Least Upper Bound - Supremum

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1 Axiom of Completeness

Axiom of Completeness - Every nonempty set of real numbers that is bounded above has a least upper bound.

Definition (Upper bound)**.** *Set $A \subseteq R$ is bounded above if there exists a real number b such that it is greater than all the values in A. ($\exists\ b \in \mathbb{R} : a \leq b, \forall\ a \in A$)*

Definition (Least Upper Bound - Supremum)**.** *The least upper bound is an element s in A, such that:*

1. *s is an upper bound*
2. *For any other upper bound, b, $s \leq b$*

This means that out of all the bounds, s is the smallest. The supremum is denoted as $s = \sup(A)$.

Example 1.1. *For a set A and a number $c \in \mathbb{R}$, define*

$$c + A = \{c + a : a \in A\}$$

and show $\sup(c + A) = c + \sup A$.

Proof. $\square$