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Phys 325 Exam: Question # 1

a) $$-c v^{2/3} = m \frac{dv}{dt} \Rightarrow -\frac{c}{m} dt = -v^{-2/3}\, dv$$

$$\Rightarrow \frac{c}{m}\int_0^{t_\theta} dt = -\int_{v_0}^{v_\theta} v^{-2/3}\, dv \Rightarrow \frac{c}{m} t_\theta = -3\left[v^{1/3}\right]_{v_0}^{v_\theta}$$

$$\Rightarrow -\frac{c}{3m} t_\theta = v_\theta^{1/3} - v_0^{1/3} \Rightarrow v_\theta^{1/3} = v_0^{1/3} - \frac{c}{3m} t_\theta$$

$$\boxed{v_\theta = \left(v_0^{1/3} - \frac{ct}{3m}\right)^3}$$

b) t_θ when $v_\theta = 0$

$$\therefore v_0^{1/3} = \frac{c t_\theta}{3m} \Rightarrow \boxed{t_\theta = v_0^{1/3} \cdot \frac{3m}{c}}$$

~~c)~~ $\cancel{m\frac{dv}{dt} = -c v^{2/3} \Rightarrow m \frac{dv}{dx} v = -c v^{2/3}}$

$\cancel{m\frac{dx}{dt} = -c v^{-1/3} \Rightarrow -\frac{m}{c} dx = v^{-1/3} dt}$

$\cancel{-\frac{m}{c}\int_0^{x_\theta} dx = \int_0^{t_\theta} v^{-1/3} dt = -\frac{m}{c} x_\theta = \left[\frac{3}{2} v^{2/3}\right]_0^{t_\theta}}$

$\cancel{\Rightarrow -\frac{m}{c} x_\theta = \frac{3}{2} v}$

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Solution continued:

c) $v(t) = \left(v_0^{\frac{1}{3}} - \frac{ct}{3m}\right)^3$

$$\frac{dx}{dt} = \left(v_0^{\frac{1}{3}} - \frac{ct}{3m}\right)^3$$

$$\int_0^{x_f} dx = \int_0^{t_f} \left(v_0^{\frac{1}{3}} - \frac{ct}{3m}\right)^3 dt$$

$$u = v_0^{\frac{1}{3}} - \frac{ct}{3m}$$

$$du = -\frac{c}{3m}\,dt$$

$$-\frac{3m}{c}\,du = dt$$

$$\int_0^{x_f} dx = \int_{u(0)}^{u(t_f)} -\frac{3m}{c} \cdot u^3\,du$$

$$u(0) = v_0^{\frac{1}{3}}$$

$$u(t_f) = v_0^{\frac{1}{3}} - \frac{c}{3m}\left(\frac{3m}{c} v_0^{\frac{1}{3}}\right)$$

$$u(t_f) = 0$$

$$x_f - 0 = \int_{v_0^{\frac{1}{3}}}^{0} -\frac{3m}{c} \cdot u^3\,du$$

$$x_f = -\frac{3m}{c} \cdot \frac{1}{4} u^4 \Big|_{v_0^{\frac{1}{3}}}^{0}$$

$$x_f = -\frac{3m}{4c}\left(0 - v_0^{\frac{4}{3}}\right)$$

$$\boxed{x_f = \frac{3m}{4c} \cdot v_0^{\frac{4}{3}}}$$

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Phys 325 Exam: Question # 2

a. $F = -\nabla U$

$-\frac{c}{r} = -\int F$

$\frac{c}{r} = \int F$

$F = -\frac{c}{r^2}$, multiply by m to get force, $F = \boxed{-\frac{mc}{r^2}}$

b. $r_1 v_1 \sin\theta_1 = \frac{2}{5} r_1 v_2 \sin(\theta_2)$

$\boxed{V_2 = \frac{5}{2}\frac{v_i \sin(\theta_i)}{\sin(\theta_2)}}$

c. $r(\phi) = \frac{a}{1+\epsilon\cos(\phi)}$ $\epsilon = 0$ $C = GM$

$r(\phi) = a = r_c = \frac{\ell^2}{GM}$ $\frac{L}{n} = \ell$ $\ell^2 = \frac{L^2}{m^2}$ $r_c = \frac{L^2}{m^2 C} = \frac{c^2}{mb}$ $\boxed{r_0 = \frac{2a}{b}}$

d. $e_0 = \frac{-C^2}{2\ell^2}$ $e_p = 0$

$\Delta e = \Delta T = \boxed{\frac{c^2}{2\ell^2}}$

e. This isn't a parabola because the eccentricity is 0

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Solution continued:

f. $\epsilon = \sqrt{1 + \frac{2e\ell^2}{\kappa^2}}$ $e = -\frac{\kappa^2}{4\ell^2} = \frac{1}{2} e_0$

$\epsilon = \sqrt{1 + -\frac{1}{2}}$

$\epsilon = \sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$

$r(\phi) = \frac{a}{1+\epsilon\cos(\phi)} =$

$r_a = \frac{1+\epsilon}{1-\epsilon} r_p$

$= \frac{1+\epsilon}{1-\epsilon} r_0$

$\boxed{r_p = r_0}$

$\boxed{r_a = \frac{1+\epsilon}{1-\epsilon} r_0}$

w/ $\epsilon = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

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Phys 325 Exam: Question # 3

a) $L = T - U \qquad z = rc$

$U = \frac{1}{2}k(r_0 - r)^2 + mgrc$

$T = \frac{1}{2}mv^2$

$v = \dot{r}\hat{r} + r\dot{\phi}\hat{\phi} + \dot{z}\hat{z}$

$\dot{\phi} = \Omega \qquad \dot{z} = c\dot{r}$

$v = \dot{r} + r\Omega + c\dot{r}$

$T = \frac{1}{2}m(\dot{r}^2 + r^2\Omega^2 + c^2\dot{r}^2)$

$L = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\Omega^2 + \frac{1}{2}mc^2\dot{r}^2 - \frac{1}{2}k(r_0 - r)^2 - mgrc$

b) $-\frac{1}{2}kr_0^2 + kr_0r - \frac{1}{2}kr^2$

$\frac{dL}{dr} - \frac{d}{dt}\frac{dL}{d\dot{r}} = 0$

$mr\Omega^2 + kr_0 - kr - \frac{d}{dt}(m\dot{r} + m\dot{r}c^2) - mgc = 0$

$\ddot{r}m(1+c^2) - r(m\Omega^2 + k) = kr_0 - mgc$

c) $h = \frac{dL}{d\dot{r}}\dot{r} - L$

$= m\dot{r}^2 + m\dot{r}^2c^2 - \frac{1}{2}m\dot{r}^2 - \frac{1}{2}mr^2\Omega^2 - \frac{1}{2}mc^2\dot{r}^2 + \frac{1}{2}k(r_0 - r)^2 + mgrc$

$h = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}m\dot{r}^2c^2 - \frac{1}{2}mr^2\Omega^2 + \frac{1}{2}k(r_0 - r)^2 + mgrc$

h is not the total energy because $h \neq T + U$ due to $-\frac{1}{2}mr^2\Omega^2$ not changing.

h is conserved because L does not directly depend on t.

d) $\ddot{r} = \frac{kr_0 - mgc}{m(1+c^2)} + \underbrace{\frac{r(m\Omega^2 - k)}{m(1+c^2)}}_{<0}$

$\frac{r(m\Omega^2 - k)}{m(1+c^2)} < 0$

$\Omega^2 < \frac{k}{m}$

$\Omega < \sqrt{\frac{k}{m}}$

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Solution continued:

e) $\omega_n = \sqrt{\frac{k}{m}}$

$$\ddot{r} = \frac{kr_0 - mgc}{m(1+c^2)} + \underbrace{\frac{(m\Omega^2 - k)}{m(1+c^2)}}_{\text{looks}} r$$

like

$-\frac{k}{m}$ because $\ddot{x} = -\omega^2 x + A$

$$\omega_n = \sqrt{\frac{k - m\Omega^2}{m(1+c^2)}}$$

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Phys 325 Exam: Question # 4

a) $T = 4s \Rightarrow \Omega = \frac{2\pi}{T} = \frac{\pi}{2}$

$\boxed{\Omega = \frac{\pi}{2}}$ $\boxed{a_0 = \frac{I_0}{2}}$

$\boxed{a_n = \frac{I_0}{2}\cos\left(\frac{\pi}{2}n\right) \qquad b_n = \frac{I_0}{2}\sin\left(\frac{\pi}{2}n\right)}$

b) resonance when $\frac{n\Omega}{\omega_n} = \sqrt{1 - 2\zeta^2}$

$2\zeta^2 = \frac{c^2}{2m^2\omega_n^2} \Rightarrow n\Omega = \omega_n\sqrt{1 - \frac{c^2}{2m^2\omega_n^2}}$

$\Rightarrow n\Omega = \sqrt{\omega_n^2 - \frac{c^2}{2m^2}} = \sqrt{\frac{k}{m} - \frac{c^2}{2m^2}}$

$$\boxed{n\Omega = \sqrt{\frac{k}{m} - \frac{c^2}{2m^2}}}$$

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Phys 325 Exam: Question # 8

a) $dP = F\,dt$ w/ $F = mg$

$dP = P(t+dt) - P(t)$

w/ $dm = \alpha\,dt$

$P(t) = Mv$

$P(t+dt) = (M - dm)(v + dv) + dm(v - u)$

$= Mv - dm\,v + M\,dv - \cancelto{0}{dm\,dv} + dm\,v - dm\,u$

$dP = M\,dv - dm\,u = F\,dt$

$M\,dv - dm\,u = Mg\,dt$

$dm = \alpha\,dt$, $\alpha > 0$

~~$M = M_0 - \alpha t$~~

~~$M = (1+\alpha)M_0 - \alpha t$~~

$M = M_0 - \alpha t$

$M\,dv - u\alpha\,dt = Mg\,dt$

$M\frac{dv}{dt} - u\alpha = Mg$

$M\frac{dv}{dt} = Mg + u\alpha$

$dv = \left(\frac{Mg + u\alpha}{M}\right)dt$

$dv = \left(g + \frac{u\alpha}{M_0 - \alpha t}\right)dt \Rightarrow \int_0^{v_\theta} dv = \int_0^{t_\theta}\left(g + \frac{u\alpha}{M_0 - \alpha t}\right)dt$

$v_\theta = g\int_0^{t_\theta} dt + \int_0^{t_\theta}\frac{u\alpha}{M_0 - \alpha t}\,dt = g t_\theta + u\alpha\int_0^{t_\theta}\frac{1}{M_0 - \alpha t}\,dt$

$v_\theta = g t_\theta - \frac{u\alpha}{\alpha}\ln\left(\frac{1}{M_0 - \alpha t}\right)\Big|_0^{t_\theta} = g t_\theta + u\ln\left(\frac{M_0 - \alpha t_\theta}{M_0}\right)$

$$\boxed{v_\theta = g t_\theta + u\ln\left(1 - \frac{\alpha t_\theta}{M_0}\right)}$$

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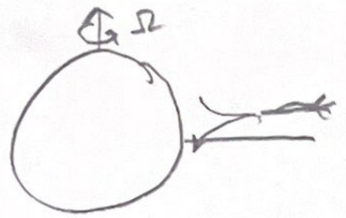
Solution continued:

b)



$\boxed{\text{aim west}}$

$\Omega > 0$ & v is down

$$F_{cor} = -2m\vec{\omega}\times\vec{v} = -2|\omega||v|(-\hat{e}) = 2|\omega||v|\hat{e}$$

aim opposite of F_{cor} to hit target

c) $M\,dv + u\,dM = Mg\,dt$ if $dm = \alpha\,dt = -dM$

$dt = \frac{dm}{\alpha}$

$$M\,dv + u\,dM = -Mg\frac{dM}{\alpha}$$

$$\left(u - \frac{Mg}{\alpha}\right)dM = M\,dv \Rightarrow dv = \left(\frac{u}{M} - \frac{g}{\alpha}\right)dM$$

$$\Rightarrow \int_0^{v_f} dv = u\int_{M_0}^{M_f}\frac{1}{M}\,dM - \frac{g}{\alpha}\int_{M_0}^{M_f} dM \Rightarrow v_f = u\ln\left(\frac{\lambda M_0}{M_0}\right) - \frac{g}{\alpha}\left(M_0(\lambda - 1)\right)$$

but $M_f = \lambda M_0$, $\lambda > 0$

$$\boxed{v_f = u\ln(\lambda) + \frac{gM_0}{\alpha}(1-\lambda)}$$