HW #1 - Joseph Specht

1) $\vec{r}(t) = (7\alpha_1 t^2 + 3\alpha_3 t)\hat{x} - 4\alpha_2 t^3 \hat{y} + 3\alpha_3 \hat{z}$

$\vec{r}'(t) = 7\alpha_1 t^2 \hat{x} - (4\alpha_2 t^3 - 6\alpha_3)\hat{y} + 4\alpha_3 \hat{z}$

a) $\vec{r}' - \vec{r} = -3\alpha_3 t\,\hat{x} - 6\alpha_3 \hat{y} + \alpha_3 \hat{z}$

$\vec{v}$ w/ S' moving to S $= \frac{d}{dt}(\vec{r}' - \vec{r}) = -3\alpha_3 \hat{x} + 0\,\hat{y} + 0\,\hat{z}$

b) $\dot{\vec{r}} = (14\alpha_1 t + 3\alpha_3)\hat{x} - 12\alpha_2 t^2 \hat{y}$

$\ddot{\vec{r}} = 14\alpha_1 \hat{x} - 24\alpha_2 t\,\hat{y}$

$\dot{\vec{r}}' = 14\alpha_1 t\,\hat{x} - 12\alpha_2 t^2 \hat{y}$

$\ddot{\vec{r}}' = 14\alpha_1 \hat{x} - 24\alpha_2 t\,\hat{y}$

c) ~~If~~ S is inertial, S is also inertial because they have the same acceleration

2) $\vec{F} = -GMm/r^2\,\hat{e}_r$

a) $\vec{F} = \frac{dv}{dt} m = -GMm/r^2 \cdot \frac{dr}{dr} \Rightarrow \frac{dv}{dt} dr = -GM/r^2\,dr \qquad \frac{dr}{dt} = v$

$\Rightarrow v\,dv = -GM/r^2\,dr \Rightarrow \int v\,dv = -GM \int \frac{1}{r^2}\,dr$

$\Rightarrow \frac{v^2}{2} = \frac{GM}{r} + C \Rightarrow v^2 = \frac{GM}{r} + C \Rightarrow v = \sqrt{\frac{GM}{r}} + C^{0}$

$\Rightarrow v(r=R) = v_0 = \sqrt{\frac{GM}{R}} = \boxed{3543.722\ m/s}$

b) $ma = -\frac{GMm}{r^2} \Rightarrow a = -\frac{GM}{r^2} \Rightarrow a\frac{dr}{dt} = -\frac{GM}{r^2}\frac{dr}{dt} \Rightarrow \dot{v}v = -\frac{GM}{r^2}\frac{dr}{dt}$

$\Rightarrow \dot{v}v\,dt = -GM/r^2\,dr \Rightarrow \int \dot{v}v\,dt = \int -GM/r^2\,dr \Rightarrow \frac{1}{2}v^2 = \frac{GM}{r}$

$\Rightarrow v = \sqrt{\frac{2GM}{r}}$ $\boxed{v_e = 5011.58\ m/s}$

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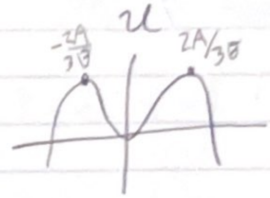
3) $U(x) = Ax^2 - Bx^3$, $A>0$ & $B>0$ constants

a) F is = ~~to~~ - change in energy $\therefore F = \frac{-dU}{dx}$



$F = -2Ax + 3Bx^2$

b) find critical points $F = x(-2A + 3Bx)$

0 @ $x = 0$ & $2A = 3Bx \Rightarrow x = \frac{2A}{3B}$

find U @ $\frac{2A}{3B}$, $U\left(\frac{2A}{3B}\right) = A\left(\frac{4A^2}{9B^2}\right) - B\left(\frac{8A^3}{27B^3}\right)$

$\Rightarrow U\left(\frac{2A}{3B}\right) = \frac{4A^3}{9B^2} - \frac{8A^3}{27B^2} = \frac{4}{27}\frac{A^3}{B^2}$

Initial kinetic energy greater than $U\left(\frac{2A}{3B}\right)$ will break you out of region, $-\infty < x < 2A/3B$

$\frac{1}{2}mv_c^2 = \frac{4}{27}\frac{A^3}{B^2} \Rightarrow v_c^2 = \frac{8A^3}{27mB^2} \Rightarrow v_c = \sqrt{\frac{8A^3}{27mB^2}} = \frac{2}{3}\frac{A}{B}\sqrt{\frac{2A}{3m}}$

$$v_c = \frac{2A}{3B}\sqrt{\frac{2A}{3m}} = \sqrt{\frac{8A^3}{27mB^2}}$$

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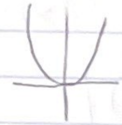
4) $F(x) = -Ax \qquad A > 0$

a) The farther from the origin, the more the force pull the mass towards the origin. This Force scales linearly

b) $F = -\frac{dU}{dx} = -Ax \Rightarrow \frac{dU}{dx} = Ax \Rightarrow dU = Ax\,dx \Rightarrow \int dU = \int Ax\,dx$

$\Rightarrow \boxed{U = \frac{1}{2}Ax^2}$



c) Follows the form of oscillator, so $\omega = \sqrt{A/m}$

$x = B\sin(\omega t + \varphi)$

$v = B\omega\cos(\omega t + \varphi)$

$a = -B\omega^2\sin(\omega t + \varphi)$

$F = -Ax = -Bm\omega^2\sin(\omega t + \varphi)$

$-Ax = -Bm\frac{A}{m}\sin(\omega t + \varphi)$

$x = B\sin(\omega t + \varphi)$

$v @ t = 0 \Rightarrow 0 = B\omega\cos(\varphi) \quad 0 = \cos(\varphi) \quad \varphi = n\pi + \frac{\pi}{2}$

Since this is a sin, we can take any phase φ $\left(\frac{\pi}{2}\right)$

$x = d @ t = 0 \Rightarrow d = B\sin\left(\frac{\pi}{2}\right) \quad B = d$

solving for t wrt x

$x = d\sin\left(\omega t + \frac{\pi}{2}\right) \Rightarrow \frac{x}{d} = \sin\left(\omega t + \frac{\pi}{2}\right) \Rightarrow \operatorname{asin}\left(\frac{x}{d}\right) = \omega t + \frac{\pi}{2}$

$\Rightarrow \operatorname{asin}\left(\frac{x}{d}\right) - \frac{\pi}{2} = \omega t \Rightarrow t = \omega^{-1}\left(\operatorname{asin}\left(\frac{x}{d}\right) - \frac{\pi}{2}\right) = \left(\frac{\sqrt{A}}{\sqrt{m}}\right)^{-1}\left(\operatorname{asin}\left(\frac{x}{d}\right) - \frac{\pi}{2}\right)$

$\Rightarrow t = \sqrt{\frac{m}{A}}\left(\operatorname{asin}\left(\frac{x}{d}\right) - \frac{\pi}{2}\right) = \sqrt{m/A}\left(\pi - \frac{\pi}{2}\right) = \frac{\pi}{2}\sqrt{m/A}$

$$\boxed{t = \frac{\pi}{2}\sqrt{m/A}}$$

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5) $v(x) = A\cosh(\beta x) = A\left(\frac{e^{\beta x} + e^{-\beta x}}{2}\right)$ $\mathbb{R} \ni A > 0$ $\beta \in \mathbb{C}$

a) A, $\mathcal{L}/t$ β, $1/\mathcal{L}$ explained below

b) $F = m\frac{dv}{dt}$ $\frac{dv}{dt} = \frac{d}{dt}\left(A\cosh(\beta x)\right) = A\beta\sinh(\beta x) \cdot dx/dt$

$\Rightarrow A\beta\sinh(\beta x) \cdot A\cosh(\beta x) \Rightarrow A^2\beta\sinh(\beta x)\cdot\cosh(\beta x)$

$$\boxed{F = m \cdot \frac{dv}{dt} = m A^2 \beta \sinh(\beta x)\cosh(\beta x)}$$

c) $F = m \cdot \mathcal{L}/t^2$

$A^2 = \mathcal{L}^2/t^2$ $\beta = \frac{1}{\mathcal{L}}$ $m = m$

$\sinh(\frac{1}{\mathcal{L}} \cdot \mathcal{L}) = 1$
$\cosh(1/\mathcal{L} \cdot \mathcal{L}) = 1$

$$\boxed{F = m \cdot \mathcal{L}^2/t^2 \cdot 1/\mathcal{L} = m \cdot \mathcal{L}/t^2}$$ ✓ works out!

a) A is multiplied by a unitless quantity cosh, so to be a velocity, A needs to be $\mathcal{L}/t$

β is multiplied by x w/ units of $\mathcal{L}$ inside of an exponential. The arguments of exponentials have to be unitless, so β needs to be $1/\mathcal{L}$