HW#5

1) 

a) $\frac{1}{v}\frac{\partial \phi}{\partial t} - \nabla\cdot(D\nabla\phi) + (\Sigma_a - \frac{1}{k}\nu\Sigma_f)\phi = 0$, w/ $\phi(x,t) = X(x)T(t)$

$$\frac{\partial\phi}{\partial t} - Dv(\nabla^2\phi) - (\frac{1}{k}\nu\Sigma_f - \Sigma_a)\phi v = 0$$

$$\Rightarrow \frac{\partial\phi}{\partial t} - Dv\nabla^2\phi + (\frac{\nu}{k}\Sigma_f - \Sigma_a)\phi v = 0$$

partials: $\frac{\partial\phi}{\partial t} = X(x)\frac{\partial T(t)}{\partial t}$, $\nabla^2\phi = T(t)\nabla^2 X(x)$

$\Rightarrow X\frac{\partial T}{\partial t} - TDv(\nabla^2 X) - (\frac{1}{k}\nu\Sigma_f - \Sigma_a)vXT = 0$ divide by XT

$\Rightarrow \underbrace{\frac{1}{T}\frac{\partial T}{\partial t}}_{-\lambda} \underbrace{- \frac{1}{X}Dv(\nabla^2 X) - (\frac{1}{k}\nu\Sigma_f - \Sigma_a)v}_{\lambda} = 0$, equal to $\pm$ same constant as no other way to be equal

$$\Rightarrow \nabla^2 X + \frac{(\frac{1}{k}\nu\Sigma_f - \Sigma_a)}{D}X = \frac{-\lambda X}{vD} \Rightarrow \nabla^2 X + \underbrace{\frac{(\frac{1}{k}\nu\Sigma_f - \Sigma_a + \frac{\lambda}{v})}{D}}_{B^2}X = 0$$

$$\Rightarrow \nabla^2 X + B^2 X = 0 \Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial X}{\partial r}\right) + B^2 X = 0$$

soln is Bessel Functions: $X = C_1 J_n(B_n r) + \cancel{C_2} Y_n(B_n r) \leftarrow$ space

now solve $\frac{1}{T}\frac{\partial T}{\partial t} = -\lambda_n \Rightarrow \frac{\partial T}{\partial t} = -\lambda_n T \Rightarrow T = C_3\exp(-\lambda_n t) \leftarrow$ time

BC) i) $-D\nabla X|_{r=0} = 0 \Rightarrow \frac{\partial X}{\partial r}|_{r=0} = 0$

$$\Rightarrow \frac{\partial X}{\partial r} = \left(\frac{n B_n C_1}{r}J_n(B_n r) - C_1 J_{n+1}(B_n r)\right) + \left(\frac{n B_n C_2}{r}Y_n(B_n r) - C_2 Y_{n+1}(B_n r)\right)$$

@ $r=0$, $Y_n \to -\infty$ $\therefore C_2 = 0$

ii) $X(\tilde{R}) = 0 \Rightarrow \sum_{n=0}^{\infty} C_n J_n(B_n\tilde{R}) = 0$, B_n = first root for all J_n

only first root for $n \neq 0$ of $J_n = 0$ $\therefore X = C_0 J_0(B_n\tilde{R})$

$\therefore B_n\tilde{R} = \overset{\text{root of } J_0}{\nu_n} \Rightarrow B_n = \frac{\nu_n}{\tilde{R}}$

HW #5

now multiply T_0 by λ_0 (all other $\lambda_n = 0$, so $\lambda_n T_n = 0$ when $n \neq 0$)

$$\boxed{\phi = \sum_{n=0}^{\infty} C_n J_0(B_n \tilde{R}) \exp(-\lambda_n t)}$$

b) say $\phi = C_0 J_0(B_0 \tilde{R}) \exp(-\lambda_0 t) + \sum_{n=1}^{\infty} C_n J_0(B_n \tilde{R}) \exp(-\lambda_n t)$

but $\lambda_0 = 0$ as it is the fundamental mode

$\Rightarrow \phi = C_0 J_0(B_0 \tilde{R}) \exp(0) + \sum_{n=1}^{\infty} C_n J_0(B_n \tilde{R}) \exp(-\lambda_n t)$

@ $t = \infty$, $\phi = C_0 J_0(B_0 \tilde{R}) + 0$ $\therefore$ $\boxed{\phi = C_0 J_0(B_0 \tilde{R})}$

c) see Buckling: $B_n^2 = (\nu_n / \tilde{R})^2$, but only interested in $n = 0$

$$\boxed{\therefore B_0^2 = (\nu_0 / \tilde{R})^2}$$

d) $B_g^2 = B_m^2$ $\Rightarrow$ $\boxed{\left(\frac{\nu_0}{\tilde{R}}\right)^2 = \frac{\frac{1}{k} \nu \Sigma_f - \Sigma_a}{D}}$ $\boxed{k = 1}$ $\boxed{\lambda_0 = 0}$

e) know $P \cdot L = \int d\forall \, \Sigma_f C_0 J_0(B_0 \tilde{R}) \cdot E_f$ ← energy / fission

$\Rightarrow P \cdot L = \int_0^L dz \int dA \, \Sigma_f C_0 J_0(B_0 \tilde{R}) \cdot E_f \Rightarrow P = \int dA \, \Sigma_f C_0 J_0(B_0 \tilde{R}) \cdot E_f$

$\Rightarrow C_0 = P / \left(\int dA \, \Sigma_f \, J_0(B_0 \tilde{R}) \cdot E_f \right)$

$$\Rightarrow \boxed{C_0 = \frac{P}{\Sigma_f J_0(B_0 \tilde{R}) \cdot E_f} \cdot \frac{1}{\int_0^{2\pi} d\varphi \int_0^R r \, dr}}$$

f) $\left(\frac{\nu_0}{\tilde{R}}\right)^2 = \frac{\frac{1}{k} \nu \sigma_f N - \sigma_a N}{1} \cdot \frac{3 \sigma_{tr} N}{1} \Rightarrow \frac{\nu_0}{R} = \sqrt{N^2 (3\sigma_{tr})(\frac{1}{k} \nu \sigma_f - \sigma_a)}$

see $\frac{1}{R} \propto \sqrt{N^2} = N$ $\boxed{\therefore \text{doubling } N \text{, halves } R \;\; \therefore R_c = 25 \text{ cm}}$

HW #5

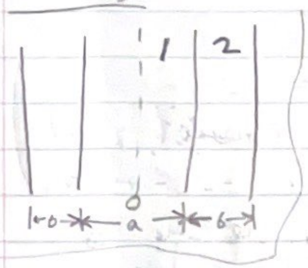
2)

a) False, geometric buckling is related to the rate of n° leakage while material buckling is related to the difference in n° production & absorption. as criticality is related to the number of neutrons, the more critical reactor will have a relatively lower leakage rate (geometric buckling) to neutron production (material buckling).

b) True, as $\lambda_n < \lambda_{n+1}$, all subsequent modes decay faster than the previous mode. as $t \to \infty$, the mode w/ the lowest λ will be the most prevelant, which is always the fundamental mode.

HW#5

3)



a) $-\nabla\cdot(D_1\nabla\phi_1)+\Sigma_a\phi_1=\nu\Sigma_f\phi\frac{1}{k}$

$\hookrightarrow \nabla^2\phi_1+\frac{\frac{1}{k}\nu\Sigma_f-\Sigma_a}{D_1}\phi_1=0$ w/ $B_1^2=\frac{\frac{1}{k}\nu\Sigma_f-\Sigma_a}{D_1}$

$\Rightarrow \nabla^2\phi_1+B_1^2\phi_1=0 \Rightarrow \frac{d^2\phi_1}{dx^2}+B_1^2\phi_1=0$

$\phi_1=C_1\cancel{\sin}(B_1x)+C_2\cos(B_1x)$

1

BC) ✓ i) $-D_1\frac{d\phi_1}{dx}\Big|_{x=0}=0$ ii) $\phi_1(x=\frac{a}{2})=\phi_2(x=\frac{a}{2})$ ✓

✓ iii) $-D_1\frac{d\phi_1}{dx}\Big|_{x=a/2}=-D_2\frac{d\phi_2}{dx}\Big|_{x=a/2}$ iv) $\phi_2(\tilde{x})=0$ ✓

2 $\begin{cases} -\nabla\cdot(D_2\nabla\phi_2)+\Sigma_a\phi_2=0 \Rightarrow \nabla^2\phi_2-\frac{1}{L_2^2}\phi_2=0 \quad ; \quad \tilde{x}=(\frac{a}{2}+b)+2D_2 \\ \Rightarrow \phi_2=C_4\sinh(\frac{x}{L_2})+C_5\cosh(\frac{x}{L_2})=C_3\sinh(\frac{\tilde{x}-x}{L_2}) \end{cases}$ ← iv) embedded

i) $-D_1\left[C_1B_1\cos(B_1x)-C_2B_1\sin(B_1x)\right]\Big|_{x=0}=0 \Rightarrow C_1B_1+0=0 \quad \therefore C_1=0$

ii) $C_2\cos(B_1\frac{a}{2})=C_3\sinh\left(\frac{\tilde{x}-a/2}{L_2}\right) \Rightarrow C_2=C_3\sinh\left(\frac{\tilde{x}-a/2}{L_2}\right)/\cos(B_1\frac{a}{2})$

iii) $D_1\left[-C_2B_1\sin(B_1\frac{a}{2})\right]=D_2\left[\frac{-C_3}{L_2}\cosh\left(\frac{\tilde{x}-a/2}{L_2}\right)\right]$

$\hookrightarrow D_1\left[\left(C_3\sinh\left(\frac{\tilde{x}-a/2}{L_2}\right)/\cos(B_1\frac{a}{2})\right)\sin(B_1\frac{a}{2})\right]=D_2\left[\frac{C_3}{L_2}\cosh\left(\frac{\tilde{x}-a/2}{L_2}\right)\right]$

$\hookrightarrow D_1\sinh\left(\frac{\tilde{x}-a/2}{L_2}\right)\tan(B_1\frac{a}{2})=\frac{D_2}{L_2}\cosh\left(\frac{\tilde{x}-a/2}{L_2}\right)$

$$\boxed{\begin{gathered}\phi_1=C_3\frac{\sinh\left(\frac{\tilde{x}-a/2}{L_2}\right)}{\cos(B_1\frac{a}{2})}\cos(B_1x) \quad , \quad \phi_2=C_3\sinh\left(\frac{\tilde{x}-x}{L_2}\right) \\ \text{criticality: } D_1\sinh\left(\frac{\tilde{x}-a/2}{L_2}\right)\tan(B_1\frac{a}{2})=\frac{D_2}{L_2}\cosh\left(\frac{\tilde{x}-a/2}{L_2}\right)\end{gathered}}$$

$\tilde{x}=(\frac{a}{2}+b)+2D_2$

$\tilde{x} = \left(\frac{a}{2}+b\right)+2\sigma_2$

HW #5

b) $P \cdot A = \int d\forall\, \Sigma_f \phi_1 E_f + \int d\forall\, \Sigma_s \phi_2 E_s$

$$P \cdot A = \left(2A\int_0^{a/2} dx\, \Sigma_f C_3 \frac{\sinh\left(\frac{\tilde{x}-a/2}{L_2}\right)}{\cos\left(B_1 \frac{a}{2}\right)} \cos(B_1 x) E_f\right) + \left(2A\int_{a/2}^{a/2+b} dx\, \Sigma_s C_3 \sinh\left(\frac{\tilde{x}-x}{L_2}\right) E_s\right)$$

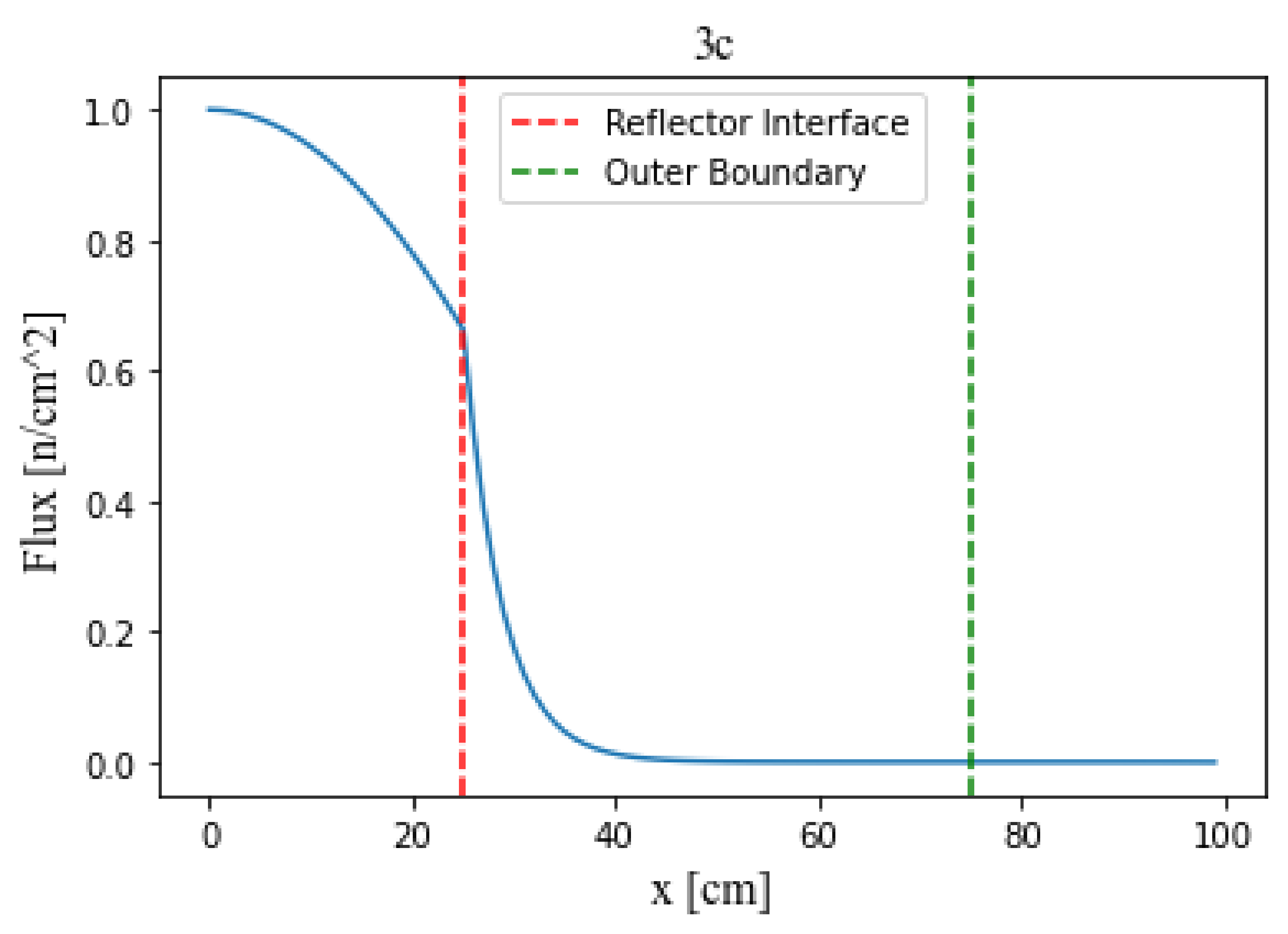
$$P = 2\Sigma_f C_3 \frac{\sinh\left(\frac{\tilde{x}-a/2}{L_2}\right)}{\cos(B_1\, a/2)} E_f \int_0^{a/2} \cos(B_1 x)\, dx + 2\Sigma_s C_3 E_s \int_{a/2}^{a/2+b} \sinh\left(\frac{\tilde{x}-x}{L_2}\right) dx$$

$$\Rightarrow \int_0^{a/2} \cos(B_1 x)\, dx = \frac{1}{B_1}\sin(B_1 x)\Big|_0^{a/2} = \frac{1}{B_1}\sin\left(\frac{aB_1}{2}\right)$$

$$\Rightarrow \int_{a/2}^{a/2+b} \sinh\left(\frac{\tilde{x}-x}{L_2}\right) dx = -\frac{1}{L_2}\cosh\left(\frac{\tilde{x}-x}{L_2}\right)\Big|_{a/2}^{a/2+b} = \frac{1}{L_2}\left(\cosh\left(\frac{b+2\sigma_2}{L_2}\right) - \cosh\left(\frac{2\sigma_2}{L_2}\right)\right)$$

Plug back in to get...

$$\boxed{C_3 = \frac{P}{\frac{2\Sigma_f E_f}{B_1} \frac{\sinh\left(\frac{b+2\sigma_2}{L_2}\right)}{\cos\left(B_1 \frac{a}{2}\right)} \sin\left(\frac{aB_1}{2}\right) + \frac{2\Sigma_s E_s}{L_2}\left[\cosh\left(\frac{b+2\sigma_2}{L_2}\right) - \cosh\left(\frac{2\sigma_2}{L_2}\right)\right]}}$$


3c
Reflector Interface
Outer Boundary
Flux [n/cm^2]
x [cm]
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100

HW #5

4) 

a) $-\nabla\cdot(D_1\nabla\phi_1)+\Sigma_a\phi_1=\frac{1}{k}\nu_1\Sigma_{f1}\phi \Rightarrow \nabla^2\phi_1+\frac{\frac{1}{k}\nu_1\Sigma_{f1}-\Sigma_a}{D_1}\phi_1=0$

$\Rightarrow \nabla^2\phi_1+B_1^2\phi=0 \Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi_1}{\partial r}\right)+B_1^2\phi=0$; say $\phi_1=\frac{u}{r}$

$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\left(\frac{u}{r}\right)\right)+B_1^2\phi=0 \Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(\frac{\partial u}{\partial r}\cdot\frac{1}{r}-\frac{u}{r^2}\right)\right)+B_1^2\phi_1=0$

$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}r-u\right)+B_1^2\phi_1=0 \Rightarrow \frac{1}{r^2}\left(\frac{\partial^2 u}{\partial r^2}r+\frac{\partial u}{\partial r}-\frac{\partial u}{\partial r}\right)+B_1^2\phi_1=0 \Rightarrow \frac{\partial^2 u}{\partial r^2}+B_1^2\phi_1=0$

$\Rightarrow u=C_1\sin(B_1r)+C_2\cos(B_1r) \Rightarrow \phi_1=\frac{C_1}{r}\sin(B_1r)+\frac{C_2}{r}\cancel{\cos}(B_1r)$ } 1

$\nabla^2\phi_2-\frac{1}{L_2^2}\phi_2=0 \Rightarrow \phi_2=\frac{C_3}{r}\sinh\left(\frac{\tilde{b}-r}{L_2}\right)$ } 2

BC i) $-D_1\frac{\partial\phi_1}{\partial r}\Big|_{r=0}=0$ ✓ ii) $\phi_1(R)=\phi_2(R)$ ✓ iii) $-D_1\frac{\partial\phi_1}{\partial r}\Big|_{r=R}=-D_2\frac{\partial\phi_2}{\partial r}\Big|_{r=R}$

i) $-D_1\left(\frac{C_1}{r}B_1\cos(B_1r)-\frac{C_2}{r}B_1\sin(B_1r)-\frac{C_1}{r^2}\sin(B_1r)-\frac{C_2}{r^2}\cos(B_1r)\right)\Big|_{r=0}=0$

$\Rightarrow \left(C_1B_1r\cos(B_1r)-C_2B_1r\sin(B_1r)-C_1\sin(B_1r)-C_2\cos(B_1r)\right)\Big|_{r=0}=0 \Rightarrow -C_2=0 \quad \therefore C_2=0$

ii) $\left[\frac{C_1}{r}\sin(B_1r)\right]\Big|_{r=R}=\left[\frac{C_3}{r}\sinh\left(\frac{\tilde{b}-r}{L_2}\right)\right]\Big|_{r=R} \Rightarrow C_1\sin(B_1R)=C_3\sinh\left(\frac{\tilde{b}-R}{L_2}\right) \quad \therefore C_1=C_3\sinh\left(\frac{\tilde{b}-R}{L_2}\right)/\sin(B_1R)$

iii) $-D_1\left(\frac{C_1}{R}B_1\cos(B_1R)-\frac{C_1}{R^2}\sin(B_1R)\right)=-D_2\left(\frac{-C_3}{RL_2}\cosh\left(\frac{\tilde{b}-R}{L_2}\right)-\frac{C_3}{R^2}\sinh\left(\frac{\tilde{b}-R}{L_2}\right)\right)$

$\Rightarrow D_1\left(C_3\sinh\left(\frac{\tilde{b}-R}{L_2}\right)/\sin(B_1R)\right)\left(\frac{B_1}{R}\cos(B_1R)-\frac{1}{R^2}\sin(B_1R)\right)=-D_2C_3\left(\frac{1}{RL_2}\cosh\left(\frac{\tilde{b}-R}{L_2}\right)+\frac{1}{R^2}\sinh\left(\frac{\tilde{b}-R}{L_2}\right)\right)$

$$\boxed{\begin{gathered}\phi_1=\frac{C_3}{r}\frac{\sinh\left(\frac{\tilde{b}-R}{L_2}\right)}{\sin(B_1R)}\sin(B_1r) \qquad \phi_2=\frac{C_3}{r}\sinh\left(\frac{\tilde{b}-r}{L_2}\right)\\ \text{criticality: } D\frac{\sinh\left(\frac{\tilde{b}-R}{L_2}\right)}{\sin(B_1R)}\left(\frac{B_1}{R}\cos(B_1R)-\frac{1}{R^2}\sin(B_1R)\right)=-D_2\left(\frac{1}{RL_2}\cosh\left(\frac{\tilde{b}-R}{L_2}\right)+\frac{1}{R^2}\sinh\left(\frac{\tilde{b}-R}{L_2}\right)\right)\end{gathered}}$$

HW # 5

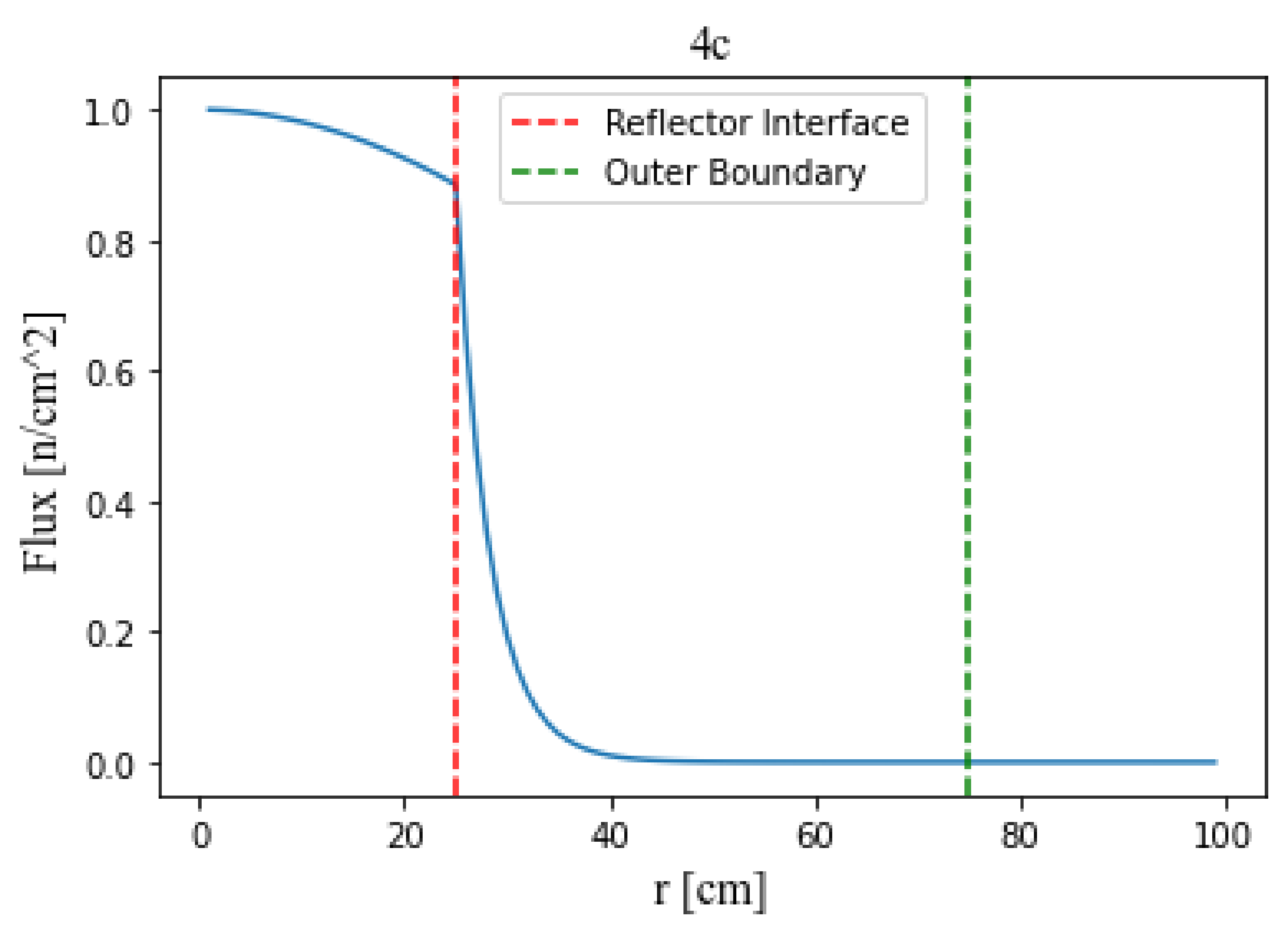
b) We would need a rxn rate, the associated cross sections, & the results of the reaction per reaction.

For example, we could have a Power, Σ_f for region 1, Σ_s for region 2, energy per fission for region 1, and energy per scatter

$$P = \int dV \, \Sigma_f \, \phi_1 \, E_f + \int dV \, \Sigma_s \, \phi_2 \, E_s$$

c) on python

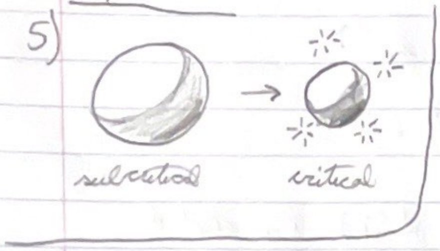
d) The shapes of the graphs are roughly the same, but the sphere has a higher fraction of max flux as the sphere has lower leakage compared to the cylinder as the shere is the est shape for minimizing leakage.



4c
Reflector Interface
Outer Boundary
Flux [n/cm^2]
r [cm]
1.0
0.8
0.6
0.4
0.2
0.0
0
20
40
60
80
100

HW#5

5)



a) This is the case as the sphere being compressed results in the same # of potential fission sites, but the SA is lower, so there will be less leakage. Same mass w/ less leakage ↑ k.

b) know $B^2 = \frac{\frac{1}{k}\nu\Sigma_f - \Sigma_a}{D}$, $N_1 V_1 = N_2 V_2 \Rightarrow N_c \frac{4}{3}\pi R_c^3 = N_0 \frac{4}{3}\pi R_0^3 \therefore N_c = N_0 \frac{R_0^3}{R_c^3}$

$$\Rightarrow \frac{\nu\Sigma_f - \Sigma_a}{D} = \left(\frac{\pi}{R_c}\right)^2 \Rightarrow 3\sigma_{tr} N_c^2 (\nu\sigma_f - \sigma_a) = \left(\frac{\pi}{R_c}\right)^2 \Rightarrow 3\sigma_{tr} N_0^2 \left(\frac{R_0}{R_c}\right)^6 (\nu\sigma_f - \sigma_a) = \left(\frac{\pi}{R_c}\right)^2$$

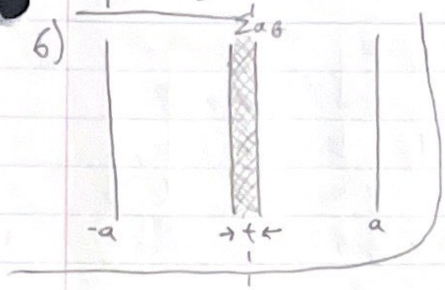
$$\Rightarrow R_c^4 = \frac{3\sigma_{tr} N_0^2 (\nu\sigma_f - \sigma_a) R_0^6}{\pi^2} = \frac{3\sigma_{tr}\sigma_a N_0^2 \left(\frac{\nu\sigma_f}{\sigma_a} - 1\right) R_0^6}{\pi^2}$$

know $k_\infty = \frac{\nu\sigma_f}{\sigma_a} \Rightarrow R_c^4 = \frac{(k_{\infty,0} - 1) R_0^6}{L_0^2 \pi^2}$

& $3\sigma_{tr}\sigma_a = 1/L_0^2$

$$\Rightarrow \boxed{R_c = \sqrt{\frac{(k_{\infty,0} - 1) R_0^6}{L_0^2 \pi^2}}}$$

HW#5

6) 

a) $-D\nabla^2\phi + \Sigma_a\phi = 0$

$\Rightarrow \nabla^2\phi - \frac{1}{L_c^2}\phi = 0 \Rightarrow \frac{d^2\phi}{dx^2} - \frac{1}{L^2} = 0$

$\boxed{\therefore \phi = C_1 \sinh\left(\frac{\tilde{a} - x}{L_c}\right)} \leftarrow \phi(\tilde{a}) = 0$ embedded

for the region: $-D\nabla^2\phi_t + \Sigma_a\phi_t = \nu\Sigma_f\phi_t \Rightarrow -D\nabla^2\phi_t = (\nu\Sigma_f - \Sigma_a)\phi_t$

integrate over x: $-D\frac{d\phi_t}{dx} = \int_0^{t/2}(\nu\Sigma_f - \Sigma_a)\phi_t\,dx = (\nu\Sigma_f - \Sigma_a)(t/2)\,\phi_t$

very thin; so $\phi_t = \phi(0)$, $x = 0 \Rightarrow \frac{d\phi}{dx} = \frac{-C_1}{L_c}\cosh\left(\frac{\tilde{a}-x}{L_c}\right) = -\frac{(\nu\Sigma_f - \Sigma_a)}{D}(t/2)\,C_1\sinh\left(\frac{\tilde{a}-x}{L_c}\right)$

$\therefore \frac{1}{L_c} = \frac{(\nu\Sigma_f - \Sigma_a)}{D}(t/2)\tanh\left(\frac{\tilde{a}-x}{L_c}\right)$

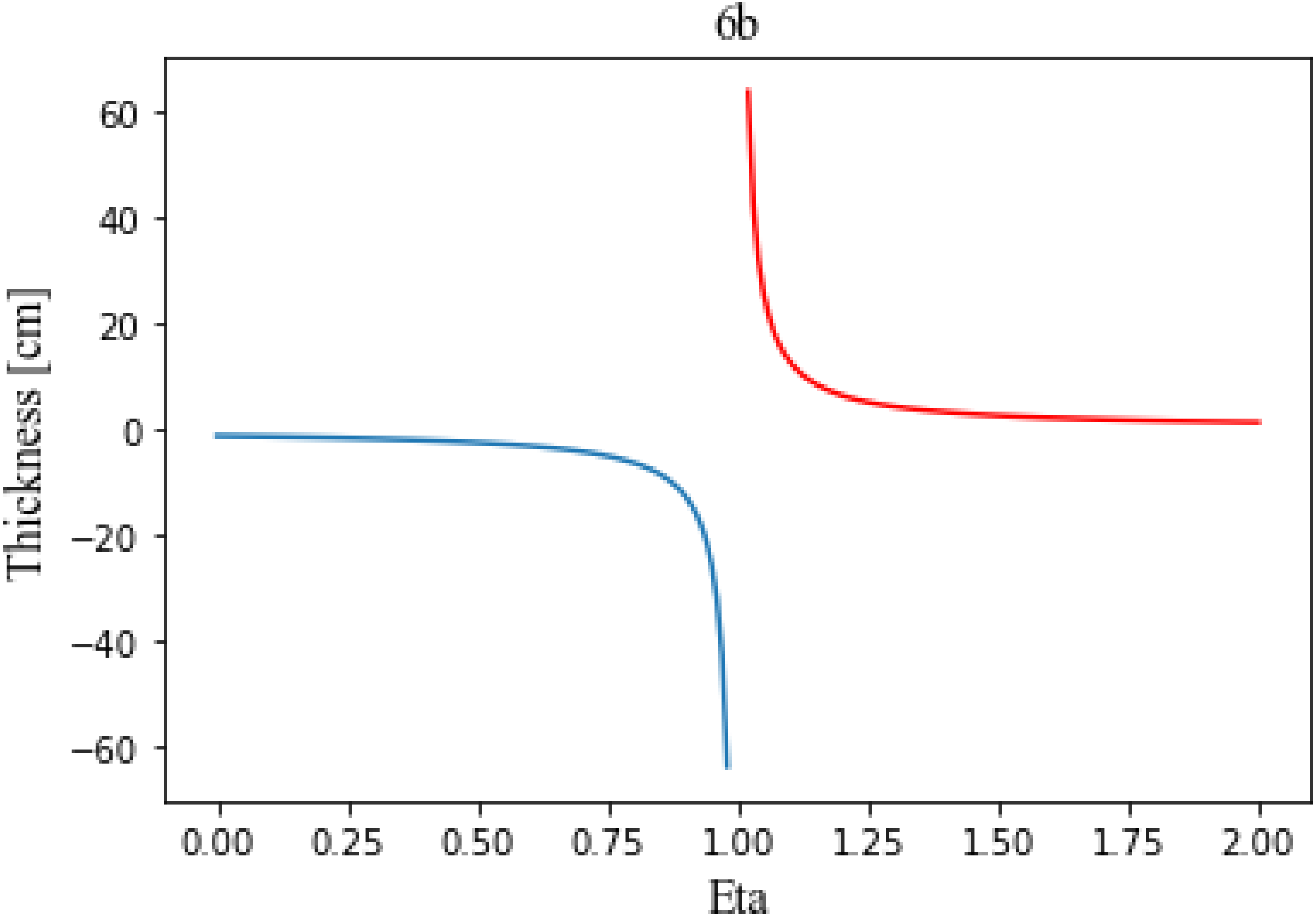
but $\frac{(\nu\Sigma_f - \Sigma_a)}{D} = \frac{\Sigma_a\left(\frac{\nu\Sigma_f}{\Sigma_a} - 1\right)}{D} = \frac{\Sigma_a}{D}(\eta - 1) = \frac{(\eta - 1)}{L^2}$

$\Rightarrow \boxed{1 = \frac{(\eta - 1)}{L}(t/2)\tanh\left(\frac{\tilde{a}-x}{L}\right)}$

b) on python

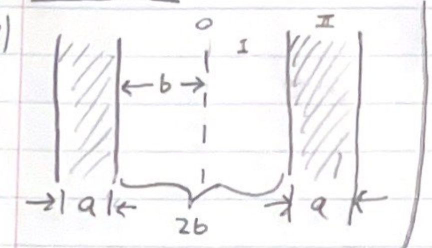
c) ~~theoretically~~ 1, but this occurs w/ ∞ thickness
max realistic is just above 1 w/ ∞ thickness too
$\lim_{\eta \to 1^-}(t)$

d) the limitations are $\eta < 1$ are not meaningful as you can't have negative thickness. also, @ $\eta = 1$, it is undefined, so need a limit.


6b
60
40
20
0
−20
−40
−60
Thickness [cm]
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
Eta

HW#5

7)



a) $-\nabla(D\nabla\phi) + \Sigma_a\phi = \nu\Sigma_f\phi$

$\Rightarrow -D\nabla^2\phi - (\nu\Sigma_f - \Sigma_a)\phi = 0$

$\Rightarrow \nabla^2\phi + B^2\phi = 0$

$\phi = C_1\sin(Bx) + C_2\cos(Bx)$, $\frac{d\phi}{dx} = C_1B\cos(Bx) - C_2B\sin(Bx)$

$\hookrightarrow \phi = C_3\sin((\tilde{a}-r)B)$, $\frac{d\phi}{dx} = -C_3B\cos((\tilde{a}-r)B)$

& BC) i) $-D\frac{d\phi}{dx}\Big|_{x=b} = 0 \Rightarrow -C_3B\cos((\tilde{a}-r)B) = 0$

$\therefore (\tilde{a}-r)B = \frac{n\pi}{2}$ odd n $\Rightarrow$ $B = \frac{n\pi}{(\tilde{a}-r)}$

critical when $\left(\frac{n\pi}{(\tilde{a}-r)}\right)^2 = \frac{\nu\Sigma_f - \Sigma_a}{D} = \frac{\Sigma_a\left(\frac{\nu\Sigma_f}{\Sigma_a} - 1\right)}{D} = \frac{k_\infty - 1}{L^2}$

fundamental mode $n=1$

$\boxed{\left(\frac{n\pi}{\tilde{a}-r}\right)^2 = \frac{k_\infty - 1}{L^2}} \Rightarrow \left(\frac{\pi}{\tilde{a}-r}\right) = \sqrt{\frac{k_\infty - 1}{L^2}}$

$\Rightarrow \tilde{a} - r = \frac{\pi}{\sqrt{\frac{k_\infty-1}{L^2}}}$ $\Rightarrow (b+a) + 2D - b = \frac{\pi}{\sqrt{\frac{k_\infty-1}{L^2}}}$ $\Rightarrow a = \frac{\pi}{\sqrt{\frac{k_\infty-1}{L^2}}} - 2D = \frac{\pi L}{\sqrt{k_\infty - 1}} - 2D$

plug in $a = \frac{\pi(56)}{2\sqrt{1.1-1}} - 2(1) = \boxed{276.17\ cm = a}$

$\hookrightarrow$ divide by 2 as we only want thickness for one region & not both

HW #5

b) The criticality condition ~~does not change~~ as a function of b, which makes sense as the neutrons that enter the vacuum do not get destroyed or eaten, so the region is effectively not there.

c) ~~moderator medium:~~ $\cancel{\phi_1 = C_4 \sinh(Bx) + C_5 \cosh(Bx)}$

$$\phi_2 = C_6 \sin((\tilde{a} - r)B)$$

BC) i) $\phi_1 |_{x=0} = 0$

ii) $-D\frac{d\phi_1}{dx}\Big|_{x=b} = -D\frac{d\phi_2}{dx}\Big|_{x=b}$

iii) $\phi_1(b) = \phi_2(b)$

iv) embedded in extrapolated length, $\phi_2|_{x=\tilde{a}} = 0$

$$\cancel{C_4 = C_6 \frac{\sin((a+2D)B)}{\sinh(Bb)}}$$

$-D\nabla^2\phi + \Sigma_a \phi_1 = \nu\Sigma_f \phi_1$ $\qquad k_\infty = \frac{\nu\Sigma_f}{\Sigma_a} = 1 \Rightarrow \nu\Sigma_f = \Sigma_a$

$\hookrightarrow -D\nabla^2\phi_1 = 0 \Rightarrow \nabla^2\phi_1 = 0 \Rightarrow \phi_1 = C_7 x + C_8$

ii) $-D\frac{d\phi_1}{dx} = 0 \Rightarrow \frac{d\phi_1}{dx} = C_7 = 0$ $\qquad \therefore C_7 = 0$ $\qquad$ & $\phi_1 = C_8$ is constant

This is the same for vacuum & $k_{\infty,2} = 1.1$, which is the same as in a), so a is the same as in a), so

$$\boxed{a = 276.17 \text{ cm}}$$

d) The answers are the same for a & c, which makes sense as k_∞ being 1 is the same as $\nu\Sigma_f = \Sigma_a$. For vacuum, both are 0, but, for a material w/ k_∞ prescribed as 1, they are equal. makes sense