

HW#5

6.2) In flow over a surface, velocity & temp profiles are of forms:

$$u(y) = Ay + By^2 - Cy^3$$

$$T(y) = T_\infty + E_y + Fy^2 - Gy^3$$

A, B are constants. Find C_f & h in terms of u_∞, T_∞ , & constants.

$$(b) \tau_s = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0} = \mu (A + 2By - 3Cy^2) \Big|_{y=0} = \mu A$$

$$C_f \equiv \frac{2\tau_s}{\rho u_\infty^2} = \frac{2\mu A}{\rho u_\infty^2} = C_f = \frac{2\nu A}{u_\infty^2}$$

$$h = \frac{-k \left. \frac{\partial T}{\partial y} \right|_{y=0}}{T_s - T_\infty} = \frac{-k (E + 2Fy - 3Gy^2) \Big|_{y=0}}{T_s - T_\infty} = \frac{-kE}{T_s - T_\infty} = h$$

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6.3) BL temp distribution $\sim \frac{T-T_s}{T_\infty-T_s} = 1 - \exp\left(-Pr \frac{u_\infty y}{\nu}\right)$

$Pr = c_p \mu / k = 0.7$, $T_\infty = 400\text{ K}$, $T_s = 300\text{ K}$, $u_\infty / \nu = 5000\text{ 1/m}$

6.3) $\Theta = T - T_s \Rightarrow \frac{\Theta}{\Theta_\infty} = 1 - \exp\left(-Pr \frac{u_\infty y}{\nu}\right) \Rightarrow \Theta = \Theta_\infty - \Theta_\infty \exp\left(-Pr \frac{u_\infty y}{\nu}\right)$

$\frac{\partial \Theta}{\partial y} = \frac{\partial T}{\partial y} \Rightarrow q_s'' = -k_f \left. \frac{\partial T}{\partial y} \right|_{y=0} = -k_f \left(\Theta_\infty Pr \frac{u_\infty}{\nu} \exp\left(-Pr \frac{u_\infty y}{\nu}\right) \right)$

$\Rightarrow q_s'' = -k_f \left(\Theta_\infty Pr \frac{u_\infty}{\nu} \right)$ Table A.4, $k_f = 26.3 \times 10^{-3}\text{ W/m}\cdot\text{K}$

$q_s'' = -(26.3 \times 10^{-3})(400 - 300\text{ K})(0.7)(5000\text{ m}^{-1}) = \boxed{-9205\text{ W/m}^2 = q_s''}$

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6.5) Laminar flow over a flat plate, $\Delta x \propto x^{-1/2}$
where x is distance from leading edge ($x=0$)

Find ratio of $\bar{\Delta x} / \Delta x$ @ some point x ?

6.14 $\bar{\Delta x} = \frac{1}{L} \int_0^L \Delta x \, dx = \frac{1}{L} \int_0^L C x^{-1/2} \, dx = \frac{C}{L} \left(2 x^{1/2} \right) \Big|_0^L = \frac{2C}{L} L^{1/2}$

say $L=x \therefore \bar{\Delta x} = \frac{2C}{\sqrt{x}} \quad \frac{\bar{\Delta x}}{\Delta x} = \frac{2C}{\sqrt{x}} \cdot \frac{\sqrt{x}}{C} = 2$

$$\boxed{\frac{\bar{\Delta x}}{\Delta x} = 2}$$

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- 8.5) An engine oil cooler consists of 25 smooth tubes, w/
 $L = 2.5 \text{ m}$ & $D = 10 \text{ mm}$.

- a) If oil @ 300 K & flow rate of 24 kg/s is fully developed, what is pressure drop & pump power?

$$A = 25 \left(\pi \frac{D^2}{4} \right) = \frac{25 \pi D^2}{4}, \quad \dot{m} = \rho A V \Rightarrow V = \frac{\dot{m}}{\rho A}$$

Table A-5

$$\text{w/ } \rho = 884.1 \text{ kg/m}^3, \quad V = \frac{24}{(884.1) \left(\frac{25}{4} \pi (10^{-3})^2 \right)} = 13.8255 \text{ m/s} = V$$

$\gamma = 550 \text{e-6}$

$$\text{say } Re = \frac{V D}{\gamma} = \frac{(13.827)(10^{-3})}{550 \text{e-6}} = 251.4 = Re$$

$$\text{w/ } f = \frac{64}{Re} = 0.2546 \quad \& \quad \Delta P = f \frac{\rho V^2 L}{2D}$$

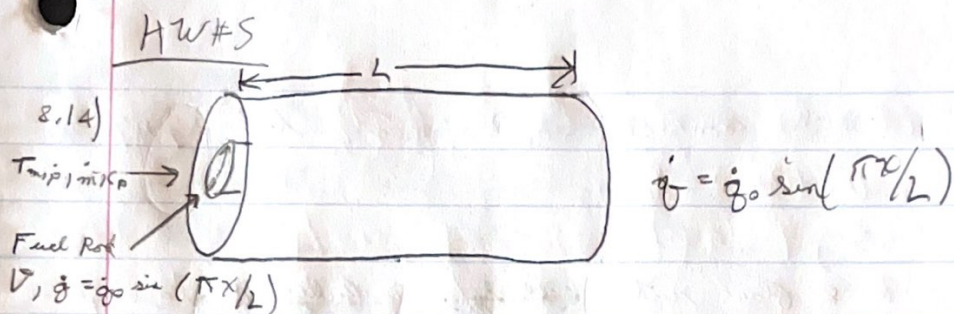
$$\Delta P = \frac{(0.2546)(884.1)(13.826)^2(2.5)}{2(10^{-3})} = 5.378 \text{e-6} \frac{\text{N}}{\text{m}^2} = \Delta P$$

$$P = \frac{\Delta P \dot{m}}{\rho} = \frac{\Delta P (24)}{884.1} = 146 \text{ kW}$$

b)

$$\Delta P = 224,090.16 \text{ in}$$
$$P = 253.47 \text{ in}^2$$

check plots



a) Find $q''(x)$ & total q from rod to water.

SS, no heat in: $\dot{E}_{gen} = \dot{E}_{out}$

$$\Rightarrow \dot{q} V = \dot{q}'' A \Rightarrow q'' = \frac{\dot{q} V}{A} = \dot{q}_0 \sin\left(\frac{\pi x}{L}\right) \frac{(L \pi \frac{D^2}{4})}{\pi D L}$$

$$q'' = \frac{D}{4} \dot{q}_0 \sin\left(\frac{\pi x}{L}\right)$$

$$q = \frac{D \dot{q}_0}{4} \int_0^L \sin\left(\frac{\pi x}{L}\right) dx = \frac{D \dot{q}_0}{\pi 4} \left[-\frac{L}{\pi} \cos\left(\frac{\pi x}{L}\right) \right]_0^L = \frac{D L \dot{q}_0}{2} = q$$

b) Ex 8.36) $dq_{rod} = \dot{m} c_p dT_m$

$$\text{say } dq = ''(\pi D dx) \Rightarrow dT_m = \frac{\pi D}{\dot{m} c_p} \left[\frac{D}{4} \dot{q}_0 \sin\left(\frac{\pi x}{L}\right) \right] dx \Rightarrow$$

$$T_m - T_{m,0} = \frac{D^2 \pi \dot{q}_0}{4 \dot{m} c_p} \left[-\frac{L}{\pi} \cos\left(\frac{\pi x}{L}\right) \right]_0^x = \frac{-L D^2 \dot{q}_0}{4 \dot{m} c_p} (\cos(\frac{\pi x}{L}) - 1)$$

$$T_m(x) = T_{m,0} + \frac{L D^2 \dot{q}_0}{4 \dot{m} c_p} (1 - \cos(\frac{\pi x}{L}))$$

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- c) Find expression for $T_s(x)$ w/ x along tube.
Expression for x -loc @ which temp is maxed.

Newton's Law of cooling: $q'' = h(T_s - T_m)$

$$\Rightarrow T_s = \frac{q''}{h} + T_m = \frac{\dot{V} \dot{q}_0 \sin\left(\frac{\pi x}{L}\right)}{4A} + T_{m,0} + \frac{L \dot{V}^2 \dot{q}_0 (1 - \cos\left(\frac{\pi x}{L}\right))}{4 \dot{m} c_p} = T_s$$

Take derivative wrt x

$$\frac{dT_s}{dx} = \frac{\dot{V} \dot{q}_0 L}{4A \pi} \cos\left(\frac{\pi x}{L}\right) + \frac{L^2 \dot{V}^2 \dot{q}_0}{4 \dot{m} c_p \pi} \sin\left(\frac{\pi x}{L}\right) = 0$$

$$\Rightarrow \frac{-L \dot{V} \sin\left(\frac{\pi x}{L}\right)}{\dot{m} c_p} = \frac{\cos\left(\frac{\pi x}{L}\right)}{L} \Rightarrow \tan\left(\frac{\pi x}{L}\right) = \frac{\dot{m} c_p}{L \dot{V}}$$

$$x = \frac{L}{\pi} \arctan\left(\frac{\dot{m} c_p}{\dot{V} L h}\right) = x_{\max}$$