Phys 427 - PS9

1a)

w/ $\left(P+\frac{N^2a}{V^2}\right)(V-Nb)=NkT \Rightarrow P=\frac{NkT}{V-Nb}-\frac{N^2a}{V^2}$

$\left.\frac{\partial P}{\partial V}\right|_T = \frac{-NkT}{(V-Nb)^2}+\frac{2N^2a}{V^3}=0 \Rightarrow \frac{NkT}{(V-Nb)^2}=\frac{2N^2a}{V^3}$

$\Rightarrow NkTV^3 = 2N^2a(V-Nb)^2 \Rightarrow kT=\frac{2Na}{V^3}(V-Nb)^2 \leftarrow$ (i)

$\left.\frac{\partial^2 P}{\partial V^2}\right|_T = \frac{2NkT}{(V-Nb)^3}-\frac{6N^2a}{V^4}=0 \Rightarrow kT=\frac{3Na}{V^4}(V-Nb)^3 \leftarrow$ (ii)

w/ (i) = (ii), $\frac{2Na}{V^3}(V-Nb)^2=\frac{3Na}{V^4}(V-Nb)^3 \Rightarrow \frac{2}{V^3}=\frac{3}{V^4}(V-Nb)$

$\Rightarrow 2V=3V-3Nb \Rightarrow \boxed{V_c=3Nb}$

Plug V_c into (i) $\quad kT=\frac{2Na}{(3Nb)^3}(3Nb-Nb)^2=\frac{2Na}{27N^3b^3}(2Nb)^2=\boxed{\frac{8a}{27b}=kT_c}$

Plug V_c, kT_c into P $\quad P=\frac{N\left(\frac{8a}{27b}\right)}{3Nb-Nb}-\frac{N^2a}{(3Nb)^2}=\frac{4a}{27b^2}-\frac{a}{9b^2}=\boxed{\frac{a}{27b^2}=P_c}$

b) $\hat{P}=\frac{P}{P_c}=\frac{27b^2}{a}\left[\frac{NkT}{V-Nb}-\frac{N^2a}{V^2}\right]=\frac{27b^2}{a}\left[\frac{8a}{27kb}\frac{Nk\hat{T}}{a(V-Nb)}-\frac{N^2}{V^2}\right]$ ($\hat{T}$ = T/T_c)

$=\frac{8N\hat{T}b}{V-Nb}-\frac{27b^2N^2}{V^2}=\frac{8\hat{T}Nb}{3Nb\hat{V}-Nb}-\frac{27b^2N^2}{(3Nb)^2N^2\hat{V}^2}=\boxed{\frac{8\hat{T}}{3\hat{V}-1}-\frac{3}{\hat{V}^2}=\hat{P}}$ ($\hat{V}$ = V/V_c)

Phys 427 - PS9

2a)

$$\hat{p} \approx \hat{p}(\hat{T}=1, \hat{V}=1) + \hat{p}_{\hat{T}}(\hat{T}-1) + \hat{p}_{\hat{V}}(\hat{V}-1) + \hat{p}_{\hat{T}^2}(\hat{T}-1)^2 + \hat{p}_{\hat{V}^2}(\hat{V}-1)(\hat{T}-1) + \hat{p}_{\hat{V}\hat{T}}(\hat{V}-1)(\hat{T}-1) + \dots$$

(i) (ii) (iii) (iv) (v) (vi)

(i) $\frac{8(1)}{3(1)-1} - \frac{3}{(1)^2} = 4-3 = \underline{1}$

(ii) $\frac{8}{3\hat{V}-1}(\hat{T}-1) = \underline{4(\hat{T}-1)}$

(iii) $\left[\frac{-24\hat{T}}{(3\hat{V}-1)^2} + \frac{6}{\hat{V}^3}\right](\hat{V}-1) = [6-6\hat{T}](\hat{V}-1) = \underline{-6(\hat{T}-1)(\hat{V}-1)}$

(iv) $\partial_{\hat{T}}^2 \hat{P} = 0$, so (iv) $= 0$, (v) neglect as this is a "low order"

(vi) $\frac{\partial}{\partial V}\left[\frac{8}{3\hat{V}-1}\right](\hat{V}-1)(\hat{T}-1) = \frac{-24}{(3\hat{V}-1)^2}(\hat{V}-1)(\hat{T}-1) = -6(\hat{V}-1)(\hat{T}-1)$

$\Rightarrow$ know $(1-\hat{T}) \propto (\hat{V}-1)^2$, w/ $(1-x)^2 = (x-1)^2$, $(\hat{T}-1) \propto (\hat{V}-1)^2 =$
$c(\hat{T}-1) = c(\hat{V}-1)^2$

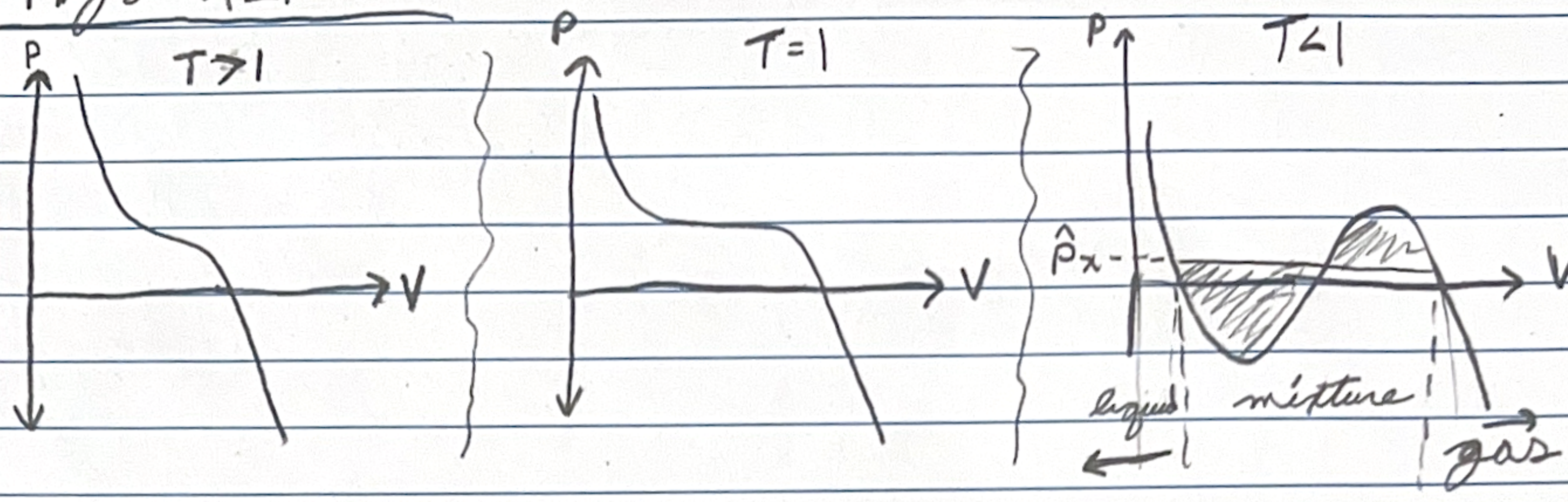
$\Rightarrow -6c(\hat{V}-1)(\hat{V}-1)^2 = -6c(\hat{V}-1)^3$, but from part c) know $c = 1/4$

$\Rightarrow$ (vi) $= \underline{-\frac{3}{2}(\hat{V}-1)^3}$

Plugging them in together

$$\boxed{\hat{p} \approx 1 + 4(\hat{T}-1) - 6(\hat{T}-1)(\hat{V}-1) - \frac{3}{2}(\hat{V}-1)^3}$$

Phys 427 - PS 9

2b) 

$\hat{P}_x$ occurs @ an inflection point

$$\frac{\partial \hat{P}}{\partial V} = -6(\hat{T}-1) - \frac{9}{2}(\hat{V}-1)^2 = 0 \Rightarrow 4(\hat{T}-1) = -3(\hat{V}-1)^2$$

$$\frac{\partial^2 \hat{P}}{\partial \hat{V}^2} = -9(\hat{V}-1) \Rightarrow \hat{V} = 1 \text{, so } \boxed{\hat{P}_x(\hat{T}, \hat{V}=1) = 1 + 4(\hat{T}-1)}$$

c) know $\hat{V}_{g,l}$ occur when $\hat{P}_x(\hat{T}, \hat{V}=1) = \hat{P}(\hat{T}, \hat{V})$, set them =

$$\Rightarrow 1 + 4(\hat{T}-1) = 1 + 4(\hat{T}-1) - 6(\hat{T}-1)(\hat{V}-1) - \frac{3}{2}(\hat{V}-1)^3$$

$$\Rightarrow -6(\hat{T}-1)(\hat{V}-1) = \frac{3}{2}(\hat{V}-1)^3 \Rightarrow -4(\hat{T}-1) = (\hat{V}-1)^2$$

$$\Rightarrow 0 = V^2 - 2V + (4T-3)$$

w/ quadratic formula, $V = \frac{2 \pm \sqrt{4 - 4(1)(4T-3)}}{2} = 1 \pm \sqrt{1 - 4T + 3} = 1 \pm 2\sqrt{1-\hat{T}}$

know $V_g > V_l$, so
$$\boxed{\begin{array}{l} V_g = 1 + 2\sqrt{1-\hat{T}} \\ V_l = 1 - 2\sqrt{1-\hat{T}} \end{array}}$$

& $V_g - V_l = 4(1-\hat{T})^{1/2}$ $\boxed{\therefore \hat{V}_g - \hat{V}_l \propto (1-T)^{1/2}}$