

# Induction

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## 1 Induction

Let  $S \subseteq \mathbb{N}$ . If  $S$  contains 1, and for any element  $n \in S$ ,  $n + 1 \in S$ , then  $S = \mathbb{N}$ .

**Proof by induction:** show the base case ( $n = 0, 1$ ). Then with the inductive hypothesis, the case where it holds for  $n$ , show it holds for  $n + 1$ .

**Example 1.1.** Let  $y_1 = 6$ , and define  $y_{n+1} = (2y_n - 6)/3$ .  
Prove  $y_n > -6, \forall n \in \mathbb{N}$ .

*Proof.* First, we prove the base case, ( $n = 1$ ). In this case, it is given.  $y_1 = 6 > -6$ . Then, with the inductive hypothesis, show it holds for  $n + 1$ . The inductive hypothesis is we claim  $y_n > -6$ , because it holds for  $n$ . Using that, derive  $y_{n+1} > -6$  as well to complete the proof.

With some scratch work:

$$\begin{aligned} y_{n+1} &= (2y_n - 6)/3 > -6 \\ \Rightarrow 2y_n - 6 &> -18 \\ \Rightarrow 2y_n &> -12 \\ \Rightarrow y_n &> -6 \end{aligned}$$

We get  $y_n > -6$  at the end, but we want to use that in the beginning. So our inductive step goes like this:

$$\begin{aligned} y_n &> -6 \\ \Rightarrow 2y_n &> -12 \\ \Rightarrow 2y_n - 6 &> -18 \\ \Rightarrow (2y_n - 6)/3 &> -6 \\ \Rightarrow (2y_n - 6)/3 &= y_{n+1} \\ \Rightarrow y_{n+1} &> -6 \end{aligned}$$

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