HW#5

1) plastic deformation starts @ 345 MPa, E = 103 GPa

a) $A = 130\ mm^2 = 130e{-6}\ m^2$, max load?

$$\sigma_{max} = F_{max}/A \Rightarrow F_{max} = \sigma_{max} A = (345\ MPa)(130e^{-6} m^2)$$

$$\boxed{F_{max} = 44.85\ kN}$$

b) $L_0 = 76\ mm$, max L?

$$\sigma = E\epsilon = E\,\frac{l - l_0}{l_0} \Rightarrow \frac{\sigma l_0}{E} = l - l_0 \Rightarrow l_{max} = l_0 + \frac{\sigma l_0}{E}$$

$$\boxed{l_{max} = 76.2546\ mm}$$

HW #5 - cont

2) $l = 120e\text{-}3\,m$, $D = 15.0e\text{-}3\,m$, $F_{max} = 35e3\,N$

- no plastic deformation or D reduction of $\underset{\Delta d}{1.2e^{-2}}$ mm or more

- for plastic deformation, check if F@ YS < 35e3 N

$$\Rightarrow \sigma_{max} = F/A \Rightarrow F = \overset{YS}{\sigma_{max}} A = (YS)\left(\tfrac{\pi}{4}D^2\right) > F_{max} = 35e3\,N$$

- for Diameter reduction, use Eq. 6.8, 6.5, 6.1

$$\Rightarrow \epsilon_x = \frac{-\Delta d}{D} \Rightarrow \epsilon_z = \frac{-\epsilon_x}{\nu} \Rightarrow \sigma = E\epsilon_z \Rightarrow F = \sigma A > F_{max} = 35e3\,N$$

Calculations done in Python: github.com/jspecht3/classes/npre330/hw5

	Al	Ti	Steel	Mg
E [GPa]	70	105	205	45
YS [MPa]	250	850	550	170
ν	0.33	0.36	0.27	0.35
$F_{max,\,plastic}$ [kN]	44.179	150.207	97.193	30.041
Passes req 1?	yes	yes	yes	no
$F_{max,\,diameter}$ [kN]	29.988	41.233	107.338	18.176
Passes req 2?	no	yes	yes	no

Where $F_{max,\,plastic}$ is the max force before plastic deformation occurs.

& $F_{max,\,diameter}$ is the force required to obtain a Δd of 1.2e-5 m

From this, we see Ti & Steel suit the task as the force required to induce plastic deformation & to cause Δd of 1.2e-5 m are $> F_{max} = 35e3$

HW#5-cont

3)

a) $E = \frac{1000 \text{ MPa}}{0.005} = \boxed{200 \text{ GPa}}$

b) proportionality limit is $\sigma = 1400$ MPa & $\epsilon = 0.006$

c) $YS_{0.2\%} = 1600$ MPa, intersection w/ line w/ slope E & x-intercept @ 0.002

d) TS = 1900 MPa, max value

e) Elastic deformation, below $\propto$ limit

$$\sigma = \frac{F}{A} = E\epsilon = E\frac{l - l_0}{l_0} \Rightarrow \underbrace{-\frac{F l_0}{EA}}_{dl} + l_0 = l$$

$l = l_0 - dl = 500\text{mm} - 4.45634\text{ mm} = 495.544\text{ mm}$

$\%\text{ elong} = \frac{l - l_0}{l_0} = -0.89\%$

$$\boxed{\begin{array}{l} \text{Elastic deformation} \\ \Delta l = -4.456\text{mm} \\ l_f = 495.544\text{mm} \\ \%\text{ elong} = -0.89\% \end{array}}$$

4 Question 4

4.a Engineering Stress-Strain Curve

The stress is given as:

$$\sigma = \frac{N}{4\pi D^2} \tag{1}$$

Where N is the load and D is the diameter.

The strain given as:

$$\epsilon = \frac{l - l_0}{l_0} \tag{2}$$

Where l is the length and l_0 is the initial length.

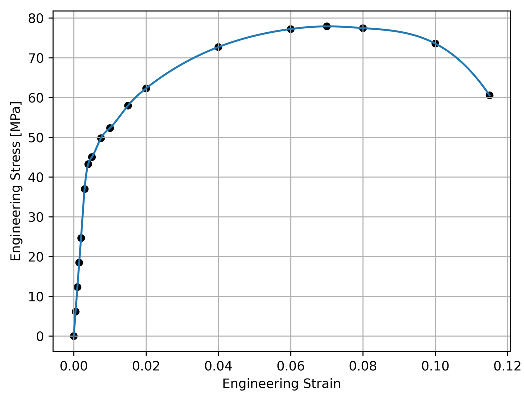


Figure 1: Engineering Stress-Strain Curve

4.b Modulus of Elasticity

To find the modulus of elasticity, find the last point where the linear relation stops. This point will be denoted with the subscript i. To calculate the elasticity modulus E use the following relation:

$$E = \frac{\sigma_i}{\epsilon_i} = 12.359\ GPa \tag{3}$$

4.c Yield Strength

The yield strength is found as the 0.02% offset yield strength where the elastic region is shifted by 0.02% strain and finding the intersection of that line with the stress-strain curve. This is given graphically as:

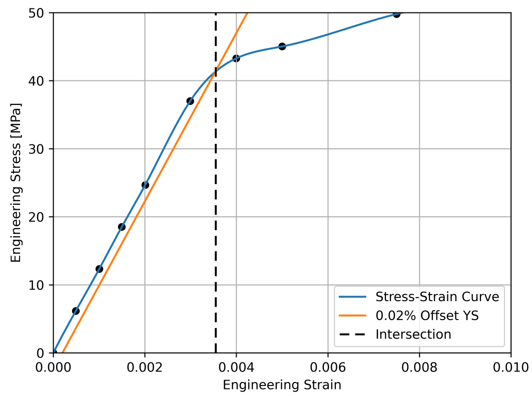


Figure 2: Yield Strength Calculation

This gives the yield strength as 41.358 MPa.

4.d Tensile Strength

The tensile strength is the ultimate strength and can be found using np.max on the stress, which gives a tensile strength of 77.907 MPa.

4.e Ductility

The ductility can be found as the max percent elongation, which is the percent elongation at the point of failure or the engineering strain at the point of failure.

$$\%_{elong} = \epsilon_f \cdot 100 = 11.5\% \tag{4}$$

4.f Modulus of Resilience

The modulus of resilience is found by integrating the stress-strain curve, which can be done with scipy. This gives $U_r = 7.934$ MPa

4.g True Stress-Strain Curve

The true stress is:

$$\sigma_t = \sigma(1 + \epsilon) \tag{5}$$

The true strain is:

$$\epsilon_t = \ln(1 + \epsilon) \tag{6}$$

Making sure to correct for the last three loads having diameters of 12.22-, 11.80-, and 10.65mm, respectively. This gives a true stress-strain curve as:

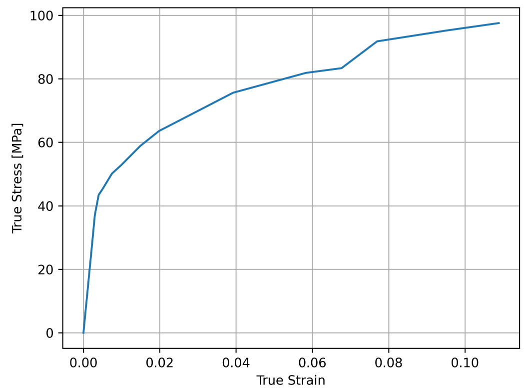
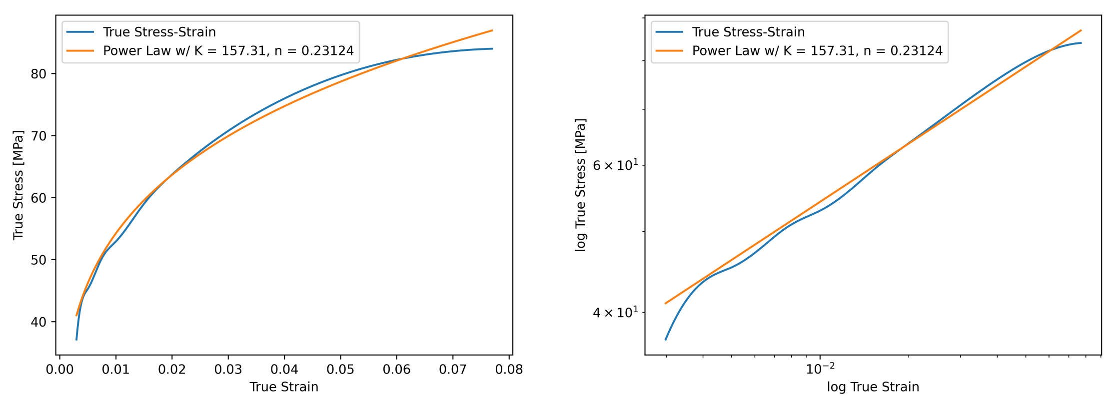


Figure 3: True Stress-Strain Curve

4.h Strain Hardening

From the true stress-strain, in the plastic deformation region before necking, we can apply a scipy.optimize.curve_fit on the true stress-strain to approximate the relation as a power law:

$$\sigma_t = K\epsilon_t^n \tag{7}$$



(a) Linear

(b) LogLog

Figure 4: Power Law Approximation

Where $K = 157.314$ MPa and $n = 0.231$. We see both graphs are linear in log-log scale.

4.i Material Comparison

From Table 6.4 in the book, there are no materials that line up perfectly with this calculation. However, we can see K is very low, concerningly low. Low enough to make me think I should have done this homework earlier and asked you at office hours. Oh well. K is very low, around a third of the lowest value for the table, so the material would have to be very weak. n is around the same as annealed low-carbon steel and annealed naval brass, which means the ductility is around that of these two materials. With the K, which corresponds to material strength, being very low and n, which corresponds to ductility, being close to annealed alloys, this material is likely something like annealed, pure aluminum.

All code for this homework can be found at https://github.com/jspecht3/classes/tree/main/npre330