frac{\Omega_0^2 k_x^6 k_z^2}{R_m^3} - 6\,\frac{\Omega_0^2 k_x^4 k_z^4 q}{R_m^3} + 12\,\frac{\Omega_0^2 k_x^4 k_z^4}{R_m^3} - 6\,\frac{\Omega_0^2 k_x^2 k_z^6 q}{R_m^3} + 12\,\frac{\Omega_0^2 k_x^2 k_z^6}{R_m^3} - 2\,\frac{\Omega_0^2 k_z^8 q}{R_m^3} + 4\,\frac{\Omega_0^2 k_z^8}{R_m^3} + k_x^2 s^5 + k_z^2 s^5 + 3\,\frac{k_x^4 s^4}{R_m} + 6\,\frac{k_x^2 k_z^2 s^4}{R_m} + 3\,\frac{k_z^4 s^4}{R_m} + 3\,\frac{k_x^6 s^3}{R_m^2} + 9\,\frac{k_x^4}{}$$

$$+\,9\,\frac{k_x^2 k_z^4 s^3}{R_m^2} + 3\,\frac{k_z^6 s^3}{R_m^2} + \frac{k_x^8 s^2}{R_m^3} + 4\,\frac{k_x^6 k_z^2 s^2}{R_m^3} + 6\,\frac{k_x^4 k_z^4 s^2}{R_m^3} + 4\,\frac{k_x^2 k_z^6 s^2}{R_m^3} + \frac{k_z^8 s^2}{R_m^3} + 2\,\frac{k_x^4 s^4}{R} + 4\,\frac{k_x^2 k_z^2 s^4}{R} + 2\,\frac{k_z^4 s^4}{R} + 6\,\frac{k_x^6 s^3}{R R_m} + 18\,\frac{k_x^4 k_z^2 s^3}{R R_m} + 18\,\frac{k_x^2 k_z^4 s^3}{R R_m} + 6\,\frac{k_z^6 s^3}{R R_m} + 6\,\frac{k_x^8 s^2}{R R_m^2} + 24\,\frac{k_x^6 k_z^2 s^2}{R R_m^2} + 36\,\frac{k_x^4}{R}$$

$$+\,24\,\frac{k_x^2 k_z^6 s^2}{R R_m^2} + 6\,\frac{k_z^8 s^2}{R R_m^2} + 2\,\frac{k_x^{10} s}{R R_m^3} + 10\,\frac{k_x^8 k_z^2 s}{R R_m^3} + 20\,\frac{k_x^6 k_z^4 s}{R R_m^3} + 20\,\frac{k_x^4 k_z^6 s}{R R_m^3} + 10\,\frac{k_x^2 k_z^8 s}{R R_m^3} + 2\,\frac{k_z^{10} s}{R R_m^3} + \frac{k_x^6 s^3}{R^2} + 3\,\frac{k_x^4 k_z^2 s^3}{R^2} + 3\,\frac{k_x^2 k_z^4 s^3}{R^2} + \frac{k_z^6 s^3}{R^2} + 3\,\frac{k_x^8 s^2}{R^2 R_m} + 12\,\frac{k_x^6 k_z^2 s^2}{R^2 R_m} + 18\,\frac{k_x^4 k_z^4 s^2}{R^2 R_m} + 12\,\frac{k_x^2 k_z^6 s}{R^2 R_m}$$

$$+\,3\,\frac{k_x^{10} s}{R^2 R_m^2} + 15\,\frac{k_x^8 k_z^2 s}{R^2 R_m^2} + 30\,\frac{k_x^6 k_z^4 s}{R^2 R_m^2} + 30\,\frac{k_x^4 k_z^6 s}{R^2 R_m^2} + 15\,\frac{k_x^2 k_z^8 s}{R^2 R_m^2} + 3\,\frac{k_z^{10} s}{R^2 R_m^2} + \frac{k_x^{12}}{R^2 R_m^3} + 6\,\frac{k_x^{10} k_z^2}{R^2 R_m^3} + 15\,\frac{k_x^8 k_z^4}{R^2 R_m^3} + 20\,\frac{k_x^6 k_z^6}{R^2 R_m^3} + 15\,\frac{k_x^4 k_z^8}{R^2 R_m^3} + 6\,\frac{k_x^2 k_z^{10}}{R^2 R_m^3} + \frac{k_z^{12}}{R^2 R_m^3}$$

In [153]:
```
expand(Eq(dd, 0))
```

Out[153]:
```
B_0**4*C**2*k_x**2*k_z**4*s + B_0**4*C**2*k_z**6*s + B_0**4*C**2*k_x**4*k_z**4/R_m + 2*B_0**4*C**2*k_x**2*k_z**6/R_m +
B_0**4*C**2*k_z**8/R_m - 2*B_0**2*C*Omega_0**2*k_z**4*q*s - 2*B_0**2*C*Omega_0**2*k_x**2*k_z**4*q/R_m -
2*B_0**2*C*Omega_0**2*k_z**6*q/R_m + 2*B_0**2*C*k_x**2*k_z**2*s**3 + 2*B_0**2*C*k_z**4*s**3 +
4*B_0**2*C*k_x**4*k_z**2*s**2/R_m + 8*B_0**2*C*k_x**2*k_z**4*s**2/R_m + 4*B_0**2*C*k_z**6*s**2/R_m +
2*B_0**2*C*k_x**6*k_z**2*s/R_m**2 + 6*B_0**2*C*k_x**4*k_z**4*s/R_m**2 + 6*B_0**2*C*k_x**2*k_z**6*s/R_m**2 +
2*B_0**2*C*k_z**8*s/R_m**2 + 2*B_0**2*C*k_x**4*k_z**2*s**2/R + 4*B_0**2*C*k_x**2*k_z**4*s**2/R + 2*B_0**2*C*k_z**6*s**2/R +
4*B_0**2*C*k_x**6*k_z**2*s/(R*R_m) + 12*B_0**2*C*k_x**4*k_z**4*s/(R*R_m) + 12*B_0**2*C*k_x**2*k_z**6*s/(R*R_m) +
4*B_0**2*C*k_z**8*s/(R*R_m) + 2*B_0**2*C*k_x**8*k_z**2/(R*R_m**2) + 8*B_0**2*C*k_x**6*k_z**4/(R*R_m**2) +
12*B_0**2*C*k_x**4*k_z**6/(R*R_m**2) + 8*B_0**2*C*k_x**2*k_z**8/(R*R_m**2) + 2*B_0**2*C*k_z**10/(R*R_m**2) -
2*Omega_0**2*k_z**2*q*s**3 + 4*Omega_0**2*k_z**2*s**3 - 6*Omega_0**2*k_x**2*k_z**2*q*s**2/R_m +
12*Omega_0**2*k_x**2*k_z**2*s**2/R_m - 6*Omega_0**2*k_z**4*q*s**2/R_m + 12*Omega_0**2*k_z**4*s**2/R_m -
6*Omega_0**2*k_x**4*k_z**2*q*s/R_m**2 + 12*Omega_0**2*k_x**4*k_z**2*s/R_m**2 - 12*Omega_0**2*k_x**2*k_z**4*q*s/R_m**2 +
24*Omega_0**2*k_x**2*k_z**4*s/R_m**2 - 6*Omega_0**2*k_z**6*q*s/R_m**2 + 12*Omega_0**2*k_z**6*s/R_m**2 -
2*Omega_0**2*k_x**6*k_z**2*q/R_m**3 + 4*Omega_0**2*k_x**6*k_z**2/R_m**3 - 6*Omega_0**2*k_x**4*k_z**4*q/R_m**3 +
12*Omega_0**2*k_x**4*k_z**4/R_m**3 - 6*Omega_0**2*k_x**2*k_z**6*q/R_m**3 + 12*Omega_0**2*k_x**2*k_z**6/R_m**3 -
2*Omega_0**2*k_z**8*q/R_m**3 + 4*Omega_0**2*k_z**8/R_m**3 + k_x**2*s**5 + k_z**2*s**5 + 3*k_x**4*s**4/R_m +
6*k_x**2*k_z**2*s**4/R_m + 3*k_z**4*s**4/R_m + 3*k_x**6*s**3/R_m**2 + 9*k_x**4*k_z**2*s**3/R_m**2 +
9*k_x**2*k_z**4*s**3/R_m**2 + 3*k_z**6*s**3/R_m**2 + k_x**8*s**2/R_m**3 + 4*k_x**6*k_z**2*s**2/R_m**3 +
6*k_x**4*k_z**4*s**2/R_m**3 + 4*k_x**2*k_z**6*s**2/R_m**3 + k_z**8*s**2/R_m**3 + 2*k_x**4*s**4/R + 4*k_x**2*k_z**2*s**4/R +
2*k_z**4*s**4/R + 6*k_x**6*s**3/(R*R_m) + 18*k_x**4*k_z**2*s**3/(R*R_m) + 18*k_x**2*k_z**4*s**3/(R*R_m) +
6*k_z**6*s**3/(R*R_m) + 6*k_x**8*s**2/(R*R_m**2) + 24*k_x**6*k_z**2*s**2/(R*R_m**2) + 36*k_x**4*k_z**4*s**2/(R*R_m**2) +
24*k_x**2*k_z**6*s**2/(R*R_m**2) + 6*k_z**8*s**2/(R*R_m**2) + 2*k_x**10*s/(R*R_m**3) + 10*k_x**8*k_z**2*s/(R*R_m**3) +
20*k_x**6*k_z**4*s/(R*R_m**3) + 20*k_x**4*k_z**6*s/(R*R_m**3) + 10*k_x**2*k_z**8*s/(R*R_m**3) + 2*k_z**10*s/(R*R_m**3) +
k_x**6*s**3/R**2 + 3*k_x**4*k_z**2*s**3/R**2 + 3*k_x**2*k_z**4*s**3/R**2 + k_z**6*s**3/R**2 + 3*k_x**8*s**2/(R**2*R_m) +
12*k_x**6*k_z**2*s**2/(R**2*R_m) + 18*k_x**4*k_z**4*s**2/(R**2*R_m) + 12*k_x**2*k_z**6*s**2/(R**2*R_m) +
3*k_z**8*s**2/(R**2*R_m) + 3*k_x**10*s/(R**2*R_m**2) + 15*k_x**8*k_z**2*s/(R**2*R_m**2) + 30*k_x**6*k_z**4*s/(R**2*R_m**2) +
30*k_x**4*k_z**6*s/(R**2*R_m**2) + 15*k_x**2*k_z**8*s/(R**2*R_m**2) + 3*k_z**10*s/(R**2*R_m**2) + k_x**12/(R**2*R_m**3) +
6*k_x**10*k_z**2/(R**2*R_m**3) + 15*k_x**8*k_z**4/(R**2*R_m**3) + 20*k_x**6*k_z**6/(R**2*R_m**3) +
15*k_x**4*k_z**8/(R**2*R_m**3) + 6*k_x**2*k_z**10/(R**2*R_m**3) + k_z**12/(R**2*R_m**3) == 0
```

In [179]:
```
aa = dd.subs(s,0)
aa
```

Out[179]:   B_0**4*C**2*k_x**4*k_z**4/R_m + 2*B_0**4*C**2*k_x**2*k_z**6/R_m + B_0**4*C**2*k_z**8/R_m –
2*B_0**2*C*Omega_0**2*k_x**2*k_z**4*q/R_m – 2*B_0**2*C*Omega_0**2*k_z**6*q/R_m + 2*B_0**2*C*k_x**8*k_z**2/(R*R_m**2) +
8*B_0**2*C*k_x**6*k_z**4/(R*R_m**2) + 12*B_0**2*C*k_x**4*k_z**6/(R*R_m**2) + 8*B_0**2*C*k_x**2*k_z**8/(R*R_m**2) +
2*B_0**2*C*k_z**10/(R*R_m**2) – 2*Omega_0**2*k_x**6*k_z**2*q/R_m**3 + 4*Omega_0**2*k_x**6*k_z**2/R_m**3 –
6*Omega_0**2*k_x**4*k_z**4*q/R_m**3 + 12*Omega_0**2*k_x**4*k_z**4/R_m**3 – 6*Omega_0**2*k_x**2*k_z**6*q/R_m**3 +
12*Omega_0**2*k_x**2*k_z**6/R_m**3 – 2*Omega_0**2*k_z**8*q/R_m**3 + 4*Omega_0**2*k_z**8/R_m**3 + k_x**12/(R**2*R_m**3) +
6*k_x**10*k_z**2/(R**2*R_m**3) + 15*k_x**8*k_z**4/(R**2*R_m**3) + 20*k_x**6*k_z**6/(R**2*R_m**3) +
15*k_x**4*k_z**8/(R**2*R_m**3) + 6*k_x**2*k_z**10/(R**2*R_m**3) + k_z**12/(R**2*R_m**3)

In [155]: `kappa = symbols('kappa', real=True)`

In [156]: `aa.subs(2*(2–q), kappa)`

Out[156]:   B_0**4*C**2*k_x**4*k_z**4/R_m + 2*B_0**4*C**2*k_x**2*k_z**6/R_m + B_0**4*C**2*k_z**8/R_m –
2*B_0**2*C*Omega_0**2*k_x**2*k_z**4*q/R_m – 2*B_0**2*C*Omega_0**2*k_z**6*q/R_m + 2*B_0**2*C*k_x**8*k_z**2/(R*R_m**2) +
8*B_0**2*C*k_x**6*k_z**4/(R*R_m**2) + 12*B_0**2*C*k_x**4*k_z**6/(R*R_m**2) + 8*B_0**2*C*k_x**2*k_z**8/(R*R_m**2) +
2*B_0**2*C*k_z**10/(R*R_m**2) – 2*Omega_0**2*k_x**6*k_z**2*q/R_m**3 + 4*Omega_0**2*k_x**6*k_z**2/R_m**3 –
6*Omega_0**2*k_x**4*k_z**4*q/R_m**3 + 12*Omega_0**2*k_x**4*k_z**4/R_m**3 – 6*Omega_0**2*k_x**2*k_z**6*q/R_m**3 +
12*Omega_0**2*k_x**2*k_z**6/R_m**3 – 2*Omega_0**2*k_z**8*q/R_m**3 + 4*Omega_0**2*k_z**8/R_m**3 + k_x**12/(R**2*R_m**3) +
6*k_x**10*k_z**2/(R**2*R_m**3) + 15*k_x**8*k_z**4/(R**2*R_m**3) + 20*k_x**6*k_z**6/(R**2*R_m**3) +
15*k_x**4*k_z**8/(R**2*R_m**3) + 6*k_x**2*k_z**10/(R**2*R_m**3) + k_z**12/(R**2*R_m**3)

In [157]: `simplify(aa)`

Out[157]:   (B_0**4*C**2*R**2*R_m**2*k_x**4*k_z**4 + 2*B_0**4*C**2*R**2*R_m**2*k_x**2*k_z**6 + B_0**4*C**2*R**2*R_m**2*k_z**8 –
2*B_0**2*C*Omega_0**2*R**2*R_m**2*k_x**2*k_z**4*q – 2*B_0**2*C*Omega_0**2*R**2*R_m**2*k_z**6*q +
2*B_0**2*C*R*R_m*k_x**8*k_z**2 + 8*B_0**2*C*R*R_m*k_x**6*k_z**4 + 12*B_0**2*C*R*R_m*k_x**4*k_z**6 +
8*B_0**2*C*R*R_m*k_x**2*k_z**8 + 2*B_0**2*C*R*R_m*k_z**10 – 2*Omega_0**2*R**2*k_x**6*k_z**2*q +
4*Omega_0**2*R**2*k_x**6*k_z**2 – 6*Omega_0**2*R**2*k_x**4*k_z**4*q + 12*Omega_0**2*R**2*k_x**4*k_z**4 –
6*Omega_0**2*R**2*k_x**2*k_z**6*q + 12*Omega_0**2*R**2*k_x**2*k_z**6 – 2*Omega_0**2*R**2*k_z**8*q + 4*Omega_0**2*R**2*k_z**8
+ k_x**12 + 6*k_x**10*k_z**2 + 15*k_x**8*k_z**4 + 20*k_x**6*k_z**6 + 15*k_x**4*k_z**8 + 6*k_x**2*k_z**10 +
k_z**12)/(R**2*R_m**3)

In [158]: `latex(expand(aa))`

Out[158]:   '\\frac{B_{0}^{4} C^{2} k_{x}^{4} k_{z}^{4}}{R_{m}} + 2 \\frac{B_{0}^{4} C^{2} k_{x}^{2} k_{z}^{6}}{R_{m}} + \\frac{B_{0}^{4}
C^{2} k_{z}^{8}}{R_{m}} – 2 \\frac{B_{0}^{2} C \\Omega_{0}^{2} k_{x}^{2} k_{z}^{4} q}{R_{m}} – 2 \\frac{B_{0}^{2} C
\\Omega_{0}^{2} k_{z}^{6} q}{R_{m}} + 2 \\frac{B_{0}^{2} C k_{x}^{8} k_{z}^{2}}{R R_{m}^{2}} + 8 \\frac{B_{0}^{2} C k_{x}^{6}
k_{z}^{4}}{R R_{m}^{2}} + 12 \\frac{B_{0}^{2} C k_{x}^{4} k_{z}^{6}}{R R_{m}^{2}} + 8 \\frac{B_{0}^{2} C k_{x}^{2} k_{z}^{8}}
{R R_{m}^{2}} + 2 \\frac{B_{0}^{2} C k_{z}^{10}}{R R_{m}^{2}} – 2 \\frac{\\Omega_{0}^{2} k_{x}^{6} k_{z}^{2} q}{R_{m}^{3}} +
4 \\frac{\\Omega_{0}^{2} k_{x}^{6} k_{z}^{2}}{R_{m}^{3}} – 6 \\frac{\\Omega_{0}^{2} k_{x}^{4} k_{z}^{4} q}{R_{m}^{3}} + 12
\\frac{\\Omega_{0}^{2} k_{x}^{4} k_{z}^{4}}{R_{m}^{3}} – 6 \\frac{\\Omega_{0}^{2} k_{x}^{2} k_{z}^{6} q}{R_{m}^{3}} + 12
\\frac{\\Omega_{0}^{2} k_{x}^{2} k_{z}^{6}}{R_{m}^{3}} – 2 \\frac{\\Omega_{0}^{2} k_{z}^{8} q}{R_{m}^{3}} + 4
\\frac{\\Omega_{0}^{2} k_{z}^{8}}{R_{m}^{3}} + \\frac{k_{x}^{12}}{R^{2} R_{m}^{3}} + 6 \\frac{k_{x}^{10} k_{z}^{2}}{R^{2}
R_{m}^{3}} + 15 \\frac{k_{x}^{8} k_{z}^{4}}{R^{2} R_{m}^{3}} + 20 \\frac{k_{x}^{6} k_{z}^{6}}{R^{2} R_{m}^{3}} + 15
\\frac{k_{x}^{4} k_{z}^{8}}{R^{2} R_{m}^{3}} + 6 \\frac{k_{x}^{2} k_{z}^{10}}{R^{2} R_{m}^{3}} + \\frac{k_{z}^{12}}{R^{2}
R_{m}^{3}}'

$$\frac{B_0^4 C^2 k_x^4 k_z^4}{R_m} + 2\frac{B_0^4 C^2 k_x^2 k_z^6}{R_m} + \frac{B_0^4 C^2 k_z^8}{R_m} - 2\frac{B_0^2 C \Omega_0^2 k_x^2 k_z^4 q}{R_m} - 2\frac{B_0^2 C \Omega_0^2 k_z^6 q}{R_m} + 2\frac{B_0^2 C k_x^8 k_z^2}{RR_m^2} + 8\frac{B_0^2 C k_x^6 k_z^4}{RR_m^2} + 12\frac{B_0^2 C k_x^4 k_z^6}{RR_m^2} + 8\frac{B_0^2 C k_x^2 k_z^8}{RR_m^2} + 2\frac{B_0^2 C k_z^{10}}{RR_m^2} - 2\frac{\Omega_0^2 k_x^6 k_z^2 q}{R_m^3} + 4\frac{\Omega_0^2 k_x^6 k_z^2}{R_m^3} - 6\frac{\Omega_0^2 k_x^4 k_z^4 q}{R_m^3} + 12$$

In [38]: `jj = (Co/(Rm*Co)**4)*((kx**2+kz**2)*Co*(((Co*Rm/R)*(kx**2 + kz**2)**2 + kz**2*Rm**2*Co**2)**2) + 2*(2–q)*Rm**2*Co**3*(kx**2+kz`

$$\frac{-2C^4 R_m^4 k_z^4 q + C^3 R_m^2 k_z^2 \left(k_x^2 + k_z^2\right)^2 (-2q+4) + C\left(k_x^2 + k_z^2\right)\left(C^2 R_m^2 k_z^2 + \frac{CR_m\left(k_x^2 + k_z^2\right)^2}{R}\right)^2}{C^3 R_m^4}$$

In [180]: `a1 = aa.subs(B0, 1)`

In [181]: `a1 = a1.subs(omega, 1)`

In [182]: `a1.expand()`

Out[182]:   C**2*k_x**4*k_z**4/R_m + 2*C**2*k_x**2*k_z**6/R_m + C**2*k_z**8/R_m – 2*C*k_x**2*k_z**4*q/R_m – 2*C*k_z**6*q/R_m +
2*C*k_x**8*k_z**2/(R*R_m**2) + 8*C*k_x**6*k_z**4/(R*R_m**2) + 12*C*k_x**4*k_z**6/(R*R_m**2) + 8*C*k_x**2*k_z**8/(R*R_m**2) +
2*C*k_z**10/(R*R_m**2) – 2*k_x**6*k_z**2*q/R_m**3 + 4*k_x**6*k_z**2/R_m**3 – 6*k_x**4*k_z**4*q/R_m**3 +
12*k_x**4*k_z**4/R_m**3 – 6*k_x**2*k_z**6*q/R_m**3 + 12*k_x**2*k_z**6/R_m**3 – 2*k_z**8*q/R_m**3 + 4*k_z**8/R_m**3 +
k_x**12/(R**2*R_m**3) + 6*k_x**10*k_z**2/(R**2*R_m**3) + 15*k_x**8*k_z**4/(R**2*R_m**3) + 20*k_x**6*k_z**6/(R**2*R_m**3) +
15*k_x**4*k_z**8/(R**2*R_m**3) + 6*k_x**2*k_z**10/(R**2*R_m**3) + k_z**12/(R**2*R_m**3)

In [174]: `jj.expand()`

Out[174]:   C**2*k_x**2*k_z**4 + C**2*k_z**6 – 2*C*k_z**4*q + 2*C*k_x**6*k_z**2/(R*R_m) + 6*C*k_x**4*k_z**4/(R*R_m) +
6*C*k_x**2*k_z**6/(R*R_m) + 2*C*k_z**8/(R*R_m) – 2*k_x**4*k_z**2*q/R_m**2 + 4*k_x**4*k_z**2/R_m**2 – 4*k_x**2*k_z**4*q/R_m**2
+ 8*k_x**2*k_z**4/R_m**2 – 2*k_z**6*q/R_m**2 + 4*k_z**6/R_m**2 + k_x**10/(R**2*R_m**2) + 5*k_x**8*k_z**2/(R**2*R_m**2) +
10*k_x**6*k_z**4/(R**2*R_m**2) + 10*k_x**4*k_z**6/(R**2*R_m**2) + 5*k_x**2*k_z**8/(R**2*R_m**2) + k_z**10/(R**2*R_m**2)

My a4 term:

$$\frac{C^2 k_x^4 k_z^4}{R_m} + 2\,\frac{C^2 k_x^2 k_z^6}{R_m} + \frac{C^2 k_z^8}{R_m} - 2\,\frac{C k_x^2 k_z^4 q}{R_m} - 2\,\frac{C k_z^6 q}{R_m} + 2\,\frac{C k_x^8 k_z^2}{R R_m^2} + 8\,\frac{C k_x^6 k_z^4}{R R_m^2} + 12\,\frac{C k_x^4 k_z^6}{R R_m^2} + 8\,\frac{C k_x^2 k_z^8}{R R_m^2} + 2\,\frac{C k_z^{10}}{R R_m^2} - 2\,\frac{k_x^6 k_z^2 q}{R_m^3} + 4\,\frac{k_x^6 k_z^2}{R_m^3} - 6\,\frac{k_x^4 k_z^4 q}{R_m^3} + 12\,\frac{k_x^4 k_z^4}{R_m^3} - 6\,\frac{k_x^2 k_z^6 q}{R_m^3} + 12\,\frac{k_x^2 k_z^6}{R_m^3} -$$

Umurhan+ a4 term:

$$C^2 k_x^2 k_z^4 + C^2 k_z^6 - 2 C k_z^4 q + 2\,\frac{C k_x^6 k_z^2}{R R_m} + 6\,\frac{C k_x^4 k_z^4}{R R_m} + 6\,\frac{C k_x^2 k_z^6}{R R_m} + 2\,\frac{C k_z^8}{R R_m} - 2\,\frac{k_x^4 k_z^2 q}{R_m^2} + 4\,\frac{k_x^4 k_z^2}{R_m^2} - 4\,\frac{k_x^2 k_z^4 q}{R_m^2} + 8\,\frac{k_z^2 k_z^4}{R_m^2} - 2\,\frac{k_z^6 q}{R_m^2} + 4\,\frac{k_z^6}{R_m^2} + \frac{k_x^{10}}{R^2 R_m^2} + 5\,\frac{k_x^8 k_z^2}{R^2 R_m^2} + 10\,\frac{k_x^6 k_z^4}{R^2 R_m^2} + 10\,\frac{k_x^4 k_z^6}{R^2 R_m^2} +$$

```
In [138]:  factor(Co**2*kx**2*kz**4 + Co**2*kz**6 - 2*Co*kz**4*q)
```

```
Out[138]:  -C*k_z**4*(-C*k_x**2 - C*k_z**2 + 2*q)
```

```
In [135]:  j1
```

```
Out[135]:  (C**5*R_m**4*k_z**4*(k_x**2 + k_z**2) - 2*C**4*R_m**4*k_z**4*q + C**3*R_m**2*k_z**2*(k_x**2 + k_z**2)**2*(-2*q +
           4))/(C**3*R_m**4)
```

```
In [136]:  simplify(j1)
```

```
Out[136]:  k_z**2*(C**2*R_m**2*k_x**2*k_z**2 + C**2*R_m**2*k_z**4 - 2*C*R_m**2*k_z**2*q - 2*k_x**4*q + 4*k_x**4 - 4*k_x**2*k_z**2*q +
           8*k_x**2*k_z**2 - 2*k_z**4*q + 4*k_z**4)/R_m**2
```

```
In [131]:  a2 = a1.subs(R, Infinity)
```

```
In [132]:  a2.subs(Rm, Infinity)
```

```
Out[132]:  0
```

```
In [ ]:
```