NPRE 412 Spring 2025
HW 3 Due 2025.02.11

- Show your work.
- This work must be submitted online as a `.pdf` through Canvas.
- Work completed with LaTeX or Jupyter earns 1 extra point. Submit source file (e.g. `.tex` or `.ipynb`) along with the `.pdf` file.
- If this work is completed with the aid of a numerical program (such as Python, Wolfram Alpha, or MATLAB) all scripts and data must be submitted in addition to the `.pdf`.
- If you work with anyone else, document what you worked on together.

1. A utility has constructed a nuclear power plant over a period of 12 years with expenditures per year as shown in the table below. (Tsoulfanidis, Question 8.6)

Year before operation	**Expenditure at end of year ($)**
12,11,10	85
9,8	110
7	115
6	200
5	220
4	230
3	245
2	280
1	200
0	120

Assume that the company has a debt ratio of 51%, the rate of return for stocks is 11%, the interest for bonds is 8.5%, and the corporate tax is 40%.

(a) (20 points) What is the total cost of the plant at the time it begins operating?

Solution: The total cost of the plant at the time it begins operating is $3139.57M. Mahmoud Eltawila and I solved the problem independently and then reconciled our differences.

Using Eq. 8.30, we obtain the effective interest rate, x, of the expenditure at the end of each year:

$$x = (i_s)(1 - r_d) + (i_b)(r_d)(1 - \tau) = 0.07991 \tag{1}$$

Where i_s is the interest rate of stocks, i_b is the interest rate of bonds, r_d is the debt ratio, $(1 - r_d)$ is the stock ratio, and τ is the corporate tax rate.

With x, we can sum over all the years to obtain the total cost using the yearly expenditure, b, as the principle:

$$Total = \sum_{n=0}^{N} b(1+x)^n \tag{2}$$

Where $Total$ is \$3139.57M with the given parameters.

(b) (20 points) If the utility decides to repay this amount by assigning 30 equal payments over the life of the plant, what should the amount of each payment be?

Solution: Assuming the lifetime of the reactor is 30 years, the cost per payment is \$278.64M. Mahmoud Eltawila and I discussed what interest rate to use for this problem.

Using Eq. 8.11, we can find the cost per year using the amortization method. We use x as the interest rate as the debt from bond interest and the debt from stock returns are still different.

$$\frac{F}{N} = \left[\frac{x(1+x)^N}{(1+x)^N - 1}\right] P \tag{3}$$

With $\frac{F}{N}$ as the price of each payment, P is the amount to be repaid, and N is the number of years the debt will be repaid. When plugging in the answer from part a for P and 15 for N, we obtain:

$$\frac{F}{15} = \left[\frac{x(1+x)^{15}}{(1+x)^{15} - 1}\right] (\$3139.57M) = \$278.64M \tag{4}$$

2. (30 points) Clinton Nuclear Power Station was finished in 1987 after a period of construction lasting about 15 years. Assuming an interest rate of 12% (during the period 1972–1987 the prime interest rate varied between 6% and a high of 21.5%, averaging in the teens most of the period), how much money did Illinois Power borrow each year for the 15 years if the final cost of the nuclear plant was \$4B? Assume that Illinois Power borrowed the same amount each year – so $\frac{1}{15}^{th}$ of the total was borrowed the first year at 12%, and additional $\frac{1}{15}^{th}$ was borrowed the second year at 12%, etc.

Solution: The money borrowed per year is \$107.297M. Mahmoud Eltawila and I checked our answers after solving it different ways.

We know the sum of each payment, P, over each year, N, converted to the future value of that payment is equal to the final cost, F:

$$F = \sum_{n=0}^{N} P(1+i)^n = P\sum_{n=0}^{N}(1+i)^n \tag{5a}$$

$$P = \frac{F}{\sum_{n=0}^{N}(1+i)^n} \tag{5b}$$

However, the sum is a geometric series, so apply series rules:

$$\sum_{n=0}^{N} ar^N = \frac{ar^{n+1} - a}{r - 1}, \quad r \neq 1 \tag{6a}$$

$$\sum_{n=0}^{N}(1+i)^n = \frac{(1+i)^N - 1}{(1+i) - 1} = \frac{(1+i)^N - 1}{i} \tag{6b}$$

Substitute back into the first equation:

$$P = F\frac{i}{(1+i)^N - 1} \tag{7}$$

Which is the equation for the payment made to a future value annuity. Plugging in $N = 15$ gives \$107.297 per year.

3. (30 points) Calculate the cost of 1 kg of uranium-fabricated fuel at the time the fuel goes into the core, assuming 12.5% cost of money compounded monthly. Also assume the following:

- Enrichment (at 3.2% enrichment, 0.2% tails, and \$105 per SWU) costs approximately \$500/kg..
- Enrichment losses are 487.1%
- Conversion losses are 0.5%.
- U_3O_8 costs $\frac{\$60}{lb\ U_3O_8}$.
- Conversion costs $\frac{\$10}{kgU}$
- Fabrication-transportation costs \$220/kg.
- Loss in Fabrication is 0.8%.
- Start of operation is date zero (fuel goes into the core).
- Uranium was paid for 24 months before date zero.
- Conversion costs were paid 15 months before date zero.
- Enrichment costs were paid 10 months before date zero.
- Fabrication costs were paid 3 months before date zero.

Solution: The cost of 1 kg of uranium-fabricated fuel is \$24721.97. Mahmoud Eltawila and I solved the problem independently, discussed what we did differently, and reconciled our differences. (Don't be fooled; the derivation is on the next page!)

For the following derivation, the following nomenclature is used:

$$i = \text{monthly interest rate } c_x = \text{unit cost of } x \; [\tfrac{\$}{kg}]$$
$$l_x = \text{percent loss during process } x$$
$$t_x = \text{time before start date when } x \text{ was purchased } [mos]$$
$$m_x = \text{mass of } x \text{ [kg]}$$
$$pw_x = \text{(p)resent (w)orth of } x\text{, price } x \text{ was purchased at [\$]}$$
$$fw_x = \text{(f)uture (w)orth of } x\text{, price of } x \text{ in month 0 [\$]}$$

With the subscript x representing the different steps in the fuel fabrication process:

$$f = \text{(f)abricated fuel}$$
$$e = \text{required-(e)nrichment } UF_6$$
$$c = \text{natural-enrichment, (c)onverted } UF_6$$
$$r = \text{(r)aw yellow cake, } U_3O_8$$

The aforementioned values are tabulated below. The derivation of each value will follow the table.

x	c_x $[\frac{\$}{kg \cdot U}]$	l_x **[%]**	m_x **[kg]**	t_x **[mos.]**	pw_x **[\$]**	fw_x **[\$]**
Yellow Cake, r	143.83	n/a	5.9476	24	855.44	14449.41
Natural UF_6, c	10	0.5	5.918	15	59.48	348.04
Enriched UF_6, e	500	487.1	1.008	10	2958.98	9608.77
Fabricated Fuel, f	220	0.8	1	3	221.76	315.75

To begin, we will calculate the massed, m_x, for each of the processes:

$$m_f = 1 \tag{8a}$$

$$m_e = m_f \left(1 + l_f\right) \tag{8b}$$

$$m_c = m_e \left(1 + l_e\right) \tag{8c}$$

$$m_r = m_c \left(1 + l_c\right) \tag{8d}$$

The cost of yellow cake is given in $\frac{\$}{lbU_3O_8}$, however, the conversion from yellow cake to UF_6 is given in units of $\frac{\$}{kgU}$. Therefore, we need to convert the cost of yellow cake to $\frac{\$}{kgU}$

$$c_r = \frac{\$60}{lb\ U_3O_8} \cdot \frac{2.2\ kg}{1\ lb} \cdot \frac{M_{U_3O_8}}{3M_U} = \frac{\$132}{kg\ U_3O_8} \cdot \frac{[3(238.03) + 8(16)]U_{U_3O_8}}{3(238.03)U_U} = \frac{\$155.66}{kg\ U} \tag{9}$$

With the masses and conversion efficiencies for each process, we calculate the past worth invested to further refine x:

$$pw_r = c_r * m_r \tag{10a}$$

$$pw_c = c_c * m_r \tag{10b}$$

$$pw_e = c_e * m_c \tag{10c}$$

$$pw_f = c_f * m_e \tag{10d}$$

Next, we account for the monthly interest rate to find the current worth of the past dollars when x was purchased:

$$cw_r = pw_r \left(1 + i\right)^{t_r} \tag{11a}$$

$$cw_c = pw_c \left(1 + i\right)^{t_c} \tag{11b}$$

$$cw_e = pw_e \left(1 + i\right)^{t_e} \tag{11c}$$

$$cw_f = pw_f \left(1 + i\right)^{t_f} \tag{11d}$$

With this, we have all the values displayed in the previous table. To find the totals, sum the cost of each process to obtain \$24721.97 / kg for uranium-fabricated fuel.