Induction

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1 Induction

Let $S \subseteq \mathbb{N}$. If S contains 1, and for any element $n \in S$, $n + 1 \in S$, then $S = \mathbb{N}$.

Proof by induction: show the base case $(n = 0, 1)$. Then with the inductive hypothesis, the case where it holds for n, show it holds for $n + 1$.

Example 1.1. *Let $y_1 = 6$, and define $y_{n+1} = (2y_n - 6)/3$.*
Prove $y_n > -6, \forall n \in \mathbb{N}$.

Proof. First, we prove the base case, $(n = 1)$. In this case, it is given. $y_1 = 6 > -6$. Then, with the inductive hypothesis, show it holds for $n + 1$. The inductive hypothesis is we claim $y_n > -6$, because it holds for n. Using that, derive $y_{n+1} > -6$ as well to complete the proof.

With some scratch work:

$$
\begin{aligned}
y_{n+1} = (2y_n - 6)/3 &> -6 \\
\Rightarrow \quad 2y_n - 6 &> -18 \\
\Rightarrow \quad 2y_n &> -12 \\
\Rightarrow \quad y_n &> -6
\end{aligned}
$$

We get $y_n > -6$ at the end, but we want to use that in the beginning. So our inductive step goes like this:

$$
\begin{aligned}
y_n &> -6 \\
\Rightarrow \quad 2y_n &> -12 \\
\Rightarrow \quad 2y_n - 6 &> -18 \\
\Rightarrow \quad (2y_n - 6)/3 &> -6 \\
\Rightarrow \quad (2y_n - 6)/3 &= y_{n+1} \\
\Rightarrow \quad y_{n+1} &> -6
\end{aligned}
$$

$\square$