NPRE 412	Spring 2025
HW 2	Due 2025.02.04

- Show your work.
- This work must be submitted online as a `.pdf` through Canvas.
- Work completed with LaTeX or Jupyter earns 1 extra point. Submit source file (e.g. `.tex` or `.ipynb`) along with the `.pdf` file.
- If this work is completed with the aid of a numerical program (such as Python, Wolfram Alpha, or MATLAB) all scripts and data must be submitted in addition to the `.pdf`.
- If you work with anyone else, document what you worked on together.

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1. (20 points) Indicate the depreciation per year for an asset costing \$50,000 with a salvage value of \$2000 and a 5-yr lifetime. Consider all three methods discussed in the text. (Tsoulfanidis, Question 8.1)

Solution:

Years:	1	2	3	4	5
SL [\$/year]:	9600	9600	9600	9600	9600
SYD [\$/year]:	16000	12800	9600	6400	3200
DDB [\$/year]:	20000	12000	7199	4319	2591

2. (15 points) A person purchased an automobile in 2010 for \$10,000, paying for it in four equal payments at the beginning of each calendar year, starting on January 1, 2011. Assuming an interest rate of 3.5%, what was the worth of this money on January 1, 2014? (Tsoulfanidis, Question 8.2)

Solution: With the inflation rate on the car being 3.5 %, the worth of this money can be found using Eq. 8.10:

$$F = \$P(1 + N \cdot i) = \$10000(1 + 5 \cdot 0.035) = \$11400 \tag{1}$$

The worth of the money after the car is paid off, neglecting inflation, is \$ 11400 in 2010 dollars.

Assuming a 2 % inflation rate, the value then becomes:

$$F = \$P(1 + N \cdot i) = \$10000(1 + 5 \cdot 0.015) = \$10600 \tag{2}$$

The worth of the money after the car is paid off, including a 2 % inflation rate, is \$ 10600 in 2010 dollars.

3. (15 points) At the time of the birth of their child, a set of parents chooses to put \$7500 in the bank for the child's education, earning 8% interest. What is the worth of this money when the child graduates from high school? (Tsoulfanidis, Question 8.3)

Solution: Assuming compound interest and the child graduates high school at 18 years old, use Eq. 8.9 to find:

$$F = \$P(1+i)^N = \$7500(1.08)^{18} = \$29970.15 \tag{3}$$

After graduating high school, the money is worth \$ 29970.15 in the year the child was born (neglecting inflation).

Assuming 2% inflation rate (r), the worth of the investment is:

$$F = \$P[1+(i-r)]^N = \$P[1+(1.06)]^N = \$21407.54 \tag{4}$$

After graduating high school and assuming a 2 % inflation rate, the money is worth \$ 21407.54 in the year the child was born.

4. (25 points) How many years does it take to double your money (assume no inflation) at 1% annual interest? Also show that this value (in years) divided by the annual interest rate gives the doubling time in years for any interest rate.

Solution: Using Eq. 8.9, the doubling time with a 1 % interest rate occurs when $\frac{F}{P} = 2$:

$$F = (1+i)^N P \tag{5a}$$

$$\frac{F}{P} = (1+i)^N \tag{5b}$$

$$2 = (1+i)^N \tag{5c}$$

$$\ln(2) = N \ln(1+i) \tag{5d}$$

$$N = \frac{\ln(2)}{\ln(1+i)} \tag{5e}$$

$$N_1 = \frac{\ln(2)}{\ln(1+i)} = 69.66\ a \tag{5f}$$

With an interest rate of 1 %, an investment will double after 69.66 years.

To show the doubling time (N_x) for some interest rate (x) is the doubling time (N_1) with a 1 % interest rate (i) divided by x, set these quantities equal.

$$\frac{N_1}{x} = N_x \tag{6a}$$

Expand

$$\frac{\ln(2)}{x\ln(1+i)} = \frac{\ln(2)}{\ln(1+x)} \tag{6b}$$

$$x \ln(1+i) = \ln(1+x) \tag{6c}$$

Using a first order Taylor expansion:

$$\ln(1+a) \approx a \tag{6d}$$

Therefore,

$$x \ln(1+i) \approx x(i) \tag{6e}$$

$$\ln(1+x) \approx x \tag{6f}$$

The equation becomes:

$$x(i) = x \tag{6g}$$

When $i = 1$, this equation holds.

$$x = x \tag{6h}$$

$$\therefore \frac{N_1}{x} = N_x \tag{6i}$$

When calculating the doubling time, $i = 0.01 \neq 1$. However, the proposed relation (Eq. 6), presupposes x is in units of percent, not decimal. We can verify this presupposition by checking the value of x when the interest rate is 1 %.

$$\frac{N_1}{x} = N_1 \to x = 1 \tag{7}$$

With a real interest rate of 1 %, $x = 1$. Therefore, making the assumption in Eq. 6 (base percent not base decimal) is valid for all interest rates.

5. (25 points) Prove that the amounts of money shown in Tsoulfanidis, Table 8.2 for plans 1 and 4 are equivalent if they are time valued for the same date. (Tsoulfanidis, Question 8.5)

Solution: The present worth factor is given by the equation:

$$PW = \frac{1}{(1+i)^2} \tag{8a}$$

Plan 1, which generalized to N years by default:

$$F = P(1+i)^N \tag{8b}$$

Plan 4, for a single year w/ n payments per year:

$$F = P\left[1 + i\frac{(n+1)}{2}\right] \tag{8c}$$

For plan 4, the principal for the current calculation is the future value for the previous year,

$$F_N = P_{N-1}\left[1 + i\frac{(n+1)}{2}\right] \tag{8d}$$

$$F_N = P_{N-2}\left[1 + i\frac{(n+1)}{2}\right]\left[1 + i\frac{(n+1)}{2}\right] \tag{8e}$$

$$F_N = P_{N-3}\left[1 + i\frac{(n+1)}{2}\right]^3 \tag{8f}$$

$$F_N = P_1\left[1 + i\frac{(n+1)}{2}\right]^N \tag{8g}$$

This is the equation for the final value of the loan after N years. As the interest rate is charged at the end of each year, the number of payments per year does not matter. Therefore, we set $n = 1$ as the total payments are effectively paid in a lump sum at the end of each year. With $n = 1$, Plan 4 becomes:

$$F_N = P_1\left[1 + i\frac{(n+1)}{2}\right]^N = P_1\left[1 + i\frac{(1+1)}{2}\right]^N = P_1\left[1 + i\right]^N \tag{8h}$$

With the aforementioned assumptions, Plan 1 (Eq. 8c) and Plan 4 (Eq. 8h) are found to be equivalent.