HW#8

1) i: $\frac{d^2u}{dx^2}+\alpha^2 v=0$; ii: $\frac{d^2v}{dx^2}+\gamma^2 u=0$; iii: $\frac{du(0)}{dx}=0$; iv: $u(l)=1$; v: $\frac{dv(0)}{dx}=0$; vi: $v(l)=0$

a) assume $v=A\exp(i\omega x)$ & $u=B\exp(i\omega x)$

$v'=i\omega A\exp(i\omega x)$ $\quad$ $u'=i\omega B\exp(i\omega x)$

$v''=-\omega^2 A\exp(i\omega x)$ $\quad$ $u''=-\omega^2 B\exp(i\omega x)$

i) $-\omega^2 B\exp(i\omega t)=\alpha^2 A\exp(i\omega x) \Rightarrow \omega^2 B=\alpha^2 A \Rightarrow A=\frac{\omega^2}{\alpha^2}B$

ii) $\omega^2 A\exp(i\omega t)=\gamma^2 B\exp(i\omega x) \Rightarrow \omega^2 A=\gamma^2 B \Rightarrow \frac{\omega^4}{\alpha^2}B=\gamma^2 B \Rightarrow \omega^4=\alpha^2\gamma^2$

w/ $\omega^4=\alpha^2\gamma^2 \Rightarrow \omega^2=\pm\alpha\gamma \Rightarrow \omega=\begin{cases}+\sqrt{\alpha\gamma}=\omega_1\\-\sqrt{\alpha\gamma}=-\omega_1\\+i\sqrt{\alpha\gamma}=\omega_2\\-i\sqrt{\alpha\gamma}=-\omega_2\end{cases}$

$\Rightarrow A_5\exp(i\omega_1 x)+A_6\exp(-i\omega_1 x)+A_7\exp(i\omega_2 x)+A_8\exp(-i\omega_2 x)=v(x)$

$B_5\exp(i\omega_1 x)+B_6\exp(-i\omega_1 x)+B_7\exp(i\omega_2 x)+B_8\exp(-i\omega_2 x)=u(x)$

w/ $\sin x+\cos x=\left(\frac{1}{2}+\frac{i}{2}\right)\exp(ix)+\left(\frac{1}{2}-\frac{i}{2}\right)\exp(-ix)$

$\Rightarrow A_1\sin(\omega_1 x)+A_2\cos(\omega_1 x)+A_3\sin(\omega_2 x)+A_4\cos(\omega_2 x)=v(x)$

$B_1\sin(\omega_1 x)+B_2\cos(\omega_1 x)+B_3\sin(\omega_2 x)+B_4\cos(\omega_2 x)=u(x)$

$-\omega_1^2 A_1\sin(\omega_1 x)-\omega_1^2 A_2\cos(\omega_1 x)-\omega_2^2 A_3\sin(\omega_2 x)-\omega_2^2 A_4\cos(\omega_2 x)=v''$

$-\omega_1^2 B_1\sin(\omega_1 x)-\omega_1^2 B_2\cos(\omega_1 x)-\omega_2^2 B_3\sin(\omega_2 x)-\omega_2^2 B_4\cos(\omega_2 x)=u''$

HW #8 - cont

ii) $\omega_1^2 A_1 \sin(\omega_1 x) + \omega_1^2 A_2 \cos(\omega_1 x) + \omega_2^2 A_3 \sin(\omega_2 x) + \omega_2^2 A_4 \cos(\omega_2 x) =$
$\gamma^2 [B_1 \sin(\omega_1 x) + B_2 \cos(\omega_1 x) + B_3 \sin(\omega_2 x) + B_4 \cos(\omega_2 x)]$

as sins & cos are linearly independent, get 4 eq from 4 basis vectors

$\sin(\omega_1 x)$: $\omega_1^2 A_1 = \gamma^2 B_1$
$\cos(\omega_1 x)$: $\omega_1^2 A_2 = \gamma^2 B_2$
$\sin(\omega_2 x)$: $\omega_2^2 A_3 = \gamma^2 B_3$
$\cos(\omega_2 x)$: $\omega_2^2 A_4 = \gamma^2 B_4$

$\Rightarrow$
$$\boxed{\begin{aligned} A_1 &= \gamma^2/\omega_1^2 \, B_1 \\ A_2 &= \gamma^2/\omega_1^2 \, B_2 \\ A_3 &= \gamma^2/\omega_2^2 \, B_3 \\ A_4 &= \gamma^2/\omega_2^2 \, B_4 \end{aligned}}$$

Boundary conditions:

iii) $B_1 \omega_1 \cos(0) - B_2 \omega_1 \sin(0) + B_3 \omega_2 \cos(0) - B_4 \omega_2 \sin(0) = 0$
$\omega_1 B_1 + \omega_2 B_3 = 0$

iv) $B_1 \sin(\omega_1 l) + B_2 \cos(\omega_1 l) + B_3 \sin(\omega_2 l) + B_4 \cos(\omega_2 l) = 1$

v) $A_1 \omega_1 \cos(0) - A_2 \omega_1 \sin(0) + A_3 \omega_2 \cos(0) - A_4 \omega_2 \sin(0) = 0$
$\omega_1 A_1 + \omega_2 A_3 = 0$

vi) $A_1 \sin(\omega_1 l) + A_2 \cos(\omega_1 l) + A_3 \sin(\omega_2 l) + A_4 \cos(\omega_2 l) = 0$

$$\boxed{\begin{aligned} &\omega_1 B_1 + \omega_2 B_3 = 0 \qquad \omega_1 A_1 + \omega_2 A_3 = 0 \\ &B_1 \sin(\omega_1 l) + B_2 \cos(\omega_1 l) + B_3 \sin(\omega_2 l) + B_4 \cos(\omega_2 l) = 1 \\ &A_1 \sin(\omega_1 l) + A_2 \cos(\omega_1 l) + A_3 \sin(\omega_2 l) + A_4 \cos(\omega_2 l) = 0 \end{aligned}}$$

b) figures

c) check gov eqs to ensure they are correct, check BC
FDM.

$\frac{k-1}{k} = \rho$

HW #8

2a) $\alpha_T = \frac{\partial \rho}{\partial T} = \left(\frac{0.09445}{1.09445} - \frac{.10024}{1.10024}\right) / (600-300) = -0.0000160278$

$$\boxed{\alpha_T = -1.60278 \ \frac{pcm}{K}}$$

b) $\rho = \alpha_T \times 55K = \boxed{-88.153 \ pcm = \rho}$

3a) False, undermoderated reactors may not be able to downscatter into the thermal regieme without sufficient moderation

b) False, delayed neutrons are created from delayed neutron precursors on the order of .1 ~ 60s, which cannot be affected instantly

c) False, point reactor kinetics assumes seperable space and time component, not point distribution

HW#8 - cont

4) $M \equiv \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \|\vec{V}\|^2 \exp\left(\frac{-m\|\vec{V}\|^2}{2kT}\right)$

set $\frac{\partial M}{\partial V}=0 \Rightarrow \frac{\partial M}{\partial V}=0=\left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \left[2\|\vec{V}\| \exp\left(\frac{-m\|\vec{V}\|^2}{2kT}\right) - \frac{m\|\vec{V}\|^2}{2kT}\exp\left(\frac{-m\|\vec{V}\|^2}{2kT}\right)2\|\vec{V}\|\right]$

$\Rightarrow 2\|\vec{V}\| \exp\left(-\frac{m\|\vec{V}\|^2}{2kT}\right) = \frac{m\|\vec{V}\|^3}{kT}\exp\left(\frac{-m\|\vec{V}\|^2}{2kT}\right) \Rightarrow 2 = \frac{m\|\vec{V}\|^2}{kT}$

$\Rightarrow \boxed{kT = \frac{1}{2}m\|\vec{V}\|^2}$ most probable velocity when $\frac{1}{2}m\|\vec{V}\|^2 = E$

$\therefore$ most probable energy is kT

5a) take weighted average to obtain effective $t_{1/2}$

$$t_{1/2,\mathrm{eff}} = \sum_{n=1}^{6} t_{1/2,i} \cdot \text{Relative Yield} = 8.82705\ s$$

$\Delta = \ell/k$, $\rho = \frac{k-1}{k}$, $\lambda = \ln(2)/t_{1/2,\mathrm{eff}}$, $\beta = 0.00650$

Start from $\frac{dn}{dt} = \left[\frac{\rho-\beta}{\Delta}\right]n + \lambda C$ & $\frac{dC}{dt} = -\lambda C + \frac{\beta}{\Delta}n$

assume $n = A\exp(\omega t)$, $C = F\exp(\omega t)$ w/o imaginaries b/c neglect space & $n \in \mathbb{R} \ni C$

$\hookrightarrow A\omega\exp(\omega t) = \frac{\rho-\beta}{\Delta}A\exp(\omega t) + \lambda F\exp(\omega t) \Rightarrow A\omega = \frac{\rho-\beta}{\Delta}A + \lambda F$

$F\omega\exp(\omega t) = -\lambda F\exp(\omega t) + \frac{\beta}{\Delta}A\exp(\omega t) \Rightarrow F\omega = -\lambda F + \frac{\beta}{\Delta}A$

solve for $\omega = -\lambda + \frac{\beta}{\Delta\omega}\left(\frac{\rho-\beta}{\Delta}A + \lambda\right) \Rightarrow \omega^2 = -\lambda\omega + \frac{\beta}{\Delta}\left(\frac{\rho-\beta}{\Delta}A + \lambda\right)$

HW # 2 - cont

$$\Rightarrow \omega = \frac{-(\beta-\rho+\lambda\Lambda) \pm \sqrt{(\beta-\rho+\lambda\Lambda)^2+4\Lambda\lambda\rho}}{2\Lambda}$$ w/ $\omega_1 = \ldots + \ldots$

$\omega_2 = \ldots - \ldots$

$\Rightarrow n = A_1 \exp(\omega_1 t) + A_2 \exp(\omega_2 t)$

$C = F_1 \exp(\omega_1 t) + F_2 \exp(\omega_2 t)$

know $n(0) = 1000 = A_1 + A_2 = n_0$

& $C(0) = (\beta/\Lambda\lambda)(n_0) = F_1 + F_2$

$\Rightarrow$ back to governing eq to get last 2 eq

$$F_1\omega_1 \exp(\omega_1 t) + F_2\omega_2 \exp(\omega_2 t) = -\lambda F_1 \exp(\omega_1 t) - \lambda F_2 \exp(\omega_2 t) + \frac{\beta}{\Lambda}\left(A_1 \exp(\omega_1 t) + A_2 \exp(\omega_2 t)\right)$$

$$\Rightarrow F_1\omega_1 = -\lambda F_1 + \frac{\beta}{\Lambda}A_1 \quad \& \quad F_2\omega_2 = -\lambda F_2 + \frac{\beta}{\Lambda}A_2$$

set up sympy solver to get coefficients

assuming $k_0 = 1$ $\quad \Delta\rho = \rho_f \Rightarrow \frac{k_f - 1}{k_f} = 0.00150 \Rightarrow \underline{k} = 1.00150$

$\underline{\lambda} = \ln(2)/t_{1/2} = 0.078525\ 1/s \qquad \underline{\beta} = 0.00650 \qquad \underline{\Lambda} = \frac{l}{k} = 9.985e^{-5}$

check graph

b) $T_{asymptote} \approx .1\,s$