set-up:

small shearing box

differential rotation $\Omega(r) \propto \Omega_0 \left(\frac{r}{r_0}\right)^{-q}$ $\uparrow\uparrow\uparrow\downarrow\downarrow\downarrow$

$\therefore\ \vec{V} = -q\,U_0\,x\,\hat{y}$ (linear shear profile.)

constant B field $\vec{B} = B_0\hat{z}$

- MHD equations + perturbations in $\vec{u}$ and $\vec{B}$ + nondimensionalization $\Rightarrow$ eqs. 1

nonlin.

$$\frac{d\vec{u}}{dt} - 2\hat{z}\times\vec{u} - \underbrace{q\,u_x\,\hat{y}}_{\text{shear term}} - C\left(\vec{B}\cdot\nabla + B_0\partial_z\right)\vec{B} = -\nabla\varpi + \frac{1}{R}\nabla^2\vec{u}$$

C ← cowling # (dynamical importance of B field)

ϖ ← total pressure

$$\frac{d\vec{B}}{dt} - \left(\vec{B}\cdot\nabla + B_0\partial_z\right)\vec{u} + q\,B_x\,\hat{y} = \frac{1}{R_m}\nabla^2\vec{B}$$

- Linearize these for perturbations $e^{st + ik_x x + ik_z z}$ (no y because axisymmetry)

$\Rightarrow$ dispersion relation $a_0 s^4 + a_1 s^3 + a_2 s^2 + a_1 s + a_4 = 0$ (q, S, k_x, k_z dep.)

- But when marginal to MRI, $s = 0$ (because $s > 0$ = grows)

$$\therefore\ a_4 = \frac{\epsilon}{S^4}\left[k_T^2\,C\left(C k_T^4 P_m + k_z^2 S^2\right)^2 + k^2 S^2 C k_T^4 k_z^2 - 2q\,S^4 k_z^4\right] = 0$$

$S = R_m C$ (Elsasser #)

- Fix $k_x = K$, ask at what $k_z = Q$ the most unstable mode is critical.

a_4 and its derivative are 0:

$a_4(k_z = Q;\ K, \Pi) = 0 \qquad \frac{\partial a_4}{\partial k_z}(k_z = Q;\ K, \Pi) = 0$

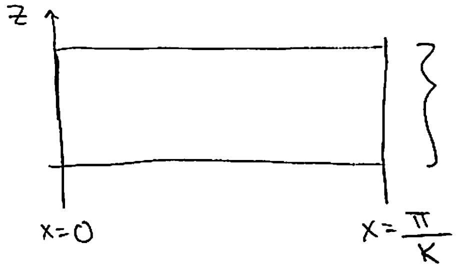
Π ← just denotes all other parameters

- Now 2 eqns, 2 unknowns (Q & q) → solve for Q & S.

↳ take asymptotic forms for $P_m \ll 1$ (eqn 5):

$$S = \frac{\sqrt{16\,C\,q\,(2-q)}\,K}{2q - CK^2} \qquad Q^2 = K^2\,\frac{2q - CK^2}{2q + CK^2}$$

• The case $CK^2 = 2q$ is ideal MRI (why?)



vertically periodic $\frac{2\pi n}{Q}$ ← $k_z = Q$

b.c.'s:

$u_x = u_y = b_x = b_y = \partial_x u_z = 0$

at $x = 0, \pi/K$ ⟶ no slip on (u_x, u_y), free slip on u_z | conducting on b_x, insulation on b_y

• Tune background field: $B_0 \to 1 - \varepsilon^2 \tilde{\lambda}$, $\varepsilon \ll 1$, $\tilde{\lambda}$ is $\mathcal{O}(1)$

• Multiscale analysis in z & t: $Z \equiv \varepsilon z$ & $T \equiv \varepsilon^2 t$

• For any dependent fluid quantity $F(x,z,t)$:

$$F(x,z,t) = \sum_{n=1}^{\infty} \varepsilon^n F_n(x,z,t)$$

* using stream & mag. flux functions, only have 4 F's:

$\Psi(x,z)$ $\Phi(x,z)$ u_y & B_y

• expand everything in multiscale terms, group by $\mathcal{O}(\varepsilon)$

• Lowest $\mathcal{O}(\varepsilon)$: make Ansatz

$$F_1(x,z,t) = \hat{F}_1 \tilde{A}(\varepsilon z, \varepsilon^2 t) e^{iQ \cdot z} \sin Kx + \text{c.c.}$$

constant ↑ envelope func. ↑ justified b/c obeys b.c.'s ?

• End result (I assume after successive solutions of each $\mathcal{O}(\varepsilon)$):
Ginzburg-Landau Eqn:

$$\partial_T A = \lambda A - \frac{1}{P_m C} A|A|^2 + D \partial_z^2 A \quad \text{for } P_m \ll 1$$

$A \equiv \sqrt{\xi} \tilde{A}$, $\lambda \equiv \zeta \tilde{\lambda}$ (coeffs UMR eq. 9)