NPRE412 Spring 2025
HW 5 Due 2025.02.25

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- Show your work.
- This work must be submitted online as a `.pdf` through Canvas.
- Work completed with LaTeX or Jupyter earns 1 extra point. Submit source file (e.g. `.tex` or `.ipynb`) along with the `.pdf` file.
- If this work is completed with the aid of a numerical program (such as Python, Wolfram Alpha, or MATLAB) all scripts and data must be submitted in addition to the `.pdf`.
- If you work with anyone else, document what you worked on together.

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1. (5 points) An enrichment plant has a throughput of 32,000 kgU/day and produces 26,000kgU as tails. What is the enrichment of the product if the feed is natural uranium and the tails are 0.25%? (Tsoulfanidis, Question 3.1)

**Solution:** The enrichment of the product would be 2.65%. For all calculations, the conversion from isotope percent to mass percent has been ignored for all heavy Z elements (U). This is done because $x_w \simeq x_a$ if Z is higher enough. This was not omitted for the Boron question. I checked my answers with Mauhmoud Eltawila and Nathan Glaser.

We find the mass of the product by taking the difference of the feed mass and waste mass

$$m_p = m_f - m_t \tag{1a}$$

We then find the mass of U235 for each quantity ($x$) by multiplying the mass of that quantity by the enrichment of that quantity.

$$m_{x,U^{235}} = m_x * x_x \tag{1b}$$

We also find the mass of $U^{235}$ in the product.

$$m_{p,U^{235}} = m_{f,U^{235}} - m_{w,U^{235}} \tag{1c}$$

Then find the product enrichment.

$$x_p = \frac{m_{p,U^{235}}}{m_p} = 2.65\% \tag{1d}$$

```python
# data
mf = 32_000  # mass feed, kgU/day
mt = 26_000  # mass tails, kg tails/day
```

```
xf = 0.7 / 100  # feed enrichment
xt = 0.25 / 100  # tails enrichment

# calculations
uf = mf * xf  # u mass feed
ut = mt * xt  # u mass tails

mp = mf - mt  # mass enriched
up = uf - ut  # u enriched

xp = ue / me  # product enrichment

print(f"Product Assay: {round(xp, 4) * 100}%")
assert(mp * xp + mt * xt == mf * xf)
```

2. (5 points) A gaseous diffusion plant uses natural uranium feed and enriches it to 2.9% of $^{235}U$ If the feed stream is 35,000kgU/day and the product is 6000kgU/day, what is the tails assay and the amount of uranium per day going into the tails? (Tsoulfanidis, Question 3.2)

**Solution:** The tails assay is 0.24% and the uranium per day going into tails is 29,000 kg. I checked my answers with Mauhmoud Eltawila and Nathan Glaser.

Find the mass of the waste/tails.

$$m_w = m_f - m_p = 29,000 \frac{kg}{day} \tag{2a}$$

Find the mass of U235 in each quantity $x$.

$$m_{x,U^{235}} = m_x * x_x \tag{2b}$$

Find the mass of U235 in the tails.

$$m_{w,U^{235}} = m_{f,U^{235}} - m_{p,U^{235}} \tag{2c}$$

Find the enrichment of the tails.

$$x_w = \frac{m_{w,U^{235}}}{m_w} = 0.24\% \tag{2d}$$

```
# data
xf = 0.7 / 100
xp = 2.9 / 100

mf = 35000
mp = 6000
```

```python
# calculations
# find tail mass and tail enrichment
uf = mf * xf
up = mp * xp

mt = mf - mp
ut = uf - up
xt = ut / mt

print(f"Tails Mass: {mt} kg")
print(f"Tails Assay: {round(xt, 4) * 100}%")

assert(ut + up == uf)
assert(mt + mp == mf)
assert(mp * xp + mt * xt == mf * xf)
```

3. (10 points) The gaseous diffusion method has been proposed for use in production of $BF_3$ enriched to 90% in $^{10}B$. How many kilograms of $BF_3$ feed (natural boron) are needed to produce 1 kg of enriched product with 8% tails? (Tsoulfanidis, Question 3.5)

**Solution:** Assuming a natural 20% enrichment of B10, the feed mass to obtain 1 kg of 90% B10 product is 6.83 kg. I checked my answers with Mauhmoud Eltawila and Nathan Glaser.

Find the mass percent given two isotopes from given isotopic enrichment.

$$x_{m1} = \frac{x_{i1}M_1}{x_{i1}M_2 + x_{i2}M_2} x_{m2} = \frac{x_{i1}M_2}{x_{i1}M_2 + x_{i2}M_2} \tag{3a}$$

Then, calculate the feed mass assuming 1 kg of product.

$$m_f = m_p \frac{x_p - x_w}{x_f - x_w} \tag{3b}$$

```python
# data
m_b10 = openmc.data.atomic_mass("B10")
m_b11 = openmc.data.atomic_mass("B11")

[(_, xi_b10), (_, xi_b11)] = openmc.data.isotopes("B")

def mass_percent(xi1, u1, u2):
    m1 = u1 * xi1
    xi2 = 1 - xi1
    m2 = u2 * xi2
```

```
    xm1 = m1 / (m1 + m2)
    xm2 = m2 / (m1 + m2)
    return xm1

# calculations
xf = 20 / 100 # assuming 20% mass enrichment
xp = 90 / 100
xt = 8 / 100

xf = mass_percent(xf, m_b10, m_b11)
xp = mass_percent(xp, m_b10, m_b11)
xt = mass_percent(xt, m_b10, m_b11)

mp = 1

# calculations
mf = (xp - xt) / (xf - xt) * mp
print(f"Feed Mass: {round(mf, 2)} kg")
```

(a) (1 point (bonus)) Do some research. Why would anyone want to enrich boron to 90% $^{10}B$?

**Solution:** Boron 10 is useful for boron coated straw detectors. These are useful for neutron multiplicity counting. This is a method to determine the enrichment of a mass of uranium by time correlating the neutrons detected in separate straws around the same instant.

(b) (1 point (bonus)) Do some research. What could one use the tails for?

**Solution:** One could use the tails to dope semiconductors made of silicon as the doping makes junction depths easy to control.

4. (10 points) Assume that Highly Enriched Uranium (HEU) from weapons dismantlement containing 90% $^{235}U$ is blended with depleted uranium with 0.2% $^{235}U$. Under these conditions, how many kg of 4% enriched fuel can be made per kg of HEU? (Tsoulfanidis, Question 3.6)

**Solution:** From 1 kg of HEU, you can create 23.63 kg of down-blended fuel. I checked my answers with Mauhmoud Eltawila and Nathan Glaser.

As down-blending is the opposite of enrichment, we can use inverted equation for the ratio of feed mass to product mass. In this equation, the fuel (F) acts like the feed, the HEU (H) acts like the product, and the DU (D) acts like the waste.

$$F = P\frac{x_p - x_w}{x_f - x_w} \tag{4a}$$

$$F = H\frac{x_h - x_d}{x_f - x_d} = 23.63\ kg \tag{4b}$$

```
# data
xh = 90 / 100    # acts like product
xd = 0.2 / 100   # acts like waste
xf = 4 / 100     # acts like feed

# calculations
mf = ((xh - xd) / (xf - xd))

print(f"Mass Fuel: {round(mf, 2)} kg")
```

5. (10 points) Natural uranium ore has not always been 0.711%$^{235}U$ And, many years from now, the percent will be even lower. Calculate (and show your work) the $^{235}U$ percentage in natural uranium that would have been common 1.8 billion years ago. In Oklo, Gabon, it was enough to sustain a natural fission reactor. (Tsoulfanidis, Question 3.7)

**Solution:** The natural enrichment of U235 1.8 billion years ago was 3.05%. I checked my answers with Mauhmoud Eltawila and Nathan Glaser.

We calculate the decay constants based on the half lives give.

$$\lambda = \frac{\ln(2)}{t_{1/2}} \tag{5a}$$

We know looking into the future is exponential decay, so looking into the past is exponential growth with the same form as radionuclide decay. We can assume a mass of 1kg of current U, which allows us to use $x$ as the current mass.

$$m_{past} = x_{current}e^{\lambda t} \tag{5b}$$

We can then find the enrichment by taking a ratio.

$$x_{past,U235} = \frac{m_{past,U235}}{m_{past,U238}} \cdot 100 = 3.05\% \tag{5c}$$

```
# data
x235 = 0.711 / 100
x238 = 1 - x_u235

# from Q6, ignoring U234, U236
th235 = 7.1e8 # years
th238 = 4.51e9 # years
```

```
# decay constants
lam235 = np.log(2) / th235
lam238 = np.log(2) / th238

# past mass
time = 1.8e9
pm235 = x235 * np.exp(lam235 * time)
pm238 = x238 * np.exp(lam238 * time)

# past enrichment
past_x235 = pm235 / (pm235 + pm238) * 100

print(f"Past U235 Enrichment: {round(past_x235, 2)}%")
```

6. (20 points) Calculate the natural uranium feed and SWU factors one billion years into the future. Assume tails of 0.20% and 4.5% enriched product. Note that $\tau_{\frac{1}{2},{}^{235}U} = 7.1 \times 10^8 y$ and $\tau_{\frac{1}{2},{}^{238}U} = 4.51 \times 10^9 y$.

**Solution:** The SWU based on the enrichment 1 billion years into the future is 14.09 SWU. I checked my answers with Mauhmoud Eltawila and Nathan Glaser.

Find the decay constant.

$$\lambda = \frac{\ln(2)}{t_{1/2}} \tag{6a}$$

Find the future mass using the equation for radionuclide decay. Assume 1 kg of U today, so $m_x = x_x$

$$m_{future} = x_{current} \exp -$$

(6b)

Find the abundance of U235 in the future.

$$x_{future,U235} = \frac{m_{future,U235}}{m_{future,U235} + m_{future,U238}} \tag{6c}$$

Find the separation potential as a function of $x$.

$$V(x) = (2x - 1) \ln \frac{x}{1-x} \tag{6d}$$

Find the mass of F and W assuming P = 1. Where $x_p$ and $x_w$ are given and $x_f$ is the future enrichment of U235.

$$F = P\frac{x_p - x_w}{x_f - x_w} W = P\frac{x_p - x_f}{x_f - x_w} \tag{6e}$$

Find the SWU value with the future conditions.

$$SWU = P \cdot V(x_p) + W \cdot V(x_w) - F \cdot V(x_f) = 14.09\ SWU \tag{6f}$$

```python
# current enrichments
x235 = 0.711 / 100
x238 = 1 - x_u235

# ignoring U234, U236
th235 = 7.1e8 # years
th238 = 4.51e9 # years

# decay constants
lam235 = np.log(2) / th235
lam238 = np.log(2) / th238

# future mass
time = 1e9
fm235 = x235 * np.exp(-lam235 * time)
fm238 = x238 * np.exp(-lam238 * time)

# future enrichments
future_x235 = fm235 / (fm235 + fm238)
future_x238 = fm238 / (fm235 + fm238)

# seperation potential, swu
def sep_pot(x):
    return (2 * x - 1) * np.log(x / (1 - x))

# swu
def swu(xf, xp, xw):
    p = 1
    f = (xp - xw) / (xf - xw)
    w = (xp - xf) / (xf - xw)

    vp = sep_pot(xp)
    vf = sep_pot(xf)
    vw = sep_pot(xw)

    return 1 * (p * vp + w * vw - f * vf)

future_swu = swu(future_x235, xp=4.5/100, xw=0.2/100)
print(f"Future SWU: {round(future_swu, 2)} SWU")
```

7. (20 points) Assuming that the price per SWU is \$80 and the cost of conversion is \$4/kgU,

what is the price of the $U_3O_8$ in $\frac{\$}{lb_{U_3O_8}}$ beyond which it will cost less to enrich the already mined, purified, and converted (to UF 6 ) tails that contain 0.2% $^{235}U$ rather than mine new uranium? Assume the product will be 3% enriched in U in either case and the new tails will be 0.1% (when the old tails are enriched). Tails stored as $UF_6$ cost nothing. (Tsoulfanidis, Question 3.13)

**Solution:** For enriching converted tails to be profitable, the cost of yellowcake needs to be above \$334.38 per lb. I checked my answers with Mauhmoud Eltawila and Nathan Glaser.

Find the cost of converting the tails $c_{tails}$ as the product of the price per SWU and the SWU of the tails.

$$p_{tails} = p_{SWU} \cdot SWU_{tails} \tag{7a}$$

To calculate the cost of yellowcake required, find the mass fraction of U in yellowcake.

$$mf = \frac{3M_U}{3M_U + 8M_O} \tag{7b}$$

We know the cost of new fuel is given by the cost of enrichment plus the cost of conversion.

$$p_{new} = p_{SWU} SWU_{new} + \frac{p_{U3O8}}{mf} \tag{7c}$$

We can rearrange for the cost of yellowcake.

$$p_{U3O8} = mf\left(p_{new} - p_{SWU} SWU_{new}\right) \tag{7d}$$

For the cost of new yellowcake to be more expensive than refining old tails, $p_{new}$ needs to be the cost of enriching old tails

$$p_{U3O8} = mf\left(p_{tails} - p_{SWU} SWU_{new}\right) \tag{7e}$$

Want the price in \$ per lb, so convert.

$$p_{U3O8}/ = 2.2 = \$334.38 \; per \; lb \tag{7f}$$

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```
# prices
```

8. It is stated in Section 3.6 that in the US $700,000t$ of $UF_6$, tails of the enrichment process, are stored. (Tsoulfanidis, Question 3.16, **note correction**)

   (a) (10 points) How many kilograms of fuel enriched to 4.5% in $^{235}U$ can be produced if these tails are reinserted into the enrichment process? Assume that the tails contain **0.25%** $^{235}U$ and the new tails will go down to 0.15% $^{235}U$.

   **Solution:** The mass of fuel that could be produced at these variables is 16.092 million kg. I checked my answers with Mauhmoud Eltawila and Nathan Glaser.

Directly calculate the mass that can be produced.

$$m_f = m_t / \frac{x_f - x_w}{x_t - x_w} = 16.092e6\ kg \tag{8}$$

```python
# part a
m_tails = 7e8

# enrichments
x_tails = 0.25 / 100
x_fuel = 4.5 / 100
x_waste = 0.15 / 100

# mass of product
# tails are feed, fuel is product, waste is waste
mp = m_tails / ((x_fuel - x_waste) / (x_tails - x_waste))
print(f"Mass of Product: {round(mp * 1e-6, 3)}M kg")
```

(b) (10 points) At current prices, what would be the cost of one kg of such enriched fuel (leave out fabrication; consider just enriched fuel).

**Solution:** One kg of fuel would cost $1521.40. I checked my answers with Mauhmoud Eltawila and Nathan Glaser.

Assume the price of one SWU is $80 and use the equation for SWU from before.

$$p_{fuel} = SWU(x_{tails}, x_{fuel}, x_{waste}) p_{SWU} = \$1521.4\ per\ kg \tag{9}$$

```python
# part b
p_swu = 80
swu_tails = swu(x_tails, x_fuel, x_waste)

p_fuel = swu_tails * p_swu
print(f"Cost per kg: ${round(p_fuel, 2)}/kg")
```