DS-288 NUMERICAL METHODS
Assignment–2 [1]

Due date: 16, November, 2017 (16:00 Hrs IST)
Instructor: Prof. Sashikumaar Ganesan.

1. Derive the Adams-Bashforth four-step method by using Newton's backward-difference form of the interpolating polynomial.

2. Analyze the stability of the equation
$$\frac{dy(t)}{dt} = -5y(t), \quad y(0) = 0.1, \quad 0 \le y \le 5 \tag{1}$$
for (i) Backward Euler, (ii) Trapezoidal, (iii) second-order Runge-Kutta method (use the Midpoint Rule as your choice), (iv) Adams- Bashforth two-step method and (v) Adams-Moulton two-step method. Determine the maximum allowable step-size that will still maintain stability for each method and rank these schemes from the most to least stable method for this problem. Verify your analysis by programming all the five schemes and solving the ODE (use analytical solution to produce enough values to get the method started) with two different step-sizes one of which violates and the other which obeys the stability criterion. Plot your solution as a function of time in each case thereby providing graphical evidence of either stability or instability. Also show a case where the time-step size is such that the solution is stable, but not very accurate.

3. Use fourth-order Runge-Kutta method to approximate the solution of the second-order differential equation
$$y'' - 2y' + y = te^t - t, \quad 0 \le t \le 1, \quad y(0) = y'(0) = 0, \quad h = 0.1. \tag{2}$$
Plot the approximate solution and compare the results with the exact solution
$$y(t) = \frac{1}{6}t^3e^t - te^t + 2e^t - t - 2. \tag{3}$$

4. Use linear shooting method to approximate the solution of the following boundary-value problem
$$y'' = y' + 2y + \cos x, \quad 0 \le x \le \frac{\pi}{2}, \quad y(0) = -0.3, \quad y(\frac{\pi}{2}) = -0.1 \tag{4}$$
with (i) $h = \frac{\pi}{4}$, (ii) $h = \frac{\pi}{8}$, (iii) $h = \frac{\pi}{16}$. Plot the approximate solutions and compare the results with the exact solution
$$y(x) = \frac{-1}{10}\left(\sin x + 3\cos x\right). \tag{5}$$

5. Find u such that
$$-0.005u''(x) + u'(x) = 0, \quad 0 \le x \le 1, \quad u(0) = 0, \quad u(1) = 1. \tag{6}$$
Use central difference approximation for the second derivative, but for the first derivative examine the following two choices (i) forward difference and (ii) backward difference. Solve the boundary value problem with N=128 and 256, where N is the number of intervals in [0,1]. Plot the approximate solution and compare with the exact solution in each case. Discuss and justify which choice of difference formula for the first derivative is appropriate for this problem.

[1]Posted on: November 2, 2017.

6. Find the minimum of the function

$$F = 100(y - x^2)^2 + (1 - x)^2 \tag{7}$$

with Powell's method starting at the point (-1,1).

7. Use the downhill simplex method to minimize

$$F = 10x_1^2 + 3x_2^2 - 10x_1x_2 + 2x_1. \tag{8}$$

The coodinates of the vertices of the starting simplex are (0,0), (0,-0.2) and (0.2,0). Show graphically the first four moves of the simplex.