HW#5

1) 

a) $\frac{1}{v}\frac{\partial \phi}{\partial t} - \nabla\cdot(D\nabla\phi) + \left(\Sigma_a - \frac{1}{k}\nu\Sigma_f\right)\phi = 0$, w/ $\phi(x,t) = \mathcal{X}(x)T(t)$

$\frac{\partial\phi}{\partial t} - Dv(\nabla^2\phi) - \left(\frac{1}{k}\nu\Sigma_f - \Sigma_a\right)\phi v = 0$

$\Rightarrow \frac{\partial\phi}{\partial t} - Dv\,\nabla^2\phi + \left(\frac{\nu}{k}\Sigma_f - \Sigma_a\right)\phi v = 0$

partials: $\frac{\partial\phi}{\partial t} = \mathcal{X}(x)\frac{\partial T(t)}{\partial t}$, $\nabla^2\phi = T(t)\,\nabla^2\mathcal{X}(x)$

$\Rightarrow \mathcal{X}\frac{\partial T}{\partial t} - T Dv(\nabla^2\mathcal{X}) - \left(\frac{1}{k}\nu\Sigma_f - \Sigma_a\right)v\,\mathcal{X}T = 0$ divide by $\mathcal{X}T$

$\Rightarrow \underbrace{\frac{1}{T}\frac{\partial T}{\partial t}}_{-\lambda} \underbrace{- \frac{1}{\mathcal{X}}Dv(\nabla^2\mathcal{X}) - \left(\frac{1}{k}\nu\Sigma_f - \Sigma_a\right)v}_{\lambda} = 0$, equal to ± same constant as no other way to be equal

$\Rightarrow \nabla^2\mathcal{X} + \frac{\left(\frac{1}{k}\nu\Sigma_f - \Sigma_a\right)}{D}\mathcal{X} = \frac{-\lambda\mathcal{X}}{vD} \Rightarrow \nabla^2\mathcal{X} + \underbrace{\frac{\left(\frac{1}{k}\nu\Sigma_f - \Sigma_a + \frac{\lambda}{v}\right)}{D}}_{B^2}\mathcal{X} = 0$

$\Rightarrow \nabla^2\mathcal{X} + B^2\mathcal{X} = 0 \Rightarrow \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\mathcal{X}}{\partial r}\right) + B^2\mathcal{X} = 0$

soln is Bessel Functions: $\mathcal{X} = C_1 J_n(B_n r) + \cancel{C_2} Y_n(B_n r)$ ← space

now solve $\frac{1}{T}\frac{\partial T}{\partial t} = -\lambda_n \Rightarrow \frac{\partial T}{\partial t} = -\lambda_n T \Rightarrow T = C_3\exp(-\lambda_n t)$ ← time

BC) i) $-D\nabla\mathcal{X}\big|_{r=0} = 0 \Rightarrow \frac{\partial\mathcal{X}}{\partial r}\big|_{r=0} = 0$

$\Rightarrow \frac{\partial\mathcal{X}}{\partial r} = \left(\frac{n B_n C_1}{r}J_n(B_n r) - C_1 J_{n+1}(B_n r)\right) + \left(\frac{-B_n C_2}{r}Y_n(B_n r) - C_2 Y_{n+1}(B_n r)\right)$

@ $r=0$, $Y_n \to -\infty$ $\therefore C_2 = 0$

ii) $\mathcal{X}(\tilde{R}) = 0 \Rightarrow \sum_{n=0}^{\infty} C_n J_n(B_n\tilde{R}) = 0$, B_n = first root for all J_n

but first root for $n \neq 0$ of $J_n = 0$ $\therefore \mathcal{X} = C_0 J_0(B_n\tilde{R})$

$\therefore B_n\tilde{R} = \underset{\text{root of } J_0}{\nu_n} \Rightarrow B_n = \frac{\nu_n}{\tilde{R}}$

HW #5

now multiply T_0 by λ_0 (all other $\lambda_n = 0$, so $\lambda_n T_n = 0$ when $n \neq 0$)

$$\boxed{\phi = \sum_{n=0}^{\infty} C_n J_0(B_n \tilde{R}) \exp(-\lambda_n t)}$$

b) say $\phi = C_0 J_0(B_0 \tilde{R}) \exp(-\lambda_0 t) + \sum_{n=1}^{\infty} C_n J_0(B_n \tilde{R}) \exp(-\lambda_n t)$

but $\lambda_0 = 0$ as it is the fundamental mode

$\Rightarrow \phi = C_0 J_0(B_0 \tilde{R}) \exp(0) + \sum_{n=1}^{\infty} C_n J_0(B_n \tilde{R}) \exp(-\lambda_n t)$

@ $t = \infty$, $\phi = C_0 J_0(B_0 \tilde{R}) + 0$ $\therefore$ $\boxed{\phi = C_0 J_0(B_0 \tilde{R})}$

c) see Buckling: $B_n^2 = (\nu_n / \tilde{R})^2$, but only interested in $n = 0$

$$\boxed{\therefore B_0^2 = (\nu_0 / \tilde{R})^2}$$

d) $B_g^2 = B_m^2 \Rightarrow \boxed{\left(\frac{\nu_0}{\tilde{R}}\right)^2 = \frac{\frac{1}{k}\nu \Sigma_f - \Sigma_a}{D}}$ $\boxed{k = 1}$ $\boxed{\lambda_0 = 0}$

e) know $P \cdot L = \int dV \, \Sigma_f C_0 J_0(B_0 \tilde{R}) \cdot E_f \leftarrow$ energy / fission

$\Rightarrow P \cdot L = \int_0^L dz \int dA \, \Sigma_f C_0 J_0(B_0 \tilde{R}) \cdot E_f \Rightarrow P = \int dA \, \Sigma_f C_0 J_0(B_0 \tilde{R}) \cdot E_f$

$\Rightarrow C_0 = P / \left(\int dA \, \Sigma_f \, J_0(B_0 \tilde{R}) \cdot E_f\right)$

$$\Rightarrow \boxed{C_0 = \frac{P}{\Sigma_f J_0(B_0 \tilde{R}) \cdot E_f} \cdot \frac{1}{\int_0^{2\pi} d\varphi \int_0^R r \, dr}}$$

f) $\left(\frac{\nu_0}{\tilde{R}}\right)^2 = \frac{\frac{1}{k}\nu \sigma_f N - \sigma_a N}{1} \cdot \frac{3\sigma_{tr} N}{1} \Rightarrow \frac{\nu_0}{R} = \sqrt{N^2 (3\sigma_{tr})(\frac{1}{k}\nu\sigma_f - \sigma_a)}$

see $\frac{1}{R} \propto \sqrt{N^2} = N$ $\boxed{\therefore \text{doubling } N, \text{ halves } R \;\therefore\; R_c = 25 \text{ cm}}$

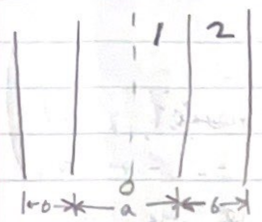
HW #5

2)

a) False, geometric buckling is related to the rate of n° leakage while material buckling is related to the difference in n° production & absorption. as criticality is related to the number of neutrons, the more critical reactor will have a relatively lower leakage rate (geometric buckling) to neutron production (material buckling).

b) True, as $\lambda_n < \lambda_{n+1}$, all subsequent modes decay faster than the previous mode. as $t \to \infty$, the mode w/ the lowest λ will be the most prevalent, which is always the fundamental mode.

HW#5

3) 

a) $-\nabla\cdot(D_1\nabla\phi_1) + \Sigma_a\phi_1 = \nu\Sigma_f\phi\frac{1}{k}$

↳ $\nabla^2\phi_1 + \frac{\frac{1}{k}\nu\Sigma_f - \Sigma_a}{D_1}\phi_1 = 0$ w/ $B_1^2 = \frac{\frac{1}{k}\nu\Sigma_f - \Sigma_a}{D_1}$

$\Rightarrow \nabla^2\phi_1 + B_1^2\phi_1 = 0 \Rightarrow \frac{d^2\phi_1}{dx^2} + B_1^2\phi_1 = 0$

$\phi_1 = C_1\cancel{\sin}(B_1x) + C_2\cos(B_1x)$

1

BC) ✓ i) $-D_1\frac{d\phi_1}{dx}\Big|_{x=0} = 0$ ii) $\phi_1(x=\frac{a}{2}) = \phi_2(x=\frac{a}{2})$ ✓

✓ iii) $-D_1\frac{d\phi_1}{dx}\Big|_{x=a/2} = -D_2\frac{d\phi_2}{dx}\Big|_{x=a/2}$ iv) $\phi_2(\tilde{x}) = 0$ ✓

2 $\left\{ \begin{array}{l} -\nabla\cdot(D_2\nabla\phi_2) + \Sigma_a\phi_2 = 0 \Rightarrow \nabla^2\phi_2 - \frac{1}{L_2^2}\phi_2 = 0 \quad ; \quad \tilde{x} = (\frac{a}{2}+b) + 2D_2 \\ \Rightarrow \phi_2 = C_4\sinh(\frac{x}{L_2}) + C_5\cosh(\frac{x}{L_2}) = C_3\sinh(\frac{\tilde{x}-x}{L_2}) \end{array} \right.$ ← iv) embedded

i) $-D_1\left[C_1B_1\cos(B_1x) - C_2B_1\sin(B_1x)\right]\Big|_{x=0} = 0 \Rightarrow C_1B_1 + 0 = 0 \quad \therefore C_1 = 0$

ii) $C_2\cos(B_1\frac{a}{2}) = C_3\sinh(\frac{\tilde{x}-a/2}{L_2}) \Rightarrow C_2 = C_3\sinh(\frac{\tilde{x}-a/2}{L_2}) / \cos(B_1\frac{a}{2})$

iii) $D_1\left[-C_2B_1\sin(B_1\frac{a}{2})\right] = D_2\left[\frac{-C_3}{L_2}\cosh(\frac{\tilde{x}-a/2}{L_2})\right]$

↳ $D_1\left[\left(C_3\sinh(\frac{\tilde{x}-a/2}{L_2})/\cos(B_1\frac{a}{2})\right)\sin(B_1\frac{a}{2})\right] = D_2\left[\frac{C_3}{L_2}\cosh(\frac{\tilde{x}-a/2}{L_2})\right]$

↳ $D_1\sinh(\frac{\tilde{x}-a/2}{L})\tan(B_1\frac{a}{2}) = \frac{D_2}{L_2}\cosh(\frac{\tilde{x}-a/2}{L_2})$

$$\boxed{\begin{array}{l} \phi_1 = C_3\frac{\sinh(\frac{\tilde{x}-a/2}{L_2})}{\cos(B_1\frac{a}{2})}\cos(B_1x) \quad , \quad \phi_2 = C_3\sinh(\frac{\tilde{x}-x}{L_2}) \\ \text{criticality: } D_1\sinh(\frac{\tilde{x}-a/2}{L_2})\tan(B_1\frac{a}{2}) = \frac{D_2}{L_2}\cosh(\frac{\tilde{x}-a/2}{L_2}) \end{array}}$$

$\tilde{x} = (\frac{a}{2}+b) + 2D_2$

$$\tilde{x} = \left(\frac{a}{2}+b\right)+2\sigma_2$$

HW#5

b) $P \cdot A = \int d\forall\, \Sigma_f \phi_1 E_f + \int d\forall\, \Sigma_s \phi_2 E_s$

$$P \cdot A = \left(2A \int_0^{a/2} dx\, \Sigma_f\, C_3 \frac{\sinh\left(\frac{\tilde{x}-a/2}{L_2}\right)}{\cos\left(B_1 \frac{a}{2}\right)} \cos(B_1 x) E_f\right) + \left(2A \int_{a/2}^{a/2+b} dx\, \Sigma_s\, C_3 \sinh\left(\frac{\tilde{x}-x}{L_2}\right) E_s\right)$$

$$P = 2\Sigma_f C_3 \frac{\sinh\left(\frac{\tilde{x}-a/2}{L_2}\right)}{\cos(B_1\, a/2)} E_f \int_0^{a/2} \cos(B_1 x)\, dx + 2\Sigma_s C_3 E_s \int_{a/2}^{a/2+b} \sinh\left(\frac{\tilde{x}-x}{L_2}\right) dx$$

$$\Rightarrow \int_0^{a/2} \cos(B_1 x)\, dx = \frac{1}{B_1} \sin(B_1 x)\Big|_0^{a/2} = \frac{1}{B_1} \sin\left(\frac{aB_1}{2}\right)$$

$$\Rightarrow \int_{a/2}^{a/2+b} \sinh\left(\frac{\tilde{x}-x}{L_2}\right) dx = -\frac{1}{L_2} \cosh\left(\frac{\tilde{x}-x}{L_2}\right)\Big|_{a/2}^{a/2+b} = \frac{1}{L_2}\left(\cosh\left(\frac{b+2\sigma_2}{L_2}\right) - \cosh\left(\frac{2\sigma_2}{L_2}\right)\right)$$

Plug back in to get...

$$\boxed{C_3 = \frac{P}{\frac{2\Sigma_f E_f}{B_1} \frac{\sinh\left(\frac{b+2\sigma_2}{L_2}\right)}{\cos\left(B_1 \frac{a}{2}\right)} \sin\left(\frac{aB_1}{2}\right) + \frac{2\Sigma_s E_s}{L_2}\left[\cosh\left(\frac{b+2\sigma_2}{L_2}\right) - \cosh\left(\frac{2\sigma_2}{L_2}\right)\right]}}$$

HW#5

4) 

a) $-\nabla\cdot(D_1\nabla\phi_1)+\Sigma_a\phi_1=\frac{1}{k}\nu_1\Sigma_{f1}\phi \Rightarrow \nabla^2\phi_1+\frac{\frac{1}{k}\nu_1\Sigma_{f1}-\Sigma_a}{D_1}\phi_1=0$

$\Rightarrow \nabla^2\phi_1+B_1^2\phi=0 \Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\phi_1}{\partial r}\right)+B_1^2\phi=0$, say $\phi_1=\frac{u}{r}$

$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\left(\frac{u}{r}\right)\right)+B_1^2\phi=0 \Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(\frac{\partial u}{\partial r}\cdot\frac{1}{r}-\frac{u}{r^2}\right)\right)+B_1^2\phi_1=0$

$\Rightarrow \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}r-u\right)+B_1^2\phi_1=0 \Rightarrow \frac{1}{r^2}\left(\frac{\partial^2 u}{\partial r^2}r+\frac{\partial u}{\partial r}-\frac{\partial u}{\partial r}\right)+B_1^2\phi_1=0 \Rightarrow \frac{\partial^2 u}{\partial r^2}+B_1^2\phi_1=0$

$\Rightarrow u=C_1\sin(B_1r)+C_2\cos(B_1r) \Rightarrow \phi_1=\frac{C_1}{r}\sin(B_1r)+\cancel{\frac{C_2}{r}\cos(B_1r)}$ $\Big\}$ 1

$\nabla^2\phi_2-\frac{1}{L_2^2}\phi_2=0 \Rightarrow \phi_2=\frac{C_3}{r}\sinh\left(\frac{\tilde{b}-r}{L_2}\right)$ $\Big\}$ 2

<u>BC</u> i) $-D_1\frac{\partial\phi_1}{\partial r}\Big|_{r=0}=0$ ii) $\phi_1(R)=\phi_2(R)$ iii) $-D_1\frac{\partial\phi_1}{\partial r}\Big|_{r=R}=-D_2\frac{\partial\phi_2}{\partial r}\Big|_{r=R}$

i) $-D_1\left(\frac{C_1}{r}B_1\cos(B_1r)-\frac{C_2}{r}B_1\sin(B_1r)-\frac{C_1}{r^2}\sin(B_1r)-\frac{C_2}{r^2}\cos(B_1r)\right)\Big|_{r=0}=0$

$\Rightarrow \left(C_1B_1r\cos(B_1r)-C_2B_1r\sin(B_1r)-C_1\sin(B_1r)-C_2\cos(B_1r)\right)\Big|_{r=0}=0 \Rightarrow -C_2=0 \quad \therefore C_2=0$

ii) $\left[\frac{C_1}{r}\sin(B_1r)\right]\Big|_{r=R}=\left[\frac{C_3}{r}\sinh\left(\frac{\tilde{b}-r}{L_2}\right)\right]\Big|_{r=R} \Rightarrow C_1\sin(B_1R)=C_3\sinh\left(\frac{\tilde{b}-R}{L_2}\right) \quad \therefore C_1=C_3\sinh\left(\frac{\tilde{b}-R}{L_2}\right)/\sin(B_1R)$

iii) $-D_1\left(\frac{C_1}{R}B_1\cos(B_1R)-\frac{C_1}{R^2}\sin(B_1R)\right)=-D_2\left(\frac{-C_3}{RL_2}\cosh\left(\frac{\tilde{b}-R}{L_2}\right)-\frac{C_3}{R^2}\sinh\left(\frac{\tilde{b}-R}{L_2}\right)\right)$

$\Rightarrow D_1\left(C_3\sinh\left(\frac{\tilde{b}-R}{L_2}\right)/\sin(B_1R)\right)\left(\frac{B_1}{R}\cos(B_1R)-\frac{1}{R^2}\sin(B_1R)\right)=-D_2C_3\left(\frac{1}{RL_2}\cosh\left(\frac{\tilde{b}-R}{L_2}\right)+\frac{1}{R^2}\sinh\left(\frac{\tilde{b}-R}{L_2}\right)\right)$

$$\boxed{\begin{array}{l}\phi_1=\frac{C_3}{r}\frac{\sinh\left(\frac{\tilde{b}-R}{L_2}\right)}{\sin(B_1R)}\sin(B_1r) \qquad \phi_2=\frac{C_3}{r}\sinh\left(\frac{\tilde{b}-r}{L_2}\right)\\[2ex] \text{criticality: } D\frac{\sinh\left(\frac{\tilde{b}-R}{L_2}\right)}{\sin(B_1R)}\left(\frac{B_1}{R}\cos(B_1R)-\frac{1}{R^2}\sin(B_1R)\right)=-D_2\left(\frac{1}{RL_2}\cosh\left(\frac{\tilde{b}-R}{L_2}\right)+\frac{1}{R^2}\sinh\left(\frac{\tilde{b}-R}{L_2}\right)\right)\end{array}}$$

HW#5

b) We would need a rxn rate, the associated cross sections, & the result of the reaction per reaction.

For example, we could have a Power, Σ_f for region 1, Σ_s for region 2, energy per fission for region 1, and energy per scatter

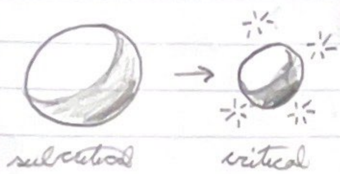
$$P = \int dV \, \Sigma_f \, \phi_1 \, E_f + \int dV \, \Sigma_s \, \phi_2 \, E_s$$

c) on python

d) The shapes of the graphs are roughly the same, but the sphere has a higher fraction of max flux as the sphere has lower leakage compared to the cylinder as the shere is the eest shape for minimizing leakage.

HW#5

5)



a) This is the case as the sphere being compressed results in the same # of potential fission sites, but the SA is lower, so there will be less leakage. Same mass w/ less leakage ↑ k.

b) know $B^2 = \frac{\frac{1}{k}\nu\Sigma_f - \Sigma_a}{D}$, $N_1 V_1 = N_2 V_2 \Rightarrow N_c \frac{4}{3}\pi R_c^3 = N_0 \frac{4}{3}\pi R_0^3 \therefore N_c = N_0 \frac{R_0^3}{R_c^3}$

$$\Rightarrow \frac{\nu\Sigma_f - \Sigma_a}{D} = \left(\frac{\pi}{R_c}\right)^2 \Rightarrow 3\sigma_{tr} N_c^2 (\nu\sigma_f - \sigma_a) = \left(\frac{\pi}{R_c}\right)^2 \Rightarrow 3\sigma_{tr} N_0^2 \left(\frac{R_0}{R_c}\right)^6 (\nu\sigma_f - \sigma_a) = \left(\frac{\pi}{R_c}\right)^2$$

$$\Rightarrow R_c^4 = \frac{3\sigma_{tr} N_0^2 (\nu\sigma_f - \sigma_a) R_0^6}{\pi^2} = \frac{3\sigma_{tr}\sigma_a N_0^2 \left(\frac{\nu\sigma_f}{\sigma_a} - 1\right) R_0^6}{\pi^2}$$

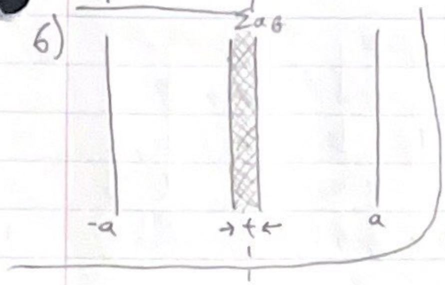
know $k_\infty = \frac{\nu\sigma_f}{\sigma_a} \Rightarrow R_c^4 = \frac{(k_{\infty,0} - 1) R_0^6}{L_0^2 \pi^2}$

& $3\sigma_{tr}\sigma_a = 1/L_0^2$

$$\Rightarrow \boxed{R_c = \sqrt{\frac{(k_{\infty,0} - 1) R_0^6}{L_0^2 \pi^2}}}$$

HW#5 :

6)



a) $-D\nabla^2\phi + \Sigma_a \phi = 0$

$\Rightarrow \nabla^2\phi - \frac{1}{L_c^2}\phi = 0 \Rightarrow \frac{d^2\phi}{dx^2} - \frac{1}{L^2} = 0$

$\boxed{\therefore \phi = C_1 \sinh\left(\frac{\tilde{a}-x}{L}\right)} \leftarrow \phi(\tilde{a}) = 0$ embedded

for the region: $-D\nabla^2\phi_t + \Sigma_a\phi_t = \nu\Sigma_f\phi_t \Rightarrow -D\nabla^2\phi_t = (\nu\Sigma_f - \Sigma_a)\phi_t$

integrate over x: $-D\frac{d\phi_t}{dx} = \int_0^{t/2}(\nu\Sigma_f - \Sigma_a)\phi_t dx = (\nu\Sigma_f - \Sigma_a)(t/2)\phi_t$

very thin; so $\phi_t = \phi(0)$, $x=0 \Rightarrow \frac{d\phi}{dx} = \frac{-C_1}{L_c}\cosh\left(\frac{\tilde{a}-x}{L_c}\right) = -\frac{(\nu\Sigma_f - \Sigma_a)}{D}(t/2)C_1\sinh\left(\frac{\tilde{a}-x}{L_c}\right)$

$\therefore \frac{1}{L} = \frac{(\nu\Sigma_f - \Sigma_a)}{D}(t/2)\tanh\left(\frac{\tilde{a}-x}{L}\right)$

but $\frac{(\nu\Sigma_f - \Sigma_a)}{D} = \frac{\Sigma_a\left(\frac{\nu\Sigma_f}{\Sigma_a} - 1\right)}{D} = \frac{\Sigma_a}{D}(\eta - 1) = \frac{(\eta-1)}{L^2}$

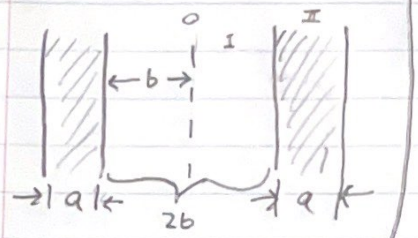
$$\Rightarrow \boxed{1 = \frac{(\eta - 1)}{L}(t/2)\tanh\left(\frac{\tilde{a}-x}{L}\right)}$$

b) on python

c) ~~theoretically~~ 1, but this occurs w/ ∞ thickness
max realistic is just above 1 w/ ∞ thickness too
$\lim_{\eta \to 1^-}(t)$

d) the limitations are $\eta < 1$ are not meaningful as you can't have negative thickness. also, @ $\eta = 1$, it is undefined, so need a limit.

HW#5

7) 

a) $-\nabla(D\nabla\phi) + \Sigma_a\phi = \nu\Sigma_f\phi$

$\Rightarrow -D\nabla^2\phi - (\nu\Sigma_f - \Sigma_a)\phi = 0$

$\Rightarrow \nabla^2\phi + B^2\phi = 0$

$\phi = C_1 \sin(Bx) + C_2\cos(Bx)$, $\frac{d\phi}{dx} = C_1 B\cos(Bx) - C_2 B\sin(Bx)$

$\hookrightarrow \phi = C_3\sin((\tilde{a}-r)B)$, $\frac{d\phi}{dx} = -C_3 B\cos((\tilde{a}-r)B)$

& BC) i) $-D\frac{d\phi}{dx}\Big|_{x=b} = 0 \Rightarrow -C_3 B\cos((\tilde{a}-r)B) = 0$

$\therefore (\tilde{a}-r)B = \frac{n\pi}{2}$ odd n $\Rightarrow$ $B = \frac{n\pi}{(\tilde{a}-r)}$

critical when $\left(\frac{n\pi}{(\tilde{a}-r)}\right)^2 = \frac{\nu\Sigma_f - \Sigma_a}{D} = \frac{\Sigma_a\left(\frac{\nu\Sigma_f}{\Sigma_a} - 1\right)}{D} = \frac{k_\infty - 1}{L^2}$

fundamental mode n=1

$\boxed{\left(\frac{n\pi}{\tilde{a}-r}\right)^2 = \frac{k_\infty - 1}{L^2}} \Rightarrow \left(\frac{\pi}{\tilde{a}-r}\right) = \sqrt{\frac{k_\infty - 1}{L^2}}$

$\Rightarrow \tilde{a} - r = \frac{\pi}{\sqrt{\frac{k_\infty-1}{L^2}}}$ $\Rightarrow (b+a) + 2D - b = \frac{\pi}{\sqrt{\frac{k_\infty-1}{L^2}}}$ $\Rightarrow a = \frac{\pi}{\sqrt{\frac{k_\infty-1}{L^2}}} - 2D = \frac{\pi L}{\sqrt{k_\infty - 1}} - 2D$

plug in $a = \frac{\pi(56)}{2\sqrt{1.1-1}} - 2(1) = \boxed{276.17 \text{ cm} = a}$

↳ divide by 2 as we only want thickness for one region & not both

HW #5

b) The criticality condition ~~does not change~~ as a function of b, which makes sense as the neutrons that enter the vacuum do not get destroyed or eaten, so the region is effectively not there.

c) ~~mod medium:~~ $\cancel{\phi_1 = C_4 \sinh(Bx) + C_5 \cosh(Bx)}$

$$\phi_2 = C_6 \sin((\tilde{a}-r)B)$$

BC) i) $\phi_1 |_{x=0} = 0$

ii) $-D\frac{d\phi_1}{dx}\Big|_{x=b} = -D\frac{d\phi_2}{dx}\Big|_{x=b}$

iii) $\phi_1(b) = \phi_2(b)$

iv) embedded in extrapolated length, $\phi_2|_{x=\tilde{a}} = 0$

$\cancel{C_4 = C_6 \frac{\sin((a+2D)B)}{\sinh(Bb)}}$

$$-D\nabla^2\phi + \Sigma_a\phi_1 = \nu\Sigma_f\phi_1 \qquad k_\infty = \frac{\nu\Sigma_f}{\Sigma_a} = 1 \Rightarrow \nu\Sigma_f = \Sigma_a$$

$$\hookrightarrow -D\nabla^2\phi_1 = 0 \Rightarrow \nabla^2\phi_1 = 0 \Rightarrow \phi_1 = C_7 x + C_8$$

i) $-D\frac{d\phi_1}{dx} = 0 \Rightarrow \frac{d\phi_1}{dx} = C_7 = 0 \qquad \therefore C_7 = 0 \qquad$ & $\phi_1 = C_8$ is constant

This is the same for vacuum & $k_{\infty,2} = 1.1$, which is the same as in a), so a is the same as in a), so

$$\boxed{a = 276.17 \text{ cm}}$$

d) The answers are the same for a & c, which makes sense as k_∞ being 1 is the same as $\nu\Sigma_f = \Sigma_a$. For vacuum, both are 0, but, for a material w/ k_∞ prescribed as 1, they are equal. makes sense