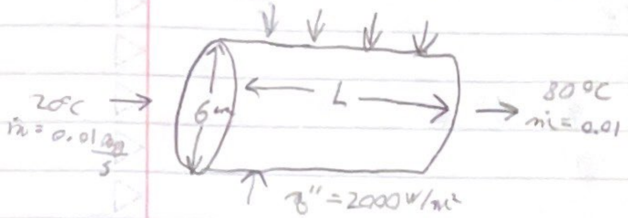
HW #2

- constant properties
- SS, inviscid, no heat gen
- fully developed single phase
- find T_s @ exit
- is fd 1ϕ a good assumption?

1) water @ atm pressure



Start w/

mass: $\cancel{\frac{\partial \rho}{\partial t}}^{0} + \frac{\partial}{\partial x}\rho v = 0 \Rightarrow G$ is constant

momentum: $\cancel{\frac{\partial \rho v}{\partial t}}^{0,\ SS} + \cancel{\frac{\partial}{\partial x}\rho v^2}^{0,\ mass\ solved} = -\frac{\partial P}{\partial z} - \tau_F \frac{\xi_w}{A} - \cancel{\rho g \sin\theta}^{\theta = 0°}$

$$\Rightarrow -\frac{\partial P}{\partial z} = \tau_F \frac{\xi_w}{A} = \frac{f \rho v^2}{2}$$

energy: $\cancel{\frac{\partial \rho h}{\partial t}}^{0,\ SS} + \frac{\partial}{\partial x}\rho h v = \frac{q'' \xi_h}{A} + \cancel{q'''}^{0,\ no}$ $\qquad h = c_p T$

$$\Rightarrow G\frac{\partial (c_p T)}{\partial x} = \frac{q'' \xi_h}{A} \Rightarrow \frac{\partial T}{\partial x} = \frac{q'' \xi_h}{G c_p A}$$

$$\Rightarrow T = \frac{q'' \xi_h}{G c_p A} x + T_0 \Rightarrow \frac{(T - T_i)\, G c_p A}{q'' \xi_h} = l$$

w/ $T = 80°C$, $T_i = 20°C$, $c_p = 4182\ J/kg\cdot K$, $q'' = 2000\ W/m^2$

$G = \rho v = \dot{m}/A \Rightarrow GA = \dot{m} = 0.01\ kg/s$, $\xi_h = \pi D = \pi(6e\text{-}2\,m)$

$$l = \frac{(60°C)(0.01\ \frac{kg}{s})(4182\ \frac{J}{kg\,K})}{\pi(6e\text{-}2\,m)(2000\ W/m^2)} = \boxed{6.656\,m = l}$$

~~yes~~, FD 1ϕ is a good approx

HW #8 - cont'd

$$T_w = \frac{q''}{h} + T_B \quad \text{need } h \Rightarrow Nu = \frac{hD}{k_f} \overset{\text{constant } q'' \text{ pipes}}{=} 4.36 \Rightarrow h = \frac{Nu\, k_f}{D}$$

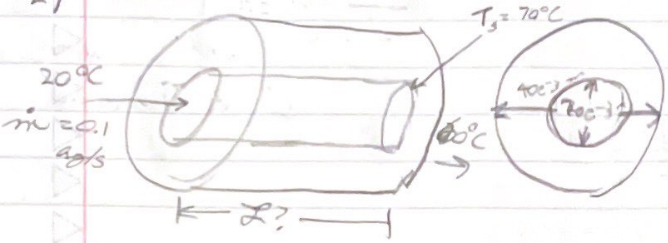
$$\text{w/ } k_f = 0.66 \qquad h = \frac{(4.36)(0.66)}{(6e-2)} = 47.96$$

$$T_w = \frac{(2000)}{(47.96)} + T_B = \boxed{121.70°C = T_w}$$

no, it is not a good assumption as $T_w > T_v$
fully developed is not a good assumption as there is a vapor film forming.

HW #2 - cont

2)



- single Φ, SS, uniform properties

start w/ same eq as 2.1

mass: $\frac{\partial}{\partial x} \rho v = 0 \Rightarrow \dot{m}$ is constant

energy: $\dot{m} \frac{\partial (c_p T)}{\partial x} = \frac{q'' \wp_a}{A} \Rightarrow \frac{\partial T}{\partial x} = \frac{q'' \wp_A}{\dot{m} c_p A} \Rightarrow T = \frac{q'' \wp_a}{\dot{m} c_p A} x + T_i$

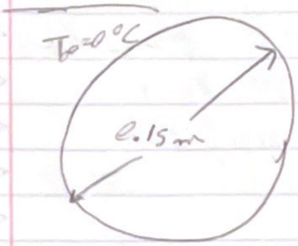
$\Rightarrow \frac{(T_o - T_i)\, \dot{m} c_p A}{q'' \wp_a} = l$ w/ $T_o = 60°C$, $T_i = 20°C$, $\dot{m}A = \dot{m} = 0.1$ kg/s, $c_p = 4182$ J/kgK, $\wp_a = \pi(20e{-3}\ m)$

$q'' A = q''' V \Rightarrow q'' = \frac{q''' V}{A} = \frac{(10^6\ W/m^3)\left[\frac{\pi}{4}\left((40e{-3})^2 - (20e{-3})^2\right) L\right]}{\pi (20e{-3}) L} = 15{,}000\ W/m^2$

$\Rightarrow l = \frac{(40°C)(0.1)(4182)}{(15000)(\pi(20e{-3}))} = \boxed{17.749\ m = l}$

$q'' = h(T_o - T_i) \Rightarrow h = \frac{1500\ W/m^2}{T_o - T_i} = \boxed{15{,}000\ W/m^2 K = h}$

HW #2

3) 

$\dot{m} = 0.05$ kg/s
$T_i = 103°C \rightarrow T_o = 77°C$
$L = 5$ m , $h = 6$ W/m²K

how much heat is lost over the length? find q'' & T_s @ L.

find heat from energy balance

$$\dot{E}_{out} = \dot{E}_{in} + \dot{E}_{gen}^{\nearrow 0} - \dot{E}_{loss} \Rightarrow \dot{m} c_p T_o = \dot{m} c_p T_i - hA(T - T_\infty)$$

$$\Rightarrow hA(T - T_\infty) = \dot{m} c_p (T_i - T_o) = \dot{E}_{loss} = (0.05 \tfrac{kg}{s})(1043 \ J/kgK)(103 - 77°C)$$

$$\boxed{\dot{E}_{loss} = 1355.9 W}$$

assume $q'' = fn(z) \Rightarrow q''(z)$

$$\Rightarrow q''(z) = h_1 (T(z) - T_s(z)) + h_2 (T_s(z) - T_\infty)$$

as thermal resistance problem $q''(z) = \dfrac{T(z) - T_\infty}{1/h_1 + 1/h_2} \Rightarrow q''(L) = \dfrac{77°C - 0°C}{1/h_1 + 1/h_2}$

know $Nu = \dfrac{hD}{k_f} \Rightarrow h = \dfrac{Nu \, k_f}{D}$ now need Nu, so find Re for flow regime

nitrogen

$\mu_{N_2} = 2.068E{-5}$ Pa·s

$$Re = \frac{\rho v D}{\mu} = \frac{\rho v D^2 \pi}{4} \cdot \frac{4}{\pi D \mu} = \frac{4\dot{m}}{\mu \pi D} = 20{,}522.88 \gg 2300 \ \therefore \text{turbulent}$$

$k_f = 0.030368$ W/mK
$c_p = 1043$ J/kgK

∴ use w/ DB correlation $\Rightarrow Nu = 0.023 Re_D^{0.8} Pr^{0.4}$ w/ $Pr = \dfrac{\mu c_p}{k_f} = 0.7103$

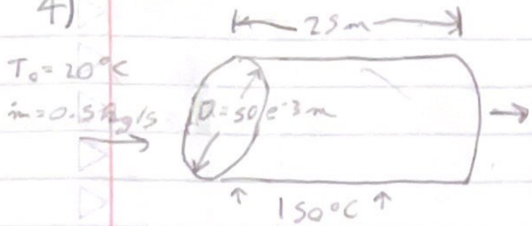
$\Rightarrow Nu = 52.77 \Rightarrow h = 11.84$ W/m²K

$$\Rightarrow q''(L) = \frac{77 K}{1/11.84 + 1/6} = \boxed{306.62 \text{ W/m}^2 = q''} = \frac{T_w(L) - 0°C}{1/h_2} \Rightarrow \boxed{T_w(L) = 51.10 K}$$

for 6, relate $q'' \to q' \to q'''$
for $T_{B,z}$ ← not the averaged one

HW # 8 - cont

4)



- assume FD, constant properties, SS

$\rho = 852$ kg/m³, $k_f = 0.132$ W/m K

$\mu = 0.032$ kg/m s, $c_p = 2131$ J/kg K

Find T_B & q

Start w/

mass: $\frac{\partial \rho}{\partial t}^{\nearrow 0,\text{ constant properties}} + \frac{\partial}{\partial x} \rho v = 0 \Rightarrow \frac{\partial \dot{m}}{\partial x} = 0$ ∴ $\dot{m}$ constant

momentum: $\frac{\partial \rho v}{\partial t}^{\nearrow 0,\text{ constant properties}} + \frac{\partial}{\partial x}\rho v^2{}^{\nearrow 0} = -\frac{\partial P}{\partial x} - \frac{\tau_s \mathscr{P}_w}{A} - \rho g \sin\theta^{\nearrow 0,\ \theta = 0°}$

energy: $\frac{\partial \rho h}{\partial t}^{\nearrow 0,\text{ SS}} + \frac{\partial}{\partial x}\rho h v = \frac{q'' \mathscr{P}_h}{A} + q'''{}^{\nearrow 0} \Rightarrow \dot{m} c_p \frac{\partial T}{\partial x} = \frac{q'' \mathscr{P}_h}{A}$

$\theta = T - T_0$, $\frac{\partial \theta}{\partial x} = \frac{\partial T}{\partial x}$

$\Rightarrow \frac{\partial T}{\partial x} = \frac{q'' \mathscr{P}_h}{c_p A \dot{m}}$ assume $q'' = fn(h, \theta) \Rightarrow \frac{\partial \theta}{\partial x} = \frac{h \mathscr{P}_h}{c_p A \dot{m}} \theta(x)$

$\Rightarrow \theta = \theta_0 \exp\left(\frac{h \mathscr{P}_h}{c_p A \dot{m}}\right) \Rightarrow T(x) = T_s - (T_s - T_0)\exp\left(-\frac{h \mathscr{P}_h x}{c_p \dot{m}}\right)$

find h w/ Nu ⇒ Re for flow regime $Re = \frac{\rho v D}{\mu} = 397.89$ ∴ Laminar

$\Rightarrow Nu = 3.66 = \frac{hD}{k_f} \Rightarrow h = \frac{3.66 k_f}{D} = 10.102$ W/m²K

$\boxed{\therefore T(L) = 24.751°C}$

find $\boxed{q = \dot{m} c_p \Delta T = 5062.14 \text{ W}}$

HW#8-cont

5) Start w/

mass: $\frac{\partial \dot{m}}{\partial z} = 0$ ∴ $\dot{m}$ is constant

momentum: $\cancel{\dot{m}^2 \frac{\partial}{\partial z}\frac{1}{\rho}} = -\frac{\partial P}{\partial z} - \frac{\tau_b \xi_w}{A} \Rightarrow -\frac{\partial P}{\partial z} = \frac{\tau_b \xi_w}{A}$

energy: $\frac{\partial T_b}{\partial z} = \frac{q'' \xi_A}{c_p \dot{m}}$ same soln as q_5^4, so

$\Rightarrow \theta(z) = \theta_0 \exp\left(\frac{-h \xi_A}{c_p \dot{m}} z\right) \Rightarrow$ solve for $z \Rightarrow \frac{\theta}{\theta_0} = \exp\left(\frac{-h \xi_A z}{c_p \dot{m}}\right)$

$\Rightarrow \ln\left(\frac{\theta_0}{\theta}\right) = \frac{h\xi}{c_p \dot{m}} L \Rightarrow L = \frac{c_p \dot{m}}{h \xi} \ln\left(\frac{\theta_0}{\theta}\right)$

$Re_D = \frac{\rho V D}{\mu} = 116415$

$Pr = \frac{\mu C_p}{k} = 6.2033$

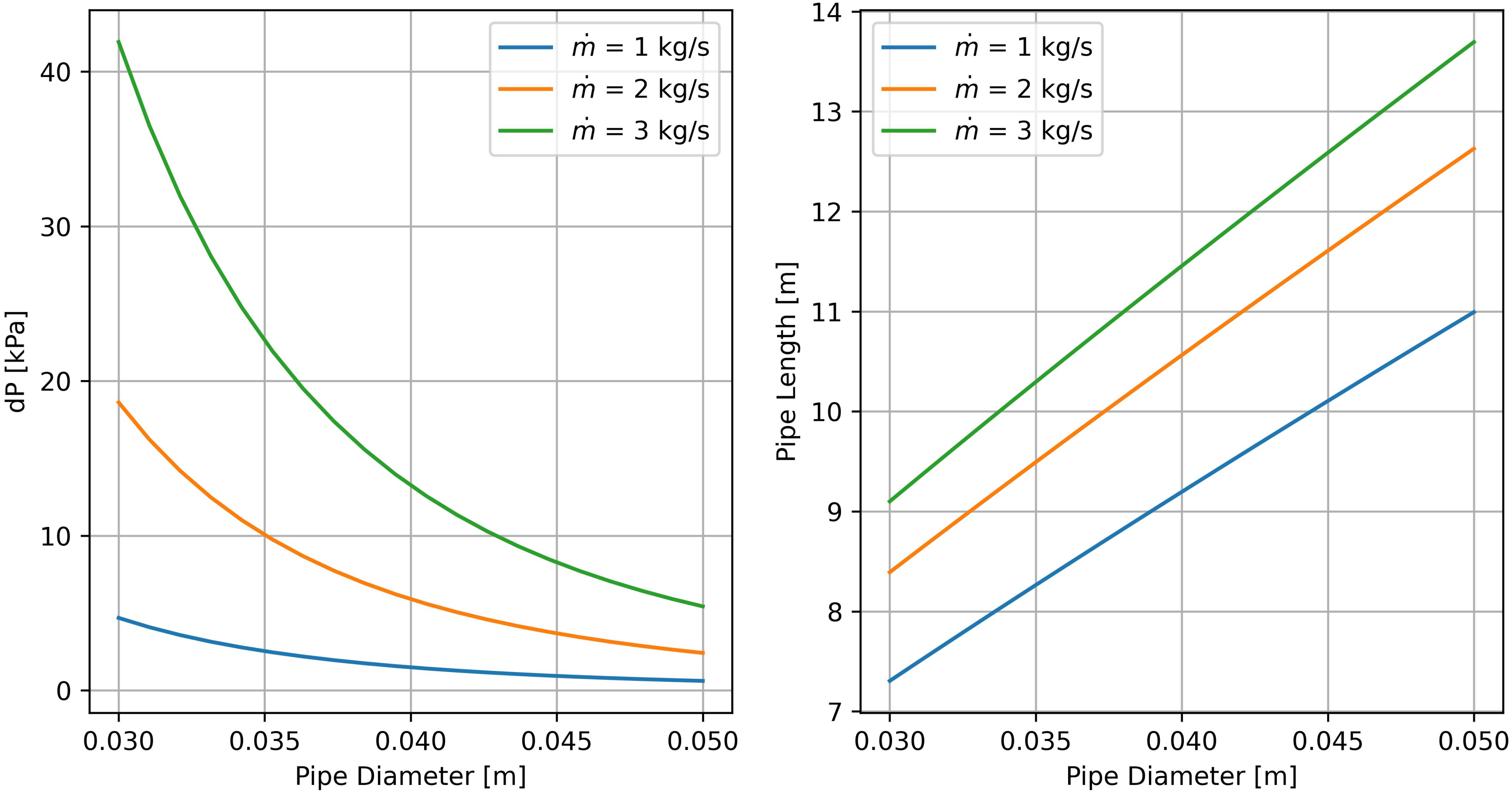
w/ $h = \frac{Nu\, k_f}{D} = \frac{0.023 Re_D^{0.8} Pr^{0.4} k}{D} = 6918.15 \text{ W/m}^2\text{K}$

$\therefore L = \frac{c_p \dot{m}}{h \xi} \ln\left(\frac{25°C - 100°C}{75°C - 100°C}\right) = \boxed{10.5696 \text{ m} = L \text{ for } 75°C}$

$\Rightarrow \Delta P = \frac{f \dot{m}^2 \xi L}{2 \rho A_b}$ w/ $f = (0.79 \ln(Re) - 1.64)^{-2} = 0.0174$

$\boxed{\Delta P = 5823.153 \text{ kPa}}$

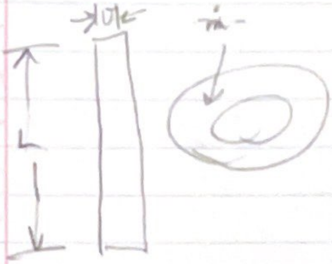
graphs shown below



dP [kPa]
0
10
20
30
40
0.030
0.035
0.040
0.045
0.050
Pipe Diameter [m]
$\dot{m}$ = 1 kg/s
$\dot{m}$ = 2 kg/s
$\dot{m}$ = 3 kg/s
Pipe Length [m]
7
8
9
10
11
12
13
14
0.030
0.035
0.040
0.045
0.050
Pipe Diameter [m]
$\dot{m}$ = 1 kg/s
$\dot{m}$ = 2 kg/s
$\dot{m}$ = 3 kg/s

HW #8 - cont'd

6)



$q'' = q_0''' \sin(\pi z/L)$ FO

find q'' fuel, $\bar{T}$ of water, max surface temp

$\therefore q'' = q'/\xi$

Start w/ CV of fuel rod

$$\dot{E}_{out} = \dot{E}_{gen} \Rightarrow \pi D q'' dz = q''' \pi \frac{D^2}{4} dz \Rightarrow q'' = \frac{4q'}{\pi D^2} \Rightarrow \pi D q'' dz = q' dz$$

next continuity

mass $\frac{\partial G}{\partial z} = 0$, constant G

momentum: $-\frac{\partial P}{\partial z} = \tau_w \frac{\xi_w}{A_x} + \rho g$

energy: $\frac{\partial T_B}{\partial z} = \frac{q'' \xi}{\dot{m} c_p}$ w/ $q'' = \frac{q'}{\xi}$ & $\frac{q'''}{A} = q'$ $\Rightarrow q'' = \frac{A q_0''' \sin\left(\frac{\pi z}{L}\right)}{\xi}$

$$\Rightarrow \frac{\partial T_B}{\partial z} = \frac{A_{xs} q_0'''}{c_p \dot{m}} \sin\left(\frac{\pi z}{L}\right) \quad \Rightarrow \int_0^z \quad \boxed{T_B = \frac{A_{xs} q_0'''}{c_p \dot{m}} \cdot \frac{L}{\pi}\left(1 - \cos\left(\frac{\pi z}{L}\right)\right) + T_{B,0}} \quad ①$$

average fuel temp found by $\frac{1}{L}\int_0^L T_B$

$$\Rightarrow \left[\frac{A_{xs} q_0'''}{c_p \dot{m}} \cdot \frac{1}{\pi}\left(z - \frac{L \sin\left(\frac{\pi x}{L}\right)}{\pi}\right) + \frac{T_{B,0}\, x}{L}\right]\Bigg|_{x=L}$$

$$\Rightarrow \boxed{\bar{T} = \frac{A_{xs} q_0''' L}{c_p \dot{m} \pi} + T_{B0}}$$

HW#8 - cont

find max surface temp take derivative of $q'' = h(T_s - T_\infty)$

$$\Rightarrow T_s = \frac{q''}{h} + T_\infty = \frac{A_{xs}\, q''}{h\, \xi_h} \sin\left(\frac{\pi x}{L}\right) + \frac{A\, q''_o L}{C_p \dot{m}\, \pi}\left[1 - \cos\left(\frac{\pi x}{L}\right)\right] + T_{B,0}$$

$\frac{d}{dx}$

$$\Rightarrow 0 = \frac{A_{xs}\, q''_o}{h\, \xi_h} \frac{\pi}{L} \cos\left(\frac{\pi x}{L}\right) + \frac{A q''_o L}{C_p \dot{m}\, \pi} \cdot \frac{\pi}{L} \sin\left(\frac{\pi x}{L}\right)$$

$$\Rightarrow -\frac{\cancel{A_{xs}}\, \cancel{q''_o}\, \pi}{h\, \xi_h L} \cos\left(\frac{\pi x}{L}\right) + \frac{\cancel{A_{xs}}\, \cancel{q''_o}}{C_p\, \dot{m}} \sin\left(\frac{\pi x}{L}\right)$$

$$\Rightarrow \frac{-\sin}{\cos}\left(\frac{\pi x}{L}\right) = \tan\left(\frac{\pi x}{L}\right) = -\frac{C_p \dot{m}\, \pi}{h\, \xi_h L} \Rightarrow \frac{\pi x}{L} = \operatorname{atan}\left(\frac{-\pi C_p \dot{m}}{h\, \xi_h L}\right)$$

c

$$\boxed{\therefore x_{max} = \operatorname{atan}\left(\frac{-\pi C_p \dot{m}}{h\, \xi_h L}\right)\frac{L}{\pi}}$$