Numerical Linear Algebra
Homework 2

Due on Wednesday, September 27, 2017

1. Let A be a matrix defined as

```
A = magic(5) - 64*eye(5);
[u v] = eig(A);
x = u(:,1);
```

where x is an eigen vector of A with eigen value 1.

$$Ax = x$$

Plot $||A^n x||_2$ for $n = 1, 2, 3, \ldots, 10$.

```
semilogy(arrayfun(@(n) norm(A^n*u(:,1)), 1:10))
```

What do you observe? Can you explain the numerical errors, if any, in the computation?

2. Let A be a matrix defined as

```
A = tril(-1*ones(N)) + diag(2*ones(N,1));
A(:,N) = ones(N,1);
```

Compute the LU decomposition of A using the built-in *lu* function. Plot $||LU - A||_F$ versus N for $N = 2, 3, 4, \ldots, 20$. Can you explain the origin of numerical errors in the factorization?

3. Let A be a $M \times N$ rectangular matrix and b a vector defined as

```
M = 2*N;

[U R] = qr(rand(M));
[V R] = qr(rand(N));
S = [diag(2.^[1:N]); zeros(M - N, N)];
A = U*S*V;
b = rand(M, 1);
```

Compute x for $Ax = b$ using the pseudoinverse of A and also using its QR factorization.

```
x = inv(A'*A) * A' * b
[Q R] = qr(A);
x = R \ Q'*b;
```

Compare accuracy (using residue $||b - Ax||_2$) and speed (using the time taken) between both solutions of x for $N = 5, 6, 7, \ldots, 50$. Can you explain the observations? Use built-in commands *tic* and *toc* to measure time.

4. Let A be a matrix defined as

```
A = rand(N);
A = A - diag(A) + diag(0.001*ones(N,1));
```

Compute the LU decomposition of A with and without partial pivoting. Plot $||LU - A||_F$ versus N for $N = 5, 6, 7, \ldots, 20$. For LU decomposition with partial pivoting, use the built-in *lu* function. For LU decomposition without pivoting, write your own function.

Note

1. Submit all your code, and a short report (in PDF format) with all your plots. Put all these files into a **folder with a name of the format "name_srno"**. For example, if your name is "Ashok" and your SR No is 10619, then the folder should be named "ashok_10619". Compress this folder into an archive (zip, .tar.gz, .tar.bz2, or .tar.xz format).

2. For embedded plots, prefer vector graphics formats such as EPS.

3. Submit your work via email (ds284@cds.iisc.ac.in). Make sure to clearly mention your name and SR No.

4. Use Matlab for all programming questions. You may also choose to use GNU Octave, a free/libre software implementation of the Matlab programming language.

5. Use logscale plots wherever appropriate to make graphs readable.