HW#2

1) $\phi(r) = \dfrac{\phi_0 \sin\left(\frac{\pi r}{R}\right)}{r}$, $\phi_0 = 13.1e13\ cm^{-1}s^{-1}$, $R = 50cm$, $\Sigma_a = 0.108\ cm^{-1}$, $\Sigma_s = 0.0727\ cm^{-1}$

a) $\phi' = \dfrac{\phi_0 \pi \cos\left(\frac{\pi r}{R}\right)}{Rr} - \dfrac{\phi_0 \sin\left(\frac{\pi r}{R}\right)}{r^2}$, max ϕ when $\phi' = 0$, so

$$\Rightarrow 0 = \frac{\phi_0 \pi \cos\left(\frac{\pi r}{R}\right)}{Rr} - \frac{\phi_0 \sin\left(\frac{\pi r}{R}\right)}{r^2} \Rightarrow \frac{\pi \cos\left(\frac{\pi r}{R}\right)}{Rr} = \frac{\sin\left(\frac{\pi r}{R}\right)}{r^2}$$

$\Rightarrow \tan\left(\frac{\pi r}{R}\right) = \frac{\pi r}{R}$, now taylor expand $\tan\left(\frac{\pi r}{R}\right)$ to get

$\Rightarrow \left(\frac{\pi r}{R}\right) + \frac{1}{3}\left(\frac{\pi r}{R}\right)^3 + \frac{2}{15}\left(\frac{\pi r}{R}\right)^5 + \ldots = \frac{\pi r}{R}$, terms w/ higher dependence on $r \neq 0$ as $r \to 0$

$\therefore$ we can say the max flux is approaching 0, so find $\lim_{r\to 0} \phi(r)$

$$\lim_{r\to 0} \phi(r) = \frac{\lim_{r\to 0}\left(\phi_0 \sin\left(\frac{\pi r}{R}\right)\right)}{\lim_{r\to 0}(r)} \xRightarrow{\text{L'Hospital}} \lim_{r\to 0}\left(\frac{\phi_0 \pi \cos\left(\frac{\pi r}{R}\right)}{R}\right) = \phi_0 \pi = \pi(13.1e13\ cm^{-1}s^{-1})$$

$$\boxed{\phi_{max} = \pi \phi_0 = 4.115\ 5e14\ cm^{-2}s^{-1}}$$

b) ...

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c) know $\phi = nv \Rightarrow n = \phi/v$, so integrate this

$$\int_{4\pi} d\hat{\Omega} \int \frac{r^2\phi}{v}\, dr = \frac{4\pi}{v}\int r^2\left(\frac{\phi_0 R \sin\left(\frac{\pi r}{R}\right)}{r}\right) dr = \frac{4\pi\phi_0 R}{v}\int_0^R \overset{u}{\overset{\shortparallel}{r}}\; \overset{dv}{\overset{\shortparallel}{\sin\left(\frac{\pi r}{R}\right) dr}}$$

$uv - \int v\,du$

integration by parts $\Rightarrow \dfrac{4\pi\phi_0 R}{v}\left[\left.-\dfrac{rR\cos\left(\frac{\pi r}{R}\right)}{\pi}\right|_0^R + \displaystyle\int_0^R \dfrac{R\cos\left(\frac{\pi r}{R}\right)}{\pi}\, dr\right]$

$= \dfrac{4\pi\phi_0 R}{v}\left[\dfrac{R^2}{\pi} + 0\right] = \dfrac{4\phi_0 R^3}{v}$, but need v, so use...

$$v = \left(\frac{2E}{m_0}\right)^{1/2} = \left(\frac{2(0.025\,eV \cdot 1.6e{-}19\,\frac{J}{eV})}{1.674928e{-}27\,kg}\right)^{1/2} = 2185.5\,m/s = 2.185e5\,cm/s$$

$$\boxed{\therefore N = \frac{4(13.1e13\,cm^{-2}s^{-1})(50\,cm)^3}{(2.185e5\,cm\,s^{-1})} = 2.997e14 \text{ neutrons}}$$

d) $V = \frac{4}{3}\pi r^3 \Rightarrow \left(\frac{3}{4\pi}V\right)^{1/3} = r$ $\quad r = \left(\frac{3}{4\pi}\,1cc\right)^{1/3} = 0.62\,cm$

from code $\phi(0.62\,cm) = 4.114e14\,cm^{-2}s^{-1} \approx \phi_{max}$

$\therefore \phi$ is spatially invariant in the region $r \leq 0.62\,cm$

$$\Rightarrow \int_{4\pi} d\hat{\Omega} \int \frac{r^2\phi}{v}\, dr = \frac{4\pi\phi_{max}}{v}\int_0^{R_1} r^2\, dr = \frac{4\pi\phi_{max}}{3v}R_1^3 = N_d$$

$$N_d = \frac{4\pi\,(4.115e14)\,[1/cm^2 s]\,(0.62\,cm)^3}{3(2185.5\,m/s)} = \boxed{1.88e9\ n^\circ}$$

HW#2 - cont

e) $P = \Sigma_f \Phi_{max} V \cdot E_f = \frac{0.0727}{cm} \cdot \frac{4.115e14}{cm^2 \cdot s} \cdot 1cc \cdot 200\ MeV$

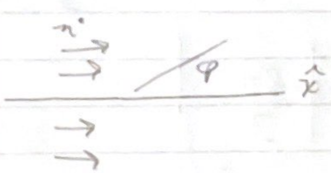
$\Rightarrow P = 5.98e15 \frac{MeV}{s} \times \frac{1.602e\text{-}13\ J}{MeV} = \boxed{958.5\ W}$

f) $P = \Sigma_f E_f \int \phi \, dV = 4\pi \Sigma_f E_f \phi_0 R \int_0^R r \sin\left(\frac{\pi r}{R}\right) = 4\pi \Sigma_f E_f \phi_0 R \left[\frac{R^2}{\pi}\right]$

$P = 4(0.0727\ cm^{-1})(3.204e\text{-}11\ J/fission)(13.1e13\ cm^{-2}s^{-1})(50\ cm)^3$

$\boxed{P = 152.57\ MW}$

g) from f) $\boxed{P = 4\Sigma_f E_f \phi_t R^3 \quad \text{or} \quad \phi_t = \frac{P}{4\Sigma_f E_f R^3}}$

2) 

a) $\vec{J} = \Psi \cdot \hat{x} = \boxed{nv\hat{x}}$

b) net rate $= \vec{J} A = (\Psi \cdot \hat{n})A = \boxed{\Psi \sin\varphi\, A}$

3a) False, change in k implies $\frac{\Delta N}{\Delta t} \neq 0$ ∴ no ss

b) False, same logic as A

c) False, P_{NL} would ↑ the better of a refector the reflector is

d) True, Σ_s for void $= 0$, but $\Sigma_{sU137} \neq 0$ ∴ k ↑

HW#2 - cont'd

4a) ...

b) These large negative spikes are caused by the resonance absorption for the given nucleus

c) Thermal as the energy is approaching 0.

5a) ...

b) Fast spectrum as no thermal neutrons are seen

6) $f(\theta,\varphi) = \theta\varphi^2$ $\quad \int_{4\pi} d\hat{\Omega}\, f(\theta,\varphi) = \int_0^{2\pi} \varphi^2 d\varphi \int_0^{\pi} \theta \sin(\theta)\, d\theta$

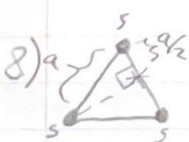
$$\Rightarrow \frac{\varphi^3}{3}\Big|_0^{2\pi} \times \int_0^{\pi} \theta \sin\theta\, d\theta = \frac{8\pi^3}{3}\int_0^{\pi} \theta \sin\theta\, d\theta$$

say $u = \theta$ $\quad du = 1$

$v = -\cos\theta$ $\quad dv = \sin\theta$

$$\Rightarrow \frac{8\pi^3}{3}\left(-\theta\cos\theta\Big|_0^{\pi} + \int_0^{\pi} \cos\theta\right) = \frac{8\pi^3}{3}(\pi + 0) = \boxed{\frac{8\pi^4}{3}}$$

7) $J = \left[\frac{n}{s\,cm^2}\right]$ $\quad J = \frac{S}{SA} = \boxed{\frac{S}{4\pi r^2} = \vec{J}(r)}$

8) 

2 sources $\frac{a}{2}$ away, so $2J\left(\frac{A}{2}\right) = \frac{2S}{4\pi\left(\frac{a}{2}\right)^2} = \frac{2S}{\pi a^2}$

another source $\sqrt{a^2 + \left(\frac{a}{2}\right)^2}$ away, so

$J\left(\sqrt{a^2 - \left(\frac{a}{2}\right)^2}\right) = \frac{S}{4\pi\left(a^2 - \frac{a^2}{4}\right)} = \frac{S}{3\pi a^2}$, now add these together

source

$$J_T = 2J\left(\frac{A}{2}\right) + J\left(\sqrt{a^2 + \left(\frac{a}{2}\right)^2}\right) = \frac{2S}{\pi a^2} + \frac{S}{3\pi a^2} = \frac{S}{\pi a^2}\left(2 + \frac{1}{3}\right) = \boxed{\frac{7}{3} \cdot \frac{S}{\pi a^2} = J}$$

HW#2 - cont

9) • UO_2 w/ $\rho = 10.5\ g/cc$ • 4.5% weight ^{235}U of Uranium
• 99.8% atoms ^{16}O find number density of each

$M_{235U} = 235.043928117\ u$ or g/mol $M_{16O} = 15.9949146196\ u$
$M_{238U} = 238.0507866936\ u$ $M_{18O} = 17.999159612414\ u$

$\underline{M_O} = \gamma_{16O} M_{16O} + \gamma_{18O} M_{18O} = 15.99892311\ u$

$\underline{M_U}$
assume 100g U, 4.5g ^{235}U & 95.5g ^{238}U

$\underline{235}$: $4.5g\ ^{235}U / M_{235U} = 0.0191453574\ mol\ ^{235}U$

$\underline{238}$: $95.5g\ ^{238}U / M_{238U} = 0.4011748973\ mol\ ^{238}U$

$\therefore \gamma_{U235} = mol_{235U} / (mol_{235U} + mol_{238U}) = 4.554945232\ \%\ ^{235}U$

& $\gamma_{U238} = 1 - \gamma_{235U} = 95.44505477\ \%\ ^{238}U$

$\Rightarrow M_U = \gamma_{235U} M_{235U} + \gamma_{238U} M_{238U} = 237.9138262\ g/mol$

$\underline{M_{UO_2}} = M_U + 2M_O = 269.9116724\ g/mol$

$\rho_N = \rho / M_{UO2} = 0.389016151\ mol/cc \xrightarrow{\times N_A} = 2.342655261e22\ \frac{atoms}{cc}$

$\underline{\rho_N}$
U235, $\rho_{N,235U} = \rho_N \gamma_{235U} = 1.067066641e21$ atoms ^{235}U/cc
U238, $\rho_{N,238U} = \rho_N \gamma_{238U} = 2.235948597e22$ atoms ^{238}U/cc
O16, $\rho_{N,16O} = 2 \cdot \rho_N \gamma_{16O} = 4.675939901e22$ atoms ^{16}O/cc
O18, $\rho_{N,18O} = 2 \cdot \rho_N \gamma_{18O} = 9.370621044e19$ atoms ^{18}O/cc