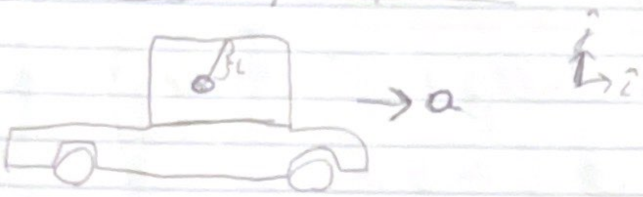
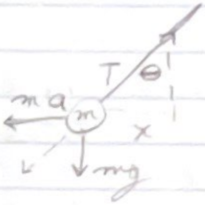
HW#9 - Joseph Specht

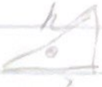
1)



$F_{elevator} = -am$

a)





$\cos\theta = x/h$

Real

Fictitious

$\Sigma F_{fict,x} = T\sin\theta - ma = 0 \Rightarrow T\sin\theta = ma \Rightarrow T = \frac{ma}{\sin\theta}$

$\Sigma F_{fict,y} = T\cos\theta - mg = 0 \Rightarrow T\cos\theta = mg \Rightarrow T = \frac{mg}{\cos\theta}$

We can equate these 2 expressions for T & solve for θ

$\frac{ma}{\sin\theta} = \frac{mg}{\cos\theta} \Rightarrow a = g\frac{\sin\theta}{\cos\theta} = g\tan\theta \Rightarrow \tan\theta = \frac{a}{g}$

$$\boxed{\therefore \theta = \text{atan}(a/g)}$$

b) We know $T = \frac{mg}{\cos\theta}$, so if we know θ, we can find T

$$\boxed{T = \frac{mg}{\cos\theta} \quad w/\ \theta = \text{atan}(a/g)}$$

HW #9 - Joseph Specht

c)

In this frame, we have a net $\vec{a}$ to the bottom left, so we can call this the g_{eff} & w/ pendulums, we know the natural frequency is $\sqrt{g/L}$

$$\boxed{\therefore \ \omega_n = \sqrt{\frac{\sqrt{g^2+a^2}}{L}} = \frac{(g^2+a^2)^{1/4}}{L^{1/2}}}$$

HW#2 - Joseph Speck

2)

We have only $F_{coriolis}$ & $F_{centrifugal}$ b/c $\dot{\omega} = 0$ & no relative translation b/w frames

a) $\vec{F}_{cor} = -2m\,\vec{\omega} \times \vec{v}'$ $\qquad$ $F_{cent} = -m\vec{\omega} \times (\vec{\omega} \times \vec{r}')$

$\vec{\omega} = (0,0,\omega)$ $\qquad$ $\vec{v}' = (v_r, 0, 0)$ $\qquad$ $\vec{r} = (r,0,0)$

$\vec{F}_{cor} = -2m(0, \omega v_r, 0) = (0, -2m\omega v_r, 0)$

$\vec{F}_{cent} = -m\vec{\omega} \times (0, \omega r, 0) = -m(-\omega^2 r, 0, 0) = (m\omega^2 r, 0, 0)$

We know $|F_s|^2 = |\vec{F}_{cent} + \vec{F}_{cor}|^2$

$$|\mu m g|^2 = |(m\omega^2 r, -2m\omega v_r, 0)|^2$$

$$(\mu m g)^2 = (m\omega^2 r)^2 + (2m\omega v_r)^2 \Rightarrow \mu^2 m^2 g^2 = m^2\omega^4 r^2 + 4m^2\omega^2 v_r^2$$

$$\Rightarrow \mu^2 g^2 = \omega^4 r^2 + 4\omega^2 v_r^2 \Rightarrow \omega^4 r^2 = \mu^2 g^2 - 4\omega^2 v_r^2$$

$$r^2 = \frac{\mu^2 g^2}{\omega^4} - \frac{4v_r^2}{\omega^2} = \frac{1}{\omega^2}\left(\frac{\mu^2 g^2}{\omega^2} - 4v_r^2\right)$$

$$r = \sqrt{\frac{1}{\omega^2}\left[\left(\frac{\mu g}{\omega}\right)^2 - (2v_r)^2\right]}$$

$$\boxed{r_{max} = \frac{1}{\omega}\sqrt{\left(\frac{\mu g}{\omega}\right)^2 - (2v_r)^2}}$$

HW # 9 - Joseph [illegible]

a)

$$\vec{F}_{cor} = -2m\vec{\omega}\times\vec{v}$$

$$\vec{F}_{cent} = -m\vec{\omega}\times(\vec{\omega}\times\vec{r})$$

i) We know the speed is the speed in the reference frame plus the relative speed & this is equal to F_c

$$F_c = \frac{m(v_r+\omega b)^2}{b} = \mu m g$$

$$\Rightarrow \frac{(v_r+\omega b)^2}{b} = \mu g \Rightarrow b\mu g = (v_r+\omega b)^2 \Rightarrow v_r + \omega b = \sqrt{b\mu g}$$

$$\boxed{v_r = \sqrt{b\mu g} - \omega b}$$

ii) We know the following

$\vec{r} = (b,0,0)$ $\quad\vec{\omega} = (0,0,\omega)$ $\quad\vec{v} = (0,v_r,0)$

$$\vec{F}_{cor} = -2m(-\omega v_r,0,0) = (2m\omega v_r,0,0)$$

$$\vec{F}_{cent} = -m\omega\times(0,\omega b,0) = -m(-\omega^2 b,0,0) = (m\omega^2 b,0,0)$$

We also know the bug has a centripetal force

$$F_f = F_{cor} + F_{cent} + F_{centripetal} \quad (F_f \text{ opposes net force})$$

$$\mu m g = 2m\omega v_r + m\omega^2 b + m\frac{v_r^2}{b} \Rightarrow \mu g = 2\omega v_r + \omega^2 b + \frac{v_r^2}{b}$$

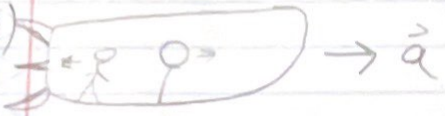
$$\Rightarrow b\mu g = 2\omega v_r b + \omega^2 b^2 + v_r^2 = (v_r+\omega b)^2$$

$$\Rightarrow \sqrt{b\mu g} = v_r + \omega b \Rightarrow \boxed{v_r = \sqrt{b\mu g} - \omega b}$$

HW #8 - Joseph Specht

iii) They match

3)



a) If we consider the astronauts, air, & balloon are different densities w/ $\rho_{ast} > \rho_{air} > \rho_{balloon}$, we see that the bulk of the spaceship has a density of ρ_{air}. This volume of air will want to stay still as it has an inertia, but it is being pushed forward by the back wall. If we assume air is a relatively incompressible fluid, we can say the bulk of the volume accelerates the same. However, if something is more dense than air (humans), it will not be pushed as hard by the air behind it & they experience less acceleration. However, the converse is true. If an ~~object~~ less dense than air (balloon) is pushed by air behind it, it will have less inertia than the air & accelerate forward more than the air.

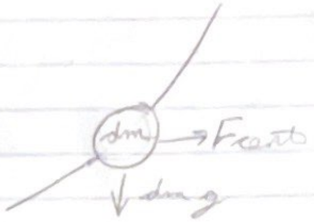
b) Vacuums have 0 density & cannot provide support to anything w/ density, so

$$\boxed{\text{the balloon will move backward.}}$$

*assuming an invincible balloon

HW#9-Joseph Specht

4)

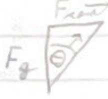


Since we are in eq, $v = \dot{w} = 0$, so coriolis & azimith are 0

$$\vec{F}_{cent} = -m\vec{\omega}\times(\vec{\omega}\times\vec{r}) = -m\vec{\omega}\times(0, \omega r, 0) = (m\omega^2 r, 0, 0)$$

& $\vec{F}_g = (0, 0, -mg)$

If these are the only 2 forces, we can make a triangle



We know $\tan\theta = \left(\frac{\omega^2 r\, dm}{g\, dm}\right)$

& this is the slope, so we know @ this point these forces are equal as we are in eq, so

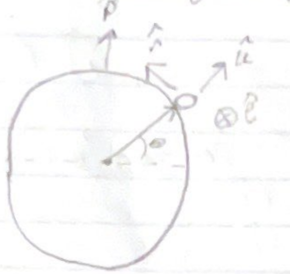
$$dm\,\omega^2 r\, dr = dm\, g\, dh \Rightarrow \omega^2 r\, dr = g\, dh$$

$$\omega^2\int r\, dr = g\int dh \Rightarrow \frac{\omega^2 r^2}{2} = gh \Rightarrow h = \frac{\omega^2 r^2}{2g}$$

$\boxed{h = \frac{\omega^2 r^2}{2g}}$, & this is equation of V, so works

HW #9 - Joseph Spaetto

5)



$\vec{r} = x\hat{e} + y\hat{n} + z\hat{u}$

a) $\hat{e} \perp \hat{p}$, so no component in $\hat{e}$ direction

If $\theta = 0°$, $\hat{p} = \hat{n}$, so want periodic function max @ 0°.
If $\theta = 90°$, $\hat{p} = \hat{u}$, so want periodic function max @ 90°

$\hat{p} = \cos\theta\,\hat{n} + \sin\theta\,\hat{u}$, so $\omega\hat{p}$ is...

$$\boxed{\vec{\omega} = \omega(\cos\theta\,\hat{n} + \sin\theta\,\hat{u})}$$

b) Since we only have F_{cor} & F_g, F_{net} is

$F_{net} = F_{cor} + F_g$

$F_g = (\overset{\hat{e}}{0}, \overset{\hat{n}}{0}, \overset{\hat{u}}{-mg})$ $\quad F_{cor} = -2m\vec{\omega} \times \vec{v}$

$F_{cor} = -2m\omega\overset{\hat{\omega}}{(\cos\theta\,\hat{n} + \sin\theta\,\hat{u})} \times \overset{\vec{v}}{(\dot{x}\hat{e} + \dot{y}\hat{n} + \dot{z}\hat{u})}$

$\hat{\omega} \times \vec{v}$

$$\begin{vmatrix} \hat{e} & \hat{n} & \hat{u} \\ 0 & \cos\theta & \sin\theta \\ \dot{x} & \dot{y} & \dot{z} \end{vmatrix} = \begin{matrix} \hat{e}(\dot{z}\cos\theta - \dot{y}\sin\theta) - \\ \hat{n}(-\dot{x}\sin\theta) + \\ \hat{u}(-\dot{x}\cos\theta) \end{matrix}$$

$F_{cor} = -2m\omega(\overset{\hat{e}}{\dot{z}\cos\theta - \dot{y}\sin\theta}, \overset{\hat{n}}{-\dot{x}\sin\theta}, \overset{\hat{u}}{-\dot{x}\cos\theta})$

HW #9 - Joseph Spectt

We know the zeroth order is just w/o coriolis, so

$\ddot{x}_0 = 0$ $\quad\ddot{y}_0 = 0$ $\quad\ddot{z}_0 = -g$ This is the general
$\dot{x}_0 = 0$ $\quad\dot{y}_0 = 0$ $\quad\dot{z}_0 = -gt + A$ one w/o coriolis
$x_0 = 0$ $\quad y_0 = 0$ $\quad z_0 = -\frac{1}{2}gt^2 + At + B$

If we apply the conditions @ $t=0$ for $\dot{z}$ & z..

$\dot{z}(t=0) = A = v$ & $z(t=0) = B = 0$, so

$x_0 = 0$ $\quad y_0 = 0$ $\quad z_0 = vt - \frac{1}{2}gt^2$

The solution for x_1, y_1, z_1 is dependent on velocities, but the only non zero v in $(\,)_0$ is

$\dot{z}_0 = v - gt$ $\qquad \dot{x}_0 = 0 = \dot{y}_0$

$\ddot{x}_1 = -2m\omega(v-gt)\cos\theta = -2m\omega(v-gt)\frac{1}{\sqrt{2}}$, $\frac{1}{\sqrt{2}} = \sqrt{2}/2$
$= -\sqrt{2}\,m\omega(v-gt) = m\omega\sqrt{2}(gt-v)$

$\dot{x}_1 = \sqrt{2}\,m\omega(\frac{1}{2}gt^2 - vt + C) = 0$ @ $t=0$, so $C=0$

$x_1 = \sqrt{2}\,m\omega(\frac{1}{6}gt^3 - \frac{1}{2}vt^2 + D) = 0$ @ $t=0$, so $D=0$

we know $\ddot{y}_1(t=0) = \ddot{z}_1(t=0)$ & $y_1(t=0) = z_1(t=0) = 0$

$$\therefore \quad x(t) = \sqrt{2}\,\omega m\left(\frac{1}{6}gt^3 - \frac{1}{2}vt^2\right)$$

$$y(t) = 0$$

$$z(t) = vt - \frac{1}{2}gt^2$$