- This work must be submitted online as a `.pdf` through Canvas.
- Work completed with LaTeX or Jupyter earns 1 extra point. Submit source file (e.g. `.tex` or `.ipynb`) along with the `.pdf` file.
- If this work is completed with the aid of a numerical program (such as Python, Wolfram Alpha, or MATLAB) all scripts and data must be submitted in addition to the `.pdf`.
- If you work with anyone else, document what you worked on together.

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1. (20 points) List and briefly describe three of the most common fuel failure modes.

Solution: From Table 4.6 on Pg. 108.

- **Grid-to-Rod Fretting :** (PWR: 83%, BWR: $\tilde{0}$%) Grid-to-rod contact point being worn down (fretted) due to vibrating fuel rods hitting the spacer grid. The constant vibration damages both the fuel rod cladding and the spacer grid, which is almost only an issue for PWRs.
- **Crud/Corrosion :** (PWR: 2%, BWR: 51%) I will be using the book's description of *stress corrosion cracking (SCC)* to describe this failure mode. Corrosion will cause crud (chalk river unidentified deposit?) to form after damaging the target. Corrosion occurs when chemicals attack a metal under. For SCC, the chemical assailant attacks a metal under stress in the atmosphere of a corrosive environment.
- **Debris Fretting :** (PWR: 5%, BWR: 20%) Solid (mostly metal) objects circulate in the coolant and collide with fuel rod cladding. Debris fretting can be mitigated by filtering the coolant for solid matter.

2. (40 points) Calculate the cost of nuclear fuel fabricated and delivered on site using the following data (Tsoulfanidis 4.3):

- Cost of natural uranium $\frac{\$60}{lb_{U_3O_8}}$
- enrichment 4.2%
- conversion $\frac{\$11.50}{kgU}$
- tails 0.25%
- price per SWU \$110
- conversion loss 0.6%
- fabrication transportation cost $\frac{\$230}{kgU}$
- fabrication loss 0.7%

Solution: The cost per kg of fabricated fuel was \$2375.35. I obtained this using equation 4.1 from our textbook. I checked this answer with Mahmoud Eltawila and Nathan Glaser.

First, find the cost of yellowcake in \$ per kgU. Multiply the given cost by the mass ratio of U_3O_8 to U_3 and convert to kg.

$$PU = \frac{\$60}{lb\ U_3O_8} \cdot \frac{3M_U + 8M_O}{3M_U} \cdot \frac{2.2046lb}{kg} = \$155.99\ per\ kgU \tag{1a}$$

We will need the function for separation potentials. Here, x is the U-235 enrichment.

$$V(x) = (2x-1)\ln\frac{x}{1-x} \tag{1b}$$

We also need the separation factor. xf, is the enrichment of the feed (natural U), xp is the enrichment of the fabricated fuel, and xw is the enrichment of the tails.

$$\frac{F}{P} = \frac{xp - xw}{xf - xw} \tag{1c}$$

$$\frac{W}{P} = \frac{xp - xf}{xf - xw} \tag{1d}$$

$$SF = V(xp) + \frac{W}{P}V(xw) - \frac{F}{P}V(xf) \tag{1e}$$

Finally, calculate FF or the fuel fabrication cost. PU was found earlier and all other costs and losses were given.

$$FF = \left\{\frac{PU}{(1-l_c)(1-l_f)} + \frac{PC}{(1-l_f)}\right\}\frac{F}{P} + \frac{PS}{(1-l_f)}SF + PF \tag{1f}$$

The code has been omitted from this section because it is long and I don't want you to have to scroll more. I included all the code at the end of question 3 because the code is identical for both questions.

3. (40 points) If the enrichment changes by 0.4% (i.e. goes from 4.2 to 4.6%) by what percentage does the cost of fuel in the previous problem (Tsoulfanidis 4.3) change?

Solution: The price difference is +10.08785% with 4.6% fuel compared to 4.2% fuel. The cost of the 4.6% fuel was \$2614.83 per kg.

Use 1f to find the new cost for 4.6% (FF_2) enriched fuel.

$$FF_2 = \left\{\frac{PU}{(1-l_c)(1-l_f)} + \frac{PC}{(1-l_f)}\right\}\frac{F}{P}_2 + \frac{PS}{(1-l_f)}SF_2 + PF = \$2614.83\ per\ kg \tag{2a}$$

But $\frac{F}{P}$, $\frac{W}{P}$, and SF change.

$$\frac{F}{P}_2 = \frac{(4.6e-2) - xw}{xf - xw} \tag{2b}$$

$$\frac{W}{P}_2 = \frac{(4.6e-2) - xf}{xf - xw} \tag{2c}$$

$$SF_2 = V(4.6e-2) + \frac{W}{P}V(xw) - \frac{F}{P}V(xf) \tag{2d}$$

With the different costs based on the the desired enrichments, we can find the percent difference.

$$\%_{diff} = 100 \cdot \frac{FF_2 - FF_1}{FF_1} = 10.08785\% \tag{2e}$$

```python
# data
# costs
c_yc = 60             # dollar per lb of yellow cake
c_con = 11.5          # dollar per kgU
c_swu = 110           # dollar per SWU
c_ft = 230            # dollar per kg U to fabricate and transport

# enrichments
x_nu = 0.711 / 100    # enrichment of natural U
x_en = 4.2 / 100      # enrichment of fuel
x_tl = 0.25 / 100     # enrichments of tails

# losses
l_con = 0.6 / 100     # conversion loss
l_fab = 0.7 / 100     # fabrication loss


# formatting data
m_o = openmc.data.atomic_weight('O')
m_u = openmc.data.atomic_weight('U')

m_yc = (3 * m_u) + (8 * m_o)
m_ratio = (3 * m_u) / m_yc

lb_per_kg = 2.2046
```

```python
# functions
def sep_pot(x):
    """returns seperation potential for an enrichment x"""
    return (2 * x - 1) * np.log(x / (1 - x))


def sep_factor(xf=x_nu, xp=x_en, xw=x_tl):
    """find the cost to enrich m [kg] of UF6"""
    fop = (xp - xw) / (xf - xw)
    wop = (xp - xf) / (xf - xw)
```

```python
    vp = sep_pot(xp)
    vf = sep_pot(xf)
    vw = sep_pot(xw)

    return vp + (wop * vw) - (fop * vf)


# functions
def fuel_fab(xp=x_en, do_print=False):
    """calculates fuel fabrication cost using Eq 4.1"""
    # calcs
    sf = sep_factor(xp=xp)
    fop = (xp - x_tl) / (x_nu - x_tl)

    # costs
    c1 = ((c_yc / (1 - l_con) / (1 - l_fab)) + (c_con / (1 - l_fab))) * fop
    c2 = (c_swu / (1 - l_fab)) * sf
    c3 = c_ft

    if do_print: print(f"{c1}\n{c2}\n{c3}")

    return c1 + c2 + c3

# question 2
cost = fuel_fab(xp=x_en)
print(f"FF w/ 4.2%: ${round(cost, 2)}/kg")

# question 3
c1 = fuel_fab(xp=x_en)
c2 = fuel_fab(xp=(4.6 / 100))

diff = (c2 - c1) / c1 * 100

print(f"FF w/ 4.2%: ${round(c1, 2)}/kg")
print(f"FF w/ 4.6%: ${round(c2, 2)}/kg")
print(f"\nPercent Diff: {round(diff, 5)}%")
```