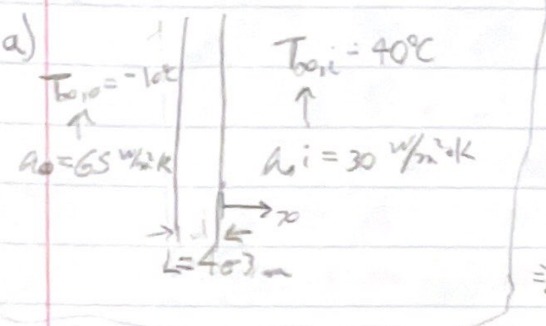
HW #3

1) 3.3 B&L

a)



Know $q = \frac{T_{\infty o} - T_{\infty i}}{R_{tot}}$

w/ $R_{tot} = \frac{1}{\frac{1}{hA} + \frac{L}{kA} + \frac{1}{h_i A}}$

A.3

$\Rightarrow q = \frac{(T_{\infty,o} - T_{\infty,i})A}{\frac{1}{h_o} + \frac{L}{k} + \frac{1}{h_i}}$ w/ $k = 1.4\ W/mK$

$\frac{q}{A} = \frac{(-10°C - 40°C)}{\frac{1}{65\ W/m^2K} + \frac{4e\text{-}3\ m}{1.4\ W/mK} + \frac{1}{30\ W/m^2K}} = q'' = \underline{-969.46\ W/m^2}$

Set equal to other q'' expression

$q'' = \frac{T_{\infty,o} - T_{s,o}}{\frac{1}{h_o}} \Rightarrow \left(\frac{1}{h_o} + \frac{L}{k}\right) q'' = T_{\infty,o} - T_{s,o} \Rightarrow T_{s,o} = T_{\infty,o} - q''\left(\frac{1}{h_o} + \frac{L}{k}\right)$

$\Rightarrow T_{s,o} = -10°C + 969.46\ W/m^2 \left(\frac{1}{65\ W/m^2K}\right)$

$\boxed{T_{s,o} = 4.915°C}$ next we find $T_{s,i}$ by similar method

$q'' = \frac{T_{s,i} - T_{\infty,i}}{\frac{1}{h_o}} \Rightarrow T_{s,i} - T_{\infty,i} = q''\left(\frac{1}{h_i}\right) \Rightarrow T_{s,i} = T_{\infty,i} + q''\left(\frac{1}{h_i}\right)$

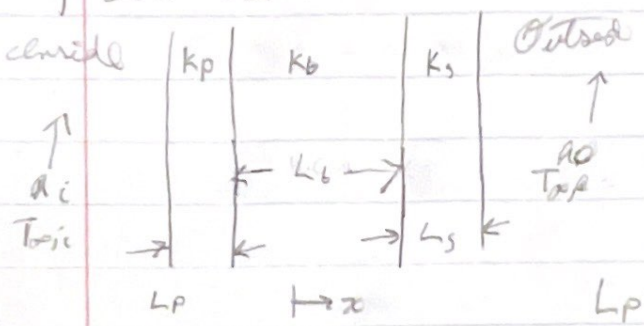
$\Rightarrow T_{s,i} = 40°C - 969.46\ W/m^2 \left(\frac{1}{30\ W/m^2K}\right)$

$\boxed{T_{s,i} = 7.685°C}$

b) ...

HW #3 - cont

2) 3.11 B&L



A.3

$k_p = 0.58\ \text{W/m·K}$, $\rho_p = 1920\ \text{kg/m}^3$

$k_b = 0.036\ \text{W/m·K}$, $\rho_b = 28\ \text{kg/m}^3$

$k_s = 0.094\ \text{W/m·K}$, $\rho_s = 640\ \text{kg/m}^3$

$T_{\infty,i} = 20°C$, $T_{\infty,o} = -15°C$

$h_i = 30\ \text{W/m}^2\text{K}$, $h_o = 60\ \text{W/m}^2\text{K}$, $A = 350\ \text{m}^2$

$L_p = 10e^{-3}\ \text{m}$, $L_b = 100e^{-3}\ \text{m}$, $L_s = 20e^{-3}\ \text{m}$

a) $$\boxed{R_{tot} = \frac{1}{h_i A} + \frac{L_p}{k_p A} + \frac{L_b}{k_b A} + \frac{L_s}{k_s A} + \frac{1}{h_o A}}$$

$$R_{tot} = \frac{1}{350\frac{W}{K}}\left(\frac{1}{30} + \frac{10e^{-3}}{0.58} + \frac{100e^{-3}}{0.036} + \frac{20e^{-3}}{0.094} + \frac{1}{60}\right) = 0.0515751\ \text{K/W}$$

b) $$q = \frac{T_{\infty i} - T_{\infty,o}}{R_{tot}} = \frac{(20+15)°C}{0.0516\ \text{K/W}} = \boxed{678.62\ \text{W} = q}$$

c) $$R_{tot,new} = \frac{1}{350\ \text{K/W}}\left(\dots + \dots + \dots + \dots + \frac{1}{300}\right) = 0.008698\ \text{K/W} \Rightarrow q_{new} = \frac{T_{\infty,i} - T_{\infty o}}{R_{tot,new}} = \frac{(20+15)°C}{0.008698\ \text{K/W}}$$

$$q_{new} = 4023.713\text{W},\ \%_{inc} = \frac{q_{new} - q}{q} = \frac{4023.71 - 672.62}{672.62} \times 100 = \boxed{492.92\%}$$

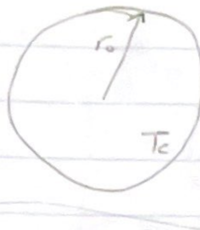
d) fiber glass as it has the highest resistance, which means it will determine q by same reasoning as secular equilibrium in decay chains

HW#3 - cont'd

4) 3.62 $\quad T_B = 37°C$

SS

q dissipation



know $q = -kA\nabla T$

$$\Rightarrow \nabla^2 T + \underbrace{\dot{q}''}_{0 \text{ in blood}} = \rho c_p \underbrace{\frac{\partial T}{\partial t}}_{0,\,SS} \Rightarrow \nabla^2 T = 0$$

$$\Rightarrow \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0 \Rightarrow \frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$$

$$\Rightarrow r^2 \frac{dT}{dr} = C_1 \Rightarrow \frac{dT}{dr} = C_1 r^{-2} \Rightarrow T = \frac{-C_1}{r} + C_2$$

i) $\lim_{r\to\infty} T = T_B \Rightarrow T_B = \frac{-C_1}{\infty} + C_2 = 0 + C_2 \Rightarrow C_2 = T_B$

ii) @ $r = r_0$, $q = -kA\nabla T\big|_{r=r_0} = -k\,4\pi r_0^2 \frac{dT}{dr}\big|_{r_0} = -k4\pi r_0^2 C_1 r_0^{-2}$

$$\Rightarrow q = -k4\pi C_1 \Rightarrow C_1 = \frac{-q}{4\pi k}$$

$$\boxed{\therefore T = \frac{q}{4\pi k r} + T_B}$$

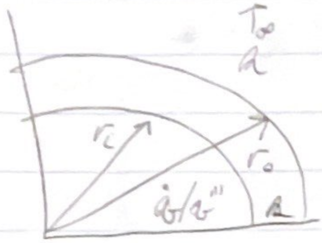
w/ $r_0 = .5e{-3}\,m$, $T_c = 42°C$, $T(5e{-3}\,m) = \overset{42°C}{T_c}$, $k = 0.5\ W/mK$

$q = (T(r) - T_b)4\pi k r$, $\quad q(5e{-3}\,m) = (42°C - 37°C)\,4\pi(0.5\,W/mK)(0.5e{-3}\,m)$

$$\boxed{q = 0.0157\,W}$$

HW # 3 - cont

3) 3.59 B&L $\qquad T(r),\ T \neq f(\theta, \varphi)$



$$k\nabla^2 T + \cancel{\dot{q}}^{\,0} = \cancel{\rho c_p \frac{dT}{dt}}^{\,0,\,ss} \Rightarrow k\nabla^2 T = 0$$

$$\Rightarrow \nabla^2 T = 0 \Rightarrow r^2 \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0$$

$$\Rightarrow \frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) = 0 \Rightarrow \frac{dT}{dr} = \frac{C_1}{r^2} \Rightarrow T = \frac{-C_1}{r} + C_2$$

Use E balance $\quad \dot{E}_g = \cancel{\dot{E}_{in}}^{\,0} - \dot{E}_{out} \Rightarrow \dot{q}\forall = -kA \left.\frac{dT}{dr}\right|_{r=r_i}$

$$\Rightarrow \dot{q}\,\frac{4}{3}\pi r_i^3 = -k\left(4\pi r_i^2\right)\left.\frac{dT}{dr}\right|_{r=r_i} \Rightarrow \left.\frac{dT}{dr}\right|_{r=r_i} = \frac{-\dot{q} r_i}{3k} = \frac{C_1}{r_i^2}$$

$$\Rightarrow C_1 = \frac{-\dot{q} r_i^3}{3k}$$

next do outer surface

$$-kA\left.\frac{dT}{dr}\right|_{r=r_o} = hA\left(T(r_o) - T_\infty\right) \Rightarrow \left.\frac{dT}{dr}\right|_{r=r_o} = \frac{-h}{k}\left(T(r_o) - T_\infty\right)$$

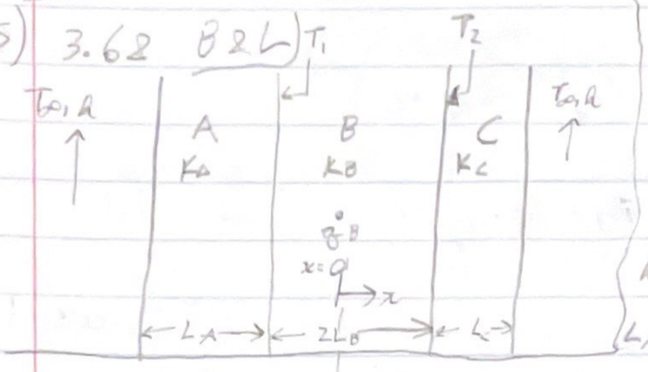
$$\Rightarrow \left.\frac{dT}{dr}\right|_{r=r_o} = \frac{-\dot{q} r_i^3}{3k r_o^2} = \frac{-h}{k}\left(\frac{\dot{q} r_i^3}{3k r_o} + C_2 - T_\infty\right) \Rightarrow \frac{\dot{q} r_i^3}{3h r_o^2} - \frac{\dot{q} r_i^3}{3k r_o} + T_\infty = C_2$$

$$\therefore T = \frac{\dot{q} r_i^3}{3kr} + \frac{\dot{q} r_i^3}{3h r_o^2} - \frac{\dot{q} r_i^3}{3k r_o} + T_\infty$$

$$\boxed{T = T_\infty + \frac{\dot{q} r_i^3}{3}\left(\frac{1}{kr} + \frac{1}{h r_o^2} - \frac{1}{k r_o}\right)}$$

HW#3-cont'd

5) 3.62 B&L



$T_\infty = 25°C$, $h = 1000\ W/m^2\cdot K$

$\nabla \dot q_B = 0$, $\dot q_A = \dot q_C = 0$

$T_1 = 261°C$, $T_2 = 211°C$

$k_A = 25\ W/mK$, $k_B = 50\ W/mK$

$L_A = 30e{-3}\ m$, $L_B = 30e{-3}\ m$, $L_C = 20e{-3}\ m$

a) $k_B \nabla^2 T_B + \dot q = \cancelto{0}{\rho c_p \frac{dT}{dt}} \Rightarrow \nabla^2 T_B = \frac{-\dot q}{k_B} \Rightarrow \frac{d^2 T_B}{dx^2} = -\frac{\dot q}{k_B}$

$\Rightarrow T_B = \frac{-\dot q}{2k_B}x^2 + C_1 x + C_2$ & $\nabla T_B = \frac{dT_B}{dx} = \frac{-\dot q}{k_B}x + C_1$

Know temp @ $\pm L_B$, so

$T_B(-L_B) = \frac{-\dot q}{2k_B}(-L_B)^2 - C_1 L_B + C_2 = T_1$

$T_B(L_B) = \frac{-\dot q}{2k_B}(L_B)^2 + C_1 L_B + C_2 = T_2$

We can also use the thermal resistance analogy to get q''

$$q_1'' = \frac{-(T_\infty - T_1)}{\frac{1}{h} + \frac{L_A}{k_A}} = \frac{-(25°C - 261°C)}{\frac{1}{1000\ W/m^2K} + \frac{30e{-3}\ m}{25\ W/mK}} = 107{,}273\ W/m^2$$

$$q_2'' = \frac{T_2 - T_\infty}{\frac{L_C}{k_C} + \frac{1}{h}} = \frac{(211°C - 25°C)}{\frac{20e{-3}}{50\ W/mK} + \frac{1}{1000\ W/m^2K}} = 132{,}857\ W/m^2$$

HW#3 - cont

BC

i) $-q_1'' = -k_\theta \nabla T_\theta = -k_\theta \left(\frac{-\dot{q}_\theta}{k_\theta}(-L_\theta) + C_1\right)$

$$\Rightarrow \frac{q_1''}{k_\theta} = \frac{\dot{q}_\theta}{k_\theta} L_\theta + C_1 \Rightarrow \frac{1}{k_\theta}\left(q_1'' - \dot{q}_\theta L_\theta\right) = C_1$$

ii & iii) plug this C_1 back into T_θ eq @ $\pm L_\theta$

$$\Rightarrow T_1 = \frac{-\dot{q}}{2k_\theta}(L_\theta)^2 + \frac{L_\theta}{k_\theta}\left(\dot{q}_\theta L_\theta - q_1''\right) + C_2 \Rightarrow C_2 = T_1 - \text{" "}$$

$$T_2 = \frac{-\dot{q}}{2k_\theta}(L_\theta)^2 + \frac{L_\theta}{k_\theta}\left(q_1'' - \dot{q}_\theta L_\theta\right) + C_2 \Rightarrow C_2 = T_2 - \text{" "}$$

set =

$$T_1 + \frac{\dot{q} L_\theta^2}{2k_\theta} - \frac{\dot{q} L_\theta^2}{k_\theta} + \frac{q_1'' L_\theta}{k_\theta} = T_2 + \frac{\dot{q} L_\theta^2}{2k_\theta} + \frac{\dot{q} L_\theta^2}{k_\theta} - \frac{q_1'' L_\theta}{k_\theta}$$

$$\Rightarrow T_1 - \frac{\dot{q} L_\theta^2}{2k_\theta} + \frac{q_1'' L_\theta}{k_\theta} = T_2 + \frac{3\dot{q} L_\theta^2}{2k_\theta} - \frac{q_1'' L_\theta}{k_\theta}$$

$$\Rightarrow T_1 - T_2 = \frac{2\dot{q} L_\theta^2}{k_\theta} - \frac{2 q_1'' L_\theta}{k_\theta} = \frac{1}{k_\theta}\left(2\dot{q} L_\theta^2 - 2 q_1'' L_\theta\right)$$

$$\Rightarrow k_\theta = \frac{2\dot{q} L_\theta^2 - 2 q_1'' L_\theta}{T_1 - T_2}$$

HW#3 - cont

iv) back to i) $-q''_2 = -k_B \nabla T_B \big|_{x=L_B} = -k_B\left(\frac{-\dot{q}_B}{k_B}(L_B) + C_1\right)$

$\Rightarrow \frac{q''_2}{k_B} = \frac{\dot{q}_B L_B}{k_B} + C_1 \Rightarrow C_1 = \frac{1}{k_B}(q''_2 - \dot{q}_B L_B)$ set result from i) & iv) =

opposite directions

$\Rightarrow \frac{1}{k_B}(q''_2 - \dot{q}_B L_B) = \frac{-1}{k_B}(q''_1 - \dot{q}_B L_B) \Rightarrow q''_2 + q''_1 = \dot{q}_B L_B + \dot{q}_B L_B$

$q''_2 + q''_1 = 2\dot{q}_B L_B \Rightarrow \dot{q}_B = \frac{q''_2 + q''_1}{2L_B} = \boxed{4.002e6 \ W/m^3 = \dot{q}_B}$

Plug back into result from ii & iii)

$k_B = \frac{2\dot{q}_B L_B^2 - 2q''_1 L_B}{T_1 - T_2} = \frac{2(4e6 \ W/m^3)(30e^{-3} m)^2 - 2(107{,}273 \ W)(30e^{-3} m)}{261°C - 211°C}$

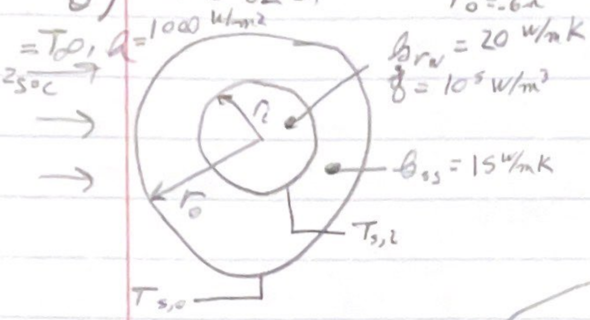
$$\boxed{k_B = 15.35 \ W/mK}$$

e) ...

c) $T_1 = 835°C$, $T_2 = 360°C$

HW#3 - cont'd

6) 3.26 B&L $\quad r_i = .5m \quad r_o = .6m$



a) Energy Balance

$$\dot{E}_{store}^{\,0} = \dot{E}_{in}^{\,0} + \dot{E}_{gen} - \dot{E}_{out}$$

$$\Rightarrow \dot{E}_{gen} = \dot{E}_{out}$$

$$\dot{q}V = hA(T_{s,o} - T_\infty)$$

$$\Rightarrow \dot{q}\frac{4}{3}\pi r_i^3 = h4\pi r_o^2 (T_{s,o} - T_\infty)$$

$$\Rightarrow T_{s,o} = T_\infty + \frac{\dot{q} r_i^3}{3hr_o^2} = 25°C + \frac{10^5\ W/m^3 \cdot (0.5m)^3}{3000\ W/m^2 \cdot K\ (.6m)^2} = \boxed{36.6°C}$$

b) $$hA(T_{s,o} - T_\infty) = \frac{4\pi k_{ss}(T_{s,i} - T_{s,o})}{\left(\frac{1}{r_i} - \frac{1}{r_o}\right)} \Rightarrow T_{si} = \frac{h\overset{4\pi r_o^2}{A}(T_{so} - T_\infty)\left(\frac{1}{r_i} - \frac{1}{r_o}\right)}{4\pi k_{ss}} + T_{so}$$

$$T_{si} = \frac{(1000)(.6m)^2(36.6°C - 25°C)\left(\frac{1}{.5} - \frac{1}{.6}\right)}{4\pi(15)} = \boxed{129.5°C}$$

c) $$k\nabla^2 T + \dot{q} = \rho c_p \cancel{\frac{\partial T}{\partial t}}^{\,0} \Rightarrow \frac{k_{rw}}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \dot{q} = 0 \Rightarrow \frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{-\dot{q}r^2}{k_{rw}}$$

$$\Rightarrow r^2\frac{\partial T}{\partial r} = \frac{-\dot{q}r^3}{3k_{rw}} \Rightarrow \underline{\frac{\partial T}{\partial r} = \frac{-\dot{q}r}{3k_{rw}} + C_1} \Rightarrow T = \frac{-\dot{q}r}{3k_{rw}} + C_1 r + C_2$$

need BC

i) $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 \Rightarrow \frac{\partial T}{\partial r} = 0 = \frac{-\dot{q}\,0}{3k_{rw}} + C_1 \Rightarrow C_1 = 0$

ii) $T|_{r_i} = T_{s,i} \Rightarrow T_{s,i} = \frac{-\dot{q}r_i}{3k_{rw}} + C_2 \Rightarrow C_2 = T_{s_i} + \frac{\dot{q}r_i}{3k_{rw}}$

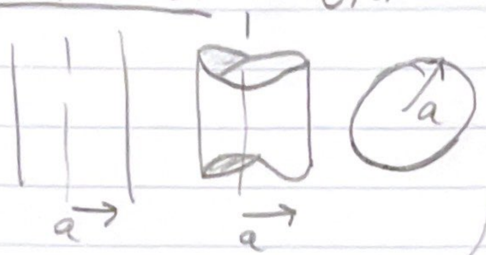
$$C_2 = 129.5°C + \frac{5e5\,(.5)^2}{6 \cdot 20} = 337.8°C$$

$$T(r=0) = C_2 = \boxed{337.8°C}$$

d) Nah

HW #3 - cont

7) 3.27 B&L $\dot{q}, k$



a) use Eq. C.22, 23, 24

$$\text{plane} = \frac{T(x)-T_s}{\dot{q}a^2/2k} = 1-\left(\frac{x}{a}\right)^2$$

$$\text{cylinder} = \frac{T(r)-T_s}{\dot{q}a^2/2k} = \frac{1}{2}\left[1-\left(\frac{r}{a}\right)\right]^2$$

$$\text{sphere} = \frac{T(r)-T_s}{\dot{q}a^2/2k} = \frac{1}{3}\left[1-\left(\frac{r}{a}\right)^2\right]$$

b) Sphere, highest SA/V ratio for any shape

c) Sphere, it can run @ a higher temperature