Range Optimal Trajectories Using GPOPS-II

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09/09/19

***Abstract*— GPOPS-II optimal control software was used To reproduce the results of a previous minimum time descent. Using the GPOPS-II software a longer duration of flight was capable of being optimized before solution divergence. This paper gives a brief overview of results obtained but is expected to be expanded upon in the future.**

I. Introduction

The purpose of this research is to optimize the maximum-range trajectories for a fixed flight time using the GPOPS-II program. The aircraft model, initial and final conditions, along with constraints will be consistent with that used in the range optimal trajectory research by Hans Seywald [1]. In his paper the approach taken used the following equations of motion:

$$\dot{E} = (\eta T - D)\frac{v}{W} \tag{1}$$

$$\dot{h} = v \sin\gamma \tag{2}$$

$$\dot{\gamma} = \frac{g}{v}\left(\frac{L}{W} - \cos\gamma\right) \tag{3}$$

$$\dot{x} = v \cos\gamma \tag{4}$$

With State Variables:

Specific Energy	E
Altitude	h
Flight Path Angle	γ
Velocity	v
Range	x

With Control Variables:

Load Factor	n
Power Setting	η

The value of initial and final velocity was added by plugging in energy and altitude into (5).

$$v = \sqrt{2g(E-h)} \tag{5}$$

All boundary conditions and constants can be found in Table I.

TABLE I
PROBLEM CONSTRAINTS AND CONSTANTS

Description	Value	Units
Mass (m)	16818	Kg
Gravity (g)	9.80665	m/s^2
Initial Specific Energy (E_0)	38029.207	m
Initial Velocity (v_0)	712.866	m/s
Initial Altitude (h_0)	12119.324	m
Initial Flight Path Angle (γ_0)	0	rad
Initial Range (x_0)	0	m
Final Specific Energy (E_f)	9000	m
Final Velocity (v_f)	397.54	m/s
Final Altitude (h_f)	942.292	m
Final Flight Path Angle (γ_f)	-0.2	Rad
Final Range (x_f)	To be optimized	m
Final Time (t_f)	60	S

The other constraint imposed is a dynamic pressure limit. The only data available for this is a graphic shown below. The q-limit can be extracted roughly from this graph.

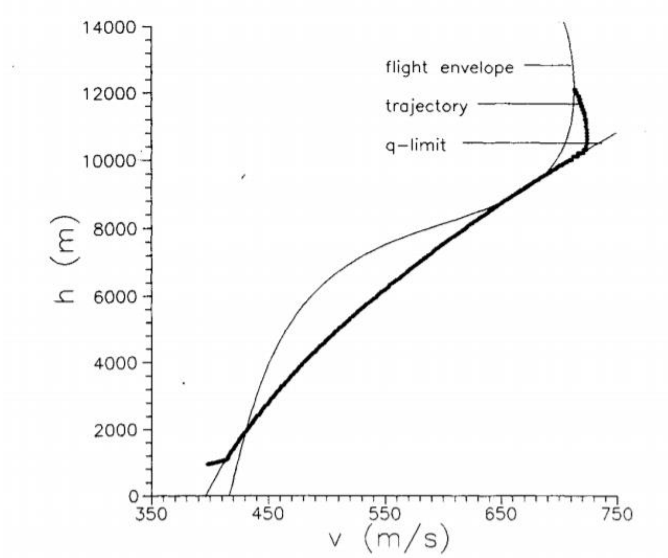

Fig. 9 Altitude-velocity chart for a typical extremal.

Extrapolating from the graph it's seen that there is a point on the q-limit line where velocity is equal to 741 m/s and altitude is equal to 12559 m. Using (6) and (7-9) a dynamic pressure limit of **74.95072 kPa** was found.

$$q = \frac{1}{2}\rho(h)v^2 \tag{6}$$

$$\rho(h) = 1.225\, e^{y}$$
$$y = -0.12122693\, 10^{-3}\, h + r - 1.0228055 \tag{7}$$

$$r = 1.0228055\, e^{-z} \tag{8}$$

$$
\begin{aligned}
z = & -3.48643241\ 10^{-5}\ h \\
& + 3.50991865\ 10^{-9}\ h^2 \\
& + -8.33000535\ 10^{-14}\ h^3 \\
& + 1.15219733\ 10^{-18}\ h^4
\end{aligned}
\tag{9}
$$