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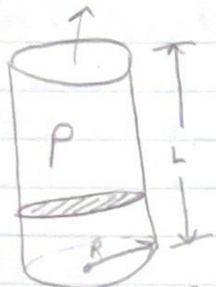
HW#3-cont

2.27)



$$V_{disk} = \frac{\sigma}{2\epsilon_0}\sqrt{R^2+z^2} - z$$

$z_0 - L/2$ ← integrate from close to far

$$\therefore V_{cylin} = \int_{z_0-L/2}^{z_0-L/2} \frac{\rho\, dz}{2\epsilon_0}\left(\sqrt{R^2+z^2} - z\right) = \frac{\rho}{2\epsilon_0}\int_{z_0-L/2}^{z_0+L/2}\left(\sqrt{R^2+z^2} - z\right) dz$$

$$\Rightarrow V_{cylinder} = \frac{\rho}{2\epsilon_0}\int_{z_0-L/2}^{z_0+L/2}\left(\sqrt{R^2+z^2} - z\right) dz$$

integral calculator

$$\Rightarrow V_{cylinder} = \frac{\rho}{4\epsilon_0}\left(R^2 \operatorname{asinh}\left(\frac{z}{R}\right) + z\sqrt{z^2+R^2} - z^2 \Big|_{z_0-L/2}^{z_0+L/2}\right)$$

$$\Rightarrow \boxed{V_{cylinder} = \frac{\rho}{4\epsilon_0}\left[\left(z_0+\tfrac{L}{2}\right)\sqrt{R^2+\left(z_0+\tfrac{L}{2}\right)^2} - \left(z_0-\tfrac{L}{2}\right)\sqrt{R^2+\left(z_0-\tfrac{L}{2}\right)^2} + R^2 \ln\left[\frac{z_0+\frac{L}{2}+\sqrt{R^2+\left(z_0+\frac{L}{2}\right)^2}}{z_0-\frac{L}{2}+\sqrt{R^2+\left(z_0-\frac{L}{2}\right)^2}}\right] - 2z_0 L\right]}$$

w/ $E = -\nabla V = -V_z \hat{z}$ by derivative calculator...

$$\boxed{\vec{E} = -\frac{\hat{z}\rho}{4\epsilon_0}\left[2\sqrt{R^2+\left(z_0+\tfrac{L}{2}\right)^2} - 2\sqrt{R^2+\left(z_0+\tfrac{L}{2}\right)^2} - 2L\right]}$$

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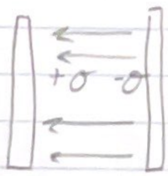
HW#3 - cont

E field from sheet

2.30 a) From 2.4, $E_{above} = \frac{\sigma}{2\epsilon_0}\hat{n}$; $E_{below} = \frac{-\sigma}{2\epsilon_0}\hat{n}$

$$\boxed{\therefore E_{above} - E_{below} = \frac{\sigma}{\epsilon_0}\hat{n}}$$

From 2.5



Field is 0 outside as ∞ sheets of charge do not depend on distance & only σ, $-\sigma$, the fields work against each other in all regions not between the two

They add like usual

$$E_{left} - E_{right} = \frac{\sigma}{2\epsilon_0}\hat{n} + \frac{\sigma}{2\epsilon_0}\hat{n} \boxed{= \frac{\sigma}{\epsilon_0}\hat{n}}$$

2.11

$E_{inside} = 0$; $E_{outside} = \frac{\sigma R^2}{\epsilon_0 r^2}\hat{r}$

From inside to outside boundaries,

$$E_{outside} - E_{inside} = \frac{\sigma R^2}{\epsilon_0 R^2}\hat{n} - 0$$

$$\boxed{\Rightarrow E_{outside} - E_{inside} = \frac{\sigma}{\epsilon}\hat{n}} \checkmark$$

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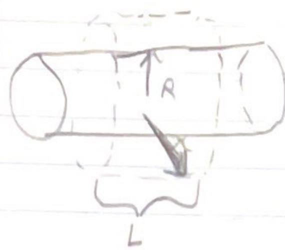
HW#3 - cont

b)



$A E_{inside} = \frac{q_{inc}}{\epsilon_0} = 0$, no charge inside

$A E_{outside} = \frac{q_{inc}}{\epsilon_0} = \frac{\overbrace{2\pi R L}^{A} \sigma}{\epsilon_0}$, so

$E_{outside} = \frac{\sigma}{\epsilon_0} \hat{n}$

$\boxed{E_{out} - E_{in} = \frac{\sigma}{\epsilon_0} \hat{n}}$ ✓

c) $V_{above}^{(out)} = V_{below}^{(in)}$ & $\Delta V_n = \frac{-\sigma}{\epsilon_0}$

$V_{out} = \frac{R^2 \sigma}{\epsilon_0 z}$, but $z = R$, so $V_{out} = \frac{R\sigma}{\epsilon_0}$

$V_{in} = \frac{R\sigma}{\epsilon_0}$ $\boxed{V_{out} = V_{in}, \; 2.34}$

V_{in} has no z dependence, so $\frac{\partial V_{in}}{\partial z} = 0$

$\frac{\partial V_{out}}{\partial z} = -\frac{R^2 \sigma}{\epsilon_0 z^2}$, but @ $z = R$, $\boxed{\frac{\partial V_{out}}{\partial z} = \frac{-\sigma}{\epsilon_0}, \; 2.36}$

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2.35)

a) $\sigma_R = \frac{q}{SA} = \boxed{\frac{q}{4\pi R^2} = \sigma_R}$

opposes charge on surface R

$\sigma_a = \frac{-q}{SA} = \boxed{\frac{-q}{4\pi a^2} = \sigma_a}$

σ_b opposes charge on surface a

$\sigma_b = \frac{q}{SA} = \boxed{\frac{q}{4\pi b^2} = \sigma_b}$

b) $V(0) = -\int_{\infty}^{0} \vec{E} \cdot d\vec{\ell} = -\int_{-\infty}^{0} \frac{q}{4\pi\epsilon_0} \cdot \frac{1}{r^2}\, dr = \frac{-q}{4\pi\epsilon_0}\left[\int_{-\infty}^{b} r^{-2} dr + \int_{b}^{a} r^{-2} dr + \int_{a}^{R} r^{-2} dr + \int_{R}^{0} r^{-2} dr\right]$

but, $\int_{b}^{a} \ldots = 0 = \int_{R}^{0} \ldots$ as inside conductor, so

$V = \frac{-q}{4\pi\epsilon_0}\left[\int_{-\infty}^{b} r^{-2} dr + \int_{a}^{R} r^{-2} dr\right] = \frac{q}{4\pi\epsilon_0}\left[r^{-1}\Big|_{-\infty}^{b} + r^{-1}\Big|_{a}^{R}\right] = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{b} + \frac{1}{R} - \frac{1}{a}\right]$

$$\boxed{V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{b} + \frac{1}{R} - \frac{1}{a}\right]}$$

c) now $\boxed{\sigma_b = 0}$ as it is equipotential w/ earth

& V does not have a $\frac{1}{b}$ term as theres no field there, so...

$$\boxed{V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{R} - \frac{1}{a}\right]}$$

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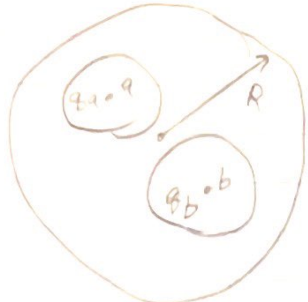
2.36



a) σ_a cancels out charge q_a, so

$$\boxed{\sigma_a = \frac{-q_a}{4\pi a^2}}$$

σ_b also cancels charge q_b, so

$$\boxed{\sigma_b = \frac{-q_b}{4\pi b^2}}$$

This charge migrates from the edge, so the edge will have $+q_a$ & $+q_b$ on it, so...

$$\boxed{\sigma_R = \frac{q_a + q_b}{4\pi R^2}}$$

b) outside, looks like the superposition of q_a & q_b @ $r=0$

$$\boxed{\therefore \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q_a + q_b}{r^2} \hat{r}}$$

c) Coulomb's Law, $E = \frac{1}{4\pi\epsilon_0} \frac{q_{enc}}{r^2} \hat{r}$

$$\boxed{E_a = \frac{1}{4\pi\epsilon_0} \frac{q_a}{r_a^2} \hat{a}} \qquad \boxed{E_b = \frac{1}{4\pi\epsilon_0} \frac{q_b}{r_b^2} \hat{b}}$$

d) 0, definition of conductors

e) only the outermost charge density $\boxed{\sigma_R}$ & $\boxed{E_{out}}$ will change