ECE 205: Lab 5

RMS and AC Simulation

University of Illinois at Urbana-Champaign

Department of Electrical and Computer Engineering

Authors

Nathan Glaser   Joseph Specht

nglaser3   jspecht3

Professor

Jonathon Schuh

July 27, 2024

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1 Statement of Purpose

The purpose of this lab is to determine the resistance limit and capacitance limit in a simple AC circuit, in which we are attempting to power a lamp, which will consume, on average 5 W. SCRAN!

2 PreLab Deliverables

For the first prelab, we calculated peak power for US outlets using $V_{rms} = 120V$, $f = 60hz$, and assuming voltage follows the form of a sine function. From the calculations below, we found V_{max} to be $\sqrt{2}V_{rms} = 120\sqrt{2}\ V = 169.70563\ V$.

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} V^2(t)dt} \tag{1}$$

Square both sides and multiply by the period T.

$$V_{rms}^2 T = \int_{t_0}^{t_0+T} V^2(t)dt \tag{2}$$

Assume $V(t)$ is a sine function with no phase shift and a max amplitude of V_{max}.

$$V_{rms}^2 T = \int_{t_0}^{t_0+T} V_{max}^2 \sin^2(\omega t)dt \tag{3}$$

Find ω using the following relation.

$$\omega = 2\pi f = 2\pi \cdot 60hz = 120\pi \frac{rad}{s} \tag{4}$$

Find T using the following relation.

$$f = 60hz = \frac{1}{T} \quad \rightarrow \quad T = \frac{1}{60hz} = \frac{1}{60}s \tag{5}$$

Evaluate the integral.

$$V_{rms}^2 T = V_{max}^2 \left[\frac{\omega t - \frac{\sin(2\omega t)}{2}}{2\omega}\right]_{t_0}^{t_0+T} \tag{6}$$

Plug in: $t_0 = 0,\ T = \frac{1}{60}s$, and $\omega = 120\pi\frac{rad}{s}$.

$$V_{rms}^2 \frac{1}{60} = V_{max}^2 \left[\frac{(120\pi)t - \frac{\sin(2(120\pi)t)}{2}}{2(120\pi)}\right]_0^{\frac{1}{60}} \tag{7}$$

Evaluate integral at bounds of integration.

$$V_{rms}^2 \frac{1}{60} = V_{max}^2 \left[\frac{(120\pi)\frac{1}{60}}{2(120\pi)} - \frac{\sin\left(2(120\pi)\frac{1}{60}\right)}{4(120\pi)} - 0 \right] = \frac{V_{max}}{120} \tag{8}$$

Rearrange and solve for V_{max}.

$$V_{max} = \sqrt{120 \cdot V_{rms}^2 \cdot \frac{1}{60}} = \sqrt{2}V_{rms} = 120\sqrt{2}V = 169.70563V \tag{9}$$

For the second prelab deliverable, we considered the following circuit and designed an indicator light that shows the line as active. For this indicator light, there is a indicator light bulb that cannot have a power higher than 5 W in series with a limiting resistor, which are plugged into a wall in the United States of America.

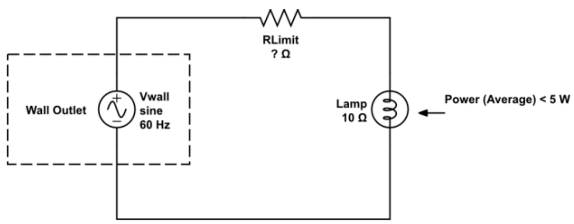

Figure 1: Prelab 2: Circuit under consideration.

To start, we know the average voltage for a wall in the US is given as:

$$\bar{V} = 0.637 \cdot (V_{max}) = 0.637 \cdot \left(\sqrt{2}V_{rms}\right) = 108.10249\ V \tag{10}$$

Next, we know the average power through the indicator light with a resistance of 10Ω cannot exceed $5W$. Therefore, the average current through the circuit is given as:

$$\bar{P}_I = 5W = \bar{I}^2 R_I = 10\bar{I}^2 \quad \rightarrow \quad \bar{I} = \sqrt{\frac{5}{10}} = \frac{\sqrt{2}}{2}A = 0.707A \tag{11}$$

With the current through the circuit, we can now relate the voltage and current via the following equation to find the limiting resistor, R_L, to be 142.90309 Ω.

$$\bar{V} = \bar{I}\,(R_L + R_I) \quad \rightarrow \quad R_L = \frac{\bar{V}}{\bar{I}} - R_I = \frac{108.10249}{0.707} - 10 = 142.90309\Omega \tag{12}$$

With the aforecalculated values, we the found the power dissipated by the current limiting resistor to be 71.45155 W via the following equation:

$$\bar{P}_L = \bar{I}^2 R_L = \left(\frac{\sqrt{2}}{2}\right)^2 142.90309 = 71.45155W \tag{13}$$

3 Procedure

To obtain the desired plots of the power dissipated across the lamp using a resistor and a capacitor, we constructed a simple circuit. We added a sinusoidal voltage source, with amplitude of 169.71 V and frequency of 60 Hz. We modeled the lamp as a 10 Ω Resistor. From here we changed one parameter to obtain the two schematics, we had two resistors for our first simulation, and one resistor and a capacitor for our second trial. The first schematic is shown below in Fig. 2, and the second in Fig. 3. To determine the appropriate current limiting capacitance to achieve 5 W dissipated across the lamp we utilized an iterative process, in which we continuously supplied guesses until one was close enough to 5 W.

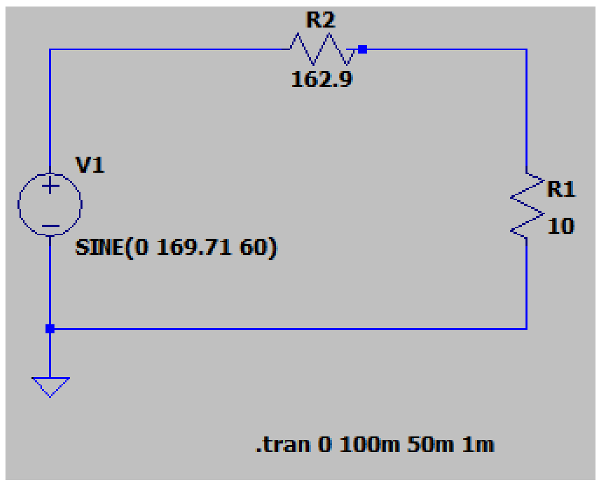

Figure 2: Schematic of current limiting resistor circuit

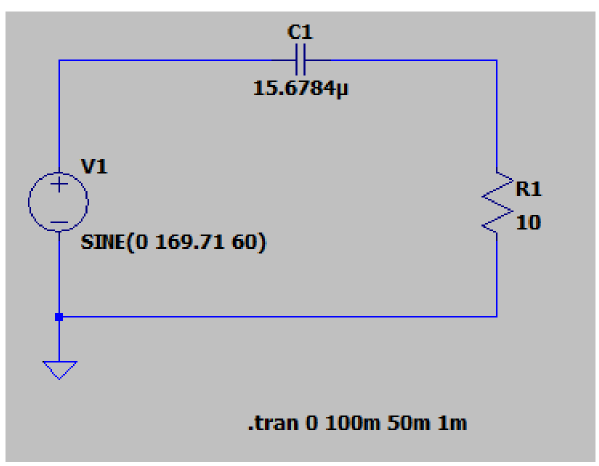

Figure 3: Schematic of current limiting capacitor circuit

4 Observation and Data

To begin, the plot and average power dissipated across the lamp from the current limiting resistor schematic are presented below in Figs. 4 and 5.

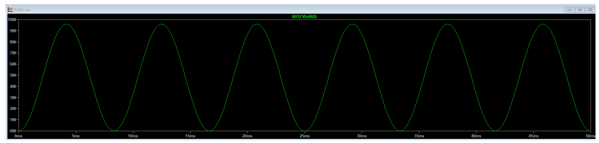

Figure 4: Power dissipated in the lamp as a function of time

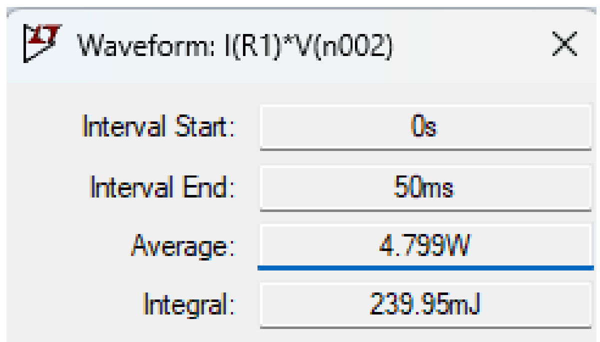

Figure 5: Average power dissipated across the lamp

Next, the plot and average power dissipated across the lamp from the current limiting capacitor schematic are presented below in Figs. 6 and 7.

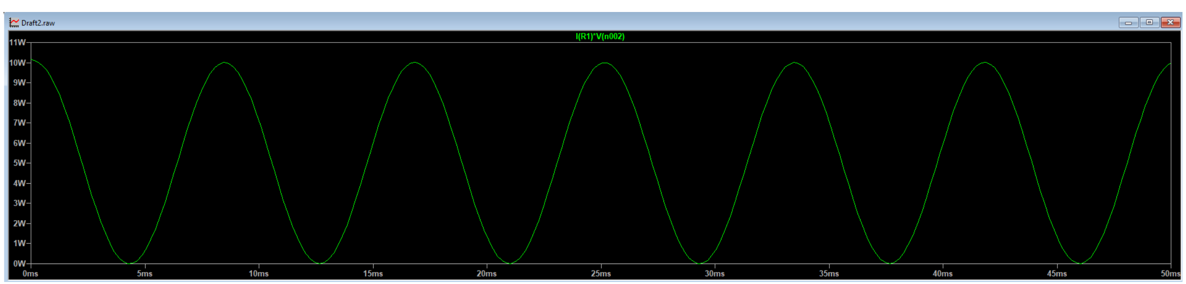

Figure 6: Power dissipated in the lamp as a function of time

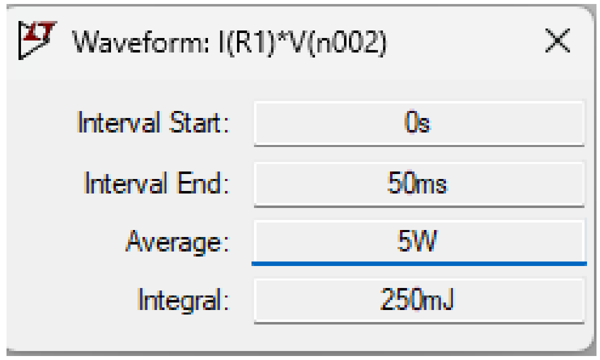

Figure 7: Average power dissipated across the lamp

5 Analysis

There are major advantages to utilizing a capacitor to limit current as opposed to a resistor. First, in the resistor circuit the non-load resistor dissipates 78 W, whereas the capacitor dissipates an average of 6 mW, basically 0. For a UK Circuit with 230 V_{RMS} and 50 Hz, the power dissipated across the lamp would be 12.753 W.

Finally, we experimentally determined the limiting capacitance to be 15.6784 μC. Analytically however, is a different story sung by different bards.

6 Conclusions

In conclusion, capacitors are a much better choice for current limiting applications. This is primarily due to its minimal power 'wasting.'