Elementary Laboratory Procedures - Full Report

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TAM 335 – Lab Section ABE

September 23, 2023

I. Introduction

This experiment serves as an initial laboratory exercise designed to introduce the fundamental principles of fluid mechanics. It aims to familiarize essential concepts, including the measurement of flow rate and pressure variations within a pipeline. Through these measurements, students become acquainted with common laboratory instruments such as manometers, the Bourdon gauge, stopwatches, and a weighing tank. Additionally, the experiment focuses on developing analytical techniques for obtaining accurate and precise data.

To determine a consistent volumetric flow rate Q, the weight-time method is employed. In this approach, the use of a tank and scale is imperative to quantify the volume of liquid collected during the flow. Furthermore, a stopwatch is essential to record the time required to accumulate a specific quantity of liquid. The calculation is based on the following relationship:

$$\dot{M} = \frac{W}{g\Delta t}, \quad (1)$$

in which $\dot{M}$ is the mass flow rate, W is the weight of liquid, g is gravitational constant, and Δt is the change in time. To obtain volumetric flow rate Q, the mass flow rate is divided by the fluid density ρ. This relationship derives the following equation of volumetric flow rate:

$$Q = \frac{\dot{M}}{\rho} = \frac{W}{\rho g \Delta t}. \quad (2)$$

It is now clear that the volumetric flow rate of a given weight of liquid W during a time interval t may be calculated through combining the rate at which mass enters the tank with fluid density. The volumetric flow rate may easily be calculated experimentally using the connection implied by Equation 2.

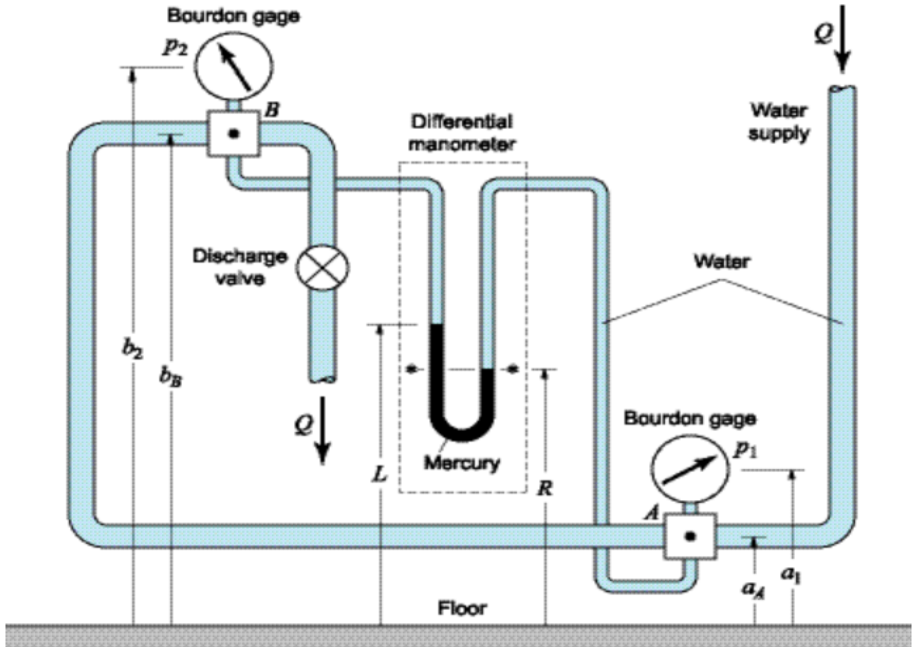

Figure 1. Instrument setup used for measuring volumetric flow rate and pressure.

The instrument setup, shown in Figure 1, can measure the volumetric flow rate as well as the pressure differential between two places in the pipe, A and B. Bourdon gauges or a differential manometer will be used in two separate experimental methods to determine this pressure difference. The more accurate measuring equipment will subsequently be identified after comparing these pressure differences.

Bourdon Gauge

It is simple to measure the pressure with the use of Bourdon gauges. They measure the difference between absolute pressure and ambient pressure since they are gauges. The pressure displayed by Bourdon gauges is centered at the gauge, therefore fundamental hydrostatic pressure calculations must also be used to calculate the water pressure in a pipe below the gauge. For instance, the equation to determine the pressure at point A:

$$p_A = p_1 + \gamma_w(a_1 - a_A), \tag{3}$$

where p_A is the pressure at point A, p_1 is the pressure measured by the gauge, γ_w *is* the specific weight of water, and $(a_1 - a_A)$ is the height difference between the Bourdon gauge (a_1) and the location to measure the water pressure (a_A).

Also, the pressure at a different point B can be calculated using Equation (3):

$$p_B = p_2 + \gamma_w(b_2 - b_B). \tag{4}$$

Ultimately, the pressure difference, $\Delta p = p_A - p_B$, can be calculated by subtracting p_B found in Equation 4 from p_A found in Equation 3.

Differential Manometer

The pressure difference Δp can also be determined using a differential manometer. By using hydrostatics principles, the manometer will deviate from equilibrium since point A has a higher pressure than point B ($b_B > a_A$ at steady flow). The pressure difference is then determined using hydrostatic analysis and the corresponding column heights, L and R. The hydrostatic relationship can be made after the different height measurements shown in Figure 1:

$$p_A - \gamma_w(R - a_A) = p_B + \gamma_w(b_B - L) + \gamma_{Hg}(L - R), \tag{5}$$

where γ_{Hg} is the specific weight of the mercury in the manometer. Rearranging to find Δp by using Equation (5) and $\Delta p = p_A - p_B$,

$$\Delta p = p_A - p_B = \gamma_w(b_B - a_A) + (\gamma_{Hg} - \gamma_w)(L - R). \tag{6}$$

A length measurement aid is positioned between the columns and confined inside the area of the differential manometer to get measurements with the differential manometer that are more precise than those obtained by taking L and R directly from the floor. The scale's new datum line is then set to be some value h_o, which its actual value is irrelevant because only the height disparity must be considered. Thus, starting with the datum h_o, h_L and h_R are used to represent the height at which the left column height increases and decreased height of the right column, respectively. The inclusion of a negative recorded value is carefully considered since h_R represents a downward displacement. When the values are recorded as such, the height difference $\Delta h = h_L - h_R$ is taken, which is the same value discovered by removing R from L. The final connection to measure the pressure difference is calculated from Equation 6 using hydrostatic and fundamental geometric principles:

$$\Delta p = p_A - p_B = \gamma_w(b_B - a_A) + (\gamma_{Hg} - \gamma_w)(h_L - h_R). \tag{7}$$

The pressure difference between points A and B can be measured using a Bourdon gauge with equation 3 and 4 or by using differential manometer with Equation 7.

II. Experimental Methods

Figure 1 shows a schematic image of the experimental setup, including the differential manometer and Bourdon gauges for measuring pressure difference as well as the discharge valve for determining volumetric flow rate.

Make sure the tank drain is open before starting the experiment. First, allow for a maximum, steady flow to be attained using the discharge valve. Both the pressure readings from the Bourdon gauges and the height difference in the differential manometer should be noted. Focusing on the volumetric flow apparatus from the introduction, the balance beam should be overbalanced to the point where the arm touches the bottom stop. Next, close the drain and wait for the tank to fill with water until the balance beam becomes balanced. At this instance, add a 100-lb weight to the balance pan and start recording the time using the timer. The length of time it takes for the balance beam to return to its balanced position is recorded. When balance is attained, open the drain to let the tank completely empty before removing the 100-lb weight. Continue the experiment in same manner for five sets, incrementally lowering the water's mass flow rate with each passing test.

Random errors are likely in this experiment since the experiment in lab was carried out as a class, with smaller groups collecting two measurements each. Erroneous measurement using the manometer height can happen due to human error by incorrect reading of the meniscus. Also, the time is measured by five individuals for each group who may have discrepancy with applying the weight on the balance pan or beginning and stopping a timer. Since the measured and calculated values tend to cluster around accurate values in both cases, random errors occur around these points, and precision is lost due to these errors.

III. Results and Discussion

a. Lab Report

Using Equation 7, the pressure difference is calculated by studying the data gathered for the differential manometer along with measurements of h_L and h_R. The recorded readings from the Bourdon gauges are similarly used with Equations 3 and 4 to get the pressure difference. An analysis of the two methods for determining the pressure difference reveals differences in pressure difference measurements and recording techniques. The comparison of the two approaches is shown in Figure 2 (pg. 6).

Figure 2 illustrates a significant correlation between the pressure difference discovered using the differential manometer and the pressure difference discovered using the Bourdon gauge technique (R2 = 0.9678). The Bourdon gauge pressure measurement is around 0.80 times that of the differential manometer, plus a fixed constant of 4.9 kPa, according to a line of best fit for the relationship. Such differences in pressure calculations are modest for low pressures. Consider a manometer pressure difference of 20 kPa as an illustration. According to the established connection, the difference in Bourdon gauge pressure would be around 20.9 kPa. It barely represents a 4.5% difference. Now, consider a much higher manometer pressure difference of 100 kPa as an illustration. According to the established connection, the difference in Bourdon gauge pressure would be around 84.9 kPa. It barely represents a 15% difference. As shown, this regression model may vary accuracy depending on the level of pressure difference being measured. In this example, the data shows increasing inaccuracy by regression relation for much higher-pressure differences beyond the range of pressure difference measured in this experiment.

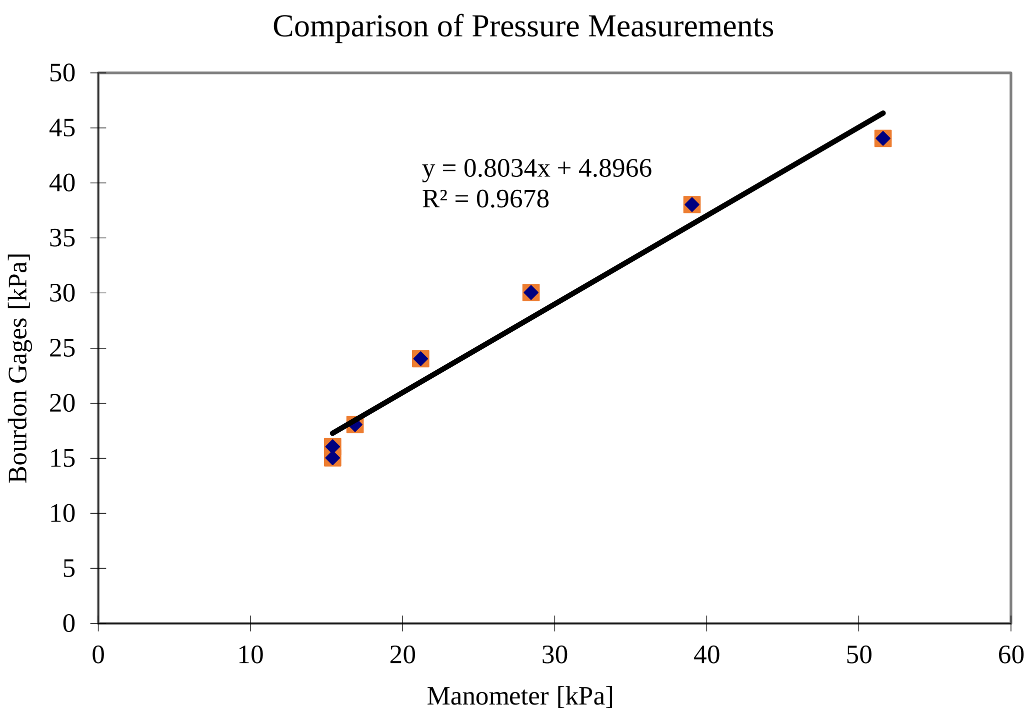

Figure 2. Comparison between the pressure difference measured using Bourdon gauges and the manometer.

The computed pressure differences of the two approaches may then be compared, along with their dependence on flow rate Q, using the calculated flow rates for the six distinct values obtained from Equation 2. This is shown in Figure 3 (next page), which also includes the R^2 values for both approaches and a correlation between them and the flow rate.

The volumetric flow and pressure difference appear to be significantly linearly related. In comparison to the manometer method which gives linear best fit R^2 value of 0.9239, the Bourdon gauges' correlation coefficient for linear relationship between flow rate and pressure differential is 0.9844. From these R^2 numbers, the data demonstrates that the Bourdon gauge is the more accurate measuring device in this experiment since it has an R^2 value that is closer to 1, indicating a stronger linear relationship between the flow rate and its impact on the pressure difference. The manometer method seems more adequate with exponential trendline based on the given data from the lab experiment.

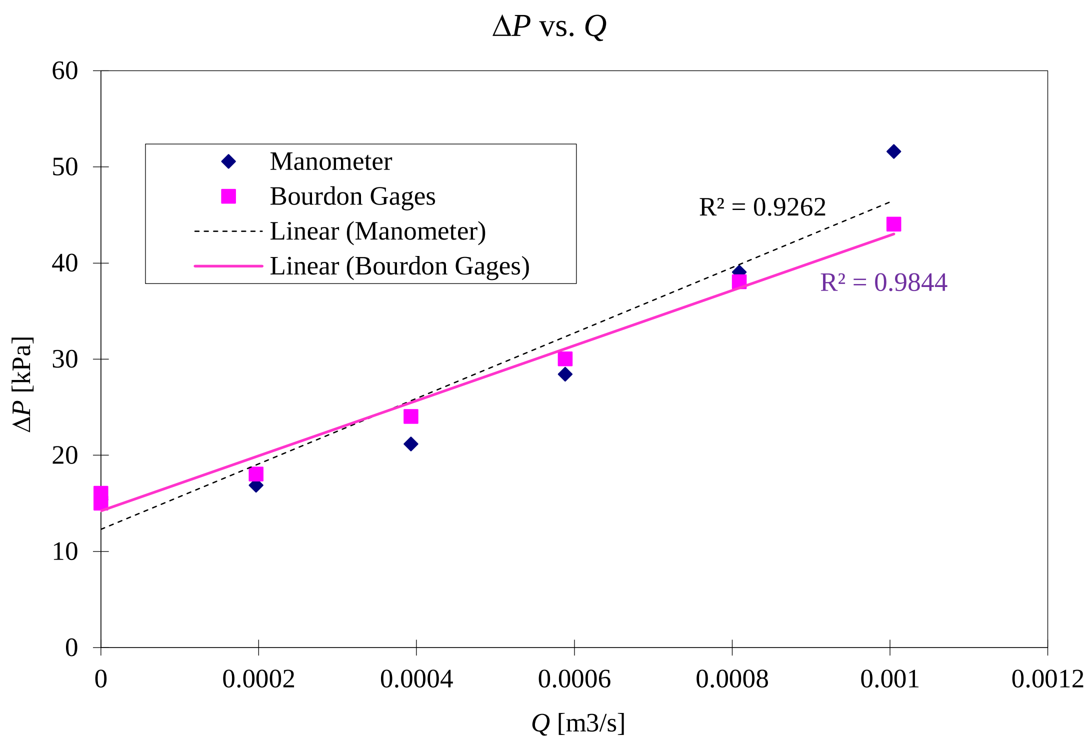

Figure 3. Plot of the pressure difference of each method as a function of volumetric flow.

It is important to note that these relationships may not hold exactly in all real-world scenarios, as factors like fluid viscosity, pipe roughness, turbulence, and compressibility can influence the behavior of fluid flow. The choice between using a manometer or a Bourdon gauge depends on the specific requirements of the application. In fact, manometers are preferred for precise measurements of low-pressure differences due to the sensitivity of the liquid column's movement and that there is a smaller number of measurement readings to be taken, while Bourdon gauges are better suited for moderate to high-pressure differences. There is some overlap in their capabilities, but each has its strengths and limitations. In fact, the specific relationship between flow rate and pressure difference will depend on the fluid system and the conditions under which it operates.

b. Questions

Identifying the accuracy of the recorded volumetric flow rates is a crucial component of the data analysis. The shortest flow rate (Q_s) and the longest flow rate (Q_l) are utilized to determine the accuracy of these measurements. The pertinent parameters are shown in Table 1, and Q_s and Q_l are calculated using Equation 2.

Table 1. Flow rate data acquisition for 5 different flow rates (shortest and longest Δt).

	Weight (lb)	Times (s)	Q (m^3/s)
Shortest Time Readings	100	43.95 51.70 72.72 113.35 227.38	1.033E-3 8.779E-4 6.241E-4 4.004E-4 1.996E-4
Longest Time Reading	100	45.16 56.10 77.12 115.55 231.02	1.005E-3 8.090E-4 5.885E-4 3.928E-4 1.965E-4

Next, using the equation,

$$e = \frac{Q_s - Q_l}{0.5(Q_s + Q_l)} \times 100\%, \tag{8}$$

the precision (e) can be calculated.

Equation 8 reveals the e for five different test readings. e_1 will denote the precision for reading 1, e_2 will denote the precision for reading 2, etc. Using Equation 8 for calculating the precision, e, with the shortest and longest flow rates, Q_s and Q_l, given in Table 1, e_1 is calculated as 2.75%, e_2 as 8.17%, e_3 as 5.87%, e_4 as 1.92%, and e_5 as 1.57%. In engineering analysis, however, it is often preferred that 95% of results fall within two standard deviations of the mean. The desired accuracy might be conceived of as being somewhere between 0-5% from the standpoint of the analysis employing e. For example, if Q_s and Q_l were closer together (more accurate), the value of e would be lower. As a result, e has a smaller value for better accuracy. Therefore, it is possible to consider the percentage e as more of a variable parameter. The shortest and longest time measurements' precision are found for all five different time readings taken in this experiment, and show relatively high precision for the intended values of e as they are mostly between 0% and 5%, except for the 2nd testing (e_2), which resulted from high divergence between the shortest and longest time difference percentage compared to other experimental settings. Thus, this calculation is useful in measuring the precision of the experiment.

In addition to addressing the precision of calculations from measurements in this experiment, it is crucial to consider the experimental accuracy. Previously, through the regression line plotted in Figure 3, the Bourdon Gauge was shown to provide more dependable pressure difference measurement than the manometer, but it is crucial to question the accuracy in the experimental procedure of both methods, as well as the weight-time method's calculation of the flow rate itself.

In our experiment, we averaged the 5 different time measurement for all 5 different experimental settings with different flow rates, therefore the precision of this experiment is increased compared to measuring the flow rate from a single time measurement which could lead to error in precision since time measurement is done by individual students and may involve deviation. To evaluate the accuracy, adding more data for each flow rate is an adjustment that has been suggested. The average of the estimated values may be determined using additional data per flow rate, getting values closer to the accurate value, and taking random error into consideration. Another recommendation is to use a different way to calculate flow rate. The other method's determined values might then be used as the anticipated values, and % error could be calculated using the weight-time method's real values, even though this wouldn't directly measure the accuracy of the weight-time approach. It is notable that both suggestions to increase accuracy use averages and method of gathering more data to evaluate accuracy.

V. Conclusion

The experiment demonstrates that there is a significant linear relationship between the volumetric flow rate and the pressure differential between the two points, A and B. The significant R^2 value of 0.9844 of the Bourdon gauges when showing the pressure differential as a function of flow rate further supports the conclusion that the Bourdon gauges yields more accurate findings than the manometer do according to the linear regression line of our measured data. Additionally, it was noted that manometers are preferred for precise measurements of low-pressure differences due to the sensitivity of the liquid column's movement and that there is a smaller number of measurement readings to be taken, while Bourdon gauges are better suited for moderate to high-pressure differences.

The method of experimentation might be greatly improved by keeping more data for each volumetric flow rate. The averages calculated with additional data for each flow rate assist improve measurement accuracy while taking random error into consideration. Additionally, it was shown that for numerous time measurements of the same flow rate, the accuracy rises.