Change in grading scheme

1) Assignments (derivations): 10 marks
2) Two lab exams: 30 marks (9th March, 6th April)
3) Project (implementation of a journal paper): 30 marks

    Paper selection: 15th March

    Project presentation: 21st April
4) Final written exam: 30 marks (28th April)

E9 285 Biomedical imaging-Inverse problems

Chapter 2.b (2)
Representative reconstruction methods for fluorescence microscopy

Discrete Wavelets (Multiscale Derivative Transforms)

A simple transform:

Input:

$$\{x(0),x(1),x(2),x(3),x(4),x(5),x(6),x(7)\}$$

Output 1:

$$\left\{\begin{array}{l}\underbrace{x(0)+x(1),x(2)+x(3),x(4)+x(5),x(6)+x(7)}_{x_A^{(1)}}, \\ \underbrace{x(1)-x(0),x(3)-x(2),x(5)-x(4),x(7)-x(6)}_{x_D^{(1)}}\end{array}\right\}$$

Output 2:

$$\left\{\underbrace{x_A^{(1)}(0)+x_A^{(1)}(1),x_A^{(1)}(2)+x_A^{(1)}(3)}_{x_A^{(2)}},\underbrace{x_A^{(1)}(1)-x_A^{(1)}(0),x_A^{(1)}(3)-x_A^{(1)}(2)}_{x_D^{(2)}},\left\{x_D^{(1)}\right\}\right\}$$

Output 3:

$$\left\{\underbrace{x_A^{(2)}(0),+x_A^{(2)}(1)}_{x_A^{(3)}},\underbrace{x_A^{(2)}(1)-x_A^{(2)}(0)}_{x_D^{(3)}},\left\{x_D^{(2)}\right\},\left\{x_D^{(1)}\right\}\right\}$$

Upsampler and downsampler

$$x(n) \longrightarrow \downarrow M \longrightarrow y(n)$$

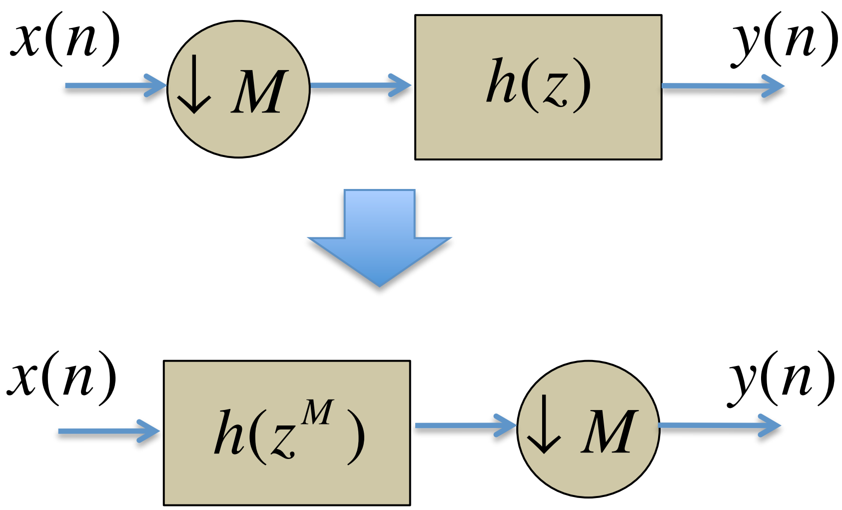
$$y(n) = x(nM)$$

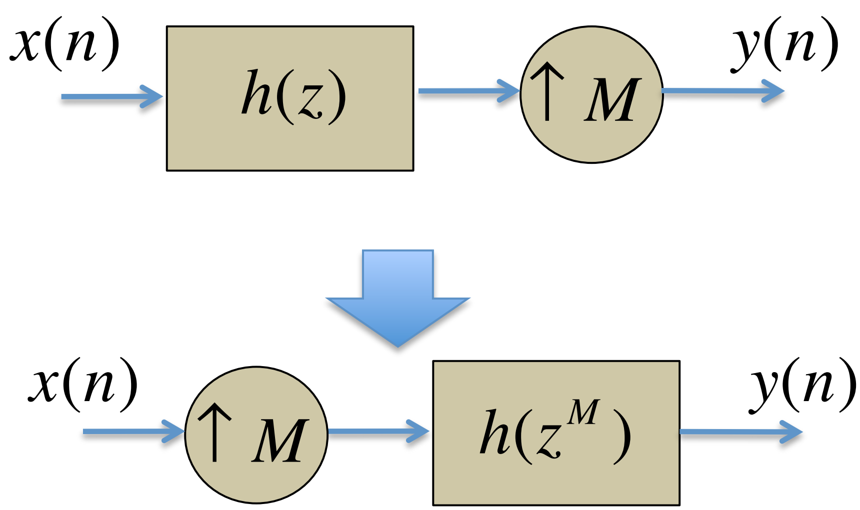
$$x(n) \longrightarrow \uparrow M \longrightarrow y(n)$$

$$y(n) = \begin{cases} x(n/M), & \text{for } n/M \text{ being an integer} \\ 0, & \text{Otherwise} \end{cases}$$

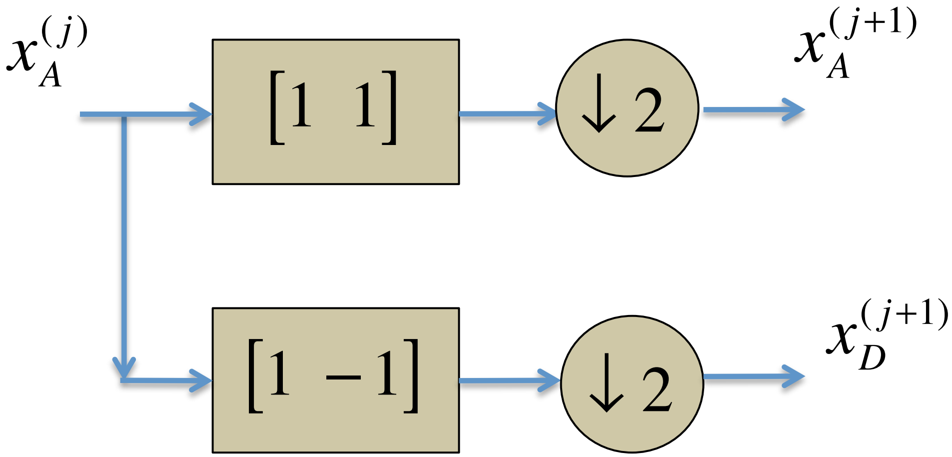
$$\Rightarrow y(z) = x(z^{M})$$

Noble (??) identities

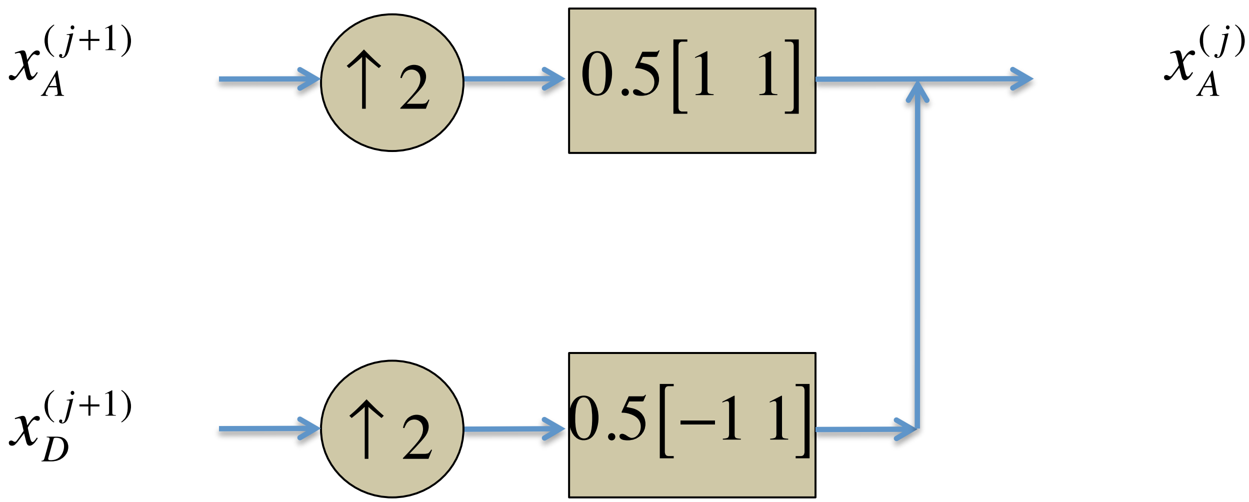




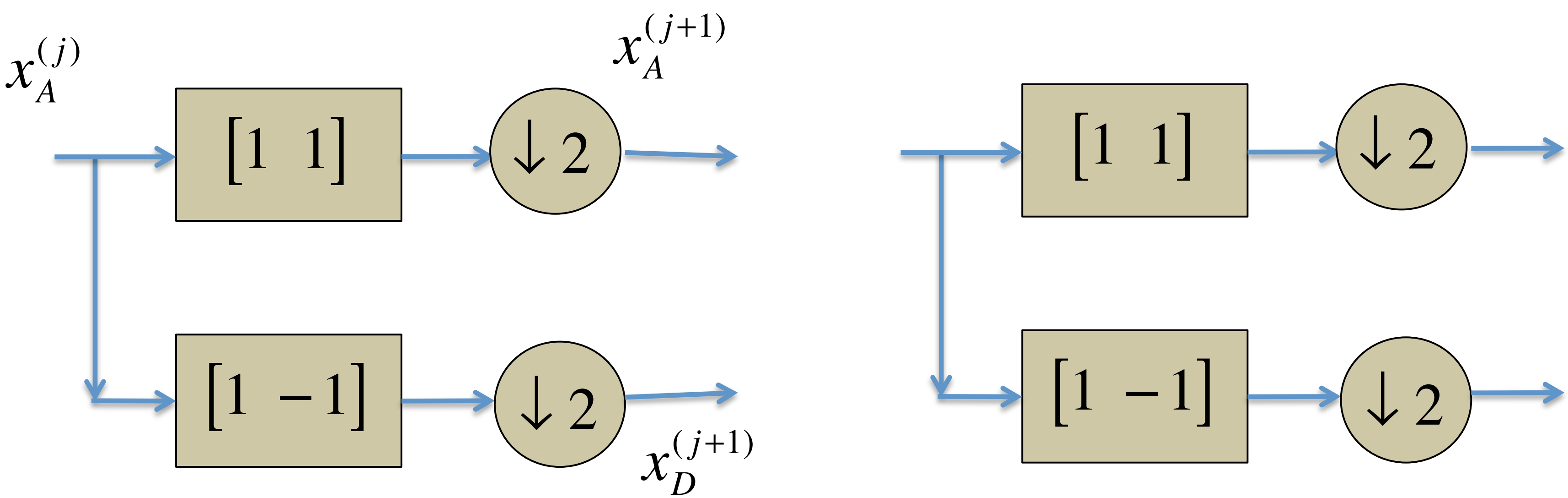
Analysis filter bank



Synthesis filter bank



Why do we call it a multiscale derivative transform ?



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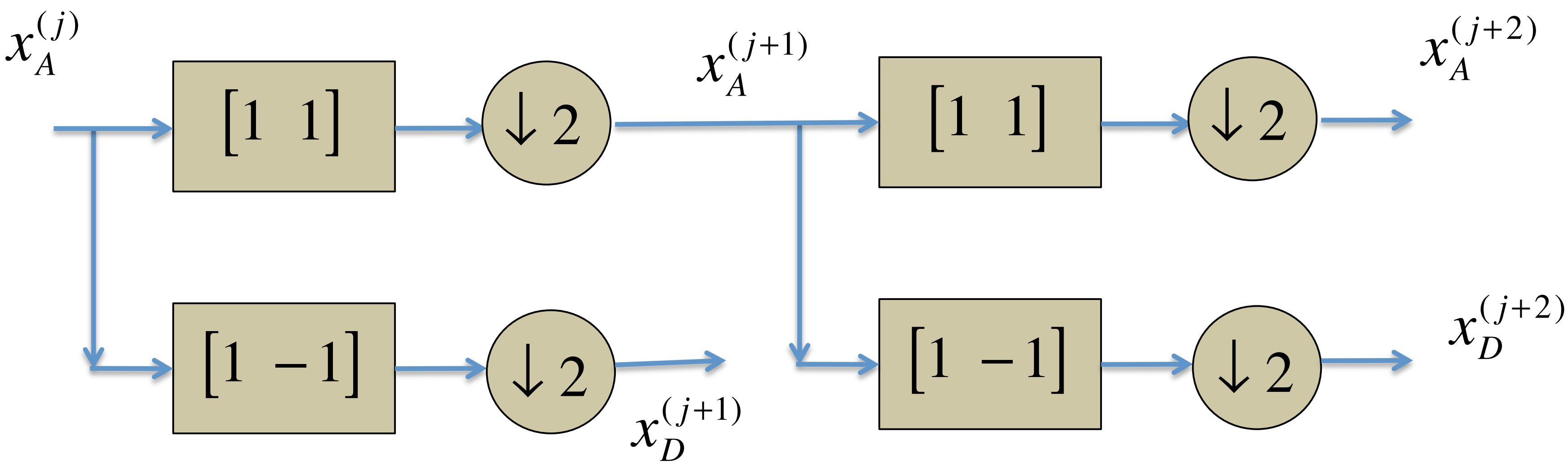

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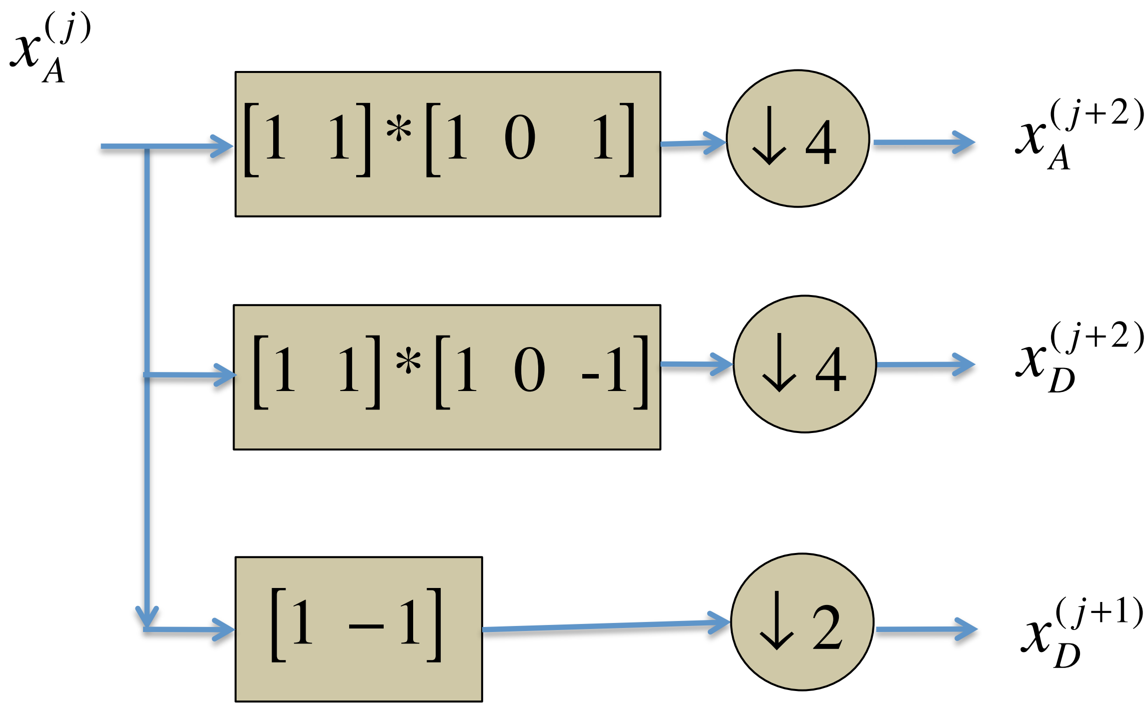

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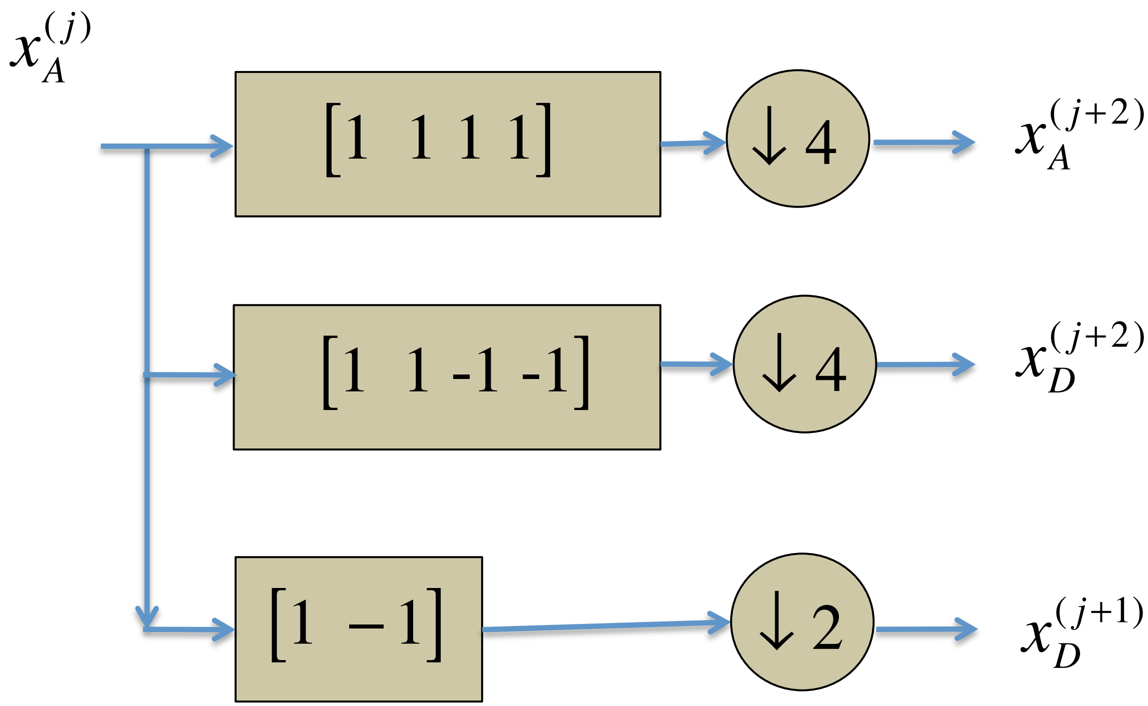

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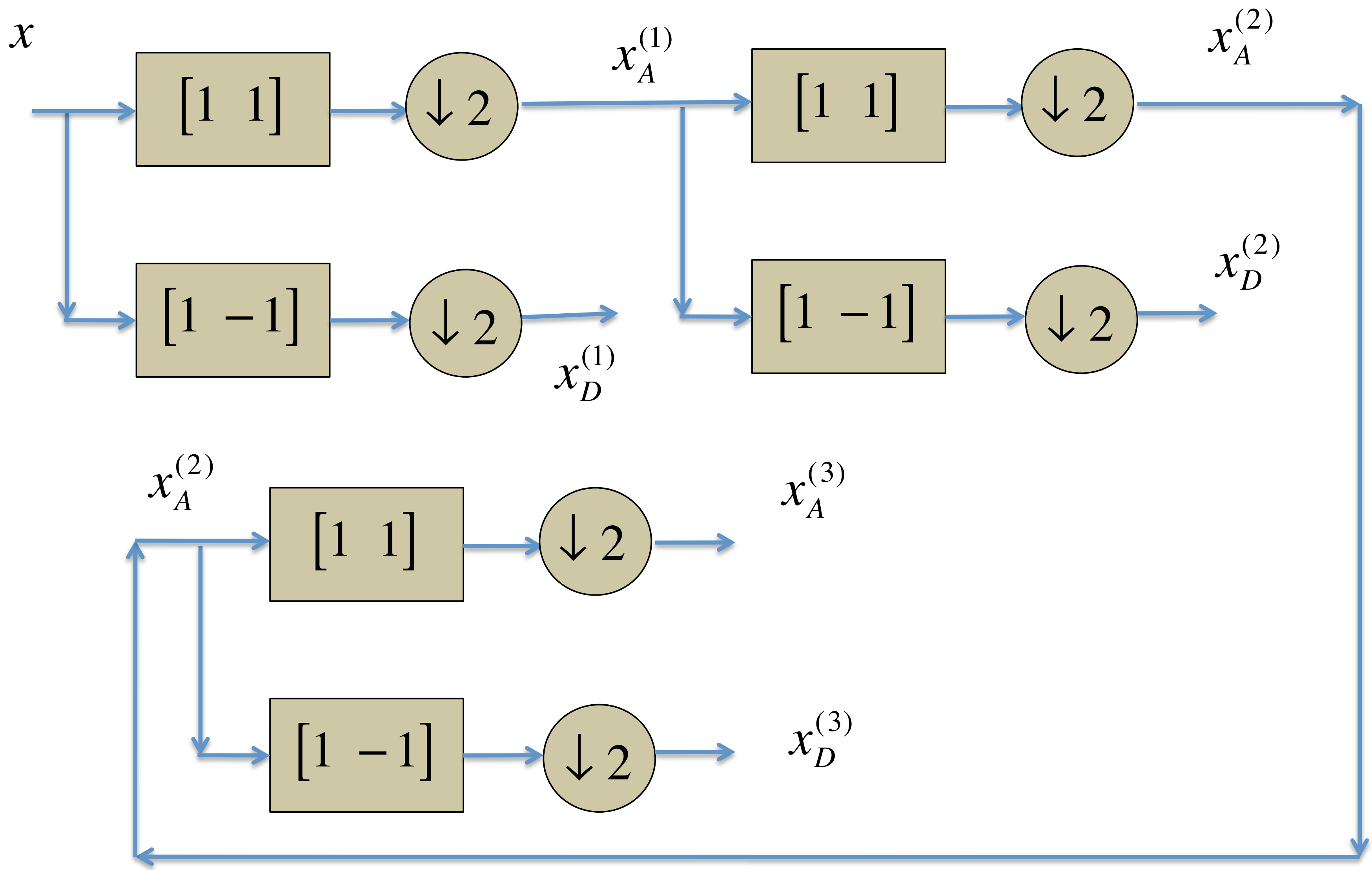

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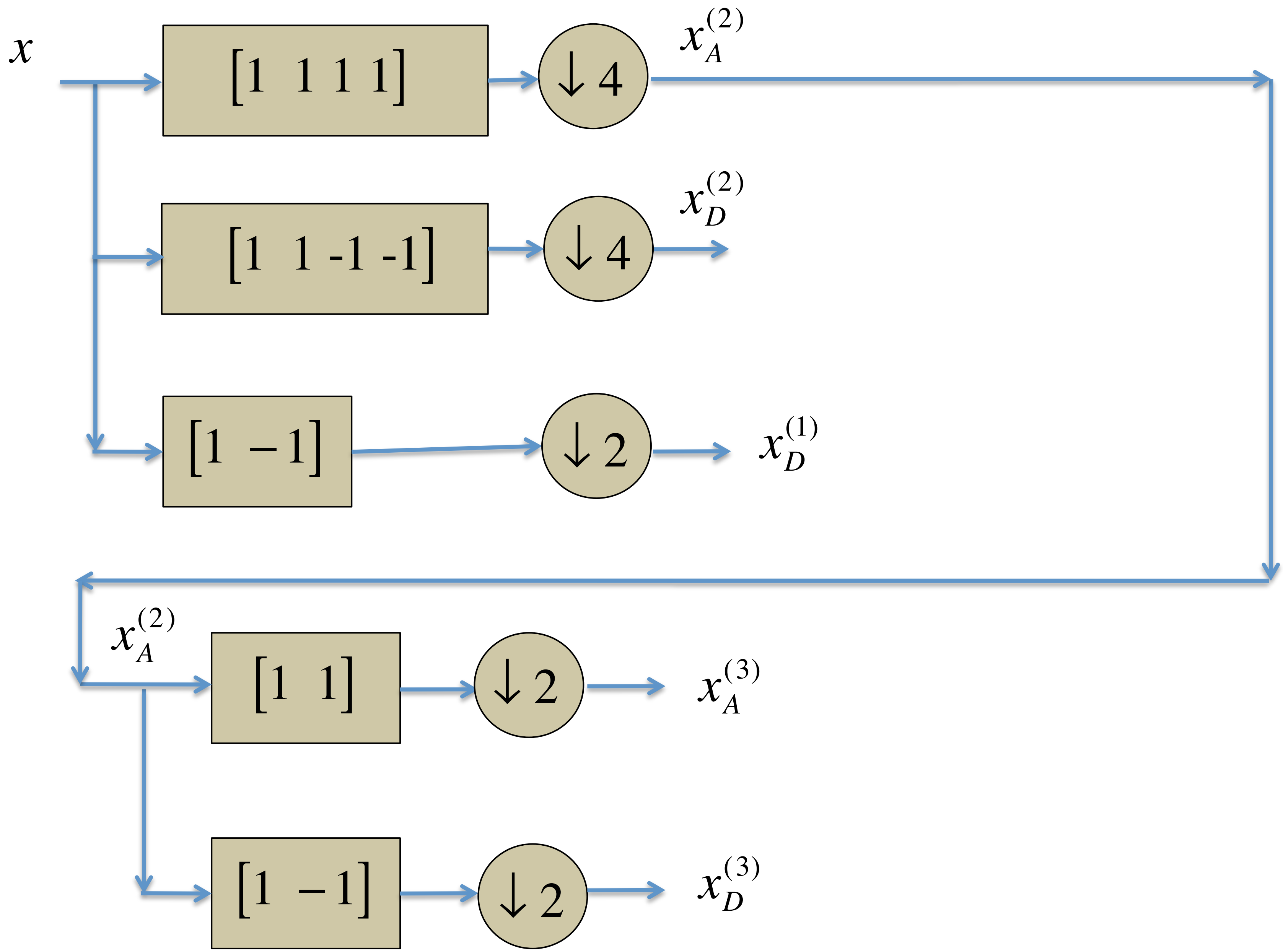

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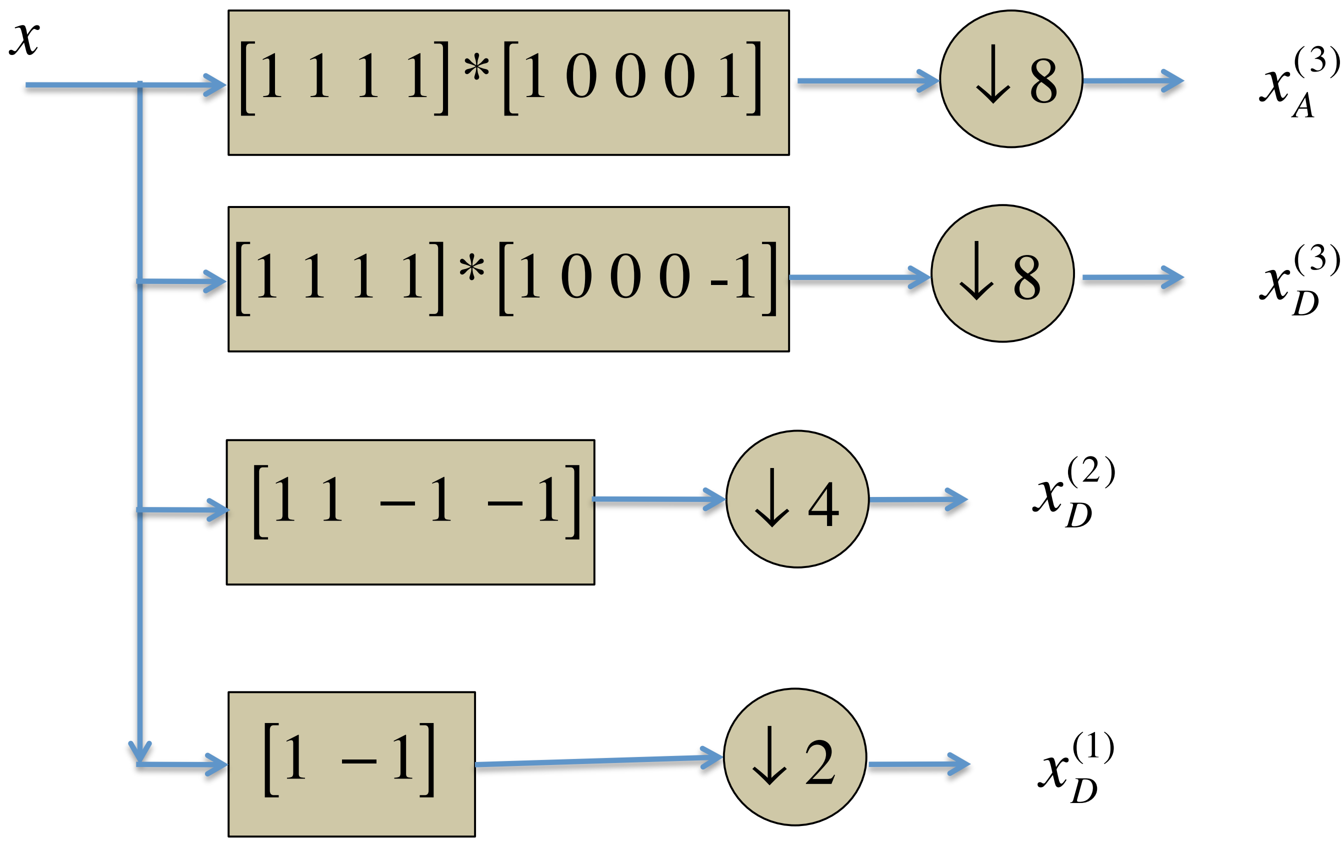

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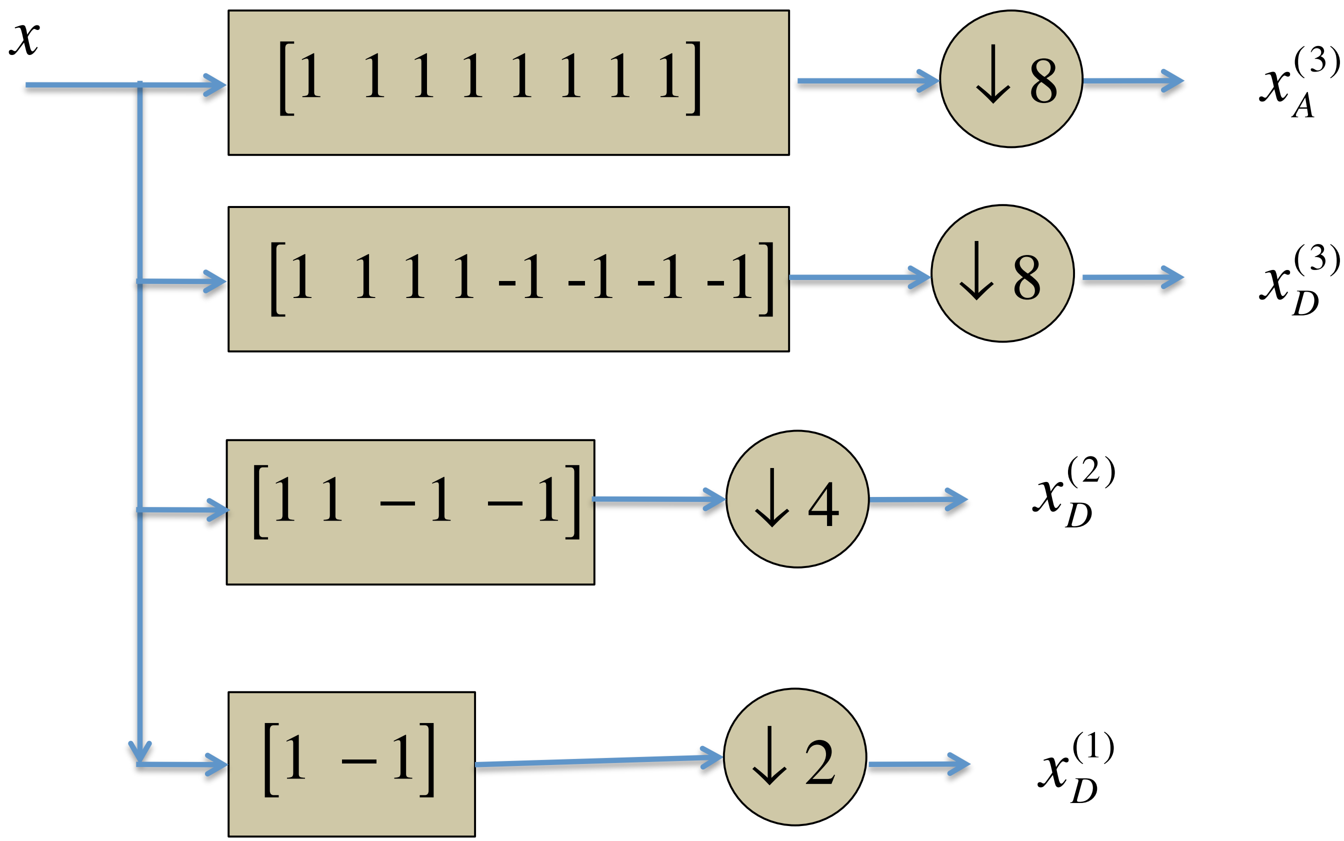
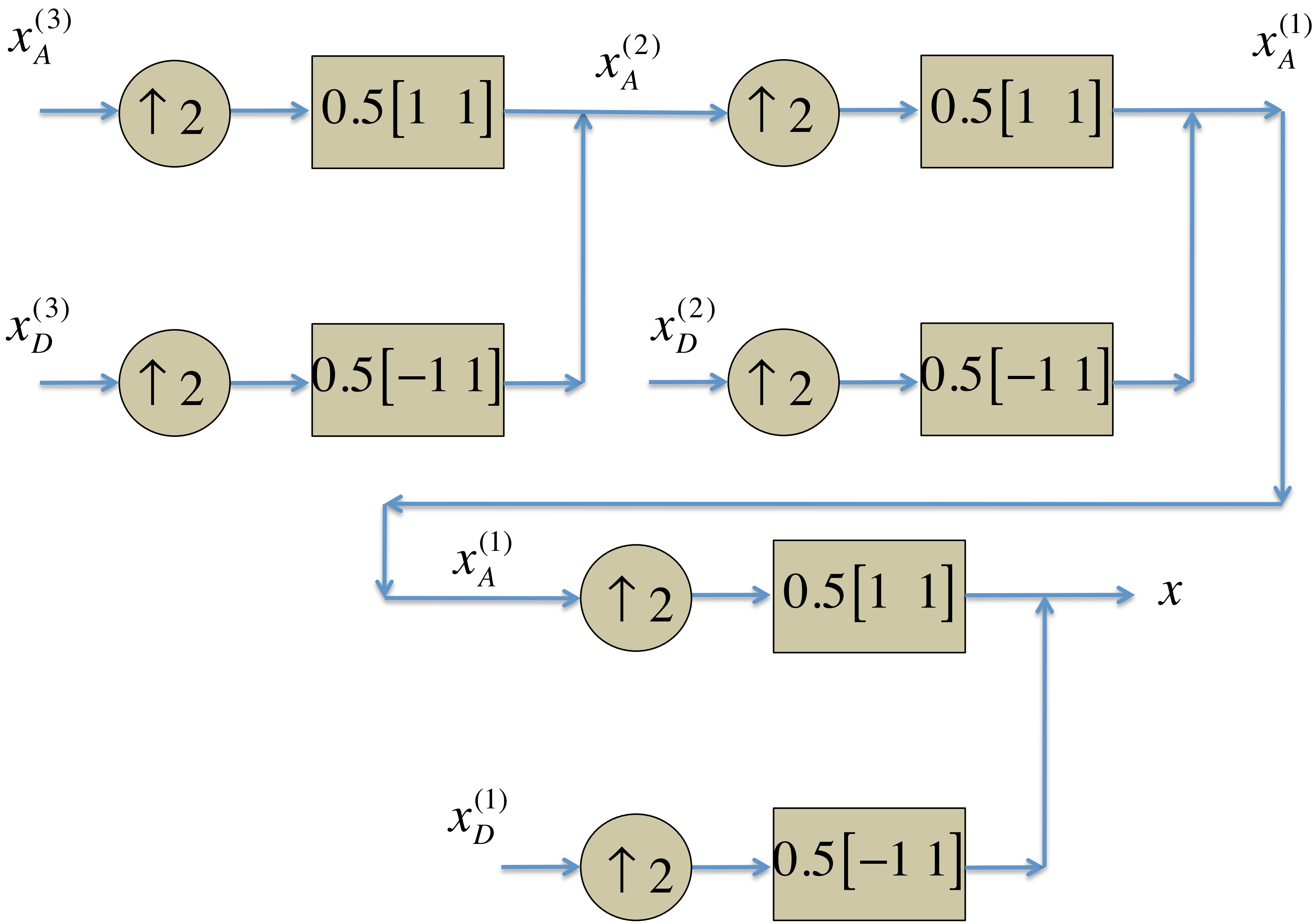
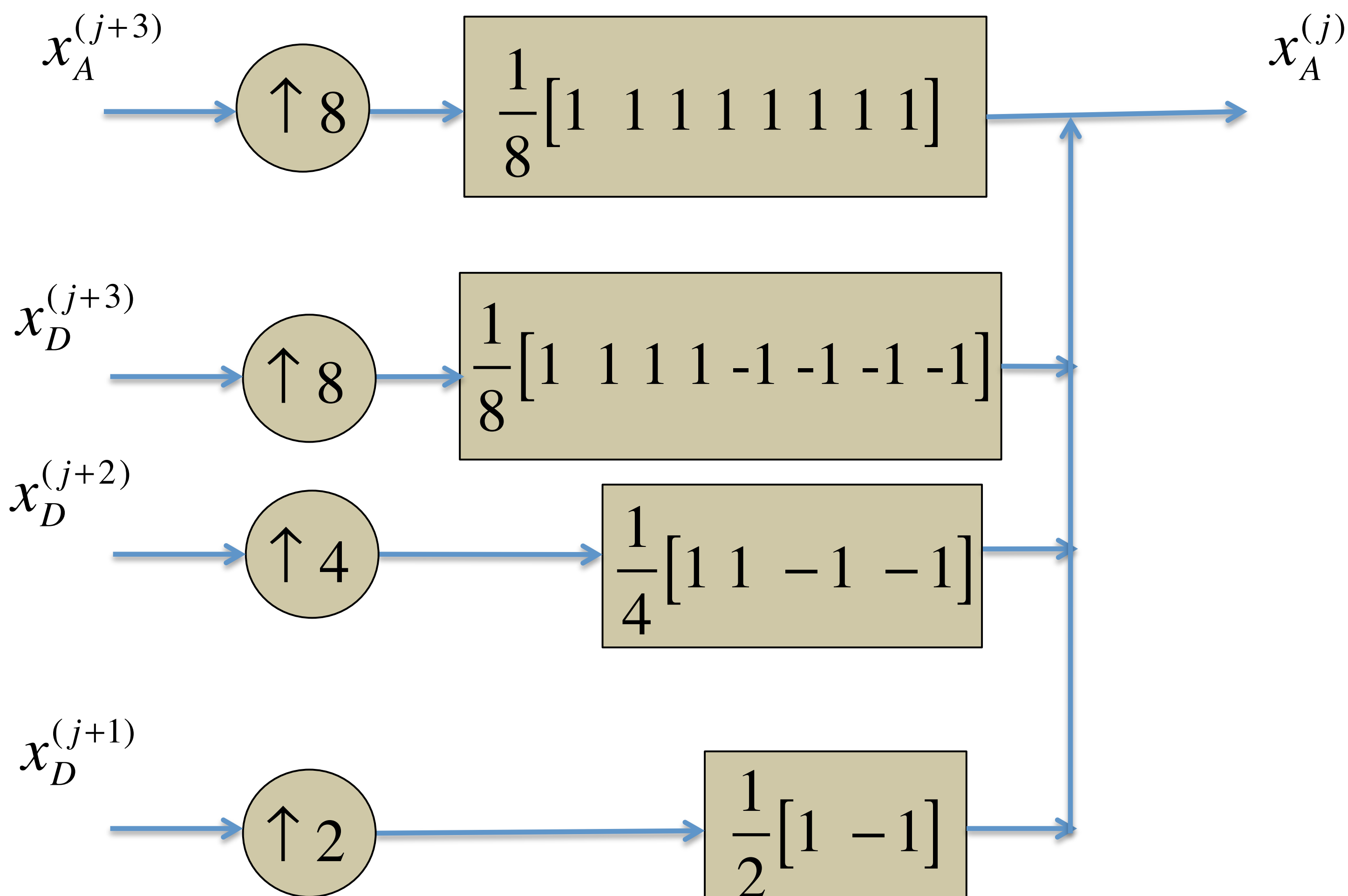


How does the analysis matrix look like ?

$$
\begin{bmatrix} \mathbf{x}_A^{(3)} \\ \mathbf{x}_D^{(3)} \\ \mathbf{x}_D^{(2)} \\ \mathbf{x}_D^{(1)} \end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\
1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \mathbf{x}
$$

The reconstruction (synthesis)



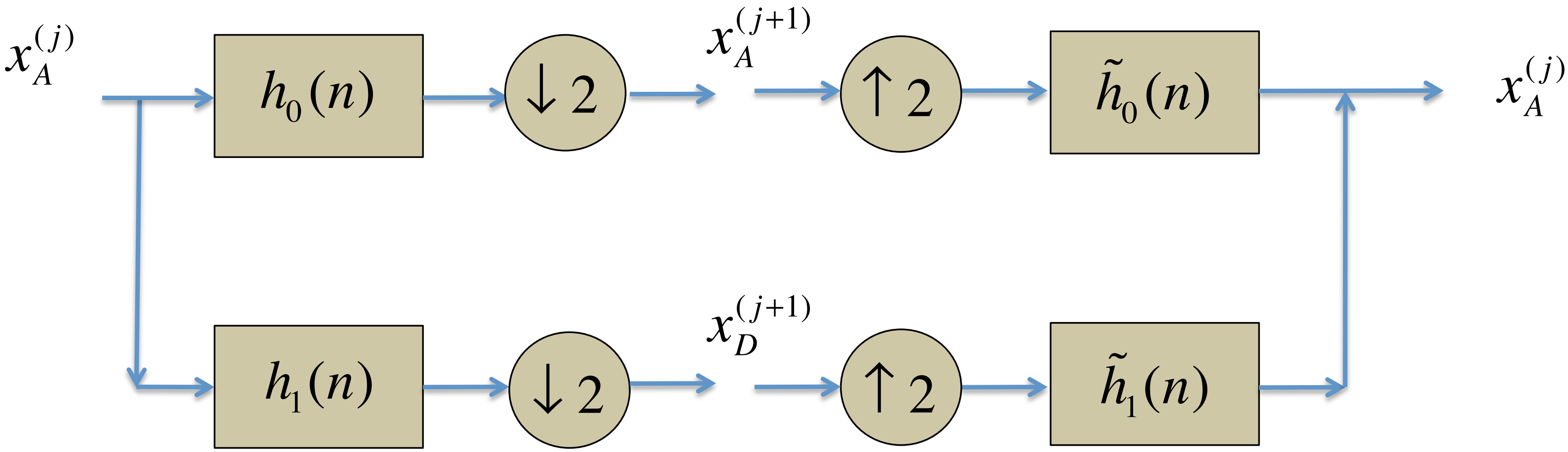


$x_A^{(j+3)}$
$\uparrow 8$
$\frac{1}{8}[1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$
$x_A^{(j)}$
$x_D^{(j+3)}$
$\uparrow 8$
$\frac{1}{8}[1\ 1\ 1\ 1\ \text{-}1\ \text{-}1\ \text{-}1\ \text{-}1]$
$x_D^{(j+2)}$
$\uparrow 4$
$\frac{1}{4}[1\ 1\ -1\ -1]$
$x_D^{(j+1)}$
$\uparrow 2$
$\frac{1}{2}[1\ -1]$

How does the analysis matrix look like ?

$$\mathbf{x} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{x}_A^{(3)} \\ \mathbf{x}_D^{(3)} \\ \mathbf{x}_D^{(2)} \\ \mathbf{x}_D^{(1)} \end{bmatrix}$$

Using other type of filters



What is the condition such that matrix equivalent of the iterated filter bank is orthogonal ?

$$\left\langle h_i(n-2l), h_j(n-2m) \right\rangle = \delta(i-j)\delta(l-m)$$

Using other type of filters

What is the condition such that matrix equivalent of the iterated filter bank is orthogonal ?

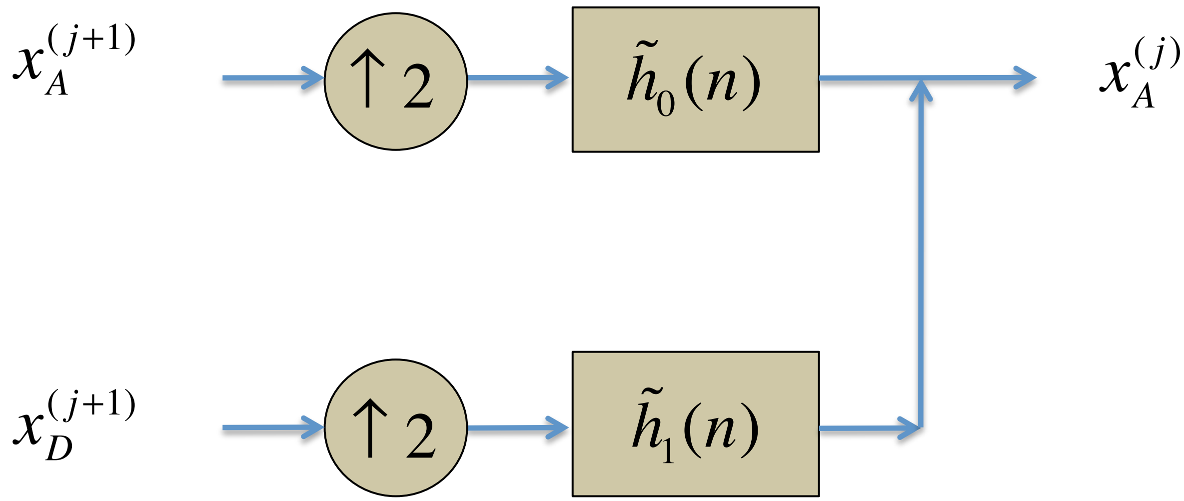
$$\left\langle h_i(n-2l), h_j(n-2m) \right\rangle = \delta(i-j)\delta(l-m)$$

$$\Downarrow$$

$$\left\langle h_0(n-2l), h_0(n-2l) \right\rangle = \delta(l-m) \Rightarrow H_0(z)H_0(z^{-1}) + H_0(-z)H_0(-z^{-1}) = 1$$

$$h_1(n) = (-1)^n h_0(n)$$

Using more other type of filters



$$\tilde{h}_0(n) = h_0(-n)$$

$$\tilde{h}_1(n) = h_1(-n)$$

Order of approximation

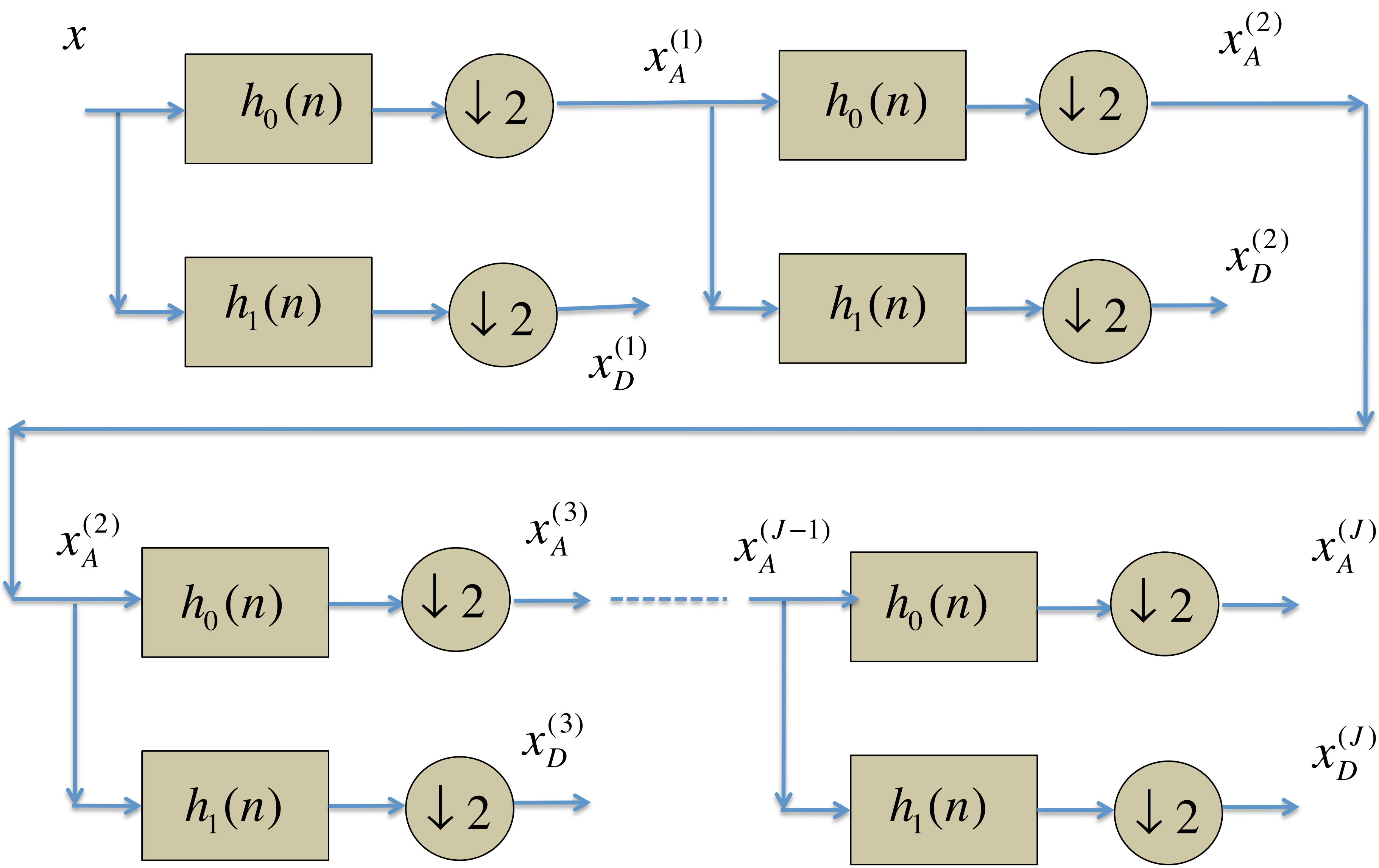
If $H_1(z)$ can be factorized as

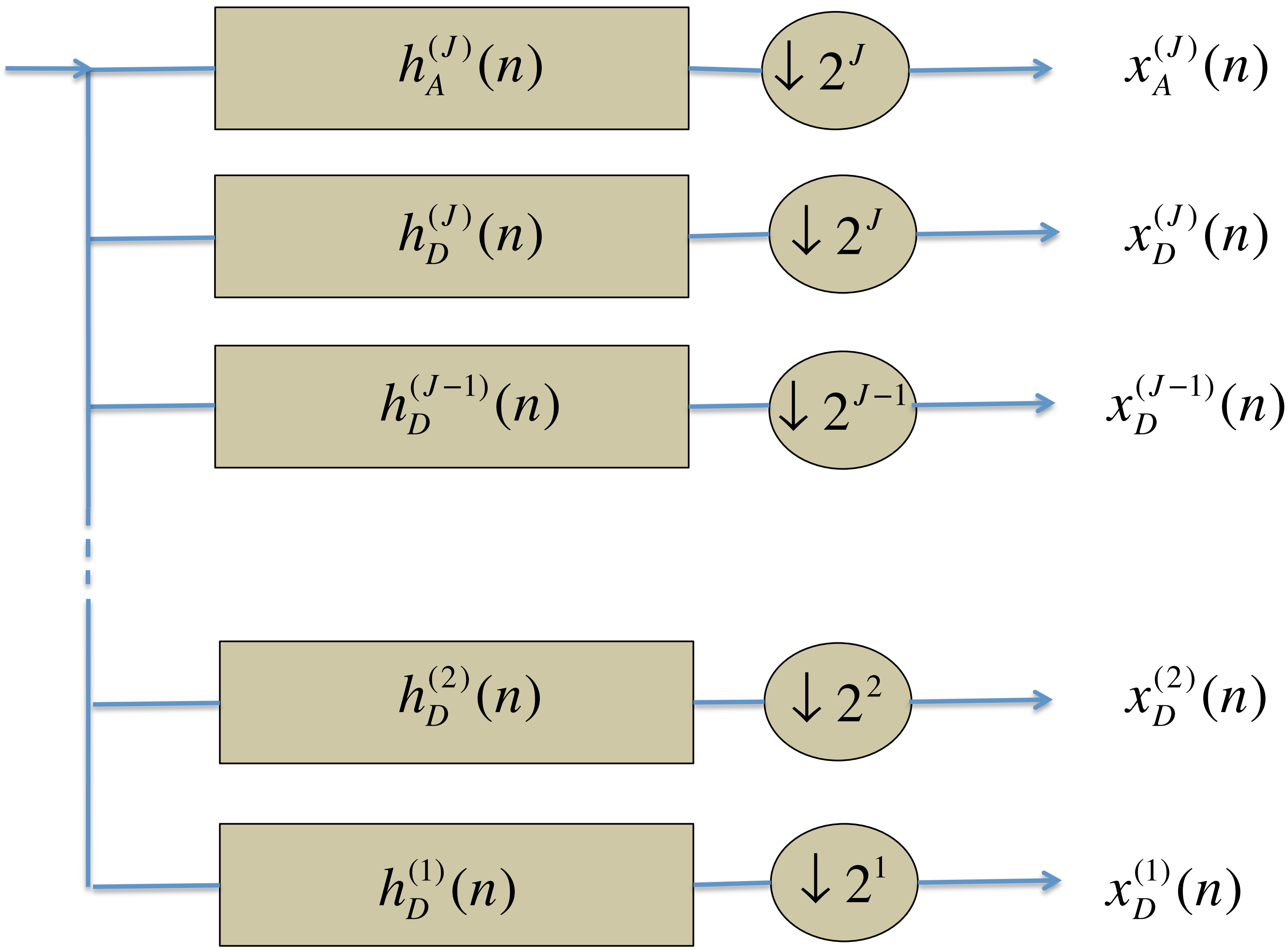
$H_1(z) = \left(1 - z^{-1}\right)^N R(z),$ then the wavelet has an order of approximation N. Then wavelet decomposition will kill polynomials of degree $N-1$.

Daubechies family of orthogonal finite length filter banks

D2 (Haar)	D4	D6	D8	D10
1	0.6830127	0.47046721	0.32580343	0.22641898
1	1.1830127	1.14111692	1.01094572	0.85394354
	0.3169873	0.650365	0.8922014	1.02432694
	-0.1830127	-0.19093442	-0.03957503	0.19576696
		-0.12083221	-0.26450717	-0.34265671
		0.0498175	0.0436163	-0.04560113
			0.0465036	0.10970265
			-0.01498699	-0.00882680
				-0.01779187
				4.71742793e-3

Why do we call it a multiscale derivative transform ?





$$x_A^{(J)}(n) = \sum_m h_A^{(J)}(m - 2^J n)x(m) \qquad x_D^{(j)}(n) = \sum_m h_D^{(j)}(m - 2^j n)x(m)$$

$$j = 1, \ldots, J$$

The discrete wavelets

$$h_D^{(1)}(z) = h_1(z)$$

$$h_D^{(2)}(z) = h_0(z)h_1(z^2)$$

$$h_D^{(3)}(z) = h_0(z)h_0(z^2)h_1(z^4)$$

$$h_D^{(j)}(z) = h_0(z)h_0(z^2)\cdots h_0(z^{2^{j-1}})h_1(z^{2^j})$$

$$h_D^{(J)}(z) = h_0(z)h_0(z^2)\cdots h_0(z^{2^{J-1}})h_1(z^{2^J})$$

$$h_A^{(J)}(z) = h_0(z)h_0(z^2)\cdots h_0(z^{2^J})$$

Orthogonality

$$\left\langle h_0(n-2l), h_0(n-2m) \right\rangle = \delta(l-m) \quad h_1(n) = (-1)^n h_0(n)$$

$$\Downarrow$$

$$\left\langle h_A^{(J)}(n-2^J l), h_A^{(J)}(n-2^J m) \right\rangle = \delta(l-m)$$

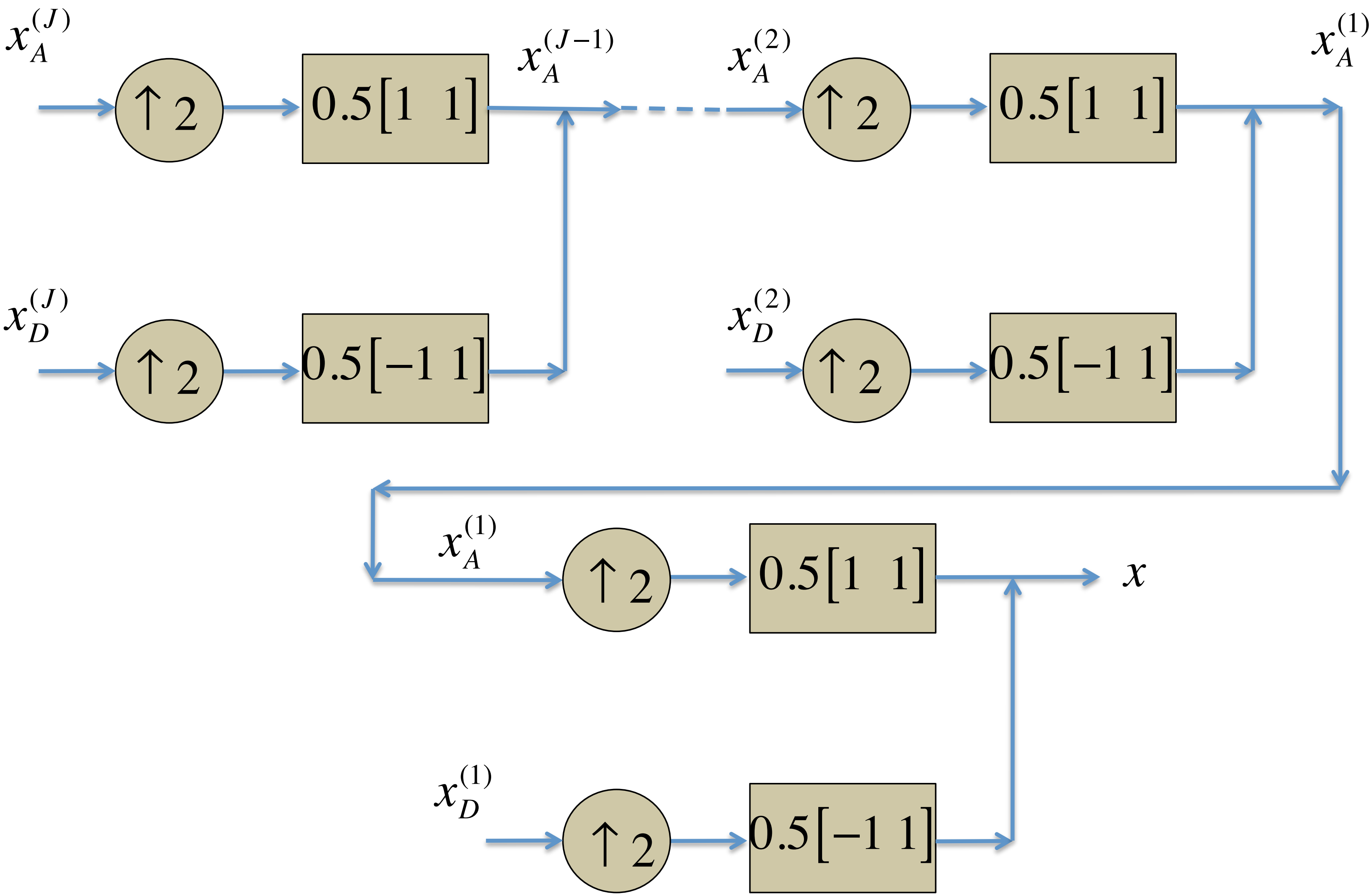
$$\left\langle h_D^{(j)}(n-2^j l), h_D^{(k)}(n-2^k m) \right\rangle = \delta(j-k)\delta(l-m)$$

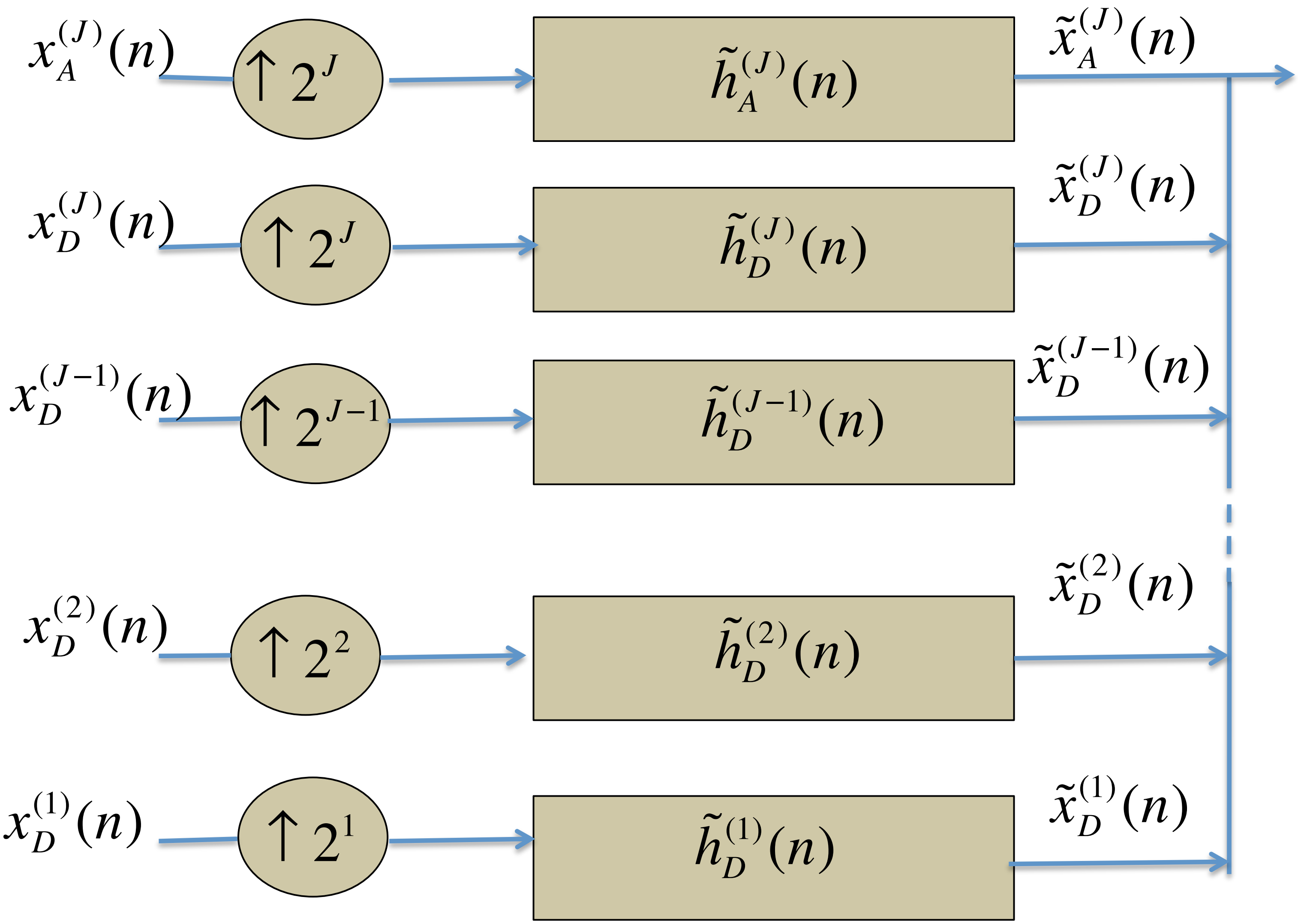
$$\left\langle h_D^{(j)}(n-2^j l), h_A^{(J)}(n-2^J m) \right\rangle = 0$$

$$\Downarrow$$

The matrix defined in the equation $\begin{bmatrix} \mathbf{x}_A^{(J)} \\ \mathbf{x}_{AD}^{(J)} \\ \mathbf{x}_D^{(J-1)} \\ \vdots \\ \mathbf{x}_D^{(2)} \\ \mathbf{x}_D^{(1)} \end{bmatrix} = \mathbf{M}\mathbf{x}$ is orthogonal.

The reconstruction (synthesis)





$$\tilde{x}_A^{(J)}(n) = \sum_m \tilde{h}_A^{(J)}(2^J m - n) x_A^{(J)}(m)$$

$$\tilde{x}_D^{(j)}(n) = \sum_m \tilde{h}_D^{(j)}(2^j m - n) x_D^{(j)}(m)$$

$$j = 1, \ldots, J$$

The discrete synthesis wavelets

$$\tilde{h}_D^{(j)}(n) = h_D^{(j)}(-n), j = 1, \ldots, J$$

$$\tilde{h}_A^{(J)}(z) = h_A^{(J)}(-n)$$