Prof. Dr. Berna Örs Yalçın   **Due Date:** 13.12.2021

EHB205 Introduction to Logic Design
Homework 3

Part 1

1) Simplify the following Boolean functions:
   a. $F(A, B, C, D) = \prod(1, 3, 5, 7, 13, 15)$
   b. $F(A, B, C, D) = \prod(1, 3, 6, 9, 11, 12, 14)$
2) Give three possible ways to express the following Boolean function with eight or fewer literals:

$$F = A'BC'D + AB'CD + A'B'C' + ACD'$$

3) Simplify the following Boolean function F, together with the don't-care conditions d:
   a. $F(x, y, z) = \sum(0, 1, 4, 5, 6) + d\sum(2, 3,7)$
   b. $F(A, B, C, D) = \sum(0, 6, 8, 13, 14) + d\sum(2, 4, 10)$
   c. $F(A, B, C, D) = \sum(5, 6, 7, 12, 14, 15) + d\sum(13, 9, 11)$
   d. $F(A, B, C, D) = \sum(4, 12, 7, 2, 10) + d\sum(0, 6, 8)$

Part 2

1) Create a new project as explained in your first homework.
2) Write a VHDL code for an AND gate.
3) Add your "*.vhd" by "Add Sources", "Add or create design sources" to your project.
4) Make sure that "*.vhd" is your top module. You can change top module by right click to the module that you would like to implement and choose "Set as Top" from the list. Produce the RTL schematic of your AND gate.
5) Write a "*_tb.vhd" as a testbench and add "*_tb.vhd" to your project by "Add Sources", "Add or create simulation sources" to your project.
6) Simulate your AND gate.
7) Write a VHDL code for a NOT gate.
8) Add your "*.vhd" by "Add Sources", "Add or create design sources" to your project.
9) Make sure that "*.vhd" is your top module. Produce the RTL schematic of your NOT gate.
10) Write a "*_tb.vhd" as a testbench and add "*_tb.vhd" to your project by "Add Sources", "Add or create simulation sources" to your project.
11) Simulate your NOT gate.
12) Add your "OR_gate.vhd" by "Add Sources", "Add or create design sources" to your project.
13) You will implement Boolean function $f(x, y, z, t, w) = \sum 0,10,13,14,15,18,24,28,29 + d \sum 2,16,31$ using AND, OR and NOT gates. Simplify the given Boolean function using either Quine-McCluskey or Karnaugh map method.
14) Draw your circuit for the simplified Boolean function.
15) Write the VHDL code for the simplified Boolean function using structural modelling. Describe your AND, OR and NOT gates as sub-components. You can find some examples for structural modelling with VHDL at the following links.
https://www.doulos.com/knowhow/vhdl_designers_guide/components_and_port_maps/
http://www.csit-sun.pub.ro/courses/Masterat/Xilinx%20Synthesis%20Technology/toolbox.xilinx.com/docsan/xilinx4/data/docs/xst/vhdl7.html

16) Make sure that the VHDL code for the simplified Boolean function is your top module. Produce the RTL schematic of your design.
17) Write the test bench file to test the VHDL code for the simplified Boolean function. Simulate your design.

References

1) Frank Vahid, **Digital design, with RTL design, VHDL, and Verilog,** Hoboken, NJ : John Wiley, 2010.
2) Perry, Douglas L, **VHDL,** New York : McGraw-Hill, c1991
3) Botros, Nazeih, **HDL with digital design : VHDL and Verilog,** Dulles, Virginia : Mercury Learning and Information, [2015]
4) Vahid, Frank, V**HDL for digital design,** Hoboken, N.J. : Wiley, c2007
5) Short, Kenneth L, **VHDL for engineers, U**pper Saddle River, NJ : Pearson Prentice Hall, c2009
6) Coelho, David R., **The VHDL Handbook,** Boston, MA : Springer US, 1989
7) Lipsett, Roger., **VHDL: Hardware Description and Design,** Boston, MA : Springer US, 1989

Part 1

1) Simplify the following Boolean functions:
   a. F(A, B, C, D) = ∏(1, 3, 5, 7, 13, 15)

1: 0001   7: 0111
3: 0011   13: 1101
5: 0101   15: 1111

	A	B	C	D		
✓ 1	0	0	0	1	⊗ 1,3	
✓ 3	0	0	1	1	✓ 1,5	
✓ 5	0	1	0	1	✓ 3, 7	
✓ 7	0	1	1	1	✓ 5, 13	
✓ 13	1	1	0	1	✓ 7, 15	
✓ 15	1	1	1	1	⊗ 13, 15	

	A	B	C	D
1,3	0	0	~	1
1,5	0	-	0	1
3,7	0	-	1	1
5,13	-	1	0	1
7,15	-	1	1	1
13, 15	1	1	-	1

	A	B	C	D	
1,5,3,7	0	-	-	1	②
5,13,7,15	-	1	-	1	④

	1	3	5	7	13	15
x	x	x				
4					x	x
2	x	x	x	x		
+			x	x	x	x

We can both use X.+ or Y.2
but I choose X.+

x : 0 0 -1
+ : - 1 - 1

F (A, B, C, D) = X . T
= (A + B + D')(B' + D')

b. F(A, B, C, D) = ∏(1, 3, 6, 9, 11, 12, 14)

1: 0001   11 : 1011
3: 0011   12: 1100
6: 0110   14: 1110
9: 1001

	A	B	C	D		
✓ 1	0	0	0	1	✓ 1, 3	
✓ 3	0	0	1	1	✓ 1, 9	
✓ 6	0	1	1	0	✓ 3, 11	
✓ 9	1	0	0	1	⊗ 6, 14	
✓ 12	1	1	0 · 0		✓ 9, 11	
✓ 11	1	0	1	1	④ 12, 14	
✓ 14	1	1	1	0		

	A	B	C	D
1, 3	0	0	~	1
1, 9	-	0	0	1
3, 11	-	0	1	1
6, 14	-	1	1	0
9, 11	1	0	-	1
12, 14	1	1	-	0

	A	B	C	D	
1,3,9,11	-	0	-	1	②

	3	6	9	11	12	14
x			x			x
y					x	x
z	x	x		x	x	

No more Optimization

$x : -110$
$y : 11-0$
$z : -0-1$

$F(A, B, C, D) = xyz$

$F(A, B, C, D) = (B' + C' + D)(A' + B' + D)(B + D')$

2) Give three possible ways to express the following Boolean function with eight or fewer literals:

$$F = A'BC'D + AB'CD + A'B'C' + ACD'$$

$F = A'BC'D + AB'CD + A'B'C'D' + A'B'C'D + ABCD' + AB'CD'$

$\quad\quad\ \underset{5}{0\ 1\ 0\ 1} \quad\quad \underset{11}{1\ 0\ 1\ 1} \quad\quad \underset{0}{0\ 0\ 0\ 0} \quad\quad \underset{1}{0\ 0\ 0\ 1} \quad\quad \underset{14}{1\ 1\ 1\ 0} \quad\quad \underset{10}{1\ 0\ 1\ 0}$

	A	B	C	D		A	B	C	D	
✓0	0	0	0	0	0, 1	0	0	0	–	x
✓1	0	0	0	1	1, 5	0	–	0	1	y
✓5	0	1	0	1	10, 11	1	0	1	–	z
✓10	1	0	1	0	10, 14	1	–	1	0	t
✓11	1	0	1	1						
✓14	1	1	1	0						

	0	1	5	10	11	14
x	x	x				
y		x	x			
z				x	x	
t				x		x

$F(A, B, C, D) = x + y + z + t$

$F(A, B, C, D) = A'B'C' + A'C'D + AB'C + ACD'$

$\quad\quad\quad\quad = A'C'(B' + D) + AC(B' + D')$ 8 literals ①

3) Simplify the following Boolean function F, together with the don't-care conditions d:
   a. $F(x, y, z) = \sum(0, 1, 4, 5, 6) + d\sum(2, 3, 7)$

	x	y	z			x	y	z			x	y	z			x	y	z
v0	0	0	0		0, 1	0	0	−		0,1,2,3	0	−	−		0,1,2,3,4,5,6,7	−	−	−
v1	0	0	1		0, 2	0	−	0		0,1,4,5	−	0	−					
v2	0	1	0		0, 4	−	0	0		0,2,4,6	−	−	0					
v4	1	0	0		1,3	0	−	1		2,3,5,7	−	−	1					
v3	0	1	1		4,5	−	0	1		2,3,6,7	−	1	−					
v5	1	0	1		2,3	0	1	−		4,5,6,7	1	−	−					
v6	1	1	0		2,6	−	1	0										
v7	1	1	1		4,5	1	0	−										
					4, 6	1	−	0										
					3,7	−	1	1										
					5,7	1	−	1										
					6,7	1	1	−										

(A)