NPRE412 Spring 2025
HW 4 Due 2025.02.18

- Show your work.
- This work must be submitted online as a `.pdf` through Canvas.
- Work completed with LaTeX or Jupyter earns 1 extra point. Submit source file (e.g. `.tex` or `.ipynb`) along with the `.pdf` file.
- If this work is completed with the aid of a numerical program (such as Python, Wolfram Alpha, or MATLAB) all scripts and data must be submitted in addition to the `.pdf`.
- If you work with anyone else, document what you worked on together.

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1. (10 points) Is the statement "The USA has 1,300,000 tons of uranium resources" complete? Explain why or why not. (Tsouldfainidis, Question 2.1)

Solution: No, this statement is not complete. Whenever specifying a resource, one must also report the price at which that resource we be gathered. Also, as the definition of "resource" depends on which authority figure, either the DOE/EIA or OECD, the term is adopted from.

In the book, the 1.3e6 tons referenced thorium, so the following should be taken with a grain of salt. However, the term "resource" had different definitions based on the organization and "resource" under consideration. The DOE/EIA uses the term resource to describe to describe quantities of materiel that are not economical to mine currently. Conversely, OECD uses the term resource to describe quantities of material that are economical to obtain (RAR), most likely exist (IR), and undiscovered (UR).

When discussing resources using the OECD nomenclature, what are known as "reserves" by the DOE/EIA are included. When discussing resources using the DOE/EIA nomenclature, what are known as "reasonably assured resources" by the OECD are not included. For this statement to be complete, the statement should include the organization whose definitions are being used and/or a more specific description of the term "resource".

2. In 1976, the U.S. nuclear industry needed about 10,000 tons of U_3O_8. (Tsouldfanidis, Question 2.4)

   (a) (15 points) Assuming that enrichment and tails requirements do not change and the industry increased at a rate of x% per year, how long would it take for the $130/kgU reserves to be exhausted? Assume that the reserves are 600,000 tons of U_3O_8. Report this equation for time as a function of x.

Solution:

$$yte = \frac{\ln\left(\frac{R}{Q}\ln(1+x)+1\right)}{\ln(1+x)} \tag{1}$$

Where yte is the years until exhaustion of the reserve [a], R is the total amount in the reserve [Tons of U_3O_8], Q is the initial yearly consumption rate [Tons of U_3O_8 per year in 1976], and x is the yearly percent growth of the demand. I checked my answer with Mahmoud Eltawila and Nathan Charles Glaser.

The above expression can be obtained using the compound interest formula.
Start by stating using the total reserves R as the future value and the yearly consumption in 1976 Q as the principle.

$$R = Q(1+x)^y \tag{2a}$$

However, the right hand side (RHS) represents the yearly consumption in year y, so we integrate the RHS to obtain the total consumption until year yte.

$$R = \int_0^{yte} Q(1+x)^y dy \tag{2b}$$

$$\frac{R}{Q} = \int_0^{yte} (1+x)^y dy \tag{2c}$$

$$\frac{R}{Q} = \frac{(1+x)^y}{\ln(1+x)}\Bigg|_0^{yte} \tag{2d}$$

$$\frac{R}{Q} = \frac{(1+x)^{yte}}{\ln(1+x)} - \frac{1}{\ln(1+x)} \tag{2e}$$

$$\frac{R}{Q}\ln(1+x) = (1+x)^{yte} - 1 \tag{2f}$$

$$\frac{R}{Q}\ln(1+x) + 1 = (1+x)^{yte} \tag{2g}$$

$$\ln\left(\frac{R}{Q}\ln(1+x)+1\right) = yte\ln(1+x) \tag{2h}$$

$$yte = \frac{\ln\left(\frac{R}{Q}\ln(1+x)+1\right)}{\ln(1+x)} \tag{2i}$$

(b) (15 points) After you develop the equation for the time, obtain numerical results for $x = 2\%$, $x = 4\%$, and $x = 6\%$.

Solution: For 2%, the years until exhaustion are 39.54 years. For 4%, the years until exhaustion are 30.85 years. For 6.0%, the years until exhaustion are 25.8 years. Just plugging in the numbers...

3. Assume that a decision is made to start ordering reactors at such a constant rate per year from 1990 until 2030, at which time orders will stop, so that all known uranium reserves of 2.3 million tons (with price up to $260/kgU) will be used up. Assume the following (Tsoulfanidis, Question 2.5):

   - All plants are identical and need 150 tons of natural uranium per year.
   - It takes 10 yr to build a plant.
   - Every plant has a 30-yr lifetime.
   - There are 120 plants operating in 1990.
   - Reactors operating in 1990 start retiring in 2000, at the rate of 10/yr. For these reactors, consider their needs only after 1990.

   (a) (15 points) Calculate how many reactors per year could be ordered.

   Solution: Up to 29 reactors could be ordered per year. I worked on this question with Mahmoud Eltawila and Nathan Charles Glaser for far, far too long.

   We can find the reactor years ry by dividing the reserves by the fuel requirements per year.

   $$ry = \frac{1.3e6\ tons}{150\ tons\ per\ reactor} \tag{3a}$$

   Next, subtract the number of reactor years the initial reactors are operating before shutting down.

   $$ry- = 10\ years \cdot 120\ reactors \tag{3b}$$

   Next, subtract the reactor years of the initial reactors while being decommissioned. This result is the area of a trapezoid of legs 120 reactors and 10 reactors long. The height of this trapezoid is 12 years.

   $$ry- = (110\ reactors \cdot 6) + (10 reactors \cdot 12) \tag{3c}$$

   Next, we can subtract the total number of reactor years found by adding x reactors per year. The value for x is progressively increased until the first instance when ry is negative, corresponding to an exhausted reserve. The additional x in front accounts for the shut down of the old head reactors.

   $$ry- = x + \sum_{n=1}^{30} x \cdot n \tag{3d}$$

   (b) (15 points) Calculate the maximum number of reactors operating at any single time.

   Solution: The maximum number of reactors is 870, which is 30 times the reactors per year.

4. The concentration of radon and its daughters in the air at an unventilated mine results in an activity that averages $43[kBq/m^3]$ (about $1.2nCi/L$). with maximal value of $160[kBq/m^3]$ (about $4.3nCi/L$). A worker in that mine works a typical workweek and spends 70% of each workday in the mine.

(a) (5 points) What is the working level in the mine?

Solution: The working level is 12 WL. These miners are going to die.
Know 100 pCi/L of radon corresponds to a WL of 1, so divide our source by this ratio to obtain the working level. One nano is equal to 1000 pico.

$$WL = \frac{1.2e3\ pCi/L}{100\ pCi/L} = 12. \tag{4a}$$

(b) (5 points) What is the worker's annual expected exposure in units of WLM?

Solution: Annual WLM of 100.8 WLM. They are trying to kill him ;-;.
Find the time spent in the mines per month.

$$t = 40\frac{hours}{week} \cdot 4.25\frac{weeks}{month} \cdot 0.7\frac{timespelunking}{workweek} \tag{5a}$$

Convert the time to working level months and scale by the working level of the mine.

$$WLM = WL \cdot \frac{t}{170 hours\ per\ WLM} \tag{5b}$$

Finally, multiply by 12 to get the annual WLM.

$$Annaul\ WLM = 12 \cdot WLM = 100.8\ WLM \tag{5c}$$

5. (20 points) The danger presented to miners from radon and its daughters is due to the α particles emitted in their rapid decay chain, pictured in the figure below. For the four most likely alpha decays in the chain, calculate the energy Q, in MeV, released (converted to kinetic energy) during the alpha decay. (Hint: begin with $^{222}Rn \xrightarrow{\alpha} {}^{218}Po$.)

Solution:

Q for α decay from Rn222 $\rightarrow$ Po218: 5.5904241424 MeV.

Q for α decay from Po218 $\rightarrow$ Pb214: 6.1147934374 MeV.

Q for α decay from Po214 $\rightarrow$ Pb210: 7.8335907169 MeV.

Q for α decay from Po210 $\rightarrow$ Pb206: 5.4075641719 MeV.

We can obtain the data from OpenMC and use the following equation for the Q value.

$$Q = \frac{931.5MeV}{AMU}\left(M_p - M_d - M_{He4}\right) \tag{6}$$

Where were are using He-4 as the alpha particle because electrons don't matter and we hate them. I'm just kidding electrons we humans appreciate you making electricity.