NPRE412 Spring 2025
HW 7 Due 2025.03.11

- This work must be submitted online as a `.pdf` through Canvas.
- Work completed with LaTeX or Jupyter earns 1 extra point. Submit source file (e.g. `.tex` or `.ipynb`) along with the `.pdf` file.
- If this work is completed with the aid of a numerical program (such as Python, Wolfram Alpha, or MATLAB) all scripts and data must be submitted in addition to the `.pdf`.
- If you work with anyone else, document what you worked on together.

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1. (30 points) Write explicitly the multigroup equations for the following conditions (Tsoulfanidis, 5.1):

   - four groups,
   - no upscattering,
   - fissions take place in the last two groups only,
   - fission neutrons appear in the first two groups only, 79% of them in the 1st group

Solution:

The general multi-group neutron diffusion equation is

$$-\nabla D_g(r)\nabla\phi_g(r) + \Sigma_{R,g}(r)\phi_g(r) = \Sigma_{g'\neq g}^{G}\Sigma_s^{g'\to g}(r)\phi_{g'}(r) + \chi_g\Sigma_{g'=1}^{G}\left(\nu\Sigma_f\right)_{g'}\phi_{g'}(r) \tag{1}$$

You were right, I really do not like how he grouped fission. Using the ridiculous formulation from the textbook, the multi-group equations for the given conditions become:

$$-\nabla D_1(r)\nabla\phi_1(r) + \Sigma_{R,1}(r)\phi_1(r) = [\chi_1 = 0.79]\Sigma_{g'=1}^{G}\left(\nu\Sigma_f\right)_{g'}\phi_{g'}(r)$$

$$-\nabla D_2(r)\nabla\phi_2(r) + \Sigma_{R,2}(r)\phi_2(r) = \Sigma_{g'\neq 2}^{G}\Sigma_s^{g'\to 2}(r)\phi_{g'}(r) + [\chi_2 = 0.21]\Sigma_{g'=1}^{G}\left(\nu\Sigma_f\right)_{g'}\phi_{g'}(r)$$

$$-\nabla D_3(r)\nabla\phi_3(r) + \Sigma_{R,3}(r)\phi_3(r) = \Sigma_{g'\neq 3}^{G}\Sigma_s^{g'\to 3}(r)\phi_{g'}(r)$$

$$-\nabla D_4(r)\nabla\phi_4(r) + \Sigma_{R,4}(r)\phi_4(r) = \Sigma_{g'\neq 4}^{G}\Sigma_s^{g'\to 4}(r)\phi_{g'}(r)$$

2. The two-group constants for a cylindrical homogeneous PWR follow. The reactor dimensions are H=3.2 m and R=1.5 m (Tsoulfanidis, 5.3).

Constant	**1**	**2**
$\nu\Sigma_f$	0.0	0.33
χ	1.0	0.00
Σ_a	0.012	0.15
D	1.25	0.38
Σ_R	0.035	0.15
$\sigma_{a,B}$	0.0	$2250b$

(a) (20 points) Calculate the value of k.

Solution: The value of k is 1.42641.

To solve this equation, use the cylinder buckling formula given in Table 5.1 in the textbook. Where J_0 is the 0th order term of the Bessel Function of the first kind.

$$B^2 = \frac{\nu_0^2}{R^2} + \frac{\pi^2}{H^2}$$

Next, use Eq. 5.42 in the textbook to find k. Also, cancel out the terms that are zero.

$$k = \frac{(\nu\Sigma_f)_f}{\Sigma_{R,f} + D_f B^2} + \frac{\Sigma_s^{1\to2}}{\Sigma_{R,f} + D_f B^2} + \frac{(\nu\Sigma_f)_{th}}{\Sigma_{a,th} + D_{th} B^2}$$

$$k = \frac{\Sigma_s^{1\to2}}{\Sigma_{R,f} + D_f B^2} + \frac{(\nu\Sigma_f)_{th}}{\Sigma_{a,th} + D_{th} B^2}$$

Plugging in the given values gives $k = 1.42641$.

(b) (20 points) Calculate the boron concentration (in ppm) that will make the reactor exactly critical.

Solution: For a critical system, you need 510.759 ppm of boron. I find it rather odd that ppm is based on the mass and not number. Checked with Mahmoud and Nathan.

Modify the equation from part a to account for the number density of boron.

$$k = \frac{\Sigma_s^{1\to2}}{\Sigma_{R,f} + D_f B^2} + \frac{(\nu\Sigma_f)_{th}}{(\Sigma_{a,th} + N_B\sigma_{a,B}) + D_{th} B^2} \tag{2}$$

Use a root finding to solve for the previous function minus 1 to find N_B where $k = 1$.

$$N_B = 2.84531e19\frac{g}{cc} \tag{3}$$

Find the mass density of boron.

$$\rho_B = \frac{N_B M_B}{N_A} \tag{4}$$

Find ppm, strangely, by taking mass density ratio boron and water and multiplying by 1 million. Checked with Mahmoud and Nathan.

$$ppm_B = 1e6 \cdot \frac{\rho_B}{\rho_w} = 510.759\ ppm \tag{5}$$

3. (30 points) Neutron average cross sections in an eight-group structure are given below. Obtain cross sections for three groups, using the group fluxes and new (broad) group boundaries indicated in the table (Tsoulfanidis, 5.5).

Group	**Energy Upper [MeV]**	**Cross Section [b]**	**Flux [$\times 10^{12}$]**	**Broad Group Energy Boundaries [MeV]**
1	10	1.5	1.3	10
2	6	3.5	1.1	
3	1	6.0	1.2	1
4	0.5	12.0	12.1	
5	0.25	12.0	15.0	
6	0.1	7.0	17.0	0.1
7	0.05	45.0	20.0	
8	1.0×10^{-7}	115.0	55.0	
Lowest Energy = 1.0×10^{-8}				

Solution: Using Eq 5.83 from the book, the broad group cross section is 2.528 b for G1, 11.387 b for G2, 27.541 b for G3. Checked with Mahmoud and Nathan.

$$\sigma_g = \frac{\Sigma_h \sigma_h(r)\phi_h(r)(\Delta E)_h}{\Sigma_h \phi_h(r)(\Delta E)_h} \tag{6}$$

Use the right bounds and you solve the problem.