HW#5

6.2) In flow over a surface, velocity & temp profiles are of forms:

$$u(y) = Ay + By^2 - Cy^3$$

$$T(y) = D + Ey + Fy^2 - Gy^3$$

A-G are constants. Find C_f & h in terms of u_∞, T_∞, & constants.

C_f) $\tau_s = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = \mu\left(A + 2By - 3Cy^2\right)\Big|_{y=0} = \mu A$

$$C_f \equiv \frac{2\tau_s}{\rho u_\infty^2} = \boxed{\frac{2\mu A}{\rho u_\infty^2} = C_f = \frac{2\nu A}{u_\infty^2}}$$

$$h = \frac{-k_f \frac{\partial T}{\partial y}\big|_{y=0}}{T_s - T_\infty} = \frac{-k_f\left(E + 2F - 3Gy^2\right)\big|_{y=0}}{D - T_\infty} = \boxed{\frac{-k_f E}{D - T_\infty} = h}$$

HW#5

6.3) BL temp distribution ~ $\dfrac{T-T_s}{T_\infty - T_s} = 1-\exp\left(-Pr\dfrac{u_\infty y}{\nu}\right)$

$Pr = c_p\mu/k = 0.7$, $T_\infty = 400K$, $T_s = 300K$, $u_\infty/\nu = 5000$ 1/m

6.3) $\theta = T - T_s \Rightarrow \dfrac{\theta}{\theta_\infty} = 1-\exp\left(-Pr\dfrac{u_\infty y}{\nu}\right) \Rightarrow \theta = \theta_\infty - \theta_\infty \exp\left(-Pr\dfrac{u_\infty y}{\nu}\right)$

$\dfrac{\partial\theta}{\partial y} = \dfrac{\partial T}{\partial y} \Rightarrow q_s'' = -k_f \left.\dfrac{\partial T}{\partial y}\right|_{y=0} = -k_f\left(\theta_\infty Pr\dfrac{u_\infty}{\nu}\exp\left(-\dfrac{Pr\,u_\infty 0}{\nu}\right)\right)$

$\Rightarrow q_s'' = -k_f\left(\theta_\infty Pr\dfrac{u_\infty}{\nu}\right)$ Table A.4, $k_f = 26.3e{-3}$ W/m·K

$q_s'' = -(26.3e^{-3})(400K-300K)(0.7)(5000\,m^{-1}) = \boxed{-9205\ W/m^2 = q_s''}$

HW #5

6.5) Laminar flow over a flat plate, $h_x \propto x^{-1/2}$
where x is distance from leading edge $(x=0)$
Find ratio of $\bar{h}_x / h_x$ @ some point x?

Eq 6.14 $\bar{h} = \frac{1}{L}\int_0^L h\,dx = \frac{1}{L}\int_0^L C x^{-1/2}\,dx = \frac{C}{L}\left(2x^{1/2}\right|_0^L = \frac{2C\,L^{1/2}}{L}$

say $L = x$ $\therefore \bar{h} = \frac{2C}{\sqrt{L}}$ $\qquad \frac{\bar{h}}{h} = \frac{2C}{\sqrt{L}} \cdot \frac{\sqrt{L}}{C} = 2$

$$\boxed{\frac{\bar{h}}{h} = 2}$$

HW#5

8.5) An engine oil cooler consists of 25 smooth tubes, w/ $L = 2.5\,m$ & $D = 10\,mm$.

a) If oil @ 300 K & flow rate of 24 kg/s is fully developed, what is pressure drop & pump power?

$A = 25\left(\pi \frac{D^2}{4}\right) = \frac{25}{4}\pi D^2$, $\dot{m} = \rho A V \Rightarrow V = \frac{\dot{m}}{\rho A}$

Table A.5

w/ $\rho = 884.1\ kg/m^3$, $\nu = 550e{-6}$; $V = \frac{24}{(884)\left(\frac{25}{4}\pi (10e{-3})^2\right)} = \underline{13.8255\,m/s = V}$

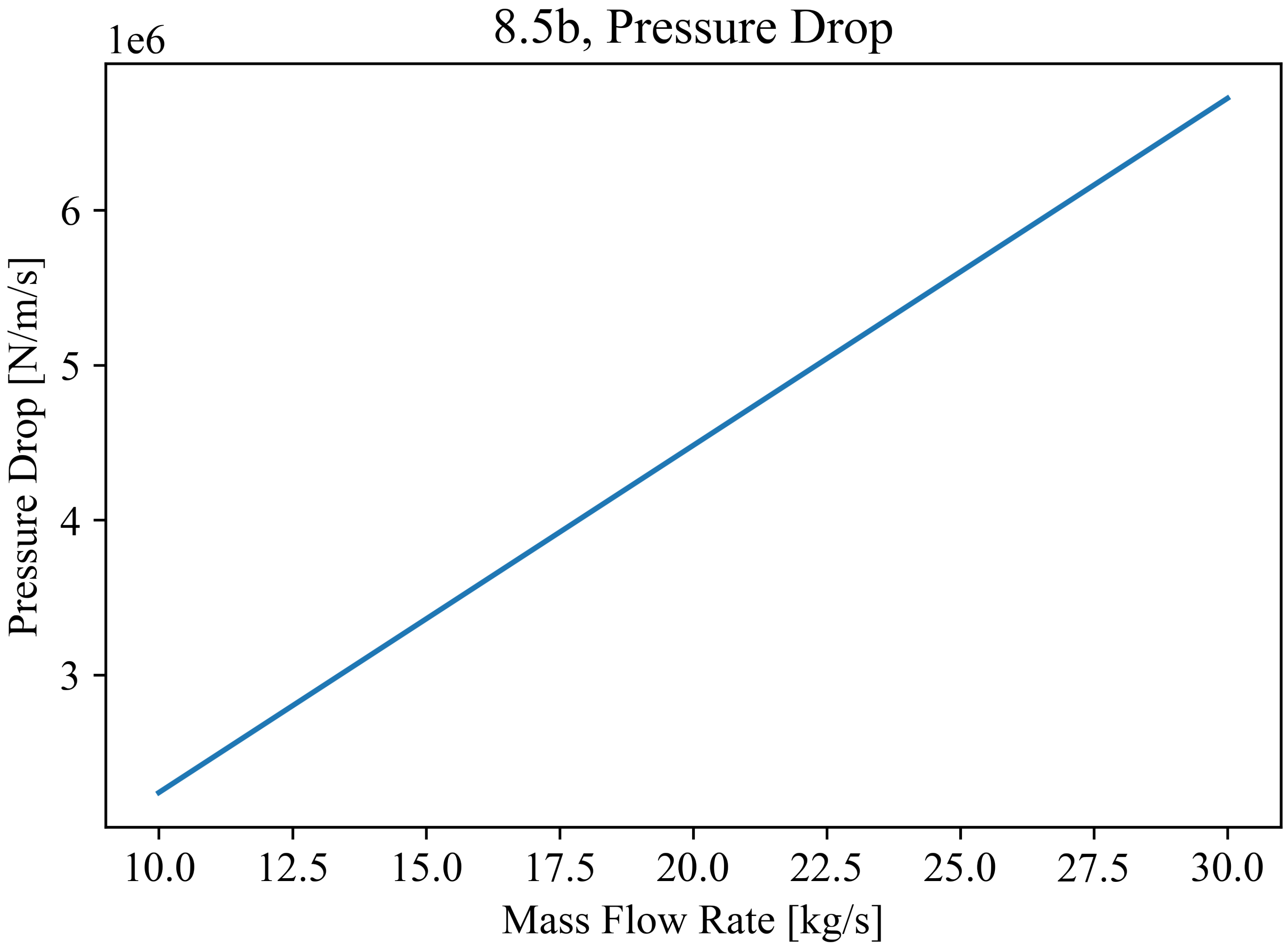
$$\text{say } Re = \frac{\overset{u_m}{\downarrow}{V} D}{\nu} = \frac{(13.827)(10e{-3})}{550e{-6}} = 251.4 = Re$$

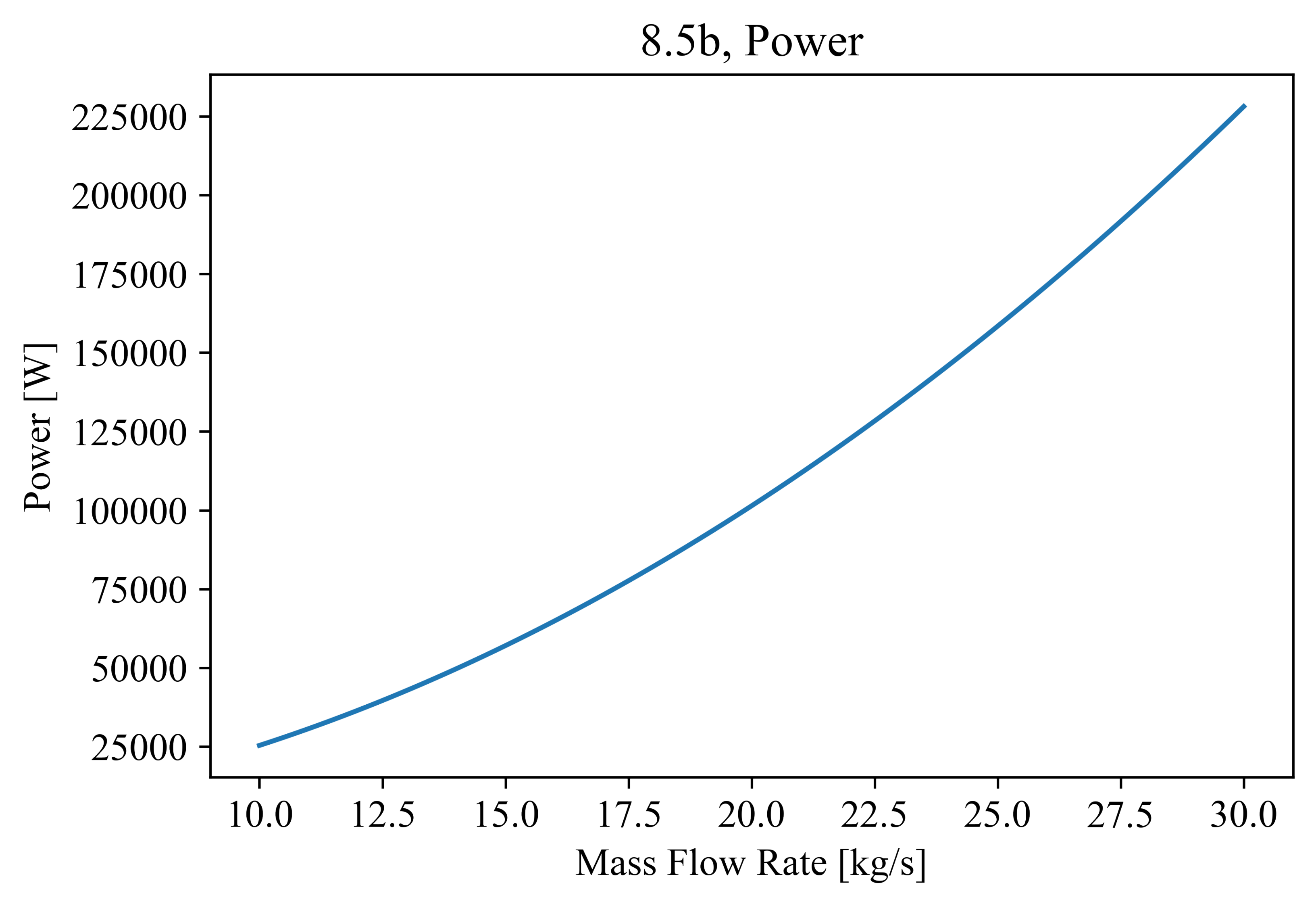
w/ $f = \frac{64}{Re} = 0.2546$ & $\Delta p = f\frac{\rho V^2 L}{2D}$

$$\Delta p = \frac{(0.2546)(884.1)(13.826)^2(2.5)}{2(10e{-3})} = \boxed{5.378e{-6}\ \frac{N}{m \cdot s} = \Delta p}$$

$$P = \frac{\Delta p\, \dot{m}}{\rho} = \frac{\Delta p(24)}{884.1} = \boxed{146\ kW}$$

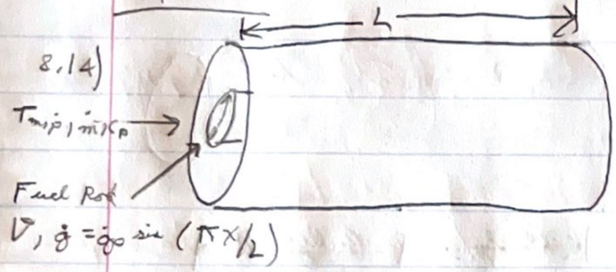
b) $\boxed{\begin{array}{l} \Delta P = 224{,}090.16\ \dot{m} \\ P = 253.47\ \dot{m}^2 \end{array}}$ check plots


8.5b, Pressure Drop
1e6
Pressure Drop [N/m/s]
3
4
5
6
10.0
12.5
15.0
17.5
20.0
22.5
25.0
27.5
30.0
Mass Flow Rate [kg/s]


8.5b, Power
Power [W]
225000
200000
175000
150000
125000
100000
75000
50000
25000
10.0
12.5
15.0
17.5
20.0
22.5
25.0
27.5
30.0
Mass Flow Rate [kg/s]

HW#5

8.14)



$\dot q = \dot q_o \sin(\pi x/L)$

a) Find $q''(x)$ & total q from rod to water.

SS, no heat in: $\dot E_{gen} = \dot E_{out}$

$$\Rightarrow \dot q \forall = q'' A \Rightarrow q'' = \frac{\dot q \forall}{A} = \dot q_o \sin\left(\frac{\pi x}{L}\right) \frac{\left(L \pi \frac{D^2}{4}\right)}{\pi D L}$$

$$\boxed{q'' = \frac{D}{4} \dot q_o \sin\left(\frac{\pi x}{L}\right)}$$

$$q = \frac{D \dot q_o}{4} \int_0^L \pi D \sin\left(\frac{\pi x}{L}\right) dx = \frac{D^2 \dot q_o}{\pi 4}\left[\frac{L}{\pi} \cos\left(\frac{\pi x}{L}\right)\Big|_0^L\right] = \boxed{\frac{D^2 L \dot q_o}{2} = q}$$

b) Eq 8.36) $dq_{conv} = \dot m c_p \, dT_m$

$$\text{say } dq = q''(\pi D dx) \Rightarrow dT_m = \frac{\pi D}{\dot m c_p}\left[\frac{D}{4} \dot q_o \sin\left(\frac{\pi x}{L}\right)\right] dx \Rightarrow \int_0^x$$

$$T_m - T_{m,0} = \frac{D^2 \pi \dot q_o}{4 \dot m c_p}\left[\frac{-L}{\pi} \cos\left(\frac{\pi x}{L}\right)\Big|_0^x\right] = \frac{-L D^2 \dot q_o}{4 \dot m c_p}\left(\cos\left(\frac{\pi x}{L}\right) - 1\right)$$

$$\boxed{T_m(x) = T_{m,0} + \frac{L D^2 \dot q_o}{4 \dot m c_p}\left(1 - \cos\left(\frac{\pi x}{L}\right)\right)}$$

HW#5

c) Find expression for $T_s(x)$ w/ x along tube.
Expression for x-loc @ which temp is max.

newton's Law of cooling: $q'' = h(T_s - T_m)$

$$\Rightarrow T_s = \frac{q''}{h} + T_m = \boxed{\frac{D\dot{q}_o}{4h}\sin\left(\frac{\pi x}{L}\right) + T_{m,i} + \frac{LD^2\dot{q}_o}{4\dot{m}c_p}\left(1-\cos\left(\frac{\pi x}{L}\right)\right) = T_s}$$

Take derivative wrt x

$$\frac{dT_s}{dx} = \frac{D\dot{q}_o L}{4h\pi}\cos\left(\frac{\pi x}{L}\right) + \frac{L^2D^2\dot{q}_o}{4\dot{m}c_p\pi}\sin\left(\frac{\pi x}{L}\right) = 0$$

$$\Rightarrow \frac{-LD}{\dot{m}c_p}\sin\left(\frac{\pi x}{L}\right) = \frac{\cos\left(\frac{\pi x}{L}\right)}{h} \Rightarrow \tan\left(\frac{\pi x}{L}\right) = \frac{-\dot{m}c_p}{LDh}$$

$$\boxed{x = \frac{L}{\pi}\,\text{atan}\left(-\frac{\dot{m}c_p}{DLh}\right) = x_{max}}$$