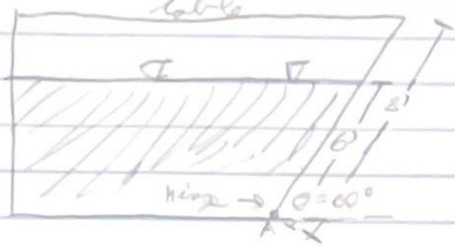
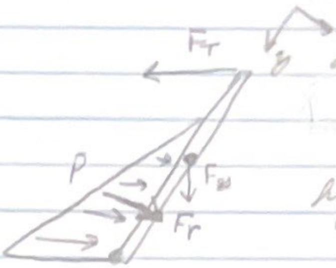
Joseph Specht
jspecht3

HW #3

2.27)





(z, y)

hinge @ $(0, 8)$

$F_W = 800$ lbs $\qquad F_T \perp \quad L = 4' \qquad F_r = \gamma_w h_c A$

$\Rightarrow h_c = \frac{6'}{2}\sin(60°)$ & $A = 4' \cdot 6' = 24\,ft^2 \Rightarrow F_r = \left(62.4 \frac{lb}{ft^3}\right) h_c A$
≈ 3220.9 lbs

Find where the force acts $\perp$ to the gate, so...

$\Rightarrow y_R = \frac{I_{xc}}{y_c A} + y_c = \frac{w l^3}{12} \cdot \frac{1}{y_c A} + y_c$ where $w = 4'$, $y_c = 3'$ & $l = 6'$, so

$y_R = \frac{4'(6')^3}{12} \cdot \frac{1}{3' \, 24\,ft^2} + 3' = 1' + 3' = \boxed{4'} = y_c$

$\Rightarrow$ Balancing moments.

$\Sigma M = \Sigma F_\perp r = F_r(y_c) + F_W \cos(60°) L - F_T \sin(60) L = 0$

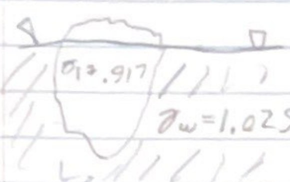
$\Rightarrow F_T\left(\frac{\sqrt{3}}{2}\right) \cdot 8' = F_r(4') + 800\text{ lbs}\left(\frac{1}{2}\right)(6' - 4') = 9381.8$ ft lbs

$\Rightarrow F_T = \frac{9381.8 \text{ ft lbs}}{(\sqrt{3}/2) \cdot 8'} = \boxed{1354.15 \text{ lbs} = F_T}$

Joseph Specht
jspecht3

HW #3-cont

2.133)

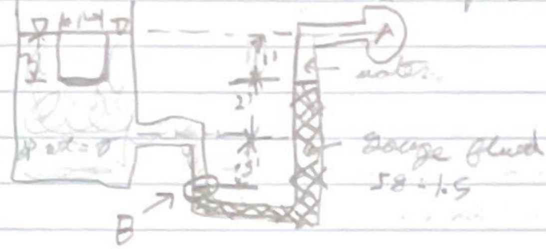


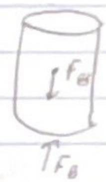
% underwater $= \gamma_i / \gamma_w = \boxed{89.46\%}$

Know from previous physics class

2.142)

$P_a = -.1\,psi$





$F_{g} = F_B \Rightarrow W = \gamma[\forall_{cylinder}]$

$W = \gamma[\pi R^2 h] = \gamma[\pi \cdot .5^2 \cdot 2']$

$\Rightarrow W = \frac{\pi}{2}\gamma$

Pressure balance @ Point B

From left side,

$P_B = \gamma(1' + 2' + .5') = 3.5'\gamma$

$SG = 1.5$

$\gamma_w = 62.4\,lbs/ft^3$

$P_A = -.1\,psi = \frac{144\,psf}{1\,psi} \cdot -.1 = -14.4\,psf$

From Right side,

$P_B = SG(\gamma_w)(2' + .5') + \gamma_w(1') + P_A = 282\,psf$

$\Rightarrow 282\,psf = 3.5'\gamma \Rightarrow \gamma = 282\,psf / 3.5' = 80.57\,lbs/ft^3$

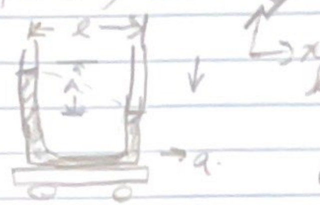
Plug back into formula for W & get

$W = \frac{\pi}{2}(80.57) = \boxed{126.56\,lbs}$

Joseph Specht
jspecht3

HW #3 - cont

2.157)



Know $\frac{\partial P}{\partial x} = -\rho a_x$ & $\frac{\partial P}{\partial y} = -\rho g$

But we can eliminate the partials as P is only a function of y

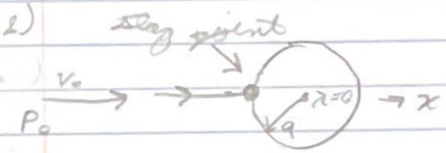
$\Rightarrow \frac{dP}{dx} = -\rho a_x$ & $\frac{dP}{dy} = -\rho g$

$\hookrightarrow dP = -\rho a_x dx$ $\qquad \hookrightarrow dP = -\rho g dy$

$\Rightarrow \rho a_x dx = \rho g dy$ where $dx = l$ & $dy = h$, so

$\rho a_x l = \rho g h \Rightarrow \boxed{a_x = gh/l}$

3.2)



$V = V_0(1 - a^2/x^2)$ for $-\infty \leq x \leq -a$

a) Know $\frac{dP}{ds} + \rho g \frac{dz}{ds} + \rho V \frac{dV}{ds} = 0$ for all points, but

$\frac{dz}{ds} = 0$ as we are along a horizontal streamline, so

$\frac{dP}{ds} + \rho V \frac{dV}{ds} = 0$; & $ds = dx$, so $\frac{dP}{dx} = -\rho V \frac{dV}{dx}$

$\Rightarrow \frac{dV}{dx} = \frac{d}{dx}\left(V_0(1 - a^2 x^{-2})\right) = V_0(2a^2 x^{-3}) = \frac{2V_0 a^2}{x^3}$

& $V \frac{dV}{ds} = V_0(1 - a^2/x^2) \frac{2V_0 a^2}{x^3} = 2V_0^2 a^2 \left[\frac{1}{x^3} - \frac{a^2}{x^5}\right]$, so

$$\boxed{\frac{dP}{dx} = -2\rho V_0^2 a^2 \left[\frac{1}{x^3} - \frac{a^2}{x^5}\right] = 2\rho V_0^2 a^2 \left[\frac{a^2}{x^5} - \frac{1}{x^3}\right]}$$

Joseph Specht
jspecht3

HW#3 - cont

b) Want to integrate, so seperate variables

$$\int_{P_0}^{P(x)} dP = \int_{-\infty}^{x} 2\rho V_0^2 a^2 \left[a^2 x^{-5} - x^{-3}\right] dx$$

$$\Rightarrow P(x) - P_0 = 2\rho V_0^2 a^2 \left[a^2 \int_{-\infty}^{x} x^{-5} dx - \int_{-\infty}^{x} x^{-3} dx\right] = 2\rho V_0^2 a^2 \left[a^2 \left(-\frac{1}{4} x^{-4}\Big|_{-\infty}^{x}\right) + \left(\frac{1}{2} x^{-2}\Big|_{-\infty}^{x}\right)\right]$$

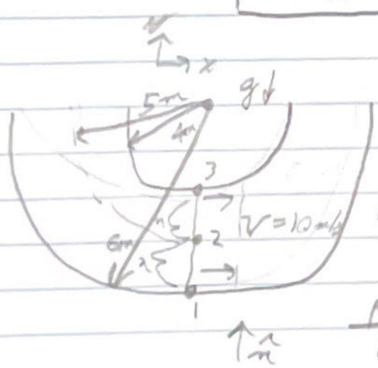
$$\Rightarrow P(x) - P_0 = 2\rho V_0^2 a^2 \left[\frac{-a^2}{4x^4} + 0 + \frac{1}{2x^2} + 0\right] = \rho V_0^2 a^2 \left[\frac{1}{x^2} - \frac{a^2}{2x^4}\right]$$

$$\boxed{\therefore P(x) = P_0 + \frac{\rho V_0^2 a^2}{x^2}\left[1 - \frac{a^2}{2x^2}\right]}$$

c) $P(-a) = P_0 + \frac{\rho V_0^2 a^2}{a^2}\left[1 - \frac{a^2}{2a^2}\right] = P_0 + \rho V_0^2 \left[1 - \frac{1}{2}\right] = P_0 + \rho V_0^2 \left[\frac{1}{2}\right]$

$$\boxed{\therefore P(-a) = P_0 + \frac{\rho V_0^2}{2}, \text{ which is expected}}$$

3.12)



$P_1 = 40\,kPa$ find P_2 & P_3

eq for $\hat{n}$ direction ($\hat{z}$)

$\frac{\rho V_0^2}{R} + \gamma \frac{dz}{dn} + \frac{dP}{dn} = 0$, but $dz = dn$, so $\frac{dz}{dn} = 1$

$\Rightarrow \frac{\rho V_0^2}{R} + \gamma = -\frac{dP}{dn}$ separate vars $\Rightarrow -\frac{\rho V_0^2}{R} dn - \gamma\, dn = dP$

but $R = 6 - n$, so $\Rightarrow -\frac{\rho V_0^2}{6-n} dn - \gamma\, dn = dP$

Joseph Specht
jspecht3

HW#3 - cont

now integrate

$$\int_{P_0}^{P} dP = -\rho v_0^2 \int_0^x \frac{dx}{6-x} - \gamma \int_0^x dx$$

$$\Rightarrow P - P_0 = \rho v_0^2 \left[\ln(x-6) \Big|_0^x \right] - \gamma x = \rho v_0^2 \ln\left(\frac{x-6}{6}\right) - \gamma x$$

General expression for P is as follows

$$P(x) = P_0 + \rho v_0^2 \ln\left(\frac{x-6}{6}\right) - \gamma x$$

$$P(x) = 40\,kpa + 1000\,\frac{kg}{m^3}\left(10\,\frac{m}{s}\right)^2 \ln\left(\frac{x-6\,m}{6\,m}\right) - 9810\,\frac{N}{m^3}\,x$$

$$P(x) = 40\,kpa - 100\,kpa\,\ln\left(\left|\frac{6}{x-6}\right|\right) - 9.81\,\frac{kpa}{m}\,x$$

$$P_2 = P(1) = 40\,kpa + 100\,kpa\,\ln\left(\left|\frac{1-6}{6}\right|\right) - 9.81 \cdot 1\,kpa$$

$$\boxed{P_2 = 11.958\ kPa}$$

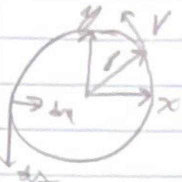
$$P_3 = P(2) = 40\,kpa + 100\,kpa\,\ln\left(\left|\frac{2-6}{6}\right|\right) - 9.81 \cdot 2\,kpa$$

$$\boxed{P_3 = -20.167\ kPa}$$

Joseph Specht
jspecht3

HW#3 - cont

π) $\vec{V} \perp \hat{r}$



need general pressure gradient in $\hat{n}$, so

$-\frac{dP}{dn} = \frac{\rho V^2}{R} + \gamma \frac{dz}{dn}$, but $\frac{dz}{dn} = 0$, so

apparently density is in [slug/ft³]

$-\frac{dP}{dn} = \frac{\rho V^2}{R}$, but $dn = -dr$, so $\frac{dP}{dr} = \frac{\rho V^2}{r}$

a) Pressure gradient w/ water, r = 3 in, & V = .8 ft/s

$$\frac{dP}{dr} = \frac{\overset{\rho_w}{1.94\ \text{slug/ft}^3} \left(.8\ \frac{ft}{s}\right)^2}{(.25\ ft)} = 4.966\ \frac{slugs}{(ft\,s)^2}$$

slug = [lbs s²/ft], so $1\ \frac{slug}{(ft\,s)^2} = \frac{1\ lbs}{ft^3}$

$$\boxed{\therefore \frac{dP}{dr} = 4.966\ lbs/ft^3} \leftarrow \text{water}$$

b) Pressure gradient w/ air, r = 300 ft, V = 200 mph = 293.33 fps

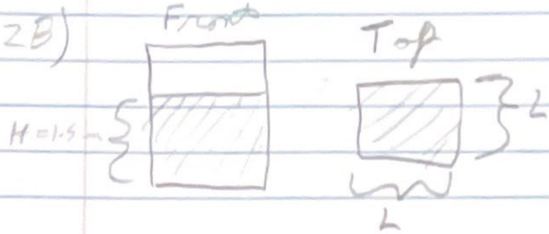
$$\frac{dP}{dr} = \frac{\overset{\rho_{air}}{.002377}\ \frac{slugs}{ft^3} (293.33\ fps)^2}{300\ ft} = .6817\ \frac{slug}{(ft\,s)^2}$$

$$\boxed{\therefore \frac{dP}{dr} = .6817\ \frac{lbs}{ft^3}} \leftarrow \text{air}$$

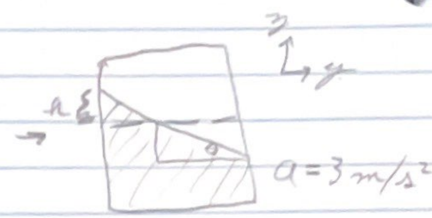
Joseph Specht
jspecht3

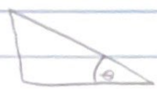
HW #3 - cont

2B) Front Top



$L = 1\,m$



d) 

$\tan\theta = \frac{-a}{g} = \frac{h}{l/2} \Rightarrow h = \frac{-al}{2g}$

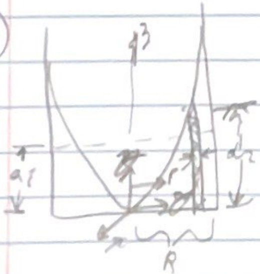
So $P_W = P_{atm} + \rho g(H + h) = P_{atm} + \rho g\left(H - \frac{al}{2g}\right)$

$\Rightarrow P_W = P_{atm} + \rho g H - \frac{\rho a l}{2} = P_{atm} + \rho\left(gH - \frac{al}{2}\right)$

$$\boxed{P_W = P_{atm} + \rho(gH - al/2)}$$

e) $P_W = 101.325\ kPa + 1000\frac{kg}{m^3}\left(9.81\frac{m}{s^2}\cdot 1.5m - 3\frac{m}{s^2}\cdot 1m/2\right)$
$= 101.325\ kPa + 13.215\ kPa$

$$\boxed{P_W = 114.54\ kPa}$$

2.159) 

$D = 1m$, $h_i = .7m$

From Example 2.12, we have

$\pi R^2 h_i = \frac{\pi \omega^2 R^4}{4g} + \pi R^2 h_o$, but $h_o = 0$, so

$\Rightarrow \pi R^2 h_i = \frac{\pi\omega^2 R^4}{4g} \Rightarrow \omega^2 = \frac{4h_i g}{R^2} = \frac{4\times .7\times 9.81}{(.5)^2} = 109.872\ \frac{rad^2}{s^2}$

$\Rightarrow \omega = \sqrt{109.872}\ \frac{rad}{s} \Rightarrow \boxed{\omega = 10.482\ \frac{rad}{s}}$