# ANALYSIS OF A SIMPLY SUPPORTED BEAM

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## ABSTRACT

The studying of the mechanics of various materials leads to the ability to theoretically find the maximum allowable load materials may withstand. In this computational study, a concrete beam of length $L = 30m$ and height $h = 6m$ is subject to an evenly distributed load $q$ which causes it to bend under the load. An increasing load creates a larger stress throughout the entire beam, until it fractures due to a small crack at $x = \frac{L}{2}$. This fracture occurs when the load distribution exceeds $F = 63833\frac{N}{m}$, causing the concrete to fail in tension.


## INTRODUCTION

The study of the *Mechanics of Materials* allows for various properties of structural components, such as beams, to be analyzed. The goal of *Mechanics of Materials* is to find mathematical relationships to describe the behavior of the structural components. In this computational experiment, the following properties are found:

1. Stress associated with the beams geometry
2. The deflection curve of the beam
3. The associated Shear / Moment diagram
4. Maximum allowable Stress
5. Load at which fracture will occur

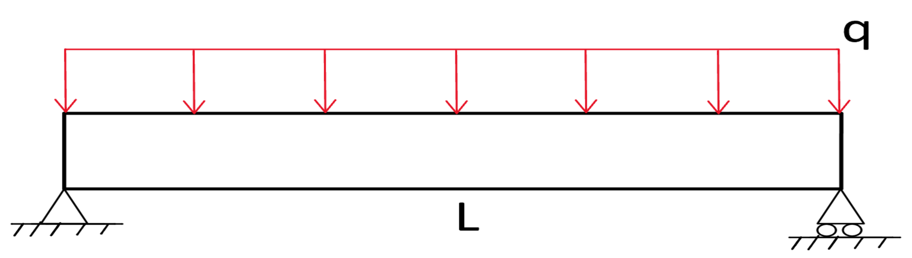


Figure 1: Simply supported beam with even load distribution.
Image created by Joseph Spear on December 10, 2019

## METHODOLOGY

In order to visualize the stresses acting on a beam under this type of load, the principles of static structures must be used by observing the beams free body diagram, and applying Newton's Second Law in Static Equilibrium:

$$\Sigma F = 0 \tag{1}$$

Once this is completed, the beam may be cut into a "section" and the internal shear force and internal moment may be found using the Left Hand - Free Body Diagram. Equations (2) and (3) are found in this manner, and Equation (4) is the relationship between the deflection ($\nu$) and the internal Moment (Goodno, 449 & 1107).

$$V(x) = \frac{qL}{2} - qx \tag{2}$$

$$M(x) = -\frac{q}{2}x^2 + \frac{qL}{2}x \tag{3}$$

$$\frac{d^2\nu}{dx^2} = \frac{M(x)}{EI} \tag{4}$$

For analysis of beam failure, a few assumptions must be made. The first is that a concrete mix with a fracture toughness of $K_{Ic} = 1.0MPa\sqrt{m}$ is used. It is also assumed that the associated geometric parameter is assumed to be $Y = 1$. Lastly, the assumed initial crack radius is $a = 0.5$. With these parameters in place, the following relation may be used to calculate the critical fracture stress (Callister,334):

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}} \tag{5}$$

## RESULTS & CONCLUSION

- Figure 2 represents the side-view of the beam, where the distributed load (q) is applied to the top face of the beam. In this situation, the symmetric load causes the concrete to bend down towards the middle which in turn causes a stress in compression on the top face (indicated with dark blue), and a stress in tension on the bottom face (yellow). The center of the beam remains a constant color, indicating what is known as the neutral axis, where the stress is zero.

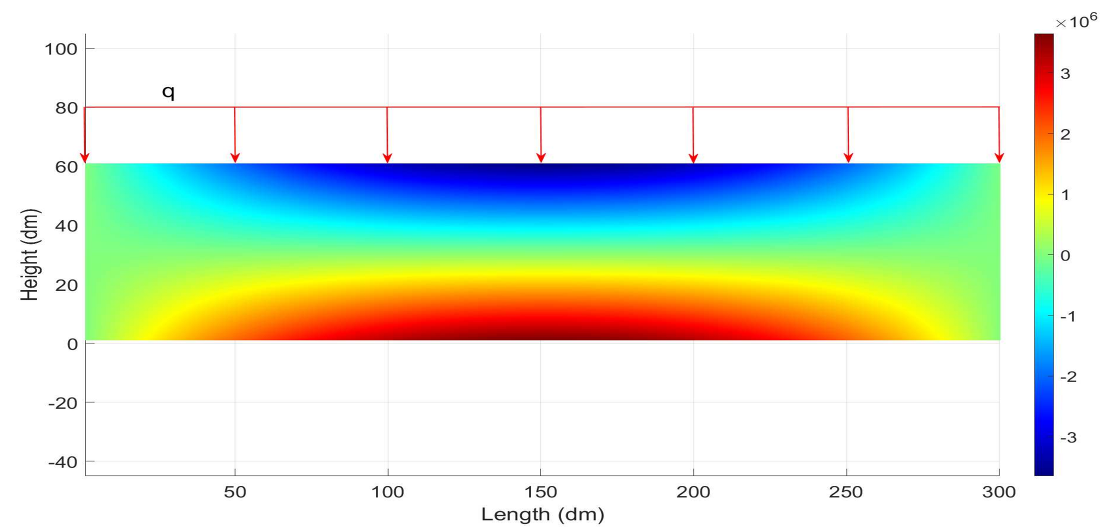


Figure 2: Stress Distribution throughout the Beam

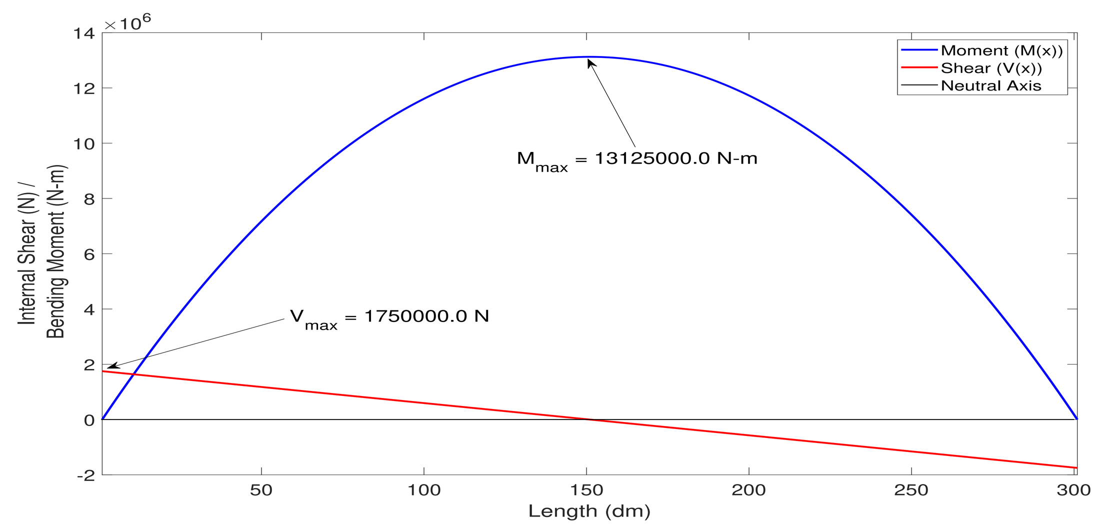


Figure 3: Shear-Moment Diagram of the Beam

- The internal Shear and bending Moment of a material is extremely important in design and in determining other properties. Equation (4) wouldn't be solvable without Equation (3)! Therefore, a useful tool to help with the analysis of materials is the *Shear-Moment Diagram*. This diagram models the internal shear and bending moments felt throughout the beam, and helps determine maximum and minimum points of stress.

- Due to the solution of Equation (4), the deflection curves for various loads may be modeled. These deflections show the neutral axis after the load is applied, and is contrasted by the load-free neutral axis in black. The deflection curve is an important property which can be used for various applications, such as understanding the ductility of a beam or designing a beam which will not fracture. Deflection is also an observable quantity making it useful in an applied setting.

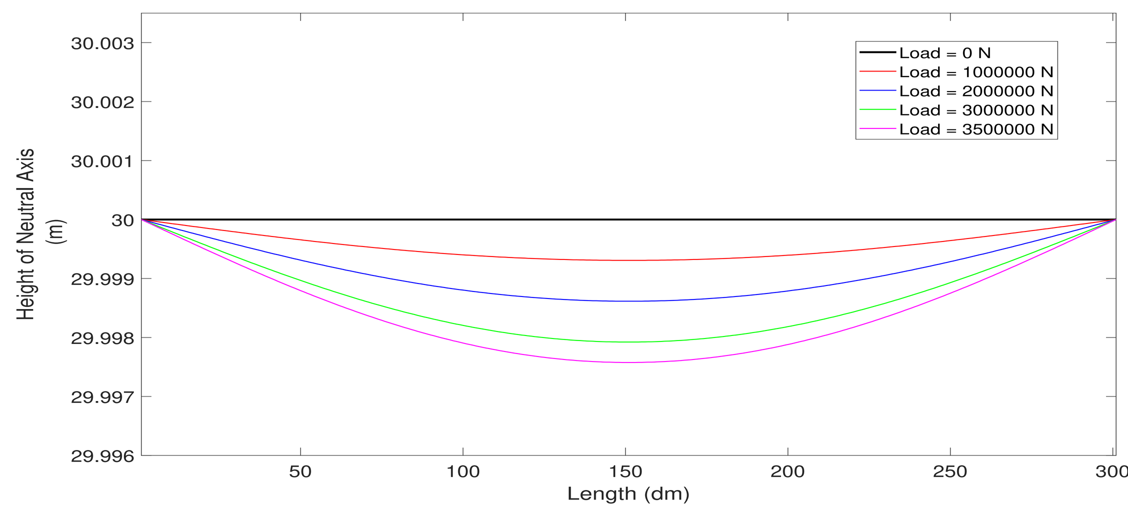


Figure 4: Deflection Relative to the Neutral Axis

Based on the previous information, specifically Equation (5), it is determined that the theoretical concrete mix would fracture in tension on the bottom face given the following conditions:

$$\sigma_{allowable} = 1784124.0913288831\ Pa\,; \quad q_{max} = 63833\ \frac{N}{m}\,; \quad \delta = 1.326259670491936\ mm; \quad a_o = 0.5\ m$$

These quantities would allow engineers to design safe structures using cheap computations instead of costly experiments. This process may also be performed with any type of load or material desired, as long as the boundary conditions are known.


## ACKNOWLEDGMENTS & FURTHER WORKS

Special thanks to Dr. Oelgoetz and Dr. Lonnghurst for their guidance in these topics!


In further pursuit of this topic, an in depth study of crack propagation over time due to fatigue cycles would be desired. This may be accomplished using a time-step approximation technique such as Euler's Method and would yield interesting and useful results. The same process used in this study may also be performed for a wide range of materials and loads to produce more intricate and realistic cases.

## REFERENCES


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