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Computer Engineering Department
Assoc. Prof. Feza BUZLUCA
Asst. Prof. Sanem KABADAYI
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BLG 231E DIGITAL CIRCUITS MIDTERM EXAM

Regulations:

1. The exam duration is **3.5 hours**. You may upload your files to Ninova until **21:30**.
2. Read the file "exam policies" in the "class-files" directory in Ninova carefully, and submit your solutions to Ninova as explained in this file. **You must submit 4 zip files.**
3. Do not send your solutions by e-mail. We will only accept files that have been uploaded to the official e-learning system Ninova before the deadline.
4. There are **4 questions**; you must **answer each question on its own sheet** as explained in the file "exam policies". Create a separate zip file for each question.
5. **You may not ask any questions during the exam.** If you think something is missing in a question, explain it, make the necessary assumption, and solve the question.
6. Any cheating or any attempt to cheat will be subject to the University disciplinary proceedings.
7. Please **show ALL work**. Answers with no supporting explanations or work will be given no partial credit. If we cannot read or follow your solution, no partial credit will be given. PLEASE BE NEAT!

QUESTION 1 (25 Points):

<u>Note:</u> Parts (a) and (b) below are not related.

a) [10 points] X is an **8-bit**, **signed** binary integer. **2's complement** is used to represent signed numbers.
If we perform the binary operation R=X+X, the 9^{th} bit (from right to left) of the result is "1". (R= 1 **** ****). We also know that an **overflow occurred**.

- **i)** What is the **largest possible** decimal X value that can yield this result? Explain your answer briefly.
- **ii)** Carry out the binary operation and show that the 9^{th} bit is 1 and overflow occurs.

b) [15 points] A and B are **8-bit** binary constant integers. Their values are not given but we know that the following two conditions hold:

- If we assume that A and B are **unsigned integers**, then A>B.
- If we assume that A and B are **signed integers**, then they have the **same sign** and B>A.

- **i)** If we carry out the binary operation R = A – B using 2's complement, what should the 9^{th} and 8^{th} bits (from right to left) of the result R be equal to, based on the information given above? Explain your answer briefly. You will get no credit for a simple 0 or 1.
- **ii)** If A and B happen to be **signed integers**, can we determine their sign? Explain.

QUESTION 2 (25 Points):
<u>Note:</u> Parts (a), (b), and (c) below are not related.

a) **[10 points]** One of the expressions of a function F(A,B,C,D) is given below:

$$\boldsymbol{F(A,B,C) = A \cdot B \cdot \overline{C} + A \cdot \overline{B} + \overline{A} \cdot C}$$

Is $\boldsymbol{A \cdot B \cdot \overline{C}}$ a prime implicant of $\boldsymbol{F(A,B,C)}$? <u>**Explain**</u> your answer using axioms and theorems of Boolean algebra. You will get no credit for a simple YES or NO.

b) [**15 points]** An expression of a function F(A,B,C,D) is given below:

$$\boldsymbol{F(A,B,C,D) = U_1(1,4) + U_{\Phi}(0,5,9,10,15)}$$

Decide how the don't cares should be treated (which don't cares should be treated as 0s, which don't cares should be treated as 1s?) so that your SOP implementation **uses the fewest number of gates**. Explain your logic and steps.
Write your final expression in SOP form. Explicitly state what you have set each don't care equal to.

QUESTION 3 (25 Points):
<u>Note:</u> Parts (a) and (b) below are not related.

a) [**10 points]** Consider the logic function f with five inputs:

$$f(a,b,c,d,e)= U_1(2, 6, 14, 18, 22, 30)$$

Find **all** prime implicants of the function f(a,b,c,d,e) using the **Quine-McCluskey** method.

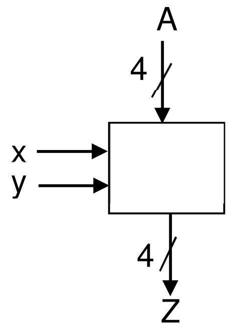
b) [**15 points]** The set of all prime implicants for the incompletely specified logic function F(A,B,C,D) in POS form is given below.

Prime implicants in **POS** form: $(A+\bar{D}), (\bar{B}+\bar{D}), (A+\bar{B})$

Don't care input combinations are ABCD = 0011 and 0110.

Find the **set of all** prime implicants for the incompletely specified logic function F(A,B,C,D) in **SOP (Sum of Products)** form using a **Karnaugh map**.

QUESTION 4 (25 Points):
<u>Note:</u> Parts (a) and (b) below are not related.



a) [**13 points]** A combinational digital circuit shown on the right performs arithmetic operations on a 4-bit integer A, based on the values of inputs x and y, as explained in the table given below:

xy	Operation
00	Z = A – 4
01	Z = A – 3
10	Z = A – 2
11	Z = A – 1

Implement and draw this circuit using only **one** parallel adder and other logic units (i.e., gates and devices that were explained in the lectures), if necessary. Use the **fewest possible** number of logic units to make your circuit design as simple as possible. Do not show the internal structure of the parallel adder; **show it only as a block**. Fully label all inputs and outputs.
Hint: For this circuit, other logic gates and devices except the parallel adder are not necessary.

b) [**12 points]** Consider the incompletely specified logic function

$$f(a,b,c,d) = U_1(7, 9, 10, 12, 15) + U_\Phi(1, 5)$$

Design and draw the circuit for the given function using **only one** 4:1 multiplexer and other necessary logic gates. Use the **fewest possible** number of logic gates to make your circuit design as simple as possible. Fully label all inputs and outputs.