NPRE 449, Hw 4

Joseph F. Specht IV

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The code associated with this homework can be found at

https://github.com/jspecht3/classes/npre449/hw4

1 Energy Release and Deposition in Thermal Reactors

1.1 Release Mechanisms

The energy release in thermal reactors all begins with fission where, most of the time, a thermal neutron is captured and 200 MeV is released from the fission event. From fission, there are three products: (1) prompt gamma-rays, (2) fast neutrons, and (3) fission daughters. The prompt gammas are either absorbed by a nucleus potentially activating the nucleus or escape the system. However, fast neutrons and fission daughters have considerably more potential interactions.

Fast neutrons can undergo scattering down to the thermal regime or undergo fast fission, but fast fission is very unlikely. In the process of thermalization, fast neutrons can release gamma-rays during their inelastic scattering. Once the fast neutron has scattered enough to become thermal, it either undergoes fission, which restarts the process, or parasitic capture. Much like gamma absorption, the parasitic capture leads to an excited nucleus. The excited nucleus will then decay and release capture gamma-rays or capture β particles. The capture radiation can then induce other reactions after interacting again.

Fission daughters come in two varieties: fission products and delayed neutron precursors. Fission products generally may release neutrons, but can also release other forms of radiation: β particles, neutrinos, and gamma-rays. The neutrinos always escape the system and can be considered unrecoverable. Delayed neutron precursors are a special type of fission product that decay and release delayed neutrons along with the other types of radiation. They have a special designation because of how important delayed neutrons are to the maintenance of a nuclear reactor. Fission daughters may also be called transuranic isotopes as they come from the decay chain of uranium in most operating reactors.

1.2 Deposition Breakdown

The various outcomes from fission all have an associated fraction of total energy from fission they release along with a location of deposition.

1. Instantaneous Energy Deposition
   (a) The kinetic energy of fission daughters accounts for 80.5% of total fission energy and is deposited locally in the fuel.
   (b) The kinetic energy of the prompt neutrons accounts for 2.5% of total fission energy and is deposited in the moderator.
   (c) The energy from gamma-rays accounts for 2.5% of total fission energy and is deposited in high Z materials like the fuel and structure.
2. Delayed Energy Deposition
   (a) The kinetic energy of the delayed neutrons accounts for only 0.02% of the total fission energy and is deposited in the moderator.
   (b) The β decay of fission daughters accounts for 2.5% of the total fission energy and is deposited in the fuel.
   (c) The neutrinos account for 5% of the total fission energy, but are considered unrecoverable.
   (d) The energy from gamma-rays produced from fission fragments accounts for 3% of the total fission energy and is deposited in the fuel and structure.
3. Parasitic Neutron Capture
   (a) Non-fission reactions together with β and gamma-decay of (n, γ) type reactions account for 3.5% of the total fission energy and are deposited in the fuel and structure. This category includes prompt and delayed non-fission reactions.

2 T-K Problem 3-1

2.1

To find the average thermal neutron flux assuming 3.5 wt% enrichment using the PWR from T-K Example 3-1, I did the following steps.

1. Assume the uranium in the fuel was only U^{235} and U^{238}. Then I found the atom percent of U^{235} by finding the number of atoms of U^{235} and U^{238}.

$$n_{U^{235}} = \frac{\%_{wt} \cdot N_A}{M_{U^{235}}} \tag{1}$$

$$n_{U^{238}} = \frac{(1 - \%_{wt}) \cdot N_A}{M_{U^{238}}} \tag{2}$$

   And then taking a ratio of the atoms of U^{235} to the total atoms

$$\%_{at} = \frac{n_{U^{235}}}{n_{U^{235}} + n_{U^{238}}} = 0.0329 \tag{3}$$

2. Next, I found the total number of atoms of the U^{235} by first finding the molar mass of the fuel.

$$M_{UO_2} = (\%_{at} \cdot M_{U^{235}}) + ((1 - \%_{at}) \cdot M_{U^{238}}) + 2M_O \tag{4}$$

   Then finding the total mass of the fuel and converting the mass of the rod from kilograms to grams.

$$m_{fuel} = n_{assemblies} \cdot m_{rod} \cdot 1000\frac{g}{kg} \tag{5}$$

   Finding the number of molecules of fuel in the assembly.

$$n_{fuel} = \frac{m_{fuel} \cdot N_A}{M_{UO_2}} \tag{6}$$

   And then multiplying the atoms of fuel by the atom percent of U_{235} to get the atoms of U_{235}.

$$n_{U^{235}} = n_{fuel} * \%_{at} = 7.331 \cdot 10^{27} atoms \tag{7}$$

3. Finally, solving for the average flux using Eq. 3-13d and making sure to include the fact that only 95%

of the power is recoverable.

$$\bar{\phi} = \frac{\gamma \cdot P}{\chi_f \cdot \sigma_{U^{235}} \cdot n_{U^{235}}} = \frac{(0.95)(3038MW \cdot 10^6 \frac{W}{MW})}{(3.04 \cdot 10^{-11} \frac{J}{fission})(350 \cdot 10^{-24} cm^2)(7.331 \cdot 10^{27} atoms)} \tag{8}$$

$$\bar{\phi} = 3.755 \cdot 10^{13} \ \frac{neutrons}{s \cdot cm^2} \tag{9}$$

2.2

To find the average power density with the assumption of 90% theoretical density, I started with the fact that UO_2 has a density of 10.97 g/cc and found the number density of U^{235}.

$$\rho_{n,U^{235}} = \frac{\rho_{UO_2} \cdot \gamma \cdot N_A \cdot \%_{at}}{M_{UO_2}} = 7.248 \cdot 10^{20} \ \frac{atoms}{cm^3} \tag{10}$$

With the number density, I then found the power density by multiplying the reaction rate and the energy per reaction.

$$q''' = \left(\rho_{n,U^{235}} \cdot 10^6 \frac{cm^3}{m^3}\right) \sigma_{U^{235}} \bar{\phi} \left(\chi_f \cdot 10^{-6} \frac{MJ}{J}\right) = 289.576 \ \frac{MW}{m^3} \tag{11}$$

2.3

To find the average linear power of the fuel, find the total length of the rods in every assembly.

$$L = n_{rods} \cdot n_{assemblies} \cdot l_{rod} \tag{12}$$

Then divide the total power by the length of the rods and remember to account for the 5% energy loss due to neutrinos.

$$q' = \frac{\left(P \cdot 1000 \frac{kW}{MW}\right) \cdot \gamma}{L} = 19.242 \frac{kW}{m} \tag{13}$$

2.4

TO find the average heat flux at the cladding, I first found the total clad area.

$$a_{clad} = (\pi D_{clad}) \cdot l_{rod} \cdot n_{rods} \cdot n_{assemblies} \tag{14}$$

Then dividing the total power by the clad area.

$$q'' = \frac{P \cdot 1000 \frac{kW}{MW} \cdot \gamma}{a_{clad}} = 612.484 \frac{kW}{m^2} \tag{15}$$

3 T-K Problem 3-3

This uses the Example 3-4.

3.1

3.1.1 Gamma Power

To find the total power generation in the thermal shield if it is 4.0m high, first start by constructing a volume integral and substituting Eq. 3-57 in for q”’.

$$\dot{Q}_\gamma = \iiint q_\gamma''' dV = \iiint SB_{Fe}\mu_a E_0 e^{-\mu x} dV = SB_{Fe}\mu E_0 \iiint e^{-\mu x} dV \tag{16}$$

We are integrating over a cylinder, so convert the volume integral to cylindrical coordinates.

$$\dot{Q}_\gamma = SB_{Fe}\mu_a E_0 \int_0^h \int_0^{2\pi} \int_{r_i}^{r_o} e^{-\mu x} r dr d\theta dz \tag{17}$$

The attenuation is only a function of radius, so integrate over the polar angle and the pitch.

$$\dot{Q}_\gamma = 2\pi h SB_{Fe}\mu_a E_0 \int_{r_i}^{r_o} e^{-\mu x} r dr \tag{18}$$

To account for the thickness properly, we need to convert x to r such that $x = 0$ corresponds to r_i. Assume x and r scale by the same linear rate.

$$x = 0 = r_i \longrightarrow x = r - r_i \tag{19}$$

Apply the transformation of coordinates on the integral. The constants at the front have been omitted for now.

$$\int_{r_i}^{r_o} e^{-\mu x} r dr = \int_{r_i}^{r_o} e^{-\mu(r-r_i)} r dr \tag{20}$$

Breaking up the integrand.

$$\int_{r_i}^{r_o} e^{-\mu(r-r_i)} r dr = \int_{r_i}^{r_o} e^{-\mu r} e^{\mu r_i} r dr = e^{\mu r_i} \int_{r_i}^{r_o} e^{-\mu r} r dr \tag{21}$$

Do integration by parts assuming $u = r$ and $dv = e^{-\mu r}$.

$$e^{\mu r_i} \int_{r_i}^{r_o} e^{-\mu r} r dr = e^{\mu r_i} \left[\frac{-re^{-\mu r}}{\mu} + \int \frac{e^{-\mu r}}{\mu} dr \right]_{r_i}^{r_o} = e^{\mu r_i} \left[\frac{-e^{\mu r}}{\mu} \left(r + \frac{1}{\mu} \right) \right]_{r_i}^{r_o} \tag{22}$$

Neglecting the constant, plugging in the bounds of integration, and evaluating making sure to convert meters to cm so the units of radii and absorption coefficient can be canceled.

$$\left[\frac{-e^{\mu r}}{\mu}\left(r+\frac{1}{\mu}\right)\right]_{r_i}^{r_o} = \left[\frac{-e^{\mu r_o}}{\mu}\left(r_o+\frac{1}{\mu}\right)\right] - \left[\frac{-e^{\mu r_i}}{\mu}\left(r_i+\frac{1}{\mu}\right)\right] = 1.322 \cdot 10^{-15} = A \tag{23}$$

Using Eq. 18 and 23, we can find the final expression for the power generation in the shield from gamma-rays. Making sure to convert all energies to J, lengths to cm,

$$\dot{Q}_\gamma = 2\pi h S B_{Fe} \mu_a E_0 e^{\mu r_i} A \tag{24}$$

$$\dot{Q}_\gamma = (2\pi)(400cm)\left(10^{14}\frac{n^o}{cm^2 \cdot s}\right)(4.212)(0.333\ cm^{-1})\left(2\ MeV \cdot 1.602 \cdot 10^{-13}\frac{J}{MeV}\right) \tag{25}$$
$$\left(2.762 \cdot 10^{17}\right)\left(1.322 \cdot 10^{-15}\right) = 23.057\ MW$$

3.1.2 Scattering Power

Start with Eq. 3-63, 3-64.

$$q'''_{el} = \xi E \Sigma_{s,el} \phi_{fast} \tag{26}$$

$$q'''_{il} = f(E) E \Sigma_{s,il} \phi_{fast} \tag{27}$$

Find α for iron.

$$\alpha = \left(\frac{A-1}{A+1}\right)^2 = 0.931 \tag{28}$$

Find the lethargy of the neutrons.

$$\xi = \frac{1}{2}(1-\alpha) = 0.0346 \tag{29}$$

Find removal cross section.

$$\Sigma_T = \frac{\rho_{FeA_v}}{M_{Fe}}\sigma_T = 0.254\ cm^{-1} \tag{30}$$

Assume each scattering cross section is half of the removal cross section.

$$\Sigma_{s,il} = \Sigma_{s,el} = \frac{1}{2}\Sigma_T = 0.127\ cm^{-1} \tag{31}$$

Find the volumetric heat generation for each scattering.

$$q'''_{el} = \xi E \Sigma_{s,el} \phi_{fast} = (0.0346)\,(0.6MeV)\,(0.127\ cm^{-1})(10^{14}\ n^o/cm^2 \cdot s) = 0.26 \cdot 10^{12}\ \frac{MeV}{cm^3 \cdot s} \tag{32}$$

$$q'''_{il} = f(E)E\Sigma_{s,il}\phi_{fast} = (0.1)(0.6MeV)(0.127\ cm^{-1})(10^{14}\ n^o/cm^2 \cdot s) = 0.76 \cdot 10^{12}\ \frac{MeV}{cm^3 \cdot s} \tag{33}$$

Add the two together.

$$q'''_s = q'''_{il} + q'''_{el} = 1.02 \cdot 10^{-12} \frac{MeV}{cm^3 s} \tag{34}$$

Find the volume of the thermal shield.

$$V = h\pi \left(r_o^2 - r_i^2\right) = 4.052m^3 = 4.052 \cdot 10^6\ cm^3 \tag{35}$$

Multiply the volumetric heat generation from scattering by the volume to get the power generated from scattering and convert to MJ.

$$\dot{Q}_s = V \cdot q'''_s \cdot 1.602 \cdot 10^{-19} \frac{MJ}{MeV} = 0.662\ MW \tag{36}$$

3.1.3 Answer

Add the heat generation from the gamma and scattering.

$$\dot{Q} = \dot{Q}_\gamma + \dot{Q}_s = 23.056MW + 0.662MW = 23.719MW \tag{37}$$

3.2

If the shield thickness was increased to 15 cm, the outer radius would become.

$$r_o = 1.206 + 0.15 = 1.356m \tag{38}$$

Then the power generation from gamma is integrated over the new r_o.

$$\left[\frac{-e^{\mu r}}{\mu}\left(r + \frac{1}{\mu}\right)\right]_{r_i}^{r_o} = \tag{39}$$

And the gamma power generation becomes.

$$\dot{Q}_\gamma = 23.741MW \tag{40}$$

Next, the new volume is

$$V = h\pi \left(r_o^2 - r_i^2\right) = 4.829m^3 = 4.829 \cdot 10^6\ cm^3 \tag{41}$$

Multiplying the new volume by the volumetric heat generation rate.

$$\dot{Q}_s = V \cdot q_s''' = 0.789MW \tag{42}$$

Adding the two together gets the final answer.

$$\dot{Q} = \dot{Q}_\gamma + \dot{Q}_s = 23.741MW + 0.789MW = 24.530MW \tag{43}$$

4 T-K Problem 3-4

4.1

To find the power after shutdown, use Eq. 3-70c.

$$\frac{P}{P_0} = 0.066\left[(\tau - \tau_s)^{-0.2} - \tau^{-0.2}\right] = 1.334 \cdot 10^{-15} \tag{44}$$

Rearranging.

$$P = 0.066 P_0 \left(\int (\tau - \tau_s)^{-0.2} d\tau - \int \tau^{-0.2} d\tau \right) \tag{45}$$

Integrating.

$$E = 0.066 P_0 = \left(\frac{5}{4}(\tau - \tau_s)^{4/5} - \frac{5}{4}\tau^{4/5} \right) \tag{46}$$

Rearranging. Add $\frac{3}{4}$ term as the reactor is operating at 75% power and convert to TW.

$$E = \left(\frac{5}{4}\right)\left(\frac{3}{4}\right)(0.066)\left(10^{-12}\frac{TJ}{J}\right)\left(3000 \cdot 10^6 W\ \right)\left[\tau_s^{4/5} + (\tau - \tau_s)^{4/5}\,\tau^{4/5}\right] \tag{47}$$

Now that we have Eq. 47, we can find the energy generated after: 1 hour, 1 day, and 1 month by plugging in the respective times converted into seconds to cancel the 1/s in W.

Table 1: Decay Energy After Shutdown

Part	Time After	Decay Energy [TW]
a	1 Hour	0.133
b	1 Day	1.246
c	1 Month	13.123

4.2

If we used Eq. 3-71, we would expect the energy to be high because of the inclusion of U^{239}, Np^{289}, decay of U^{235}, and neutron capture in fission products. All the aforementioned processes serve to increase the total decay heat generation.