HW #28

For the cases $n=2, n=3$ prove sum product & sum are equal

$n=2$

product: $1-\prod_{n=1}^{N}[1-P(A_n)] = 1-(1-P(A_1))(1-P(A_2)) = 1-(1-P(A_1)-P(A_2)+P(A_1)P(A_2))$

$= P(A_1)+P(A_2)-P(A_1)P(A_2) = P(A_1)+P(A_2)$ ($P(A_1)P(A_2) \to 0$, small chance)

sum: $\sum_{n=1}^{2} P(A_n) = P(A_1)+P(A_2)$

product & sum for $n=2$ are the same

$n=3$

product: $1-\prod_{n=1}^{N}[1-P(A_n)] = 1-(1-P(A_1))(1-P(A_2))(1-P(A_3))$

$= 1-(1-P(A_1)-P(A_2)+P(A_1)P(A_2))(1-P(A_3))$ ($P(A_1)P(A_2) \to 0$)

$= 1-(1-P(A_3)+P(A_1)P(A_3)+P(A_2)P(A_3))-P(A_2)-P(A_3))$

$= P(A_1)+P(A_2)+P(A_3)$

sum: $\sum_{n=1}^{3} P(A_n) = P(A_1)+P(A_2)+P(A_3)$

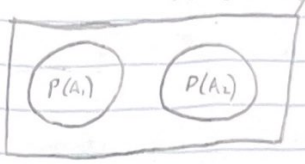
product & sum for $n=3$ are the same

HW#28 - cont

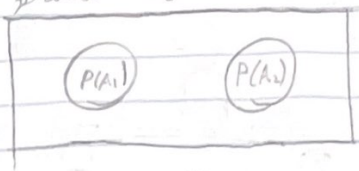
assuming four probabilities

$n=2$

product:

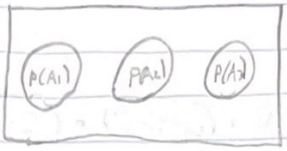


sum:

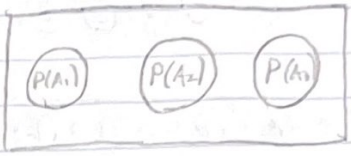


$n=3$

product:



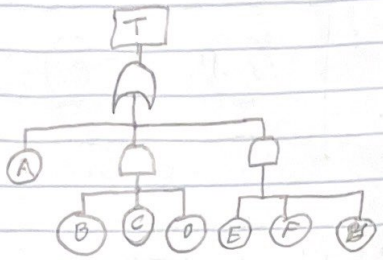
sum:



L #28 - unit

Fault tree $T = A + (B \cdot C \cdot D) + (E \cdot F \cdot G)$

1)

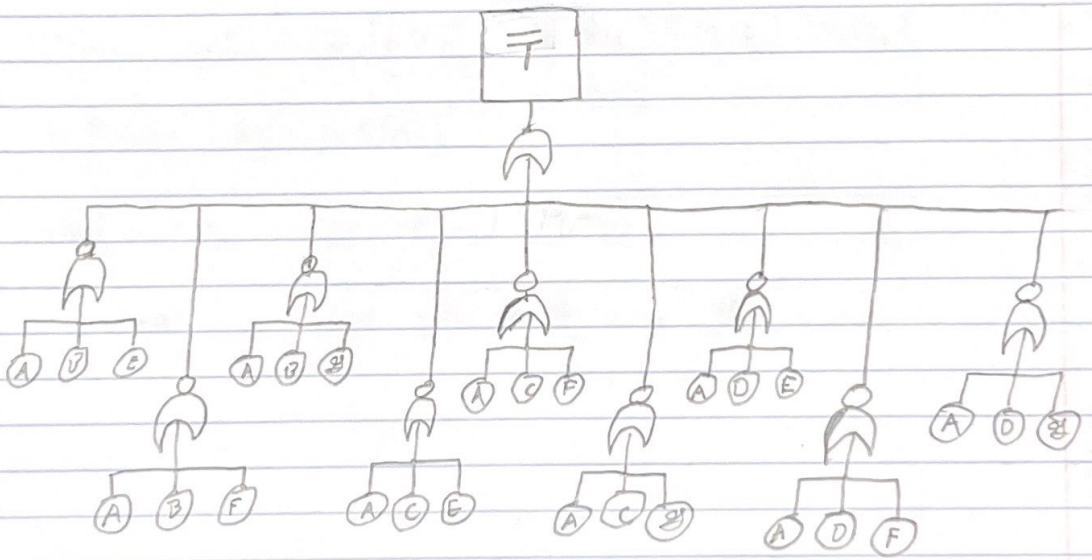


2) $\overline{T} = \overline{(A + (B \cdot C \cdot D) + (E \cdot F \cdot G)} = \overline{A} \cdot \overline{(B \cdot C \cdot D)} \cdot \overline{(E \cdot F \cdot G)}$

$= \overline{A} \cdot (\overline{B} + \overline{C} + \overline{D}) \cdot (\overline{E} + \overline{F} + \overline{G}) = (\overline{A} \cdot \overline{B} + \overline{A} \cdot \overline{C} + \overline{A} \cdot \overline{D}) \cdot (\overline{E} + \overline{F} + \overline{G})$

$= \overline{A}\overline{B}\overline{E} + \overline{A}\overline{B}\overline{F} + \overline{A}\overline{B}\overline{G} + \overline{A}\overline{C}\overline{E} + \overline{A}\overline{C}\overline{F} + \overline{A}\overline{C}\overline{G} + \overline{A}\overline{D}\overline{E} + \overline{A}\overline{D}\overline{F} + \overline{A}\overline{D}\overline{G}$

$= \overline{A+B+E} + \overline{A+B+F} + \overline{A+B+G} + \overline{A+C+E} + \overline{A+C+F} + \overline{A+C+G} + \overline{A+D+E} + \overline{A+D+F} + \overline{A+D+G}$



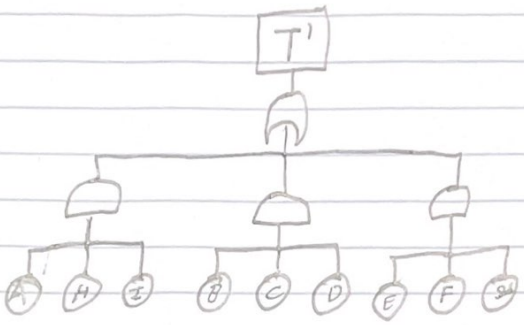
L#28 - cont

3) Say $X = 10^{-2}$

$\Rightarrow T = X + X^3 + X^3 = X + 2X^3 = 0.010002$

4) add redundancies for A $\Rightarrow T' = (A \cdot H \cdot I) + (B \cdot C \cdot D) + (E \cdot F \cdot G)$

$T' = X^3 + X^3 + X^3 = 3X^3 = 0.000003 \ll 0.010002$



5) $P(T) = 0.010002 \gg P(T') = 0.000003$