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# Data Structures, Algorithms & Data Science Platforms

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# L5: Algorithm Types

Algorithms

*Some slides courtesy:*
*Venkatesh Babu & Sathish Vadhiyar, CDS, IISc*

# Algorithm classification

- Algorithms that use a *similar problem-solving approach* can be grouped together
  - A classification scheme for algorithms
- Classification is neither exhaustive nor disjoint
- The purpose is not to be able to classify an algorithm as one type or another, but to *highlight the various ways in which a problem can be attacked*

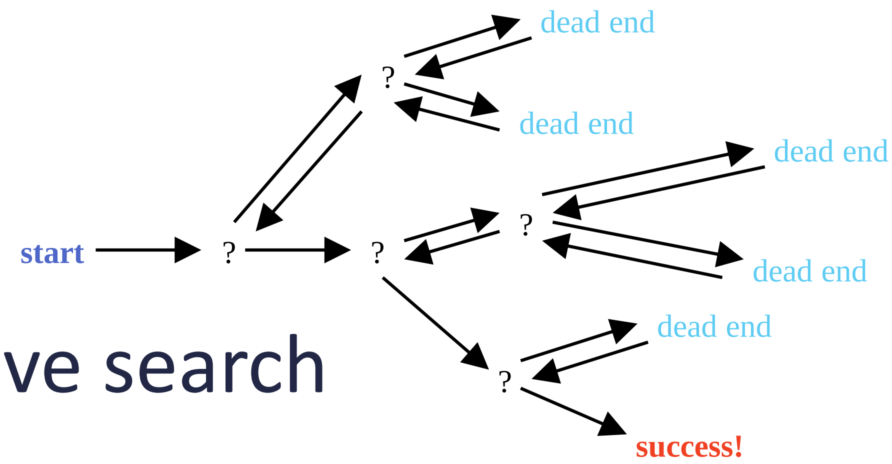
# A short list of categories

- Algorithm types we will consider include:
  1. Simple recursive algorithms
  2. Backtracking algorithms
  3. Divide and conquer algorithms
  4. Dynamic programming algorithms
  5. Greedy algorithms
  6. Branch and bound algorithms
  7. Brute force algorithms
  8. Randomized algorithms

# Simple Recursive Algorithms

- A simple recursive algorithm:
  1. Solves the base cases directly
  2. Recurs with a simpler subproblem
  3. Does some extra work to convert the solution to the simpler subproblem into a solution to the given problem
- These are “simple” because several of the other algorithm types are inherently recursive
- *Any seen so far?*
  - Tree traversal
  - Binary search over sorted array

# Backtracking algorithms

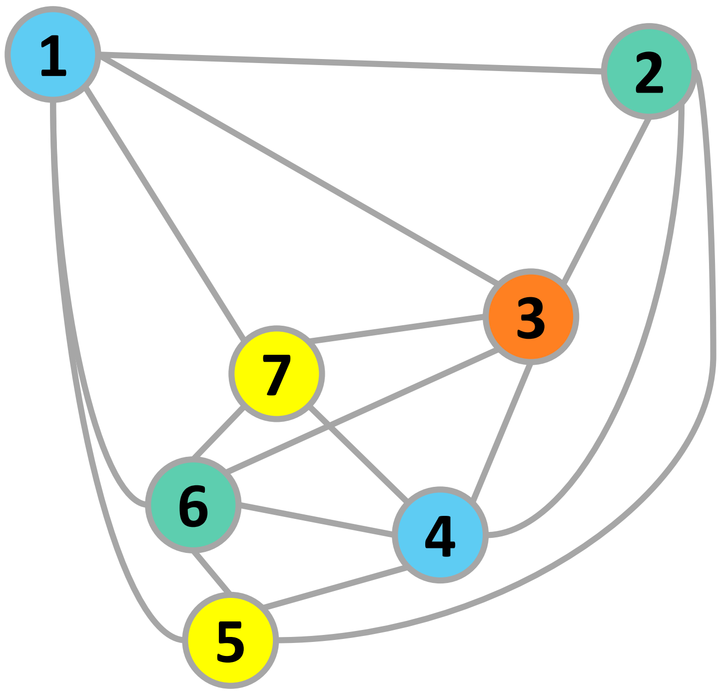



- Uses a depth-first recursive search over solution space
  - Test to see if a solution has been found, and if so, returns it; otherwise
  - For each choice that can be made at this point,
    - Make that choice
    - Recurse
    - If the recursion returns a solution, return it
  - If no choices remain, return failure

- *Any seen so far?*
  - DFS traversal

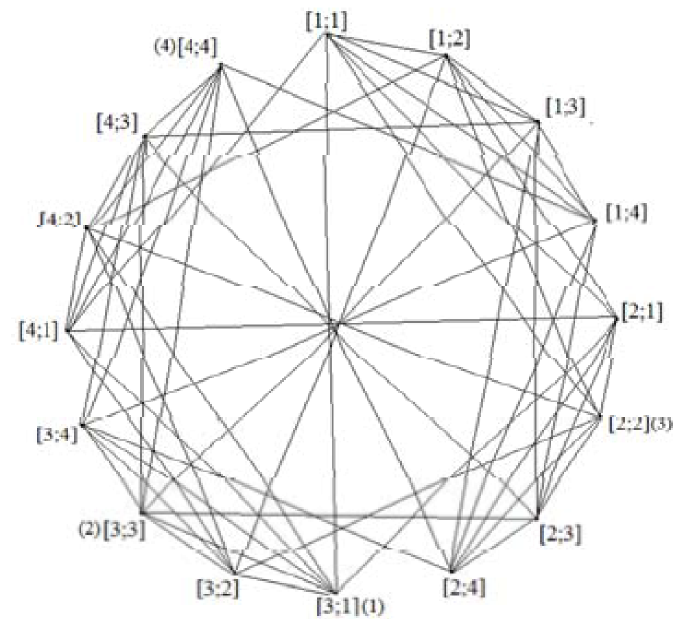
# Sample backtracking algo.

***Graph coloring:*** *Color the vertices of a graph such that no two adjacent vertices have the same color*




**4x4 Sudoku**

| | | | |
|---|---|---|---|
| [1;1] | [1;2] | [1;3] | [1;4] |
| [2;1] | [2;2] 3 | [2;3] | [2;4] |
| [3;1] 1 | [3;2] | [3;3] 2 | [3;4] |
| [4;1] | [4;2] | [4;3] | [4;4] 4 |




The above mentioned graph has 16 vertices and 56 edges.

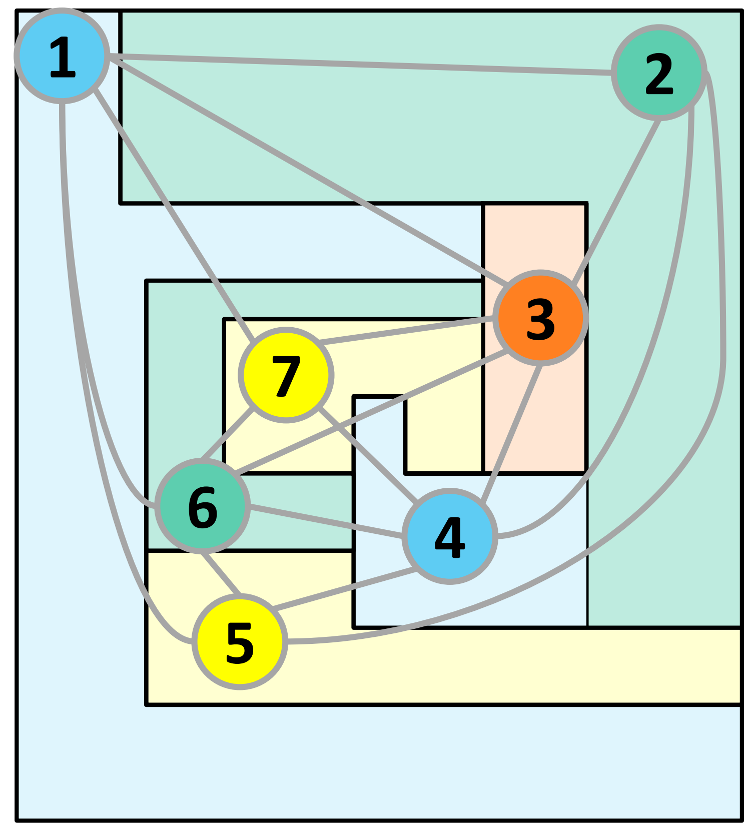
# Graph Coloring

*The **Four Color Theorem** states that any map on a plane can be colored with no more than four colors, so that no two countries with a common border are the same color*

- m-color problem
  - Given a graph, find out if its vertices can be colored with no more than m colors
  - `O(m^v)`

# Sample backtracking algo.

```
boolean explore(int ctry, int col){
 if (ctry >= map.size) return true;
 if (okToColor(ctry, col)) {
 map[ctry] = col;
   for (int c=RED; c<=BLUE; c++){
     if (explore(ctry+1, c))
       return true;
   }
 } else
   return false;
}
```



# Divide and Conquer

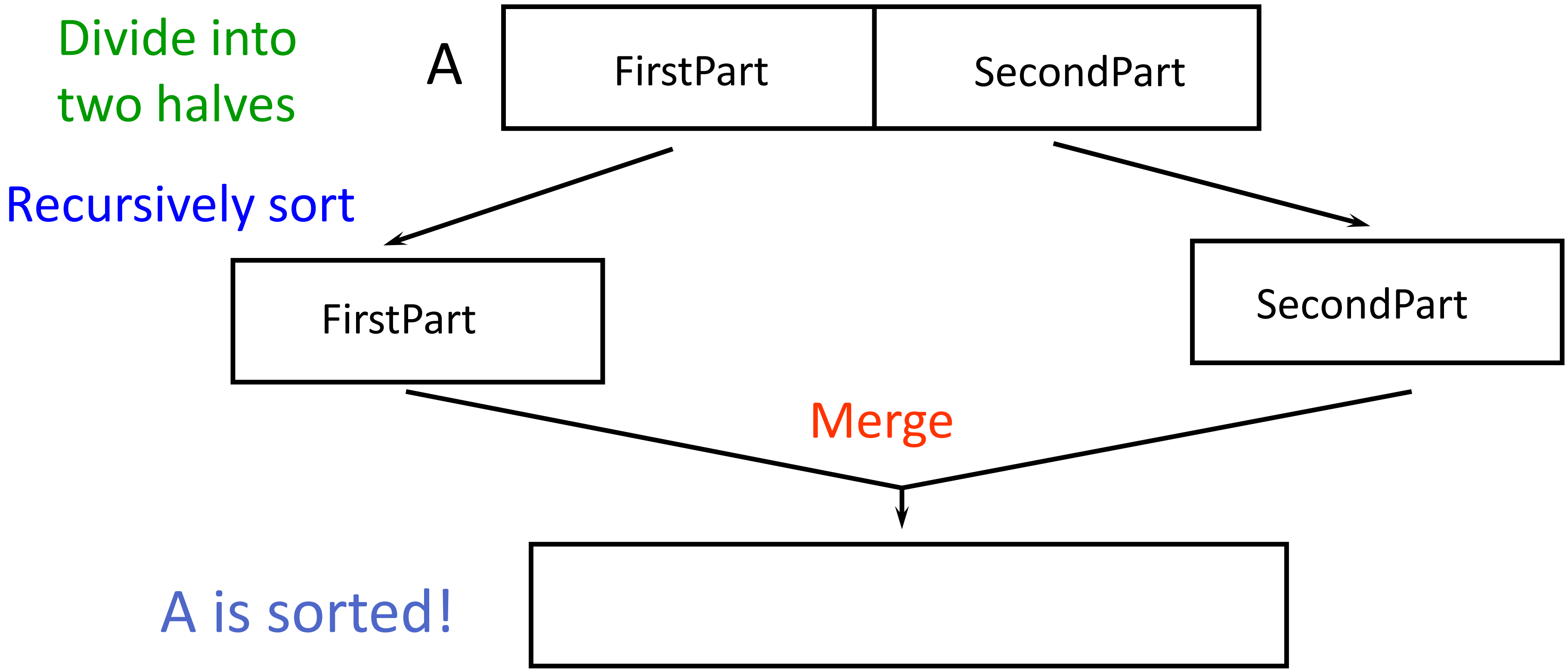
- A divide and conquer algorithm consists of two parts:
  - *Divide* the problem into smaller subproblems of the same type, and solve these subproblems recursively
  - *Combine* the solutions to the subproblems into a solution to the original problem
- *Traditionally, an algorithm is only called "divide and conquer" if it contains at least two recursive calls*

# Binary search tree lookup?

- Compare the key to the value in the root
  - If the two values are equal, report success
  - If the key is less, search the left subtree
  - If the key is greater, search the right subtree

- This is *not* a divide and conquer algorithm because, although there are two recursive calls, only one is used at each level of the recursion
- *E.g. Recursive binary search over an unsorted array. Search all elements.*
- *E.g. Merge Sort, Quick Sort*

# Merge Sort: Idea



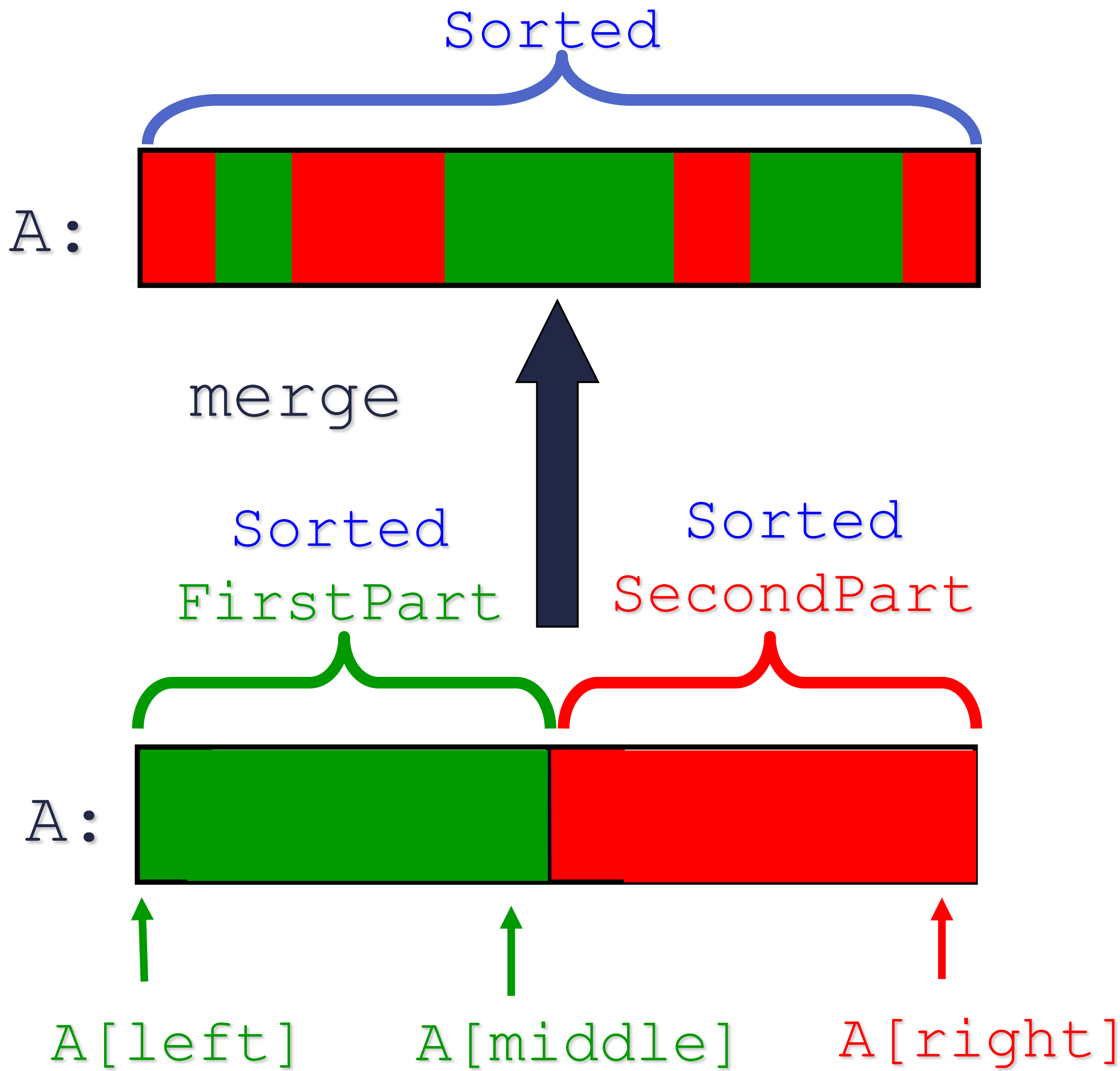
# Merge Sort: Algorithm

```
MergeSort (A, left, right)
  if (left >= right) return
  else {
     middle = Floor(left+right/2)
     MergeSort(A, left, middle)
     MergeSort(A, middle+1, right)
     Merge(A, left, middle, right)
   }
}
```

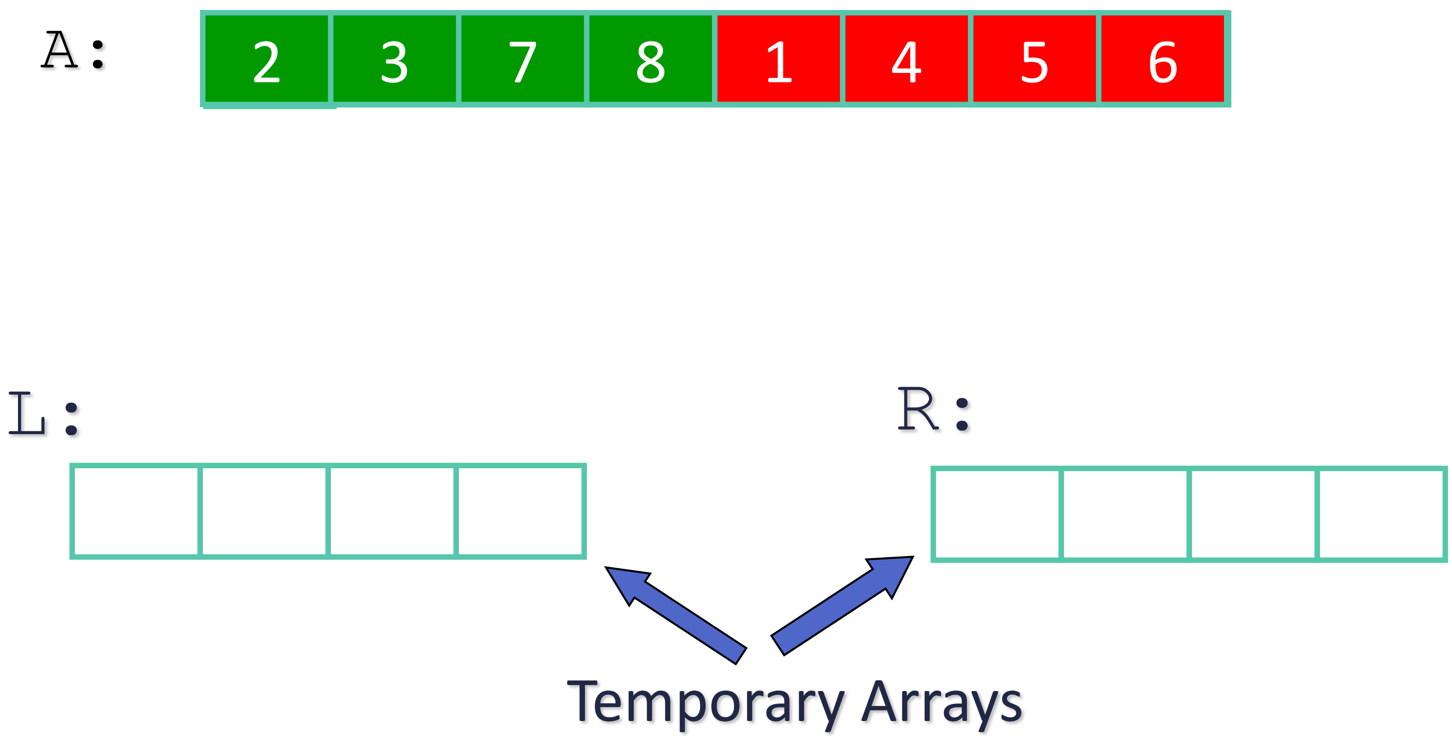
Recursive Call

**Merge**: Given two sorted arrays, merges them into a single sorted array

# Merge-Sort: Merge



# Merge-Sort: Merge



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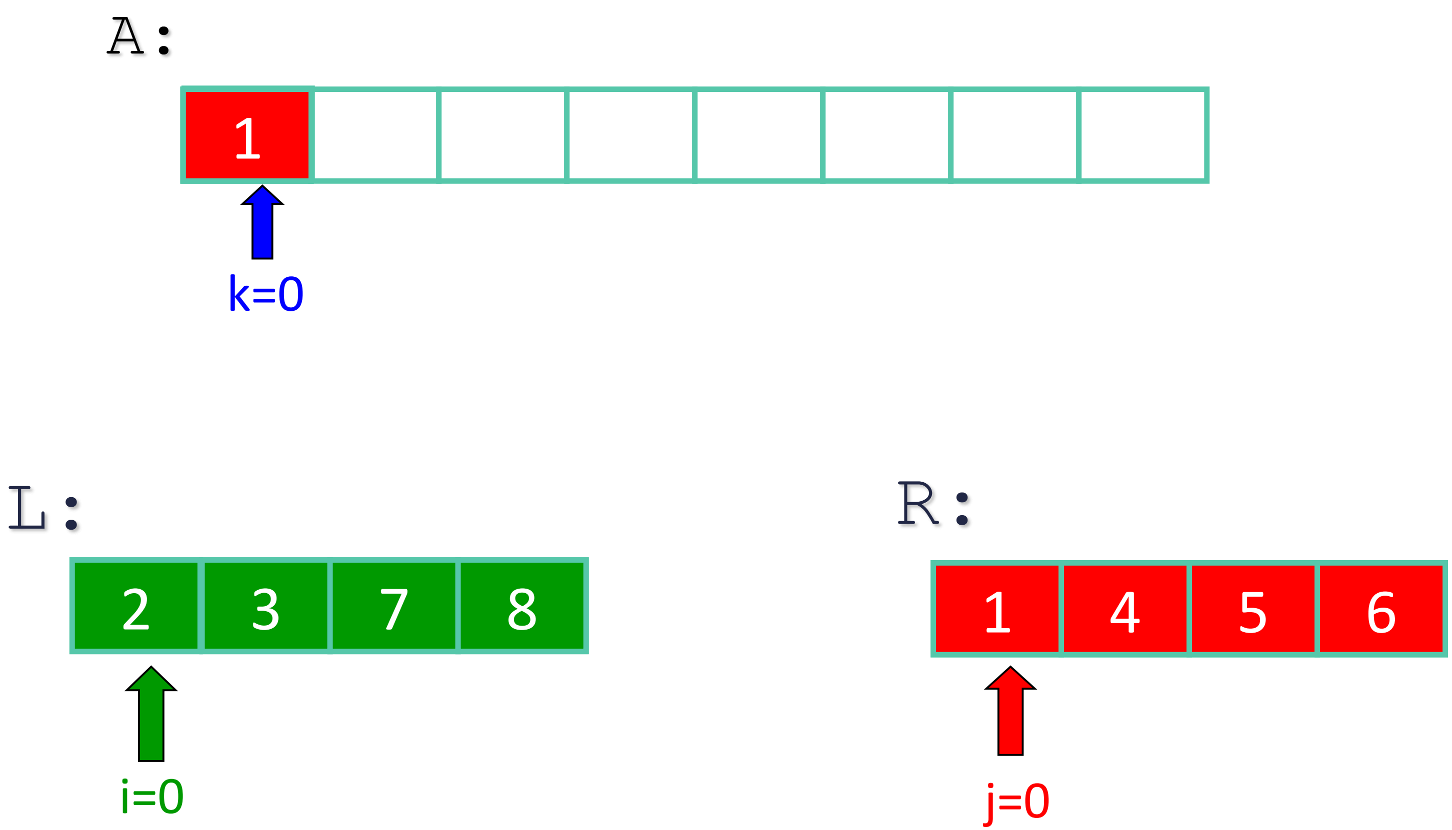

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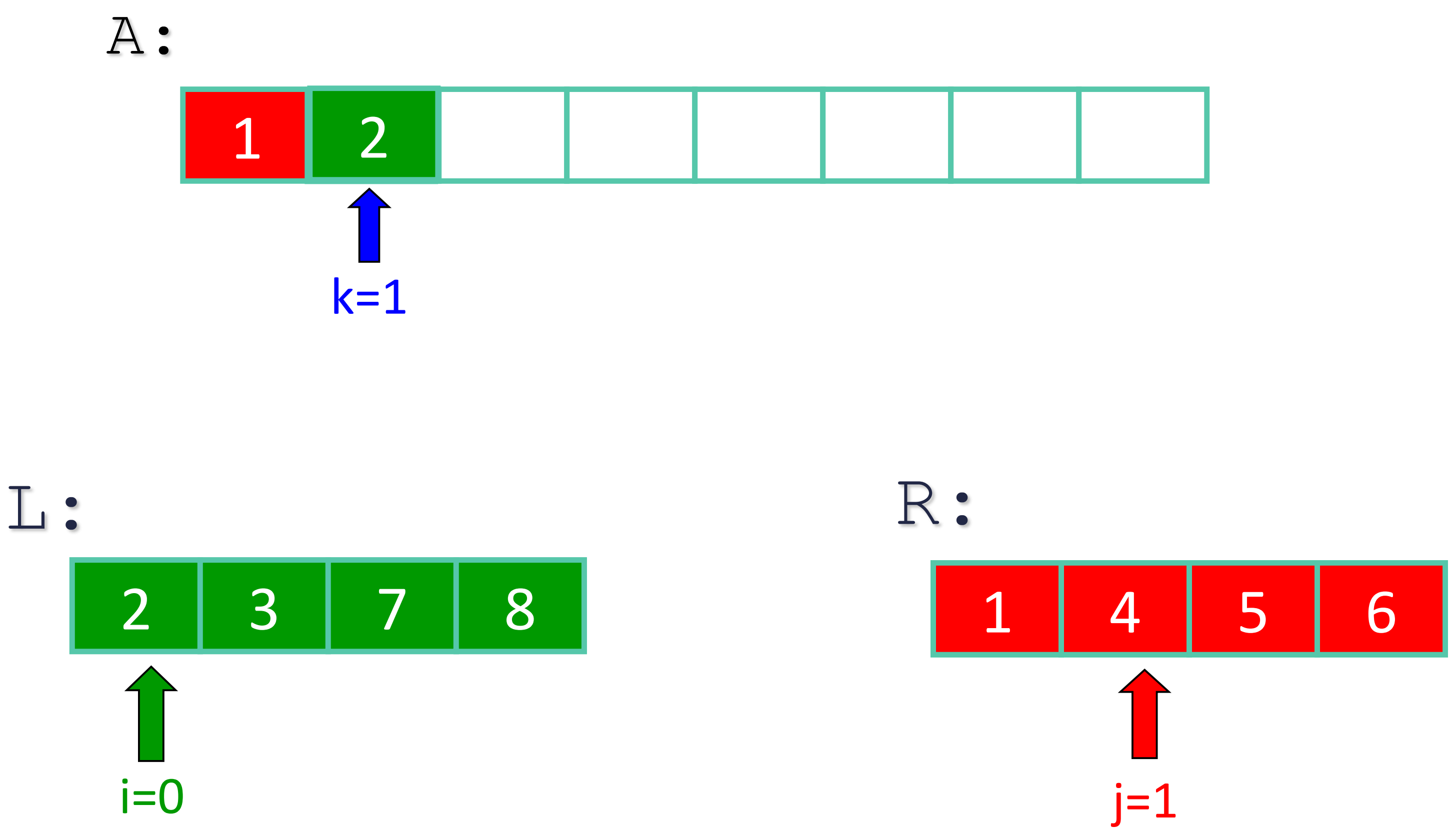

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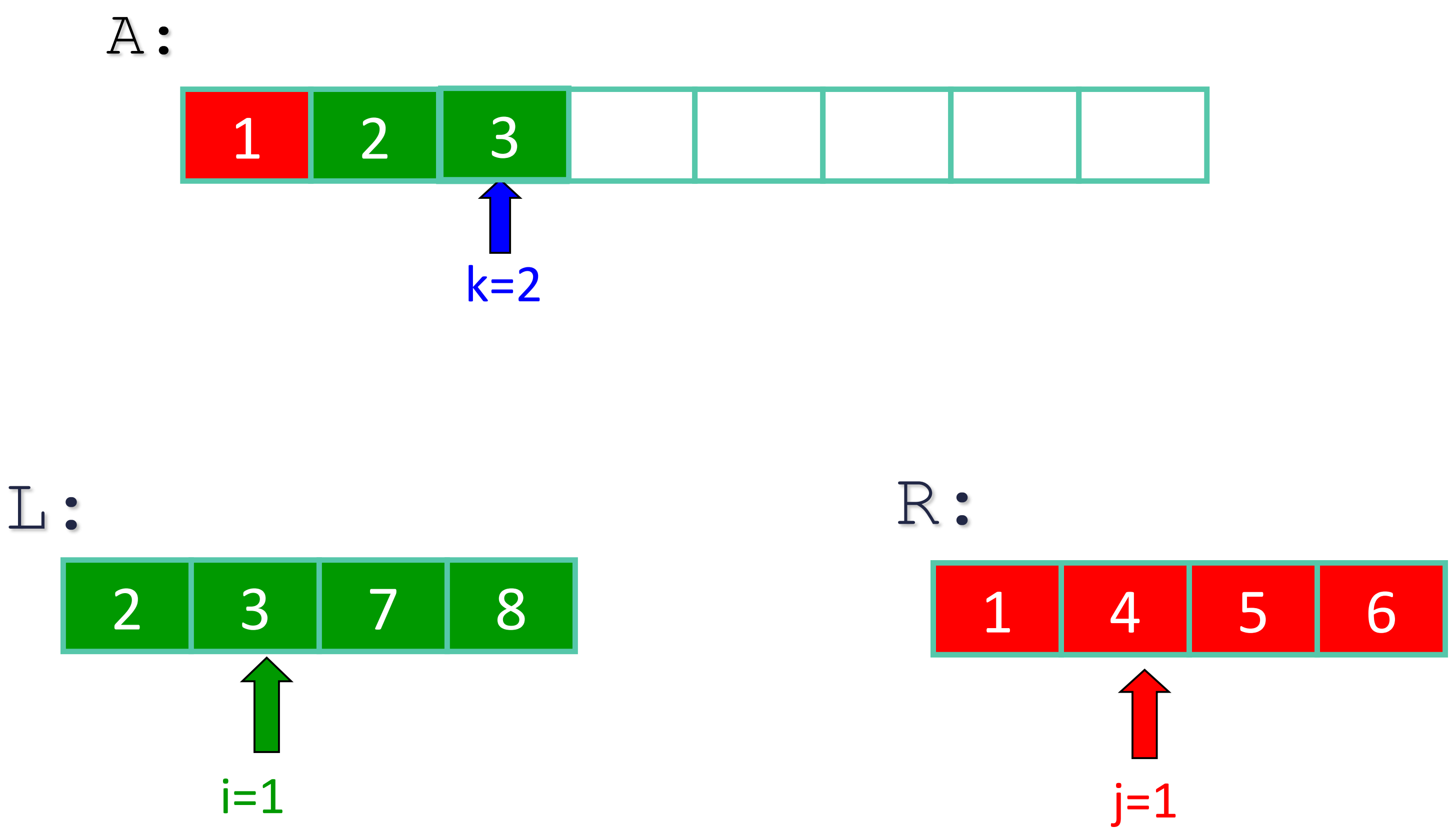

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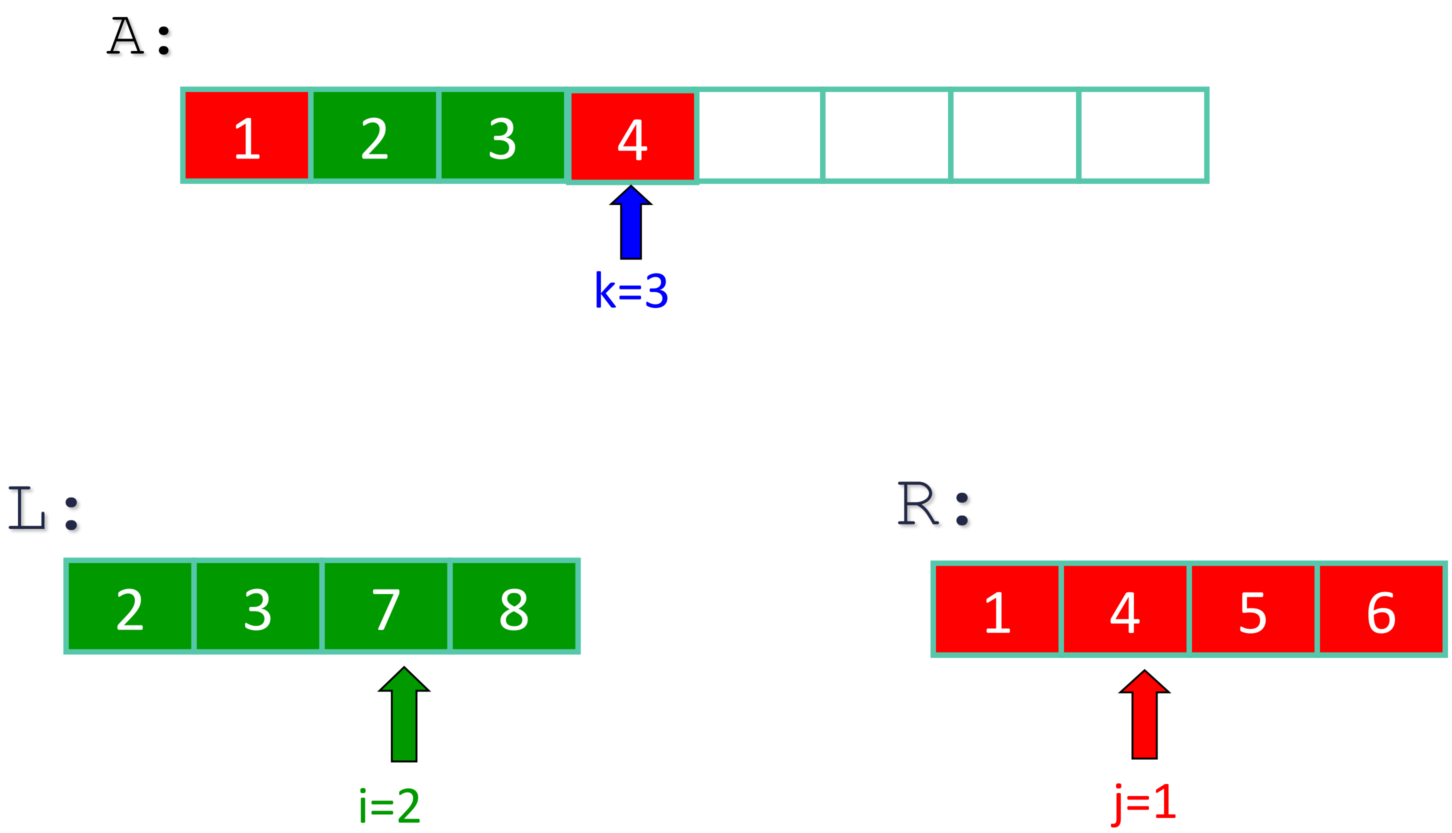

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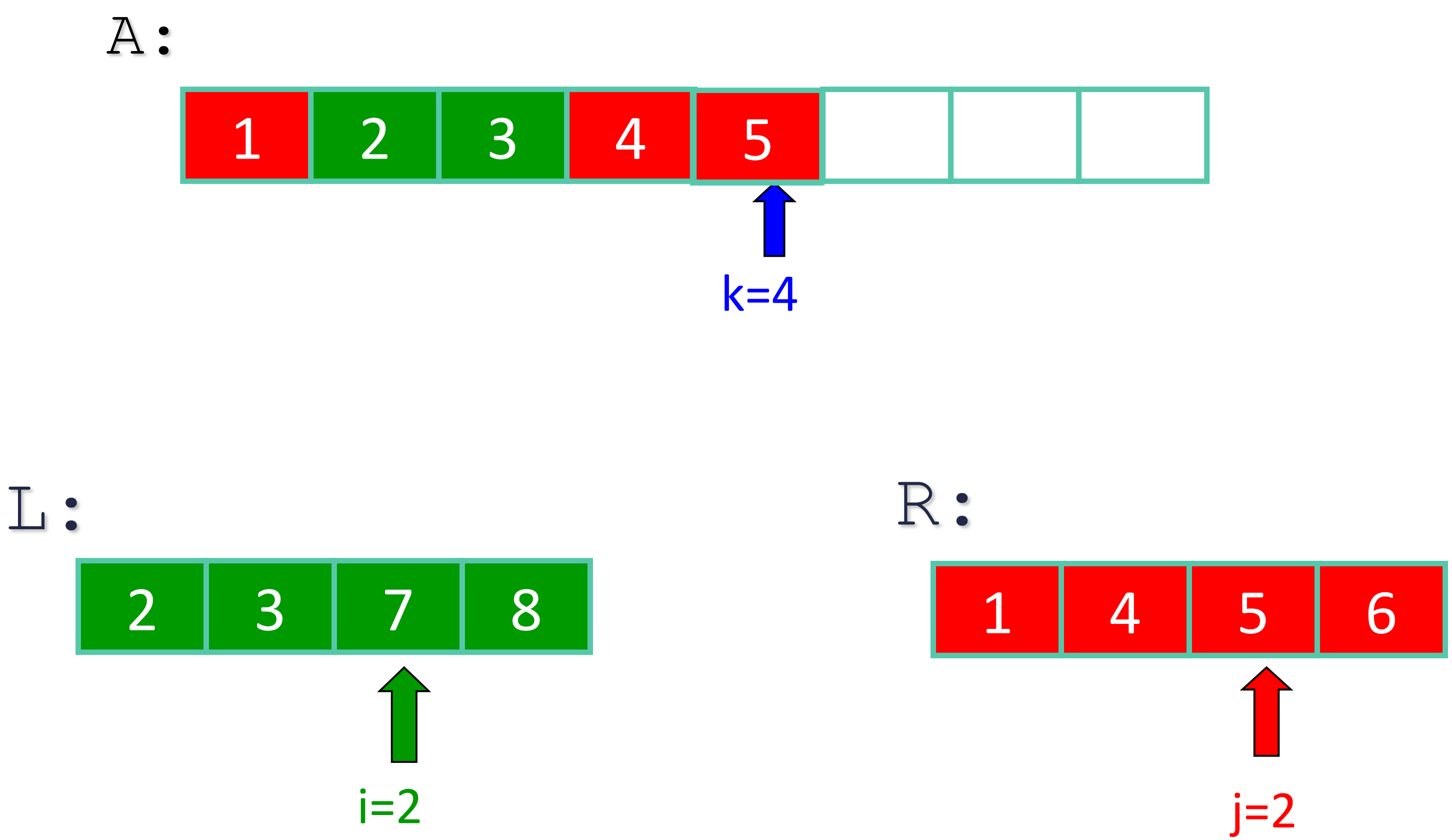

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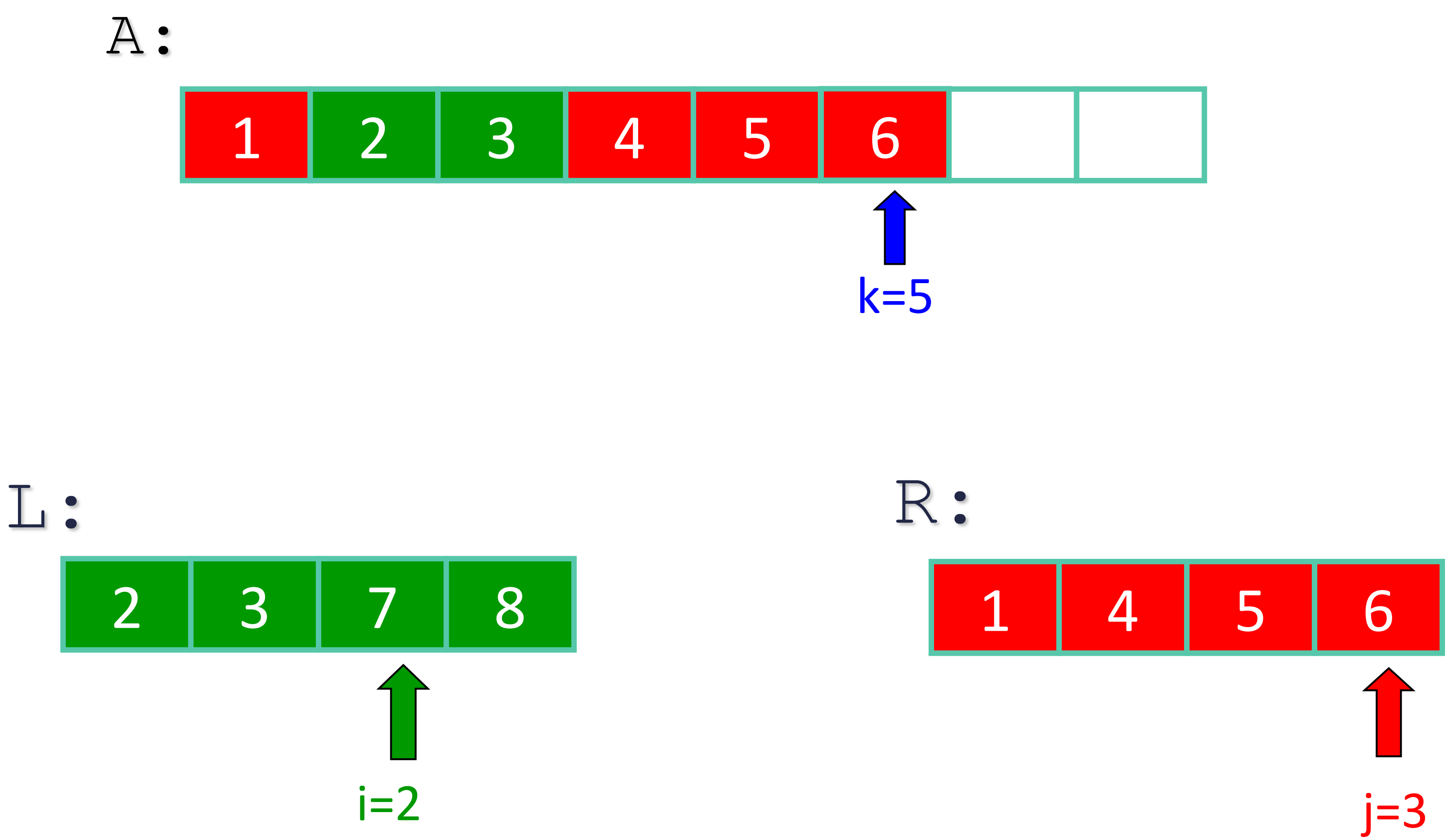

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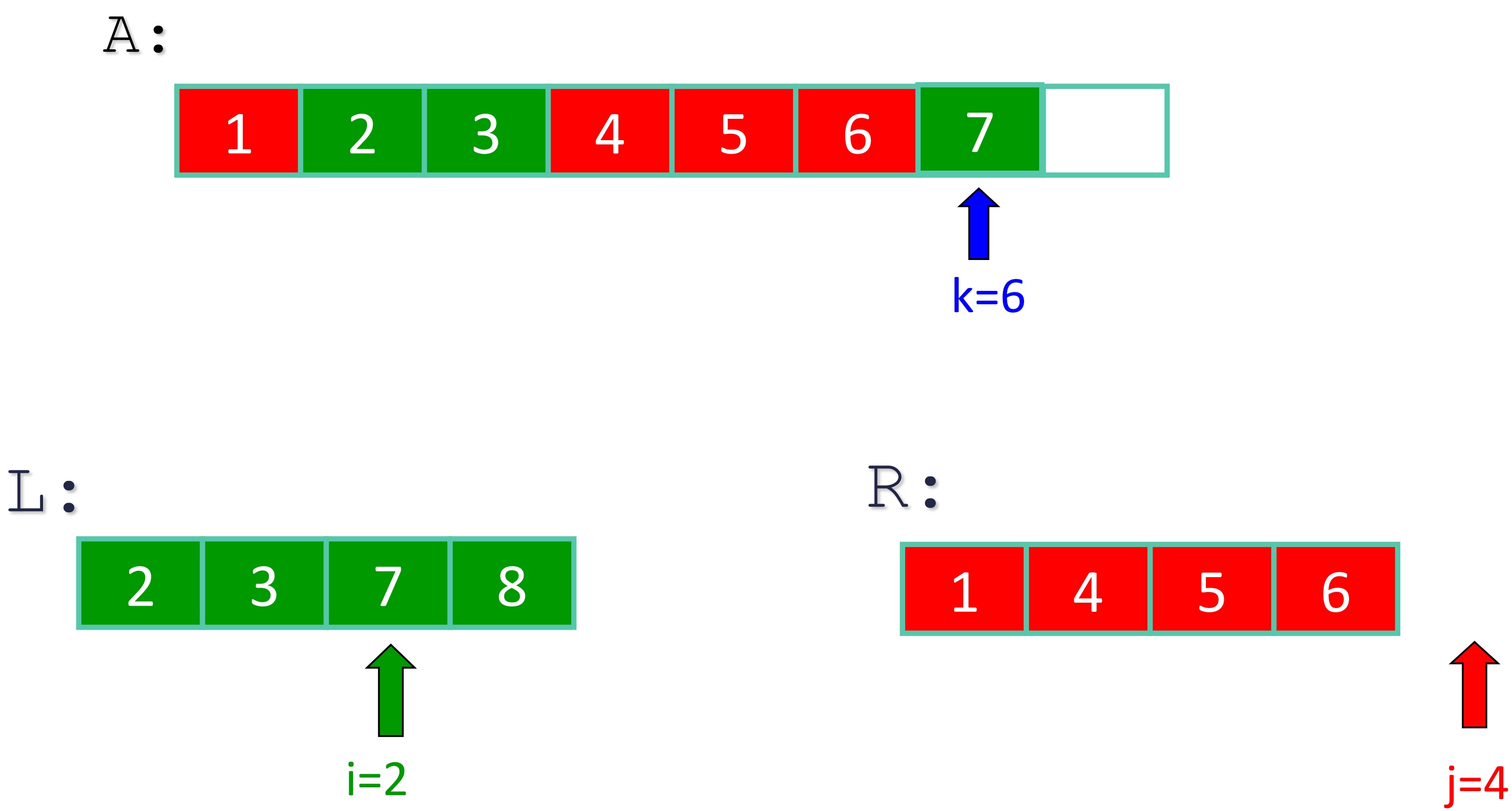


# Merge-Sort: Merge

A:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

k=7

L:

| | | | |
|---|---|---|---|
| 2 | 3 | 7 | 8 |

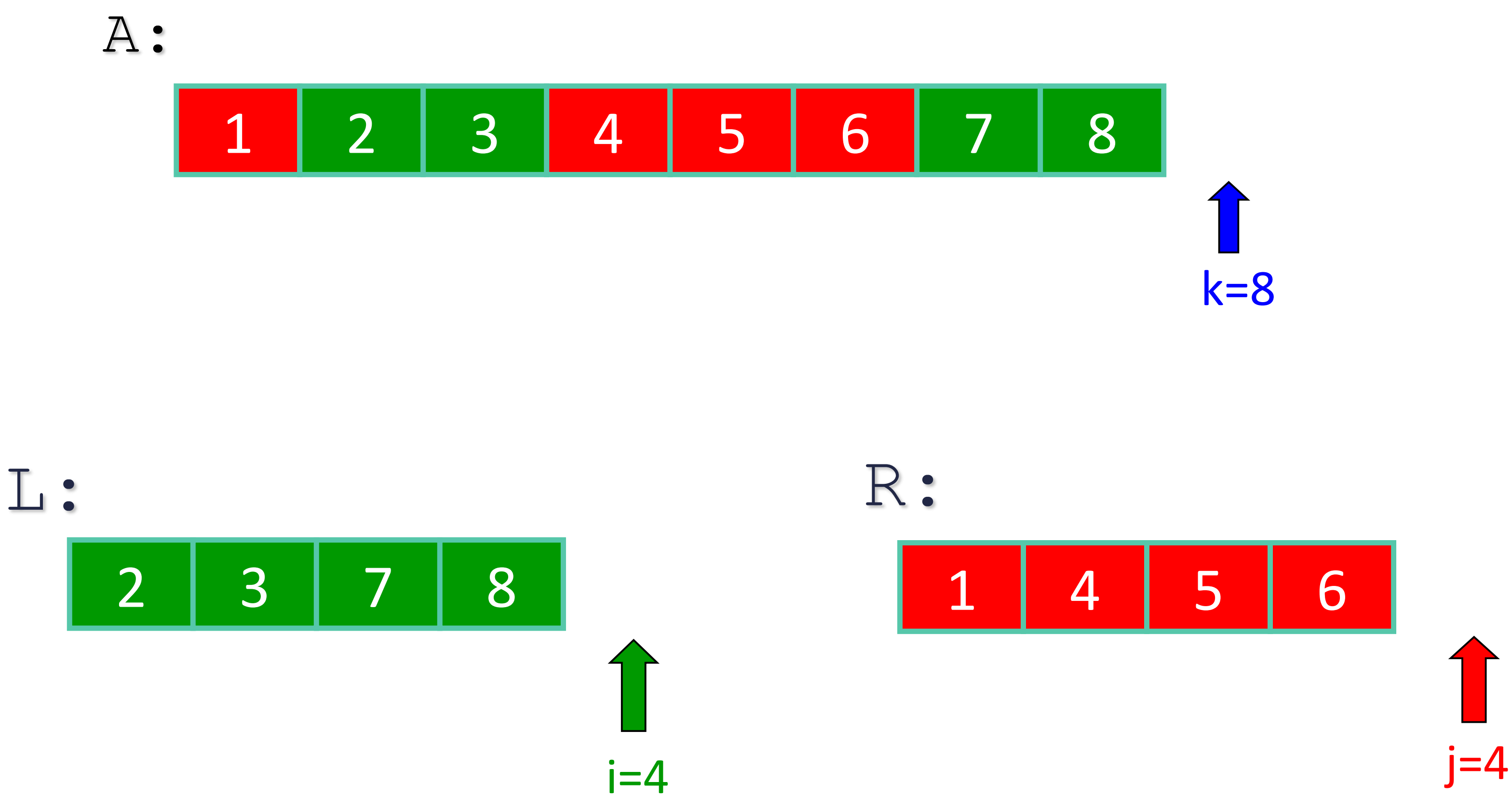
i=3

R:

| | | | |
|---|---|---|---|
| 1 | 4 | 5 | 6 |

j=4

# Merge-Sort: Merge

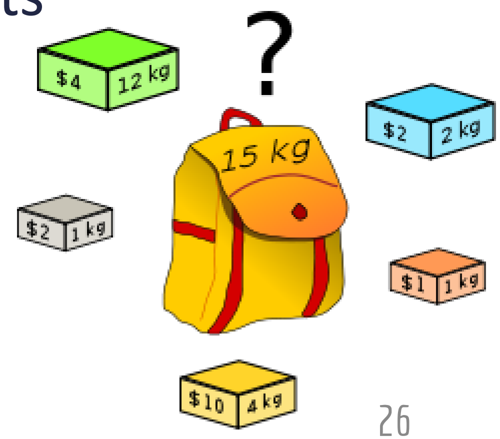


# Greedy algorithms

- An optimization problem is one in which you want to find, not just *a* solution, but the *best* solution
- A "greedy algorithm" sometimes works well for optimization problems
- A greedy algorithm works in phases: At each phase:
  - You take the best you can get right now, without regard for future consequences
  - You hope that by choosing a *local* optimum at each step, you will end up at a *global* optimum
- *Any seen so far?*
- Djikstra's Shortest path problem
  - Greedily pick the shortest among the vertices touched so far

# Knapsack Problem

- We are given a set of $n$ items, where each item $i$ is specified by a size $s_i$ and a value $v_i$. We are also given a size bound $S$ (the size of our knapsack).
- The goal is to find the subset of items of maximum total value such that sum of their sizes is at most $S$ (they all fit into the knapsack).
  - Exponential time to try all possible subsets
  - O(n.S) using DP



# Knapsack Problem

- 0-1 Knapsack:
  - *n* items (can be the same or different)
  - Have **only one** of each
  - Must **leave or take** (i.e. 0-1) each item (e.g. bars of gold)
  - DP works, greedy does not

- Fractional Knapsack:
  - *n* items (can be the same or different)
  - Can take **fractional part** of each item (e.g. gold dust)
  - Greedy works and DP algorithms work



http://www.radford.edu/~nokie/classes/360/greedy.html

# Greedy Solution 1

- From the remaining objects, select the object with maximum value that fits into the knapsack
- *Does not guarantee an optimal solution*
- E.g., n=3, s=[100,10,10], v=[20,15,15], S=105

# Greedy Solution 2

- Select the one with minimum size that fits into the knapsack
- *Also, does not guarantee optimal solution*
- E.g., n=2, s=[10,20], v=[5,100], S=25

# Greedy Solution 3

- Select the one with the maximum value density $v_i/s_i$ that fits into the knapsack
- E.g., n=3, s=[20,15,15], v=[40,25,25], S=30
- Greedy works…if fractional items possible!

# Dynamic Programming (DP)

- A dynamic programming algorithm "remembers" past results and uses them to find new results
  - *Memoization*
- Dynamic programming is generally used for optimization problems
  - Multiple solutions exist, need to find the "best" one
  - Requires "optimal substructure" and "overlapping subproblems"
    - **Optimal substructure**: Optimal solution can be constructed from optimal solutions to subproblems
    - **Overlapping subproblems**: Solutions to subproblems can be stored and reused in a bottom-up fashion
- *This differs from Divide and Conquer, where subproblems generally need not overlap*

# Fibonacci numbers

- $n_i = n_{(i-1)} + n_{(i-2)}$
- 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, ...
- To find the $n^{th}$ Fibonacci number:
  - If n is zero or one, return 1; otherwise,
  - Compute fibonacci(n-1) and fibonacci(n-2)
  - Return the sum of these two numbers

- This is a *recursive* algorithm
- This is also an *expensive* algorithm
  - It requires O(fibonacci(n)) time
  - This is equivalent to exponential time, that is, $O(2^n)$
    - *Binary tree of height 'n' with f(n) having two children, f(n-1), f(n-2)*

# Fibonacci numbers again

- To find the $n^{th}$ Fibonacci number:
  - If *n* is zero or one, return one; otherwise,
  - Compute, *or look up in a table*, fibonacci(n-1) and fibonacci(n-2)
  - Find the sum of these two numbers
  - *Store the result in a table* and return it
- Since finding the $n^{th}$ Fibonacci number involves finding all smaller Fibonacci numbers, the second recursive call has little work to do
- The table may be preserved and used again later
- Other examples: *Floyd–Warshall All-Pairs Shortest Path (APSP)* algorithm, *Towers of Hanoi*, …

# DP for 0-1 Knapsack

```
// Recursive algorithm: either we use the last element or we don't.
Value(n,S)    // S = space left, n = # items still to choose from
{
   if (n == 0) return 0;
   if (arr[n][S] != unknown) return arr[n][S];  // <- added this
   if (s_n > S) result = Value(n-1,S);
   else result = max{v_n + Value(n-1, S-s_n), Value(n-1, S)};
   arr[n][S] = result;                          // <- and this
   return result;
}
```

# Brute force algorithm

- A brute force algorithm simply tries *all* possibilities until a satisfactory solution is found
- Such an algorithm can be:
  - Optimizing: Find the *best* solution. This may require finding all solutions, or if a value for the best solution is known, it may stop when any best solution is found
    - Example: Finding the best path for a traveling salesman
  - Satisficing: Stop as soon as a solution is found that is *good enough*
    - Example: Finding a traveling salesman path that is within 10% of optimal

# Improving brute force algorithms

- Often, brute force algorithms require exponential time
- Various *heuristics* and *optimizations* can be used
  - Heuristic: A "rule of thumb" that helps you decide which possibilities to look at first
  - Optimization: In this case, a way to eliminate certain possibilities without fully exploring them

# Randomized algorithms

- A randomized algorithm uses a random number at least once during the computation to make a decision
  - Example: In Quicksort, using a random number to choose a pivot
  - Example: Trying to factor a large number by choosing random numbers as possible divisors
- *E.g. k-means clustering*