Numerical Linear Algebra
Assignment 1

Due on Sunday, September 3, 2017

1. Derive expressions for the ∞-norm and the 1-norm of a matrix. The p-norm of a matrix is defined as

$$||A||_p = \sup \frac{||Ax||_p}{||x||_p}$$

2. Generate random vectors x and c of length N.

```
x = rand(N, 1);
c = rand(N, 1);
```

Let A be a $N \times N$ Vandermonde matrix defined as

$$A_{ij} = c_i^{j-1}$$

and b be defined as

$$b = Ax$$

Solve the system $Ax' = b$ to compute x'.

```
xdash = A \ b;
```

Compute the relative error $\frac{||x-x'||}{||x||}$. Average the relative error over a large number of experiments. Plot the averaged relative error against N in log scale for $N = 5, \ldots, 20$. Use the plotting command *semilogy* to achieve this.

3. Implement classical Gram-Schmidt (*cgs.m*) and modified Gram-Schmidt (*mgs.m*) algorithms. Using these routines, try the following numerical experiment.

```
[Q1, R] = qr(rand(100))
[Q2, R] = qr(rand(100))
A = Q1 * diag(2.^[-1:-1:-100]) * Q2
[Qc, Rc] = cgs(A)
[Qm, Rm] = mgs(A)
```

Plot diagonal entries of R matrices as *log2(diag(Rc))* and *log2(diag(Rm))* versus the index 1 to N.

```
plot(log2(diag(Rc)))
plot(log2(diag(Rm)))
```

What do you observe?

Note

1. Submit all your code, and a short report (in PDF format) with all your plots. Put all these files into a **folder with a name of the format "name_srno"**. For example, if your name is "Ashok" and your SR No is 10619, then the folder should be named "ashok_10619". Compress this folder into an archive (zip, .tar.gz, .tar.bz2, or .tar.xz format).

2. For embedded plots, prefer vector graphics formats such as EPS.

3. Submit your work via email. Make sure to clearly mention your name and SR No. The email address to send your finished assignments to will be provided to you shortly.

4. Use Matlab for all programming questions. You may also choose to use GNU Octave, a free/libre software implementation of the Matlab programming language.