HW#6

1) PW pebble-bed, bare spheres w/ $D = 1''$, $P = 2500$ psia
$q''' = 1e7$ Btu/hr ft^3, $k = 10$ Btu/hr·ft°F, $T_{sat} = 668°F$

assume: - no boiling $-\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial \phi} = 0$ - uniform properties

find max temp in fuel element

$\Rightarrow \nabla k \nabla T + q''' = \rho c_p \frac{\partial T}{\partial t}$ ← 0, SS $\Rightarrow \nabla^2 T = -\frac{q'''}{k}$ ← uniform k

$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial \phi} = 0$

$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + 0 + 0 = -\frac{q'''}{k} \Rightarrow \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = \frac{-q''' r^2}{k} \Rightarrow r^2 \frac{dT}{dr} = \frac{-q''' r^3}{3k} + C_1$

$\Rightarrow \frac{dT}{dr} = \frac{-q''' r}{3k} + \frac{C_1}{r^2} \Rightarrow T = \frac{-q''' r^2}{6k} - \frac{C_1}{r} + C_2$

BC) 1) $T(0) \neq \infty$ $\Rightarrow T(0) = 0 + \infty + C_2$ $\therefore C_1 = 0$
2) $T(r = \frac{D}{2}) = T_{sat}$ $\Rightarrow T(\frac{D}{2}) = \frac{-q''' D^2}{24 k} + C_2 = T_{sat}$

$\Rightarrow C_2 = T_{sat} + \frac{q''' D^2}{24 k} = 668°F + \frac{(1e7)\left(\frac{1}{12}\right)^2}{24\,(10)} = 957.352°F$

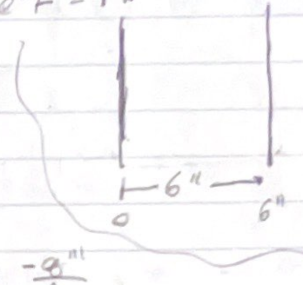
$$\boxed{T_{max} = T(r=0) = C_2 = 957.352°F}$$

HW # 6 - cont

2) $D = 6"$ thick steel, $\Phi = 10^{14}\ \gamma/s\,cm^2$ @ $S = 5\,MeV/\gamma$

cooled by $h = 1000\ BTU/hr\,ft^2\,°F$ water @ $300°F = T_w$

$k = 26\ BTU/hr\,ft\,°F$, $\mu = 0.245\ \frac{1}{cm}$, $B = 1$



a) find surface temps

$$\Rightarrow \nabla k \nabla T + q''' = \cancel{\rho c_p \frac{\partial T}{\partial t}}^{\,0,\ SS} \Rightarrow \text{uniform } k \Rightarrow \nabla^2 T = \frac{-q'''}{k}$$

$$\Rightarrow T = fn(x) \Rightarrow \frac{d^2T}{dx^2} = \frac{-q'''}{k}$$

$\hookrightarrow$ w/ $q''' = q'''_0 \exp(-\mu x)$

$\hookrightarrow q'''_0 = B\Phi S\mu = (1)\left(0.245\ \frac{1}{cm}\right)\left(10^{14}\ \frac{\gamma}{cm^2 s}\right)\left(\frac{5\,MeV}{\gamma}\right)\left(1.6e{-13}\ J/MeV\right) \overset{\times 3600\ s/h}{=} 70{,}560\ J/cc$

$$\Rightarrow \frac{d^2T}{dx^2} = \frac{-q'''_0}{k}\exp(-\mu x) \Rightarrow \frac{dT}{dx} = \frac{q'''_0}{k\mu}\exp(-\mu x) + C_1$$

$$\Rightarrow T = \frac{-q'''_0}{k\mu^2}\exp(-\mu x) + C_1 x + C_2$$

BC) 1) $-k\frac{dT}{dx}\Big|_{x=0} = h(T_w - T(0))$

2) $-k\frac{dT}{dx}\Big|_{x=0} = h(T(D) - T_w)$

1) $$-k\left(\frac{q'''_0}{k\mu}\exp(0) + C_1\right) = h\left(T_w - \left(\frac{-q'''_0}{k\mu^2}\exp(0) + C_2\right)\right)$$

$$\Rightarrow -\frac{q'''_0}{\mu} - kC_1 = hT_w + \frac{h q'''_0}{k\mu^2} - hC_2$$

$$\Rightarrow C_1 = \frac{hC_2}{k} - \frac{hT_w}{k} - \frac{h q'''_0}{k^2\mu^2} - \frac{q'''_0}{k\mu}$$

2) $$-k\left(\frac{q'''_0}{k\mu}\exp(-\mu D) + C_1\right) = h\left(\left(\frac{-q'''_0}{k\mu^2}\exp(-D\mu) + C_1 D + C_2\right) - T_w\right)$$

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$$-k\left(\frac{q_0'''}{k\mu}\exp(-\mu L) + \left(\frac{hC_2}{k} - \frac{hT_w}{k} - \frac{hq_0'''}{k^2\mu^2} - \frac{q_0'''}{k\mu}\right)\right)$$

$$= h\left(\left(-\frac{q_0'''}{k\mu^2}\exp(-L\mu) + \left(\frac{hC_2}{k} - \frac{hT_w}{k} - \frac{hq_0'''}{k^2\mu^2} - \frac{q_0'''}{k\mu}\right)L + C_2\right) - T_w\right)$$

$$\Rightarrow -\frac{q_0'''}{\mu}\exp(-\mu L) - hC_2 + hT_w + \frac{hq_0'''}{k\mu^2} + \frac{q_0'''}{\mu}$$

$$= -\frac{q_0'''h}{k\mu^2}\exp(-L\mu) + \frac{Lh^2C_2}{k} - \frac{h^2T_wL}{k} - \frac{h^2q_0'''L}{k^2\mu^2} - \frac{q_0'''Lh}{k\mu} + C_2h - T_wh$$

$$\Rightarrow -2C_2h - \frac{Lh^2C_2}{k} = -\frac{q_0'''}{\mu}\exp(-\mu L) - 2T_wh - \frac{hq_0'''}{k\mu^2} - \frac{q_0'''}{\mu}$$

$$-\frac{q_0'''h}{k\mu^2}\exp(-L\mu) - \frac{h^2T_wL}{k} - \frac{h^2q_0'''L}{k^2\mu^2} - \frac{q_0'''Lh}{k\mu}$$

$$\Rightarrow C_2\left(-2h - \frac{Lh^2}{k}\right) = -\frac{q_0'''}{\mu}\left(1 + \exp(-\mu L) + \frac{Lh}{k}\right) - \frac{q_0'''h}{k\mu^2}\left(1 + \exp(-\mu L) + \frac{Lh}{k}\right)$$

$$-2T_wh - \frac{h^2T_wL}{k}$$

$$\Rightarrow C_2 = \left(-2h - \frac{Lh^2}{k}\right)^{-1}\left[\left(1 + \exp(-\mu L) + \frac{Lh}{k}\right)\left(-\frac{q_0'''}{\mu} - \frac{q_0'''h}{k\mu^2}\right) - T_wh\left(2 + \frac{hL}{k}\right)\right]$$

Parameters

$h = 1000$ BTU/[hr ft^2 °F], $q_0''' = 70{,}560 \frac{J}{cc} \cdot \frac{26.8392\ \text{BTU/ft}^3}{1\ \text{J/cc}} = 1.8938e6 \frac{\text{BTU}}{\text{ft}^3}$

$k = 28$ BTU/[hr ft °F]

$L = 6\text{ in} \cdot \frac{1\text{ ft}}{12\text{ in}} = 0.5$ ft, $\mu = 0.245 \frac{1}{cm} \cdot \frac{30.48\text{ cm}}{1\text{ ft}} = 7.4676 \frac{1}{ft}$

$T_w = 300$°S

HW#6 - cont

w/ these values, $\boxed{C_1 = -2574.46 \ ^\circ F/ft \quad C_2 = 1694.37 \ ^\circ F}$

$$\boxed{w/\ T(x) = \frac{-q'''_0}{k\mu^2} \exp(-\mu x) + C_1 x + C_2}$$

(a) $\boxed{T(0\ ft) = 481.5137 \ ^\circ F \quad T(0.5\ ft) = 378.1463 \ ^\circ F}$

$\boxed{T_{max} = 915.950 \ ^\circ F}$ (b)

where all the calculations were done in python

github.com/jispecht3/classes/npre449/hw6

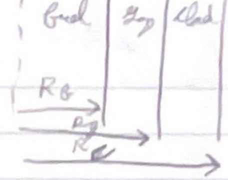
T_{max} when $\frac{dT}{dx} = 0 = \frac{q'''_0}{k\mu} \exp(-\mu x) + C_1$

=> solve for x, plug x into T, get T_{max}

i did this w/ python, so check the repo if the work is not enough

HW # 6 - cont

3) Intact uncracked w/ no relocation



a)

Parameters) $D_c = 12.52e{-3}\,m$, $t_{clad} = 0.56e{-3}\,m$, $D_{gap} = 230e{-6}\,m$, $\rho = 88\%$

$k_{clad} = 17$ W/mK, $h_g = 4300$ W/m²K, $\rho_{fuel} = 95\%$, $k_{f_1} = 2.7$ W/mK

$q' = 44$ kW/m

only radial conduction

$$\Rightarrow \nabla k \nabla T + q''' = \cancel{\rho c_p \frac{\partial T}{\partial t}}^{\,0,\,SS} \Rightarrow \text{uniform}: \nabla^2 T = -\frac{q'''}{k} \Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \cancel{\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} = -\frac{q'''}{k}$$

$$\Rightarrow \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{-q''' r}{k} \Rightarrow r\frac{\partial T}{\partial r} = \frac{-q''' r^2}{2k} + C_1 \Rightarrow \frac{\partial T}{\partial r} = \frac{-q''' r}{2k} + \frac{C_1}{r} \Rightarrow T = \frac{-q''' r^2}{4k} + C_1 \ln(r) + C_2$$

Temps) $T_f = \frac{-q''' r^2}{4 k_f} + C_1 \ln(r) + C_2$ & $T_c = C_3 \ln(r) + C_4$

BC) i) $T_f(r=0) \neq \infty$, $\therefore C_1 = 0 \Rightarrow T_f = \frac{-q''' r^2}{4 k_f} + C_2$ w/ $C_2 = T_{max}$

ii) $P_f k_f \nabla T_f = -k_c \nabla T P_c$

iii) $-k_f \nabla T_f = h\left(T_f(R_f) - T_c(R_g)\right)$

iv) $T_c(R_c) = T_o = 295°C$

apply correction for k_f

$$k_{pf} = \frac{(1 + (\alpha - 1)P_1)}{(1 - P_1)} k_{f,1} = 2.913 \text{ W/mK}$$ w/ $\alpha = 1.5$, $P_1 = 1 - .95 = 0.05$

$$\Rightarrow k_f = \frac{1 - P_2}{1 + (\alpha - 1)P_2} k_{pf} = \underline{2.4185 \text{ W/mK}}$$ w/ $P_2 = 1 - 0.88 = 0.12$

& $q''' = \frac{q'}{A_f} = \frac{q'}{\pi R_f^2} = 501$ MW/m³

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solve eqs

ii) $k_f R_f \left(\frac{-q''' R_f}{2 k_f}\right) = \frac{k_c R_g C_3}{R_g} \Rightarrow \boxed{C_3 = -\frac{q''' R_f^2}{2 k_c} = -411.93 \text{ K}}$

iv) $C_3 \ln(R_c) + C_4 = T_0 \Rightarrow C_4 = T_0 - C_3 \ln(R_c) = \boxed{-1794.96\,°C = C_4}$

iii) $-k_f \left(\frac{-q''' R_f}{2 k_f}\right) = h\left[\left(\frac{-q''' R_f^2}{4 k_f} + C_2\right) - \left(C_3 \ln(R_c) + C_4\right)\right]$

$C_2 = \frac{q''' R_f}{2h} + \frac{q''' R_f^2}{4 k_f} - C_3 \ln(R_c) - C_4 = \boxed{T_{max} = C_2 = 2111.80\,°C}$

w/ $T_f = \frac{-q''' r^2}{4 k_f} + C_2 \Rightarrow \boxed{T_f(0) = 2112.8\,°C}$

$\boxed{T_f(R_{fo}) = 664.02\,°C}$

$\Rightarrow T_c = C_3 \ln(R) + C_4 \quad \boxed{T_c(R_{ci}) = 355.88\,°C}$

b) $\bar{T} = \frac{1}{V}\iiint_V T_f \, dV = \frac{\pi h}{V}\int_0^{R_{fo}}\left(\frac{-q''' r^3}{4 k_f} + C_2\right) r\,dr \quad$ w/ $V = \pi R_{fo}^2 h$

$\Rightarrow \frac{1}{R_{fo}^2}\left[\frac{-q'''}{4 k_f}\int_0^{R_{fo}} r^3\,dr + C_2\int_0^{R_{fo}} r\,dr\right] = \frac{1}{R_{fo}^2}\left[\frac{-q'''}{4 k_f}\left(\frac{R_{fo}^4}{4}\right) + \frac{C_2 R_{fo}^2}{2}\right] = \frac{C_2}{2} - \frac{q''' R_{fo}^2}{16 k_f}$

$\bar{T} = \frac{C_2}{2} - \frac{q''' R_{fo}^2}{16 k_f} = \boxed{1387.91\,°C = \bar{T}}$

assume $\nabla^2 T = 0$ $\therefore$ T of sphere is uniform

HW#6 - cont

4) $D = 2''$, $T_0 = 850°F$, $T_\infty = 200°F$, $h = 2 \ Btu/hr\,ft^2\,°F$

$c_p = 0.122 \ Btu/lbm°F$, $\rho = 490 \ lbs/ft^3$ when $T = 300°F$?

Start from: $\dot{E}_{stored} = \overset{0,\ T_B > T_\infty}{\cancel{\dot{E}_{in}}} - \dot{E}_{out} + \overset{0,\ no\ gen}{\cancel{\dot{E}_{gen}}} \Rightarrow \dot{E}_{stored} = -\dot{E}_{out}$

we know $\dot{E}_{stored} = \rho c_p V \frac{\partial T}{\partial t}$ & $\dot{E}_{out} = hA(T - T_\infty)$

$$\Rightarrow \rho c_p V \frac{\partial T}{\partial t} = -hA(T - T_\infty) \Rightarrow \frac{\partial T}{\partial t} = \frac{-hA}{\rho c_p V}(T - T_\infty)$$

define $\tau = \frac{\rho c_p V}{hA} \Rightarrow \frac{\partial T}{\partial t} = \frac{-1}{\tau}(T - T_\infty)$. .

define $\theta = T - T_\infty \Rightarrow \frac{\partial \theta}{\partial t} = \frac{\partial T}{\partial t} \Rightarrow \frac{\partial \theta}{\partial t} = \frac{-1}{\tau}\theta$

$$\Rightarrow \theta = \theta_0 \exp(-t/\tau) \Rightarrow \frac{\theta}{\theta_0} = \exp(-t/\tau) \Rightarrow \ln\left(\frac{\theta_0}{\theta}\right) = \frac{t}{\tau}$$

$$\Rightarrow t = \tau \ln\left(\frac{\theta_0}{\theta}\right) = \tau \ln\left(\frac{T_0 - T_\infty}{T - T_\infty}\right)$$

$$w/\ \tau = \frac{\rho c_p V}{hA} = \frac{(490 \ \#/ft^3)(0.122 \frac{Btu}{\#°F})(\frac{4}{3}\pi(\frac{1}{12}ft)^3)}{(2 \frac{Btu}{hr\,ft^2\,°F})(4\pi(\frac{1}{12}ft)^2)} = \underline{0.83028 \ hr}$$

$$\Rightarrow t(T = 300) = \tau \ln\left(\frac{850°F - 200°F}{300°F - 200°F}\right) = 1.554 \ hr$$

$$\boxed{\begin{array}{l} \tau = 0.83028\ hr \quad or \quad 2989\ s \\ t = 1.5541\ hr \quad or \quad 5594.817\ s \end{array}}$$

HW #6 - cont

5) $D = 1"$, $c_p = 0.04 \frac{Btu}{\# °F}$, $\rho = 700 \ \#/ft^3$, $T_{0,fuel} = T_{0,coolant} = 100°F$

startup $\Rightarrow T_{coolant} = 500°F$, $h = 200 \frac{Btu}{hr\,ft^2\,°F}$

lumped capacitance

a) $\dot{E}_{stored} = \dot{E}_{in} - \cancel{\dot{E}_{out}}^{\,T_f < T_\infty} + \cancel{\dot{E}_{gen}}^{\,0,\ no\ gen}$ ← start w/ energy balance

w/ $\dot{E}_{stored} = \rho c_p \forall \frac{\partial T}{\partial t}$ & $\dot{E}_{in} = hA(T_\infty - T)$

$\Rightarrow \rho c_p \forall \frac{\partial T}{\partial t} = hA(T_\infty - T) \Rightarrow \frac{\partial T}{\partial t} = \frac{hA}{\rho c_p \forall}(T_\infty - T)$

w/ $\tau = \frac{\rho c_p \forall}{hA} = \frac{\left(700 \frac{\#}{ft^3}\right)\left(0.04 \frac{Btu}{\#°F}\right)\left(\frac{4}{3}\pi\left(\frac{1}{24}\right)^3\right)}{\left(200 \frac{Btu}{hr\,ft^2\,°F}\right)\left(4\pi\left(\frac{1}{24}\right)^2\right)} = 7\,s$

b) $\Rightarrow \frac{\partial T}{\partial t} = \frac{1}{\tau}(T_\infty - T)$ & $\theta = T - T_\infty$ & $\frac{\partial T}{\partial t} = \frac{\partial \theta}{\partial t}$

$\Rightarrow \frac{\partial \theta}{\partial t} = -\frac{\theta}{\tau} \Rightarrow \theta = \theta_i \exp\left(\frac{-t}{\tau}\right)$

$\Rightarrow \ln\left(\frac{\theta_0}{\theta}\right) = \frac{t}{\tau} \Rightarrow t = \tau \ln\left(\frac{\theta_0}{\theta}\right) = \tau \ln\left|\frac{100°F - 500°F}{T - 500°F}\right|$

$\Rightarrow t(T = 499°F) = \tau \ln\left|\frac{100°F - 500°F}{499°F - 500°F}\right| = 41.94\,s$

$$\boxed{\begin{array}{l} a)\ \tau = 7\,s \\ b)\ t(T = 499°F) = 41.94\,s \end{array}}$$