Engineering Design Positive Displacement Pump - Analysis - Joseph Spear

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Design Requirements

Our task is to design a pump which can produce enough pressure to transfer 2 gallons of water per minute with a pressure head of 1200ft. We are also specified to use inlet/outlet ports of the 3/4" NPT variety. We are instructed to use a reciprocating positive displacement pump as the pump design. For simplicity in calculations, units are converted to Metric.

$$\dot{V} = 2\,GPM = 1.2618 \times 10^{-4}\ \frac{m^3}{s}$$

$$H = 1200ft = 365.76m$$

$$3/4"\ NPT \rightarrow \begin{array}{l} d_i = 0.914" = 0.0232156m \\ d_o = 1.043in = 0.0264922m \end{array}$$

*Note that the diameters are for the inlet/outlet ports.

Fluid Mechanics Analysis

We must use the principles of fluid mechanics to determine the different parameters needed in the construction of the pump. The primary equation that will be used is the Bernoulli Equation:

$$P_1 + \frac{1}{2} \cdot \rho \cdot v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \cdot \rho \cdot v_2^2 + \rho g h_2 \tag{1}$$

One key take away from the Bernoulli Equation is the Equation of continuity:

$$\frac{dV_1}{dt} = \frac{dV_2}{dt} \Longrightarrow A_1 v_1 = A_2 v_2 \tag{2}$$

Eq. (2) can be used to find the v_{piston} as follows:

$$V_{outlet} = \frac{\dot{V}_{outlet}}{A_{outlet}} = \frac{1.2618 \times 10^{-4}\ \frac{m^3}{s}}{\left(\frac{\pi}{4}(0.0232156m)^2\right)}$$

$$V_{outlet} = 0.2981\ \frac{m}{s}$$

$$A_1 V_1 = A_2 V_2$$

$$A_{piston} V_{piston} = A_{outlet}\left(\overbrace{V_{outlet}}\right)$$

$$V_{piston} = \frac{A_{outlet}}{A_{piston}} \cdot V_{outlet} \quad (3)$$

This tells us how fast the piston head should be moving in order to displace the liquid at the desired v_{outlet} rate. Note that the area of the piston head (a function of the diameter) is not yet known and will be found in the Dynamics analysis.

We can divide Eq. (1) by ρg to get what is known as the *head form* of the Bernoulli Equation. When doing this at the outlet port, we get:

$$P_{outlet} = \rho g H \Rightarrow P_{outlet} = 3.588 \times 10^6\ Pa$$

$$P_{piston} + \frac{1}{2}\rho V_{piston}^2 + \cancel{\rho g h_{piston}}^{h \approx 0} = P_{outlet} + \frac{1}{2}\rho V_{outlet}^2 + \cancel{\rho g h_{outlet}}^{h=0}$$

$$P_{piston} = P_{outlet} + \frac{1}{2}\rho V_{outlet}^2 - \frac{1}{2}\rho V_{piston}^2$$

Substituting Eq. (3) into the above equation:

$$P_{piston} = P_{outlet} + \frac{1}{2}\rho V_{outlet}^2 - \frac{1}{2}\rho\left(\frac{A_{outlet}}{A_{piston}} \cdot V_{outlet}\right)^2$$

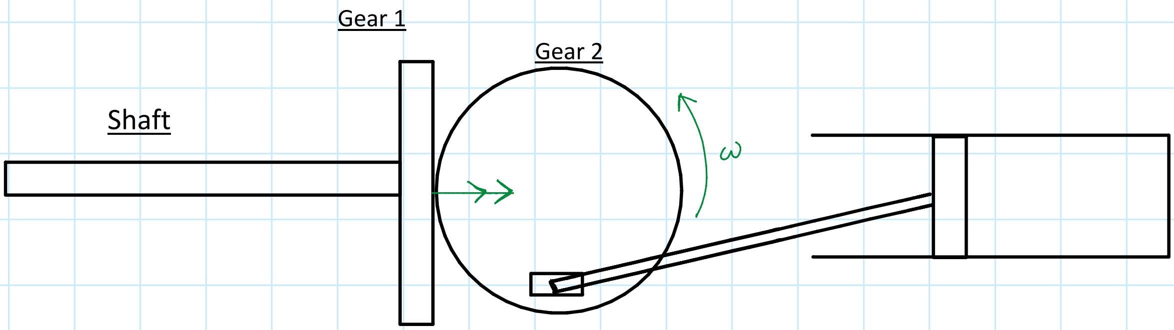
$$P_{piston} = P_{outlet} + \frac{1}{2}\rho V_{outlet}^2 - \frac{1}{2}\rho \frac{A_{outlet}^2}{A_{piston}^2} \cdot V_{outlet}^2 \qquad (4)$$

From the fluid analysis, we have obtained the velocity of the piston, and the pressure found at the piston. This information will be useful in the analysis of the dynamic processes of the mechanical system.

Dynamics Analysis

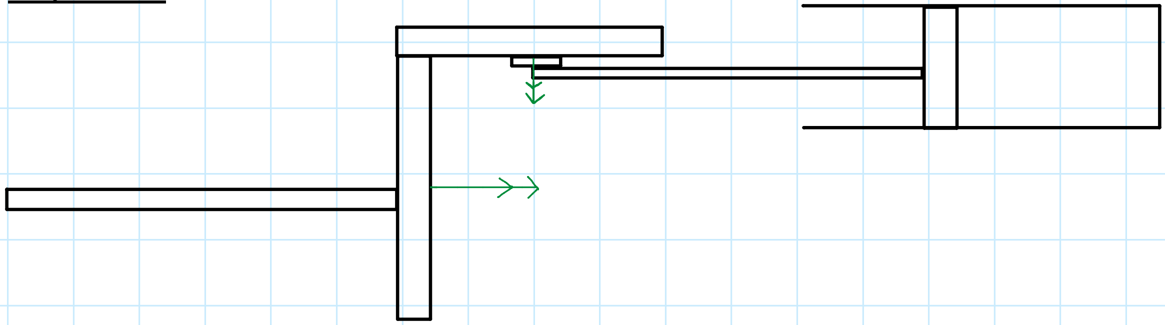
In order to make the piston actually reciprocate, we need to attach the piston to a mechanical device which will produce the back-and-forth motion. In my synthesis, I decided to attach a linkage to a gear, which is then latched into another gear at a 90° angle. This allows one gear to be attached to a crankshaft and rotate the second gear attached to the linkage without getting in the way.

Side View



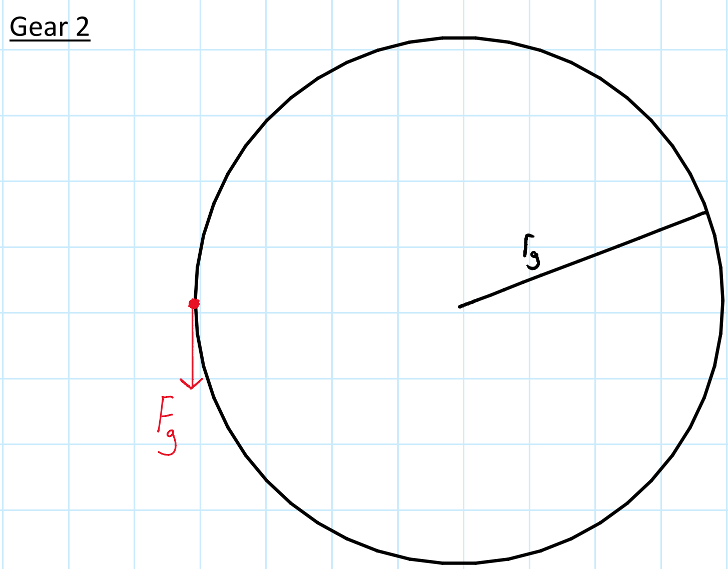
(Figure 1)

Top View



(Figure 2)

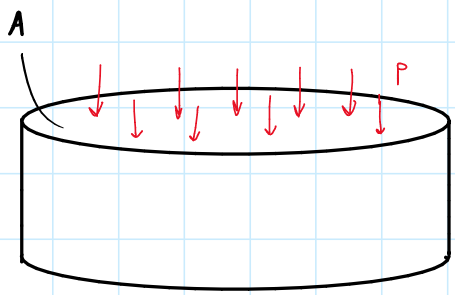
The fluids analysis of the system leads us to the understanding that we need to determine the torque of the shaft which will produce enough force to meet the design parameters. We can find this by looking at a free-body-diagram of the gear attached to the linkage and applying Newton's Second Law:



$$T = F_g \cdot r_g \qquad (5)$$

(Figure 3)

In order to move the piston in the chamber, the pressure must be counter-acted by a force. That force is the force generated from the shaft's torque, and it will equal the pressure on the piston from water multiplied by the area of the piston.



$$P = \frac{F}{A}$$

$$F_{piston} = P_{piston} \cdot A_{piston} \qquad (6)$$

(Figure 4)

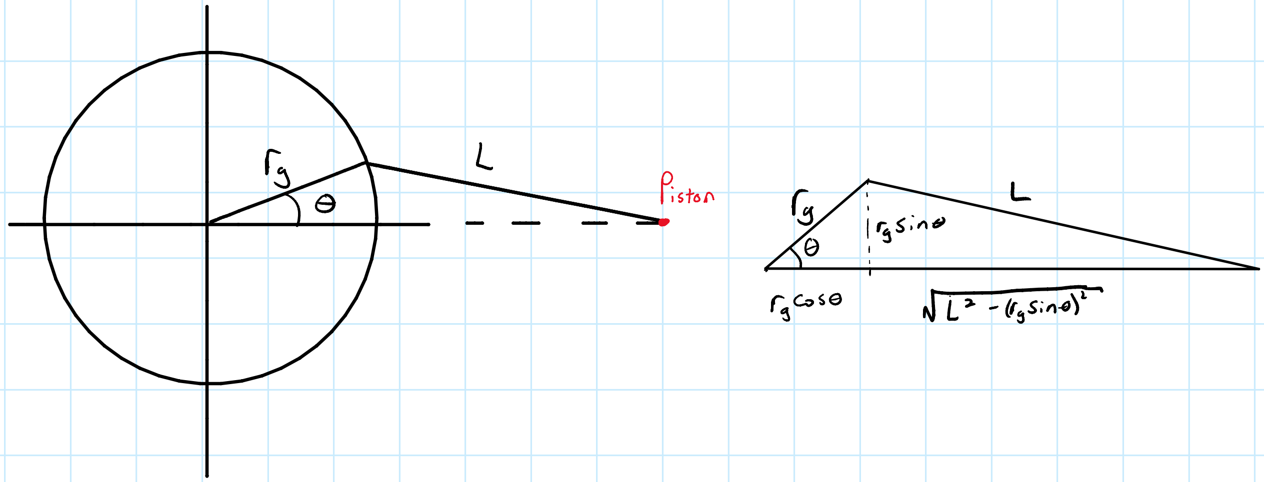
Substituting Eq. (6) into Eq. (5), recognizing that $F_g = F_{piston}$.

$$T = (P_{piston} \cdot A_{piston}) \cdot r_g \qquad (7)$$

We are also going to need to find the power needed to rotate the shaft. This can be calculated using the equation:

$$\frac{dW}{dt} = \omega T = \frac{2\pi NT}{60} \quad ; \quad [N = rpm] \qquad (8)$$

In order to find ω, we need to derive the equation from angular kinematics. This begins with the free-body diagram of the gear.



(Figure 5)

$$X(\theta) = r_g \cos\theta + \sqrt{L^2 - (r_g \sin\theta)^2}$$

$$V_x = \dot{X}(\theta) = r_g(-\sin\theta\,\dot{\theta}) + \frac{1}{\cancel{2}}\left[L^2 - (r_g \sin\theta)^2\right]^{-\frac{1}{2}}\left(-\cancel{2}(r_g \sin\theta)(r_g \cos\theta\,\dot{\theta})\right)$$

$$V_x = -r_g\,\omega \sin\theta - \frac{r_g^2\,\omega \sin\theta \cos\theta}{\sqrt{L^2 - (r_g \sin\theta)^2}} \qquad (9)$$

We also know that the volumetric displacement of a pump is related to the oscillation of the piston. From here, we can find the angular velocity in rpms.

$$\dot{V} = \frac{N \cdot A \cdot 2r_g}{60}$$

Converts from rpm to $\frac{rad}{s}$

$$\omega = \frac{2\pi\,\dot{V}}{A \cdot 2r_g} \qquad (10)$$

Substituting Eq. (7) and Eq. (10) into Eq. (8):

$$\frac{dW}{dt} = \left(\frac{\cancel{2}\pi\,\dot{V}}{\cancel{A} \cdot \cancel{2} \cdot \cancel{r_g}}\right)\left(P_{piston} \cdot \cancel{A_{piston}} \cdot \cancel{r_g}\right)$$

$$\frac{dW}{dt} = \Pi \cdot \dot{V} \cdot P_{piston}$$

This can be further simplified by substituting Eq. (4) into the above formula:

$$\frac{dW}{dt} = \Pi \cdot \dot{V} \cdot \left(P_{outlet} + \frac{1}{2}\rho V_{outlet}^{2} - \frac{1}{2}\rho \frac{A_{outlet}^{2}}{A_{piston}^{2}} \cdot V_{outlet}^{2} \right) \quad (11)$$

At this point, we see that the Power required to operate the pump (assuming ideal conditions), is dependent only on the diameter of the piston. This can be chosen based on maximum allowable stress found in the materials analysis section.

Materials Analysis

When designing what material and geometric dimensions to make the chamber out of, we must find the stress that the material will experience during this process. When analyzing the process, we can use the pressure in the chamber to find the stress that the chamber walls experience. This is done using the following equation:

$$\sigma_{normal} = \frac{Pr}{t} \Longrightarrow t = \frac{Pr}{\sigma_{normal}} \quad (12)$$

Where P is the internal pressure (from Eq. (4)), r is the radius of the cylindrical chamber, and t is the thickness of the walls.

In order to find the necessary thickness, we can assume a stress based on the typical yield strength for our selected material, Multipurpose 304 Stainless Steel, which is roughly $\sigma_{normal} = 206.8427188$ MPa. Stainless steel is chosen due to it's corrosion resistance properties. We choose Multipurpose 304 Stainless Steel that was Cold-Worked and Annealed.

Variable selection

From McMaster-Carr, a 12"-12" (0.3048m - 0.3048m) with a thickness of 0.03" (of 0.000762m) sheet of the steel can be purchased. This can be taken and rolled into a hollow cylinder which gives us a hollow opening for our piston to rest in. The diameter of this hole is 0.097 m, which can be applied to the above equations. When this is done:

$$V = \frac{A_{outlet}}{}$$

$$V_{piston} = \frac{A_{outlet}}{A_{piston}} \cdot V_{outlet} \quad (3)$$

$$P_{piston} = P_{outlet} + \frac{1}{2}\rho V_{outlet}^2 - \frac{1}{2}\rho \frac{A_{outlet}^2}{A_{piston}^2} \cdot V_{outlet}^2 \quad (4)$$

$$\frac{dW}{dt} = \Pi \cdot \dot{V} \cdot \left(P_{outlet} + \frac{1}{2}\rho V_{outlet}^2 - \frac{1}{2}\rho \frac{A_{outlet}^2}{A_{piston}^2} \cdot V_{outlet}^2 \right) \quad (11)$$

$$\boxed{\frac{dW}{dt} = 1422 \text{ Watts} = 1.9069 \text{ HP}}$$

Conclusion: Key design parameters

After analyzing the system and solving for many of the variables, we now have a list of parameters we may use when searching McMaster-Carr for suitable components. These values include a safety factor of n = 2 where they are necessary:

$$\frac{dW}{dt} = 1.907\, HP$$

$d_{piston} = 0.097m$

$P_{piston} = 3.588\, MPa$

$\sigma_{piston} = 482.6\, MPa$

$t = 0.001524\, m$

$\omega = 0.5233 \frac{rad}{s}$