Chapter 1, Section 1.3

Joseph Song

1 Subspaces

Definition (Subspace)**.** *A subset W of a vector space V over a field F is a* **subspace** *if W is a vector space over F with the operations defined on V. The subspace $\{0\}$ is called the* **zero subspace** *of V.*

The following is the theorem to prove if a subset is a subspace of V:

Theorem. *Let W be a subset of the vector space V. Then W is a subspace of V if and only if the following hold:*

(1) $0 \in W$

(2) $x + y \in W$, for $x, y \in W$

(3) $cx \in W$, for $c \in \mathbb{F}$ and $x \in W$

Example 1.1 (Polynomials)**.**

For $n \in \mathbb{N}$, let $P_n(F)$ be the set containing all the polynomials in $P(F)$ having degree less than or equal to n.
To show that it is a subspace, we use the theorem and see if the three condition holds. The zero polynomial has degree $-1 \leq n, \forall n \in \mathbb{N}$. The sum of two polynomials with degree less than or equal to n still has the degree less than or equal to n, along with scalar multiplication.

Example 1.2 (Functions)**.**

Let $C(R)$ denote the set of all continuous real-valued functions defined on $\mathbb{R}$. Claim $C(R)$ is a subspace of $F(R, R)$.
The zero function of $F(R, R)$ is just $f(t) = 0, \forall t \in \mathbb{R}$. But since constant functions are continuous, it is also an element of $C(R)$. Also notice that the sum of two continuous functions is continuous, along with the scalar multiplication of a continuous function, meaning they also belong in $C(R)$. So $C(R)$ is closed under addition and scalar multiplication, hence is a subspace of $F(R, R)$.
This is a test please work holy