HW#2

1) $\phi(r) = \frac{\phi_0 \sin\left(\frac{\pi r}{R}\right)}{r}$, $\phi_0 = 13.1e13 \text{ cm}^{-1}\text{s}^{-1}$, $R = 50\text{cm}$, $\Sigma_a = 0.108 \text{ cm}^{-1}$, $\Sigma_s = 0.0727 \text{ cm}^{-1}$

a) $\phi' = \frac{\phi_0 \pi \cos\left(\frac{\pi r}{R}\right)}{Rr} - \frac{\phi_0 \sin\left(\frac{\pi r}{R}\right)}{r^2}$, max ϕ when $\phi' = 0$, so

$$\Rightarrow 0 = \frac{\phi_0 \pi \cos\left(\frac{\pi r}{R}\right)}{Rr} - \frac{\phi_0 \sin\left(\frac{\pi r}{R}\right)}{r^2} \Rightarrow \frac{\pi \cos\left(\frac{\pi r}{R}\right)}{Rr} = \frac{\sin\left(\frac{\pi r}{R}\right)}{r^2}$$

$\Rightarrow \tan\left(\frac{\pi r}{R}\right) = \frac{\pi r}{R}$, now taylor expand $\tan\left(\frac{\pi r}{R}\right)$ to get

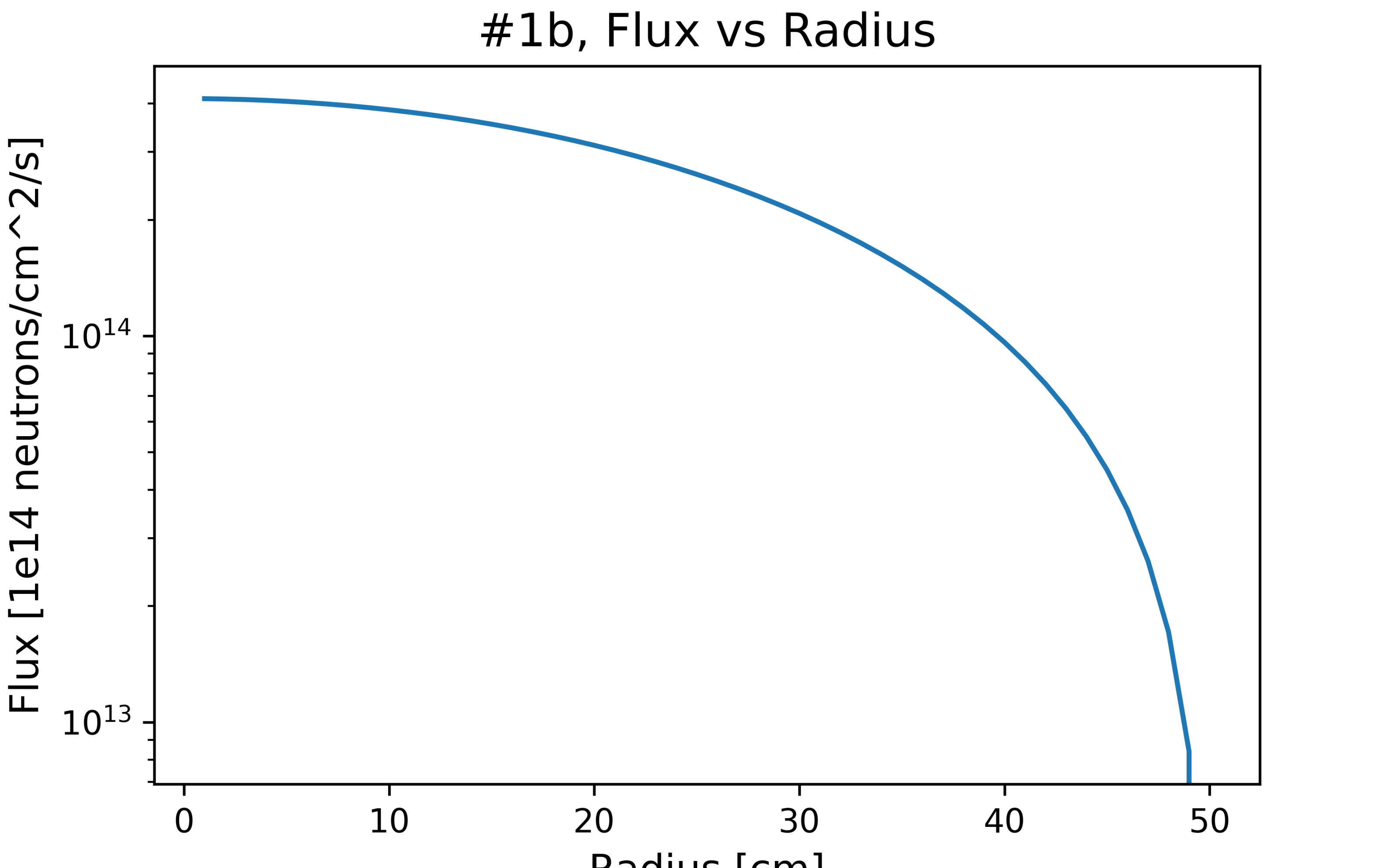
$\Rightarrow \left(\frac{\pi r}{R}\right) + \frac{1}{3}\left(\frac{\pi r}{R}\right)^3 + \frac{2}{15}\left(\frac{\pi r}{R}\right)^5 + \ldots = \frac{\pi r}{R}$, terms w/ higher dependence on $r \to 0$ as $r \to 0$

$\therefore$ we can say the max flux is approaching 0, so find $\lim_{r\to 0} \phi(r)$

$$\lim_{r\to 0} \phi(r) = \frac{\lim_{r\to 0}\left(\phi_0 \sin\left(\frac{\pi r}{R}\right)\right)}{\lim_{r\to 0}(r)} \xRightarrow{\text{L'Hospital}} \lim_{r\to 0}\left(\frac{\phi_0 \pi \cos\left(\frac{\pi r}{R}\right)}{R}\right) = \phi_0 \pi = \pi\left(13.1e13 \text{ cm}^{-1}\text{s}^{-1}\right)$$

$$\boxed{\phi_{max} = \pi \phi_0 = 4.1155e14 \text{ cm}^{-2}\text{s}^{-1}}$$

b) ...



#1b, Flux vs Radius
Flux [1e14 neutrons/cm^2/s]
10^{14}
10^{13}
0
10
20
30
40
50
Radius [cm]

HW#2 - cont

c) know $\phi = n v \Rightarrow n = \phi / v$, so integrate this

$$\int_{4\pi} d\hat{\Omega} \int \frac{r^2 \phi}{v} dr = \frac{4\pi}{v} \int r^2 \left(\frac{\phi_0 R \sin\left(\frac{\pi r}{R}\right)}{r} \right) dr = \frac{4\pi \phi_0 R}{v} \int_0^R \underbrace{r}_{u} \underbrace{\sin\left(\frac{\pi r}{R}\right) dr}_{dv}$$

$uv - \int v du$

integration by parts $\Rightarrow \frac{4\pi \phi_0 R}{v} \left[\frac{-rR\cos\left(\frac{\pi r}{R}\right)}{\pi} \Big|_0^R + \int_0^R \frac{R\cos\left(\frac{\pi r}{R}\right)}{\pi} dr \right]$

$$= \frac{4\pi \phi_0 R}{v} \left[\frac{R^2}{\pi} + 0 \right] = \frac{4\phi_0 R^3}{v}$$, but need v, so use...

$$v = \left(\frac{2E}{m_0} \right)^{1/2} = \left(\frac{2(0.025\,eV \cdot 1.6e\text{-}19\,\frac{J}{eV})}{1.674928e\text{-}27\,kg} \right)^{1/2} = 2185.5\,m/s = 2.185e5\,cm/s$$

$$\boxed{\therefore N = \frac{4(13.1e13\,cm^{-2}s^{-1})(50\,cm)^3}{(2.185e5\,cm\,s^{-1})} = 2.997e14 \text{ neutrons}}$$

d) $V = \frac{4}{3}\pi r^3 \Rightarrow \left(\frac{3}{4\pi} V\right)^{1/3} = r \qquad r = \left(\frac{3}{4\pi} 1cc\right)^{1/3} = 0.62\,cm$

from code $\phi(.62\,cm) = 4.114e14\,cm^{-2}s^{-1} \approx \phi_{max}$

$\therefore \phi$ is spatially invariant in the region $r \leq 0.62\,cm$

$$\Rightarrow \int_{4\pi} d\hat{\Omega} \int \frac{r^2 \phi}{v} dr = \frac{4\pi \phi_{max}}{v} \int_0^{R_1} r^2 dr = \frac{4\pi \phi_{max}}{3v} R_1^3 = N_d$$

$$N_d = \frac{4\pi \overset{[1/cm^2 s]}{(4.115e14)} (0.62\,cm)^3}{3(2185.5\,m/s)} = \boxed{1.88e9\ n^0}$$

HW#2 - cont

e) $P = \Sigma_f \Phi_{max} V \cdot E_f = \frac{0.0727}{cm} \cdot \frac{4.115e14}{cm^2 \cdot s} \cdot 1cc \cdot 200 \text{ MeV}$

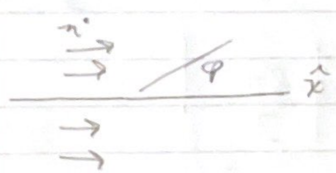
$\Rightarrow P = 5.98e15 \frac{MeV}{s} \times \frac{1.602e\text{-}13 \text{ J}}{MeV} = \boxed{958.5 \text{ W}}$

f) $P = \Sigma_f E_f \int \phi \, dV = 4\pi \Sigma_f E_f \phi_0 R \int_0^R r \sin\left(\frac{\pi r}{R}\right) = 4\pi \Sigma_f E_f \phi_0 R \left[\frac{R^2}{\pi}\right]$

$P = 4 (0.0727 \text{ cm}^{-1})(3.204e^{-11} \text{ J/fission})(13.1e13 \text{ cm}^{-2} \text{s}^{-1})(50 \text{ cm})^3$

$\boxed{P = 152.57 \text{ MW}}$

g) from f) $\boxed{P = 4\Sigma_f E_f \phi_t R^3 \quad \text{or} \quad \phi_t = \frac{P}{4\Sigma_f E_f R^3}}$

2) 

a) $\vec{J} = \Psi \cdot \hat{x} = \boxed{n v \hat{x}}$

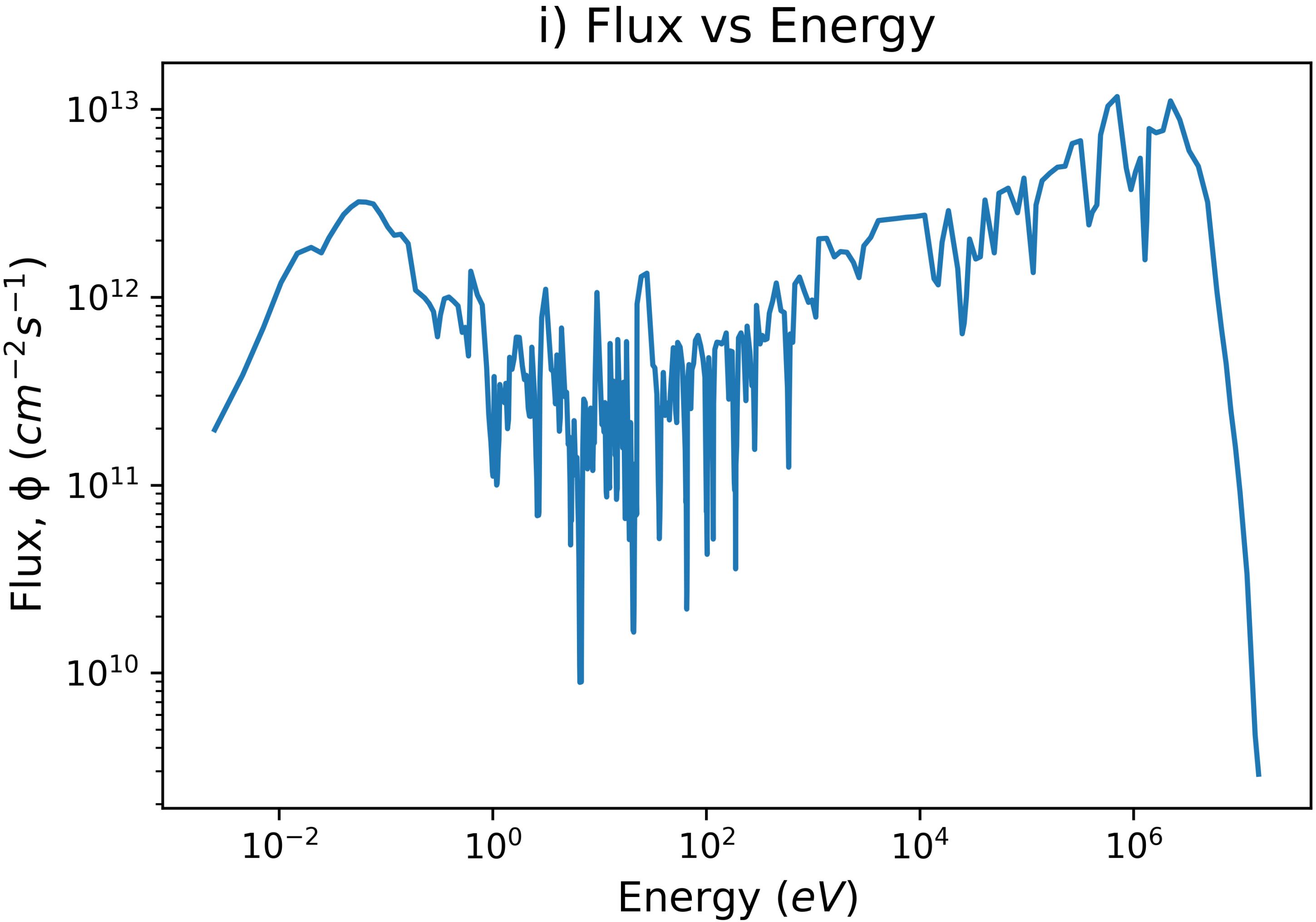
b) net rate $= \vec{J} A = (\Psi \cdot \hat{n}) A = \boxed{\Psi \sin\varphi \, A}$

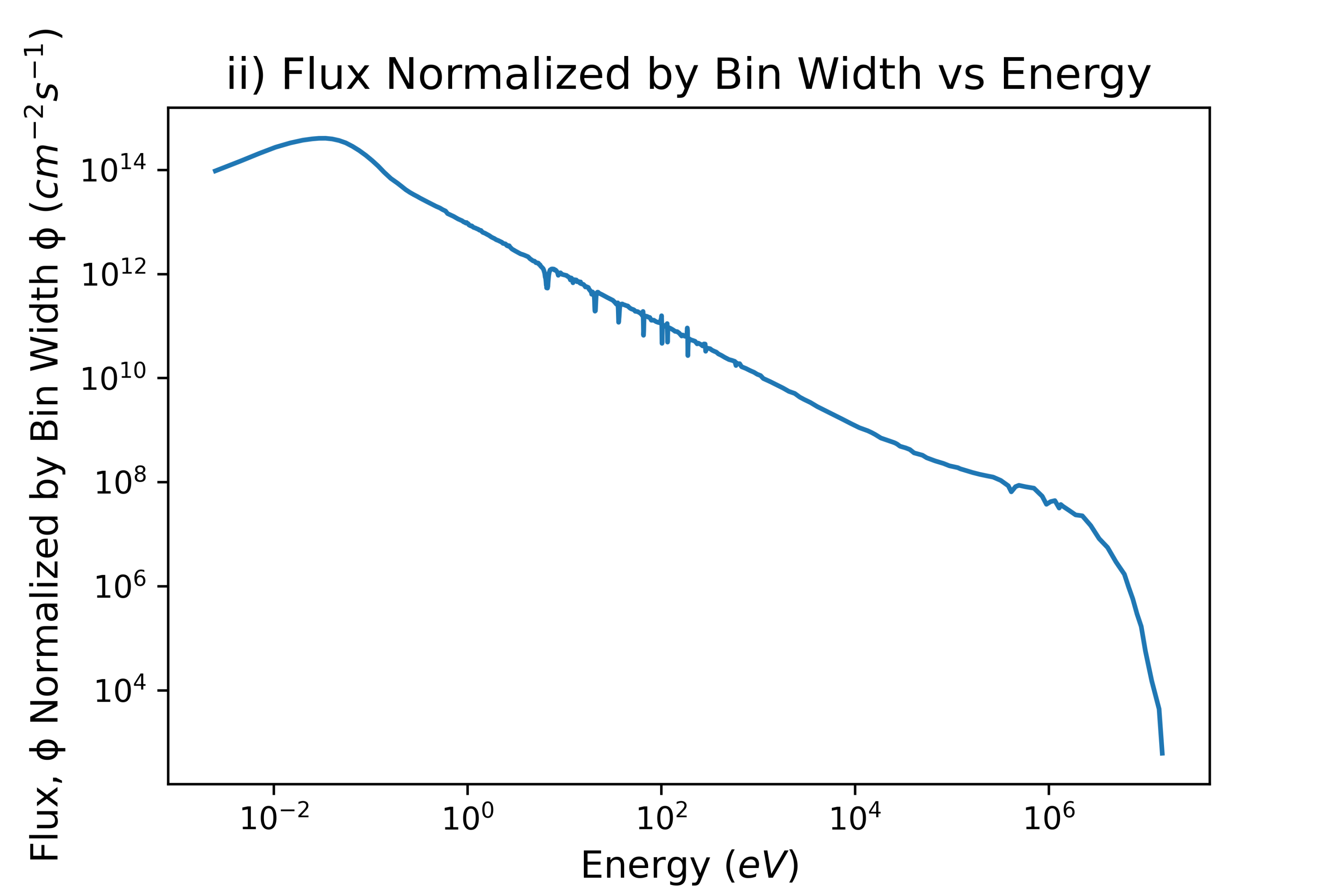
3a) False, change in k implies $\frac{\Delta N}{\Delta t} \neq 0 \therefore$ no ss

b) False, same logic as A

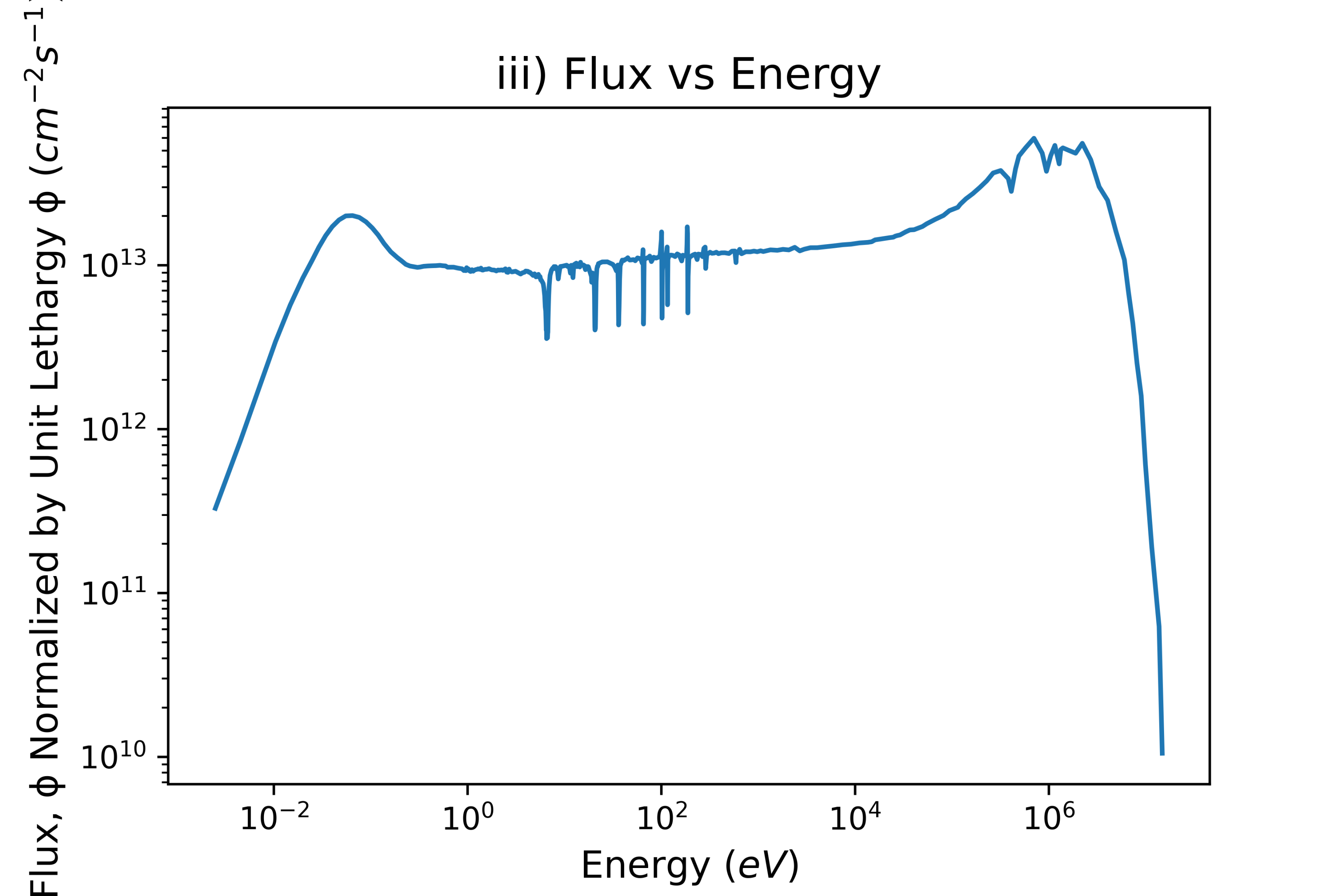
c) False, P_{NL} would ↑ the better of a reflector the reflector is

d) True, Σ_s for void $= 0$, but $\Sigma_{Ba\,137} \neq 0 \therefore k \uparrow$


i) Flux vs Energy
Flux, ϕ ($cm^{-2}s^{-1}$)
Energy (eV)
10^{13}
10^{12}
10^{11}
10^{10}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}



ii) Flux Normalized by Bin Width vs Energy
Flux, Φ Normalized by Bin Width Φ ($cm^{-2}s^{-1}$)
10^{14}
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
Energy (eV)


iii) Flux vs Energy
Flux, ϕ Normalized by Unit Lethargy ϕ ($cm^{-2}s^{-1}$)
Energy (eV)
10^{13}
10^{12}
10^{11}
10^{10}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}

HW#2 - cont'd

4a) ...

b) These large negative spikes are caused by the resonance absorption for the given nucleus

c) Thermal as the energy is approaching 0.

5a) ...

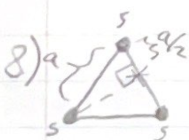
b) Fast spectrum as no thermal neutrons are seen

6) $f(\theta,\varphi) = \theta\varphi^2$ $\quad \int_{4\pi} d\hat{\Omega}\, f(\theta,\varphi) = \int_0^{2\pi} \varphi^2\, d\varphi \int_0^{\pi} \theta \sin(\theta)\, d\theta$

$\Rightarrow \frac{\varphi^3}{3}\Big|_0^{2\pi} \times \int_0^{\pi} \theta \sin\theta\, d\theta = \frac{8\pi^3}{3} \int_0^{\pi} \theta \sin\theta\, d\theta$ $\quad$ say $\underline{u} = \theta$ $\quad du = 1$, $v = -\cos\theta$ $\quad \underline{dv} = \sin\theta$

$\Rightarrow \frac{8\pi^3}{3}\left(-\theta\cos\theta\Big|_0^{\pi} + \int_0^{\pi} \cos\theta = \frac{8\pi^3}{3}(\pi + 0)\right) = \boxed{\frac{8\pi^4}{3}}$

7) $J = [\frac{n}{s\,cm^2}]$ $\quad J = \frac{S}{SA} = \boxed{\frac{S}{4\pi r^2} = \vec{J}(r)}$

8) 

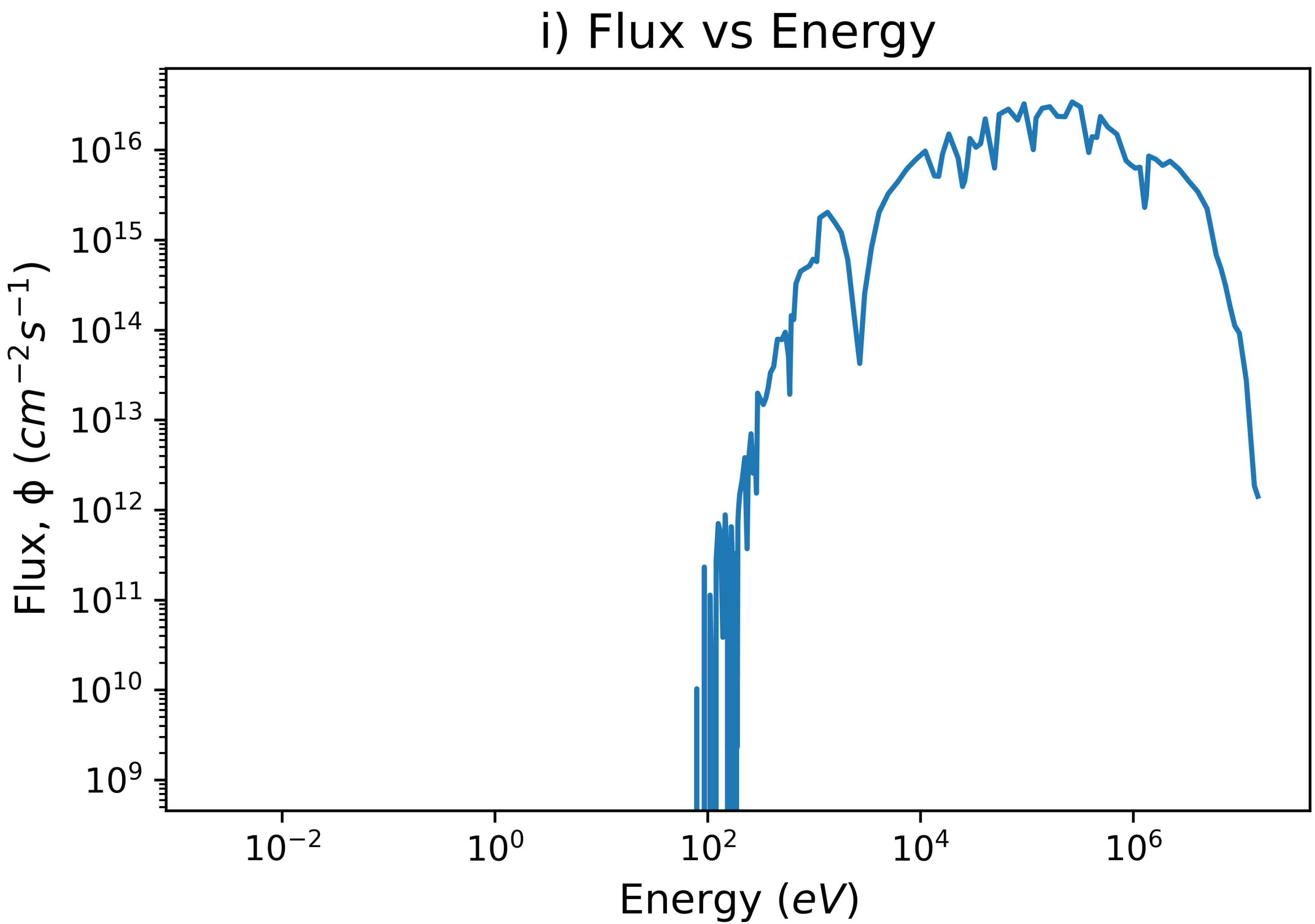
2 sources $\frac{a}{2}$ away, so $2J(\frac{A}{2}) = \frac{2S}{4\pi(\frac{a}{2})^2} = \frac{2S}{\pi a^2}$

another source $\sqrt{a^2+(\frac{a}{2})^2}$ away, so

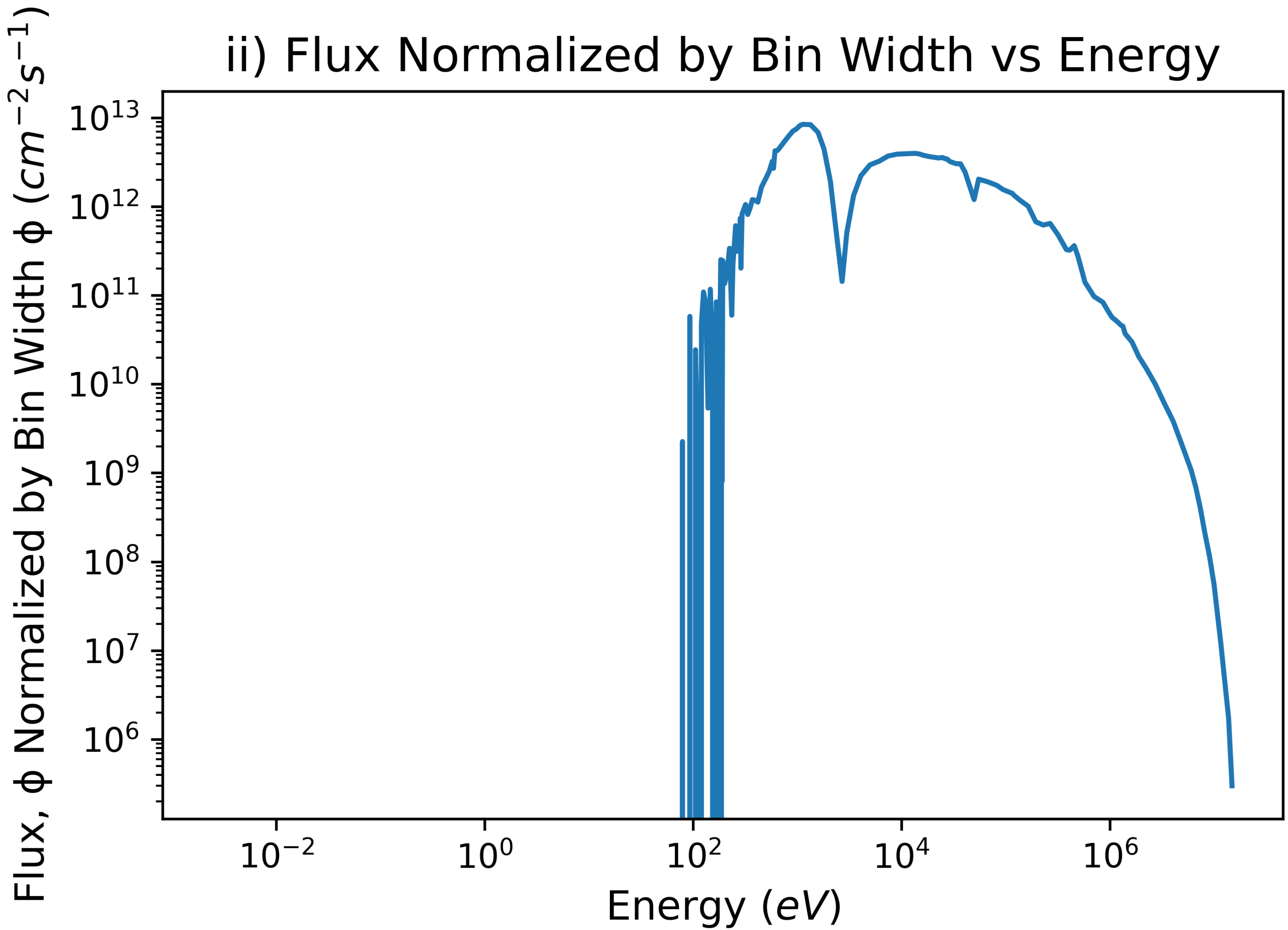
$J(\sqrt{a^2-(\frac{a}{2})^2}) = \frac{S}{4\pi(a^2-\frac{a^2}{4})} = \frac{S}{3\pi a^2}$, now add these together

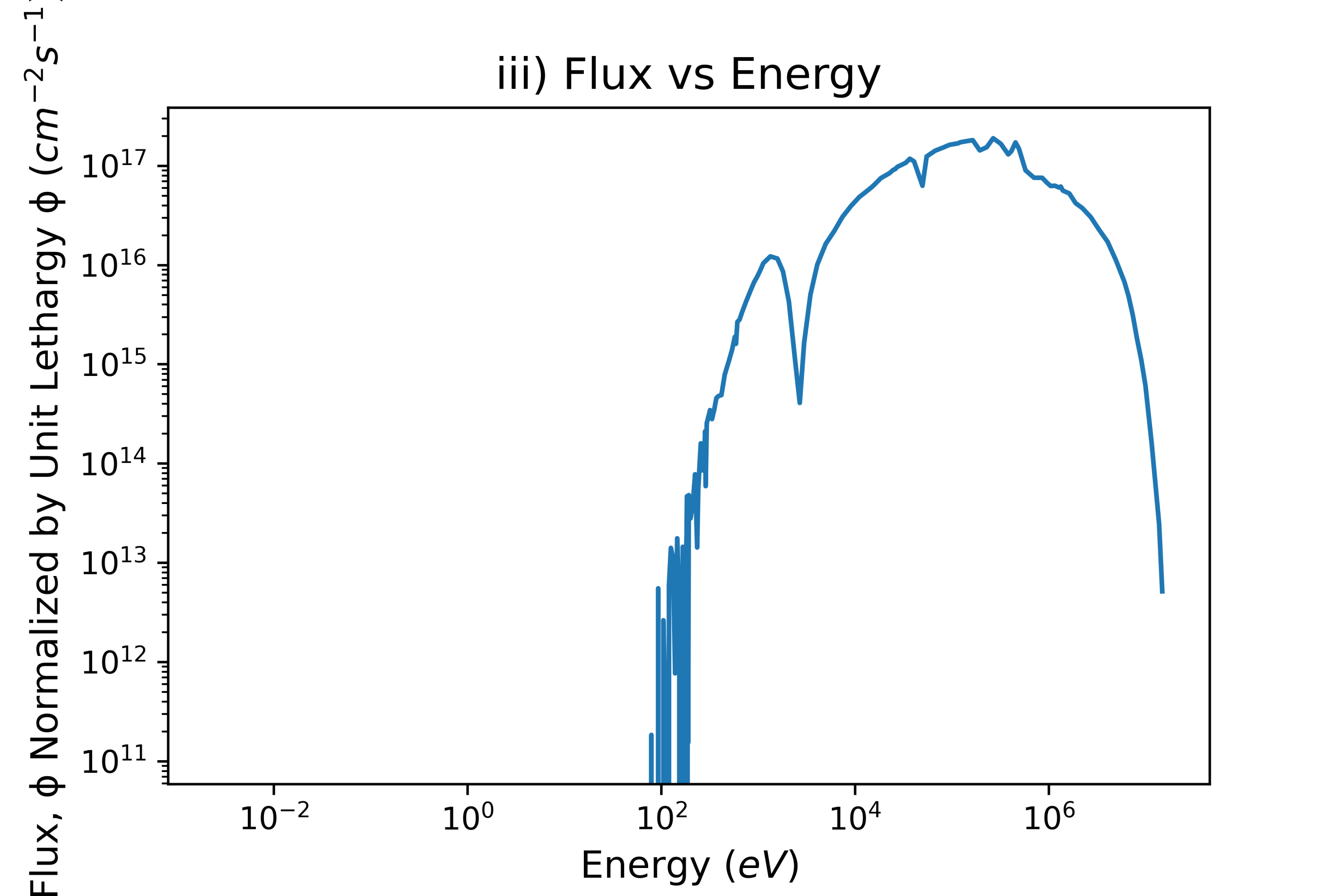
source

$J_T = 2J(\frac{A}{2}) + J(\sqrt{a^2+(\frac{a}{2})^2}) = \frac{2S}{\pi a^2} + \frac{S}{3\pi a^2} = \frac{S}{\pi a^2}(2+\frac{1}{3}) = \boxed{\frac{7}{3}\cdot\frac{S}{\pi a^2} = J}$



i) Flux vs Energy
Flux, ϕ ($cm^{-2}s^{-1}$)
10^{16}
10^{15}
10^{14}
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
Energy (eV)


ii) Flux Normalized by Bin Width vs Energy
Flux, ϕ Normalized by Bin Width ϕ $(cm^{-2}s^{-1})$
10^{13}
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
Energy (eV)



iii) Flux vs Energy
Flux, Φ Normalized by Unit Lethargy Φ ($cm^{-2}s^{-1}$)
10^{11}
10^{12}
10^{13}
10^{14}
10^{15}
10^{16}
10^{17}
10^{-2}
10^{0}
10^{2}
10^{4}
10^{6}
Energy (eV)

HW#2 - cont

9) • UO_2 w/ $\rho = 10.5\, g/cc$ • 4.5% weight ^{235}U of Uranium
• 99.8% atoms ^{16}O find number density of each

$M_{235U} = 235.043928117$ u or g/mol $M_{16O} = 15.9949146192 6$ u
$M_{238U} = 238.0507869 36$ u $M_{18O} = 17.999159612414$ u

$\underline{M_O} = \gamma_{16O} M_{16O} + \gamma_{18O} M_{18O} = 15.99892311$ u

$\underline{M_U}$
assume 100g U, 4.5g ^{235}U & 95.5g ^{238}U

$\underline{235}$: $4.5g\, ^{235}U / M_{235U} = 0.0191453574$ mol ^{235}U

$\underline{238}$: $95.5g\, ^{238}U / M_{238U} = 0.4011748973$ mol ^{238}U

$\therefore \gamma_{U235} = mol_{235U} / (mol_{235U} + mol_{238U}) = 4.554945232$ % ^{235}U

& $\gamma_{U238} = 1 - \gamma_{235U} = 95.44505477$ % ^{238}U

$\Rightarrow M_U = \gamma_{235U} M_{235U} + \gamma_{238U} M_{238U} = 237.9138262$ g/mol

$\underline{M_{UO_2}} = M_U + 2M_O = 269.9116724$ g/mol

$\rho_N = \rho / M_{UO2} = 0.389016151$ mol/cc $\xrightarrow{\times N_A}$ $= 2.342655261e22$ atoms/cc

$\underline{\rho_N}$
U235, $\rho_{N,235U} = \rho_N \gamma_{235U} = 1.067066641e21$ atoms ^{235}U/cc
U238, $\rho_{N,238U} = \rho_N \gamma_{238U} = 2.235948597e22$ atoms ^{238}U/cc
O16, $\rho_{N,16O} = 2 \cdot \rho_N \gamma_{16O} = 4.675939901e22$ atoms ^{16}O/cc
O18, $\rho_{N,18O} = 2 \cdot \rho_N \gamma_{18O} = 9.370621044e19$ atoms ^{18}O/cc