Wheeled Vehicle Mobility Assignment

16-665: Robot Mobility on Air, Land, and Sea

Assignment Parameters

This is an individual assignment. Conceptual collaboration is encouraged but students may not exchange work or code nor copy information from another source. Cite all references. This assignment is worth 28% of the total grade for this class. Please type or neatly write your solutions. Submission will be in .pdf form. Points will be given for correct work and correct solutions.

Learning Objectives

1. Determine mobility parameters using force and moment equilibrium.
2. Use analytic techniques to make informed parametric comparisons.
3. Exercise vehicle control strategies in an example maneuver.

Submission Instructions

- Create a single .pdf file with your solutions for all the sections named hw1.pdf.
- Ensure that your .pdf contains all work, explanations, images, and codes for the 5 questions.
- Submit to Canvas using the proper assignment link by the date specified in the course schedule.

Contents

1 Rigid Ground Physics

The mobile robot below has four rigid, powered wheels. It is decelerating and going down a hill. Assume that the deceleration is slow enough that you can treat the problem as constant at this point in time. Assume that the vehicle does not slip and the ground is perfectly rigid. The vehicle's mass is m.

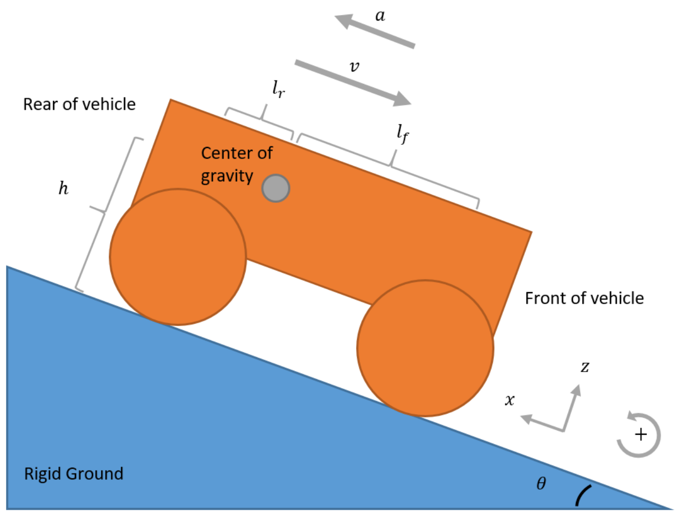

Figure 1: Vehicle with dimensions and no forces on relatively low slope.

1.1 Free Body Diagram [1 point]

Complete a full body diagram of the vehicle at the moment when the rear (uphill) wheel leaves the ground and the vehicle is tipping over (the slope will be higher than what is pictured). You may draw this by hand or use a computer program. Scale does not have to be perfect but do your best to make your work clear and accurate. Assume there are no other external forces like air resistance or internal losses like geartrain friction. Don't forget about the force due to deceleration.

1.2 Equation [1 point]

Using the free body diagram you just made, write an inequality for the maximum deceleration that this vehicle can travel without tipping over. Ignore slip limitations for this question. Use the coordinate system and moment convention depicted in the picture.

1.3 Parametric Comparison [2 points]

Now that you have an equation for maximum deceleration before tip over, we will use it to make observations about vehicle design. Create a graph showing deceleration versus center of mass position. To do this, create a graph by hand or using a computer program where deceleration is on the y-axis. Use l_f as the x-axis. Create a series of lines, one for each h. This way we can compare various combinations of center of gravity position forward-aft and vertically.

The vehicle's mass (m) is 100kg and the slope (θ) is 30°. Let the overall wheelbase (distance between contact point of front wheel and rear wheel) remain at a constant 2 meters. Vary l_f between 0.25 and 1.75 meters. Vary h between 0.5 and 2 meters. You may plot this data discretely (points) or continuously (lines), whichever you feel is more clear. If you plot this information discretely, make sure to have a discretization of no more than 0.25 m.

1.4 Discussion [2 points]

Answer each of the following in one sentence.

A. Which combination of the specified l_f and h give the greatest deceleration?

B. Is this combination what you expected? Why or why not?

C. If you were designing a vehicle, would you choose those as your l_f and h? Why or why not?

INCLUDE ANY CODE YOU WROTE AT THE END OF YOUR ASSIGNMENT

2 Vehicle Performance on Soil

Consider that you are evaluating the merits of a 4-wheeled vehicle against a 6-wheeled vehicle that is expected to operate in flat soft terrain whose cohesion is c and angle of internal friction is ϕ. For the given soil, the pressure-sinkage parameters are k_c and k_ϕ, and the exponent of sinkage is n. There is only one wheel type available; a rigid tire of diameter d and width b. Assume that gross vehicle weight is W in both cases and that the weight is evenly distributed among the wheels.

2.1 Compaction [2 points]

What is the ratio of the **total** compaction resistance of the 4-wheeled concept over the 6-wheeled concept? Given this, which concept is better to minimize compaction resistance; the one with 4 or the one with 6 wheels? Justify your answer through your work. Let n=0.5.

2.2 Thrust [2 points]

Assume that the contact area of each tire and the soil for the 4-wheeled vehicle is 20% larger than the 6-wheeled vehicle. What is the ratio of **total** thrust of the 4-wheeled concept over the 6-wheeled concept? Given this ratio, which concept is better to maximize soil thrust assuming no slip? Justify your answer through your work.

2.3 Drawbar Pull [1 point]

Derive an analytical equation for the drawbar pull of that can be generated by a single wheel of the 6-wheeled vehicle. Use the variables above. Assume that the only significant form of motion resistance is compaction.

2.4 Soil Parameters [1 point]

How would the maximum thrust change if the vehicle is operating in packed clay versus loose sand (not ideal case)? Ensure that your answer is complete in describing the change as the vehicle weight and area change.

2.5 Other Forms of Resistance [1 points]

Name three other sources of resistance that were assumed to be 0 in this problem.

2.6 MMP [2 points]

The Mean Maximum Pressure (MMP) is a semi-empirical metric of wheeled mobility. Dr. Jody Priddy's equation for MMP of a wheeled vehicle is computed given the following equation [1]:

$$MMP_{\text{wheels, P}} = \frac{W_{Gross}}{nmb^{0.8}d^{0.8}(\delta)^{0.4}} \tag{1}$$

What do do W_{Gross}, n, m, b, d, and δ represent? If you were to decide between two vehicle designs one of which had an MMP=40 and the other an MMP=100, which one would you choose?

3 Vehicle Characteristics Selection

3.1 Illustrations [1 point]

For the list of steering configurations below, draw an example vehicle, using arrows to show what portions of the vehicle (if any) can move.

A. Skid (also called Differential)

B. Ackerman

C. Articulated Body

D. Explicit (Independent) Wheel Articulation

3.2 Discussion [2 points]

For the same list of four configurations, list one distinct positive attribute and one negative attribute of that steering system.

3.3 Suspension Discussion [1 points]

Discuss 3 reasons why you would want to utilize a semi-active suspension over a passive suspension on a high-speed, all-terrain wheeled vehicle

3.4 Motor Selection [2 points]

A conservative method of sizing a motor/gearbox combination for a skid steer field robot is to select one that would enable the robot to pull itself vertically by the rim of a single wheel. This ensures that the vehicle has enough torque for most reasonable turns and slopes, even if some of the wheels are immobilized. For a robot that you want to move with a speed of at least 2 ft/s, select a real motor and gearbox from the internet that fits this criteria for a 100 lb robot with 20 in diameter wheels.

4 Kinematic Bicycle Model

In class, we derived the full kinematic bicycle model (KBM) based on Rajamani, Chapter 2. The resultant model and accompanying diagram are given below.

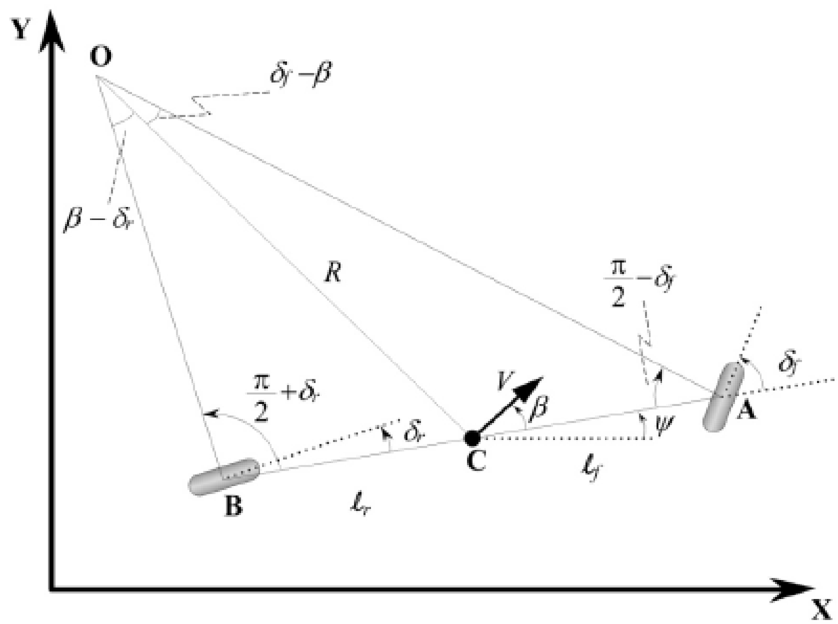

Figure 2: Kinematic Bicycle Model.

$$\dot{X} = V cos(\psi + \beta) \tag{2}$$

$$\dot{Y} = V sin(\psi + \beta) \tag{3}$$

$$\dot{\psi} = \frac{V cos\beta}{l_f + l_r}(tan\delta_f - tan\delta_r) \tag{4}$$

The KBM is usually given in simplified form in research papers. Two examples are given below from Pepy [2] and Kong [3]. The Pepy model measures the position of the vehicle at the rear wheels (Point B in the above diagram), whereas the Kong model uses the center of gravity (Point C in the above diagram). In both cases, a vehicle without steered rear wheels is considered. Derive these models, either by simplifying the Rajamani model, or from first principles. State all assumptions and show all steps.

4.1 Pepy Model [1 point]

The Pepy model:

$$\dot{X} = V cos\psi \tag{5}$$

$$\dot{Y} = V sin\psi \tag{6}$$

$$\dot{\psi} = \frac{V}{l_f + l_r} tan\delta_f \tag{7}$$

4.2 Kong Model [1 point]

The Kong Model:

$$\dot{X} = V cos(\psi + \beta) \tag{8}$$

$$\dot{Y} = V sin(\psi + \beta) \tag{9}$$

$$\dot{\psi} = \frac{V}{l_r} sin\beta \tag{10}$$

4.3 Discussion [2 points]

A. The vehicle slip angle at the center of mass β no longer appears in the Pepy model. Does this mean that the vehicle slip angle is zero? If not, write an equation for it in terms of known/measurable vehicle parameters/variables.

B. What is the difference between vehicle slip and tire slip? Are they related to one another? Can you have one without the other?

4.4 Implementation [3 points]

Implement the Pepy model in Matlab or your favorite software platform with velocity V equal to a constant and zero initial conditions for X, Y, and ψ. Let $l_r = l_f = 1.5$ meters. Use the three different steering inputs δ listed below. Provide one plot and one or two sentences to answer the questions for each part.

A δ = a non-zero constant. Try various constants. Write a mathematical expression for the relationship between the steering angle and the path radius or curvature.

B δ = a sinusoid. Before doing this, try to predict what the trajectory will look like. Did it come out as you expected? Why or why not? Try different velocities V and different amplitudes for the steering sinusoid and comment on the effects.

C δ = a square wave. Why is this unrealistic? How would you handle this lack of realism in the model?

INCLUDE YOUR CODE AT THE END OF YOUR ASSIGNMENT

5 Dynamic Bicycle Model

In class we derived the Dynamic Bicycle Model lateral dynamics based on lateral position error e_1 and yaw angle error e_2:

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2C_{\alpha f}+2C_{\alpha r}}{mV_x} & \frac{2C_{\alpha f}+2C_{\alpha r}}{m} & \frac{-2C_{\alpha f}l_f+2C_{\alpha r}l_r}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2C_{\alpha f}l_f-2C_{\alpha r}l_r}{I_zV_x} & \frac{2C_{\alpha f}l_f-2C_{\alpha r}l_r}{I_z} & -\frac{2C_{\alpha f}l_f^2+2C_{\alpha r}l_r^2}{I_zV_x} \end{bmatrix} \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2C_{\alpha f}}{m} \\ 0 \\ \frac{2C_{\alpha f}l_f}{I_z} \end{bmatrix}\delta + \begin{bmatrix} 0 \\ -\frac{2C_{\alpha f}l_f-2C_{\alpha r}l_r}{mV_x} - V_x \\ 0 \\ -\frac{2C_{\alpha f}l_f^2+2C_{\alpha r}l_r^2}{I_zV_x} \end{bmatrix}\dot{\psi}_{des} \quad (11)$$

5.1 Model Set Up [0 points]

Implement this model in Matlab or your favorite software platform. Use the following typical vehicle parameters: $V_x = 30\ m/sec$, $m = 1573\ kg$, $I_z = 2873\ kg - m^2$, $l_f = 1.1\ m$, $l_r = 1.58\ m$, $C_{\alpha f} = C_{\alpha r} = 80,000\ N/rad$.

The open-loop system, which can be written as $\dot{x} = Ax + B_1\delta + B_2\dot{\psi}_{des}$, has two eigenvalues at the origin and is therefore unstable. You can use state feedback to form the steering input $\delta(t) = -Kx$, leading to $\dot{x} = (A - B_1K)x + B_2\dot{\psi}_{des}$ for the closed-loop system.

5.2 Lane Change [2 points]

For this task, you are asked to switch from following a straight line below the global X-axis to following a straight line along the X-axis.

The vehicle's initial conditions are (X,Y) = (0,-5m) with ψ=0. Vx=30 m/sec as above and remains constant. (In other words, 5m below the origin pointing and traveling in the X-direction.) The vehicle's final conditions are (X,Y) = (100m,0m) with ψ=0. Use a diagonal line to connect the desired Y coordinates over 90 meters in the x axis. The vehicle must stay at Y = -5m until it reaches the diagonal portion of the path at X = 5m. Make sure to add a trajectory along the x-axis at the end so you can examine the maximum error and settling time.

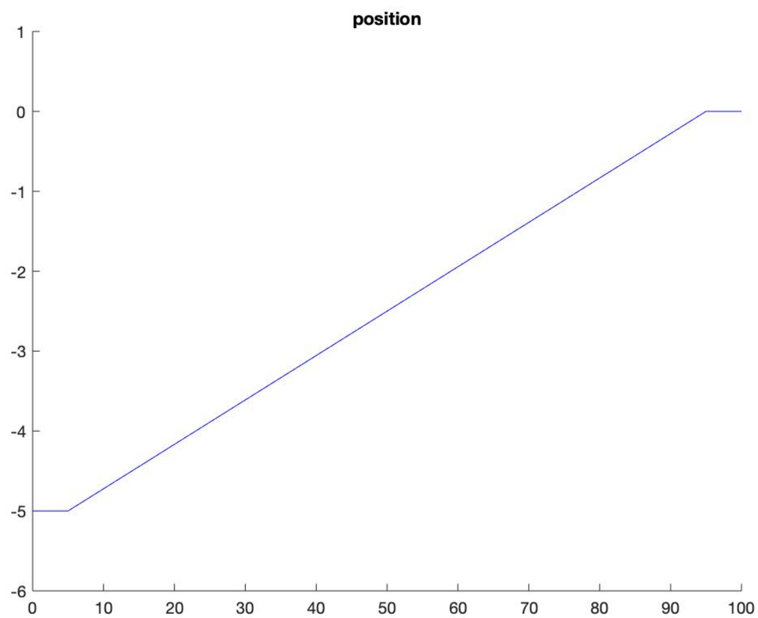

Figure 3: Desired vehicle trajectory

Create a series of inputs $\dot{\psi}_{des}$ that follows this path and use pole placement or LQR to achieve a maximum error abs(e2)$\leq$0.01 rad and reach within abs(e1)$\leq$0.002 m and abs(e2)$\leq$0.0007 rad within 1 second at the transition points (i.e., leaving the initial lane and entering the new lane). State whether you used pole placement or LQR and list the resultant poles. The time step of your simulation should be 0.01 seconds or larger, and the derivative of the steering input $\frac{\delta(t)}{dt}$ should not exceed 25 $\frac{rad}{sec}$. Please include a plot of $\frac{\delta(t)}{dt}$ over the course of the lane change maneuver.

Plot the resultant lateral position error e_1 and yaw angle error e_2 versus time, and make it clear that your simulation meets all of the specifications.

5.3 Desired and Actual Plot [1 point]

Plot the true global vehicle path in the XY plane, along with the desired vehicle path. Show separate plots zooming in on the transition points (i.e., leaving the initial lane and entering the new lane).

5.4 Curves [2 points]

Create an input $\dot{\psi}_{des}$ corresponding to following a straight path for 1 second, then a positive-curvature circular arc with radius 1000 m for 5 seconds, then a straight path for 1 second, and finally a negative-curvature circular arc with radius 500 for 5 seconds.

Use pole placement or LQR to achieve a maximum error abs(e1)$\leq$0.01 m and abs(e2)$\leq$0.01 rad. State whether you used pole placement or LQR and list the resultant poles.

Plot the resultant lateral position error e_1 and yaw angle error e_2 (these should both converge near 0 when the $\dot{\psi}_{des}$ is 0) versus time.

5.5 Desired and Actual Plot [1 points]

Plot the true global vehicle path in the XY plane, along with the desired vehicle path.

5.6 Discussion [1 point]

Create error plots as in Subsection 5.4 with two different values for V_x. Describe the effect on e_1 and e_2 in one to two sentences.

INCLUDE YOUR CODE AT THE END OF YOUR ASSIGNMENT

Bibliography

[1] Jody D. Priddy and William E. Willoughby. "Clarification of vehicle cone index with reference to mean maximum pressure". In: *Journal of Terramechanics 43.2* (Apr. 2006), pp. 85–96. URL: http://www.sciencedirect.com/science/article/pii/S0022489804001120 (visited on 06/06/2016).

[2] *Path Planning Using a Dynamic Vehicle Model*. Proceedings of the 2nd International Conference on Information & Communication Technologies. 2006. URL: https://ieeexplore.ieee.org/abstract/document/1684472/.

[3] *Kinematic and Dynamic Vehicle Models for Autonomous Driving Control Design*. Proceedings of the IEEE Intelligent Vehicles Symposium. 2015. URL: https://ieeexplore.ieee.org/document/7225830/.