Chapter 2 – Combinational Digital Circuits

Part 3 – Additional Gates and Circuits

Overview

- Part 1 – Gate Circuits and Boolean Equations
  - Binary Logic and Gates
  - Boolean Algebra
  - Standard Forms
- Part 2 – Circuit Optimization
  - Two-Level Optimization
  - Map Manipulation
  - Practical Optimization (Espresso)
  - Multi-Level Circuit Optimization
- Part 3 – Additional Gates and Circuits
  - Other Gate Types
  - Exclusive-OR Operator and Gates
  - High-Impedance Outputs

Other Gate Types

- **Why?**
  - **Implementation feasibility and low cost**
  - **Power in implementing Boolean functions**
  - **Convenient conceptual representation**
- **Gate classifications**
  - **Primitive gate - a gate that can be described using a single primitive operation type (AND or OR) plus an optional inversion(s).**
  - **Complex gate - a gate that requires more than one primitive operation type for its description**
- **Primitive gates will be covered first**

AND, OR, NOT Operations

- **They are defined by Boolean functions.**
- **They represent 3 functions out of 16 possible two variable functions.**

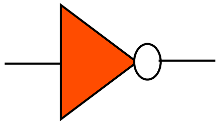
x	y	F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7
0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1

x	y	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}
0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1

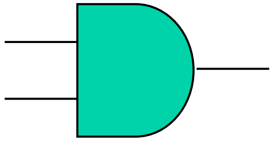
Other Logic Operations

- **Some of the two variable Boolean functions**
  - **<u>Constant functions</u>: $F_0 = 0$ and $F_{15} = 1$**
  - **<u>AND function</u>: $F_1 = xy$**
  - **<u>OR function</u>: $F_7 = x + y$**
  - **<u>ExclusiveOR function (XOR)</u>:**
    - **$F_6 = x' y + xy' = x \oplus y$ (x or y, but not both)**
  - **<u>Equivalence function (XNOR)</u>:**
    - **$F_9 = xy + x' y' = (x \oplus y)'$ (x equals to y)**
  - **<u>NOR function</u>:**
    - **$F_8 = (x + y)' = (x \downarrow y)$ (Not-OR)**
  - **<u>NAND function</u>:**
    - **$F_{14} = (x y)' = (x \uparrow y)$ (Not-AND)**

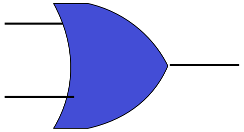
Logic Gate Symbols



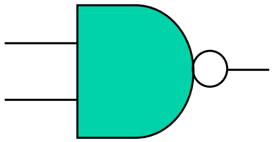
NOT



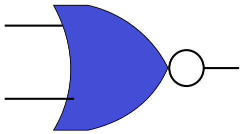
AND



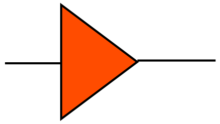
OR



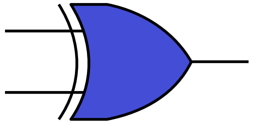
NAND



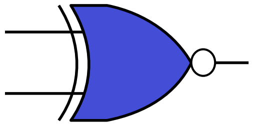
NOR



BUFFER



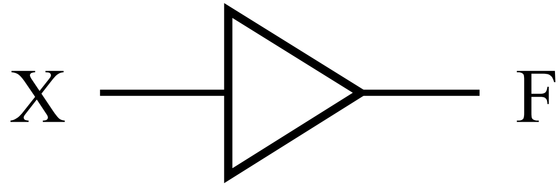
XOR



XNOR

Buffer

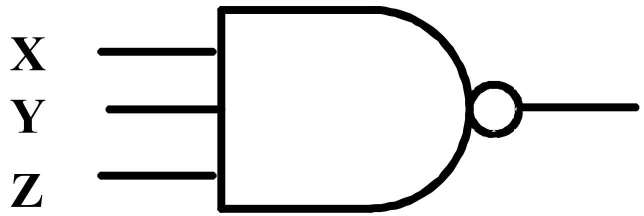
- **A buffer is a gate with the function F = X:**



- **In terms of Boolean function, a buffer is the same as a connection!**
- **So why use it?**
  - **A buffer is an electronic amplifier used to improve circuit voltage levels and increase the speed of circuit operation.**

NAND Gate

- **The basic NAND gate has the following symbol, illustrated for three inputs:**
  - **AND-Invert (NAND)**

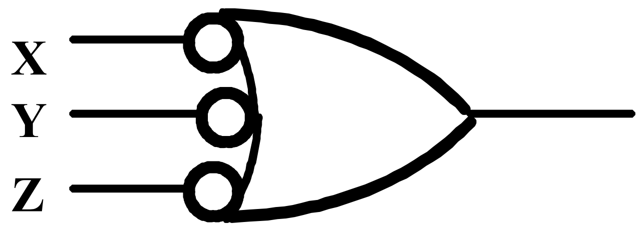


$$F(X, Y, Z) = \overline{X \cdot Y \cdot Z}$$

- **NAND represents NOT AND, i. e., the AND function with a NOT applied. The symbol shown is an AND-Invert. The small circle ("bubble") represents the invert function.**

NAND Gates (continued)

- **Applying DeMorgan's Law gives Invert-OR (NAND)**



$$F(X, Y, Z) = \overline{X} + \overline{Y} + \overline{Z}$$

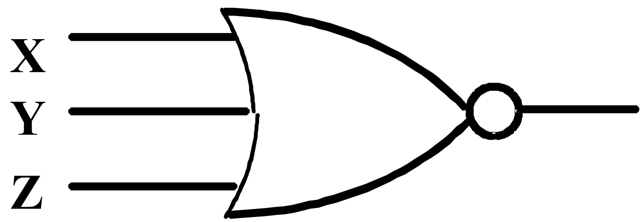
- **This NAND symbol is called Invert-OR, since inputs are inverted and then ORed together.**
- **AND-Invert and Invert-OR both represent the NAND gate. Having both makes visualization of circuit function easier.**
- **A NAND gate with one input degenerates to an inverter.**

NAND Gates (continued)

- **The NAND gate is the natural implementation for CMOS technology in terms of chip area and speed.**
- ***Universal gate*** **- a gate type that can implement any Boolean function.**
- **The NAND gate is a universal gate as shown in Figure 2-24 of the text.**
- **NAND usually does not have a operation symbol defined since**
  - **the NAND operation is not associative, and**
  - **we have difficulty dealing with non-associative mathematics!**

NOR Gate

- **The basic NOR gate has the following symbol, illustrated for three inputs:**
  - **OR-Invert (NOR)**

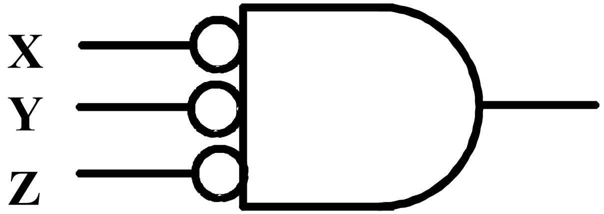


$$F(X,Y,Z) = \overline{X+Y+Z}$$

- **NOR represents <u>NOT - OR</u>, i. e., the OR function with a NOT applied. The symbol shown is an OR-Invert. The small circle ("bubble") represents the invert function.**

NOR Gate (continued)

- **Applying DeMorgan's Law gives Invert-AND (NOR)**



- **This NOR symbol is called Invert-AND, since inputs are inverted and then ANDed together.**
- **OR-Invert and Invert-AND both represent the NOR gate. Having both makes visualization of circuit function easier.**
- **A NOR gate with one input degenerates to an inverter.**

NOR Gate (continued)

- **The NOR gate is a natural implementation for some technologies other than CMOS in terms of chip area and speed.**
- **The NOR gate is a universal gate**
- **NOR usually does not have a defined operation symbol since**
  - **the NOR operation is not associative, and**
  - **we have difficulty dealing with non-associative mathematics!**

Exclusive OR/ Exclusive NOR

- **The *eXclusive OR* (*XOR*) function is an important Boolean function used extensively in logic circuits.**
- **The XOR function may be;**
  - **implemented directly as an electronic circuit (truly a gate) or**
  - **implemented by interconnecting other gate types (used as a convenient representation)**
- **The *eXclusive NOR* function is the complement of the XOR function**
- **By our definition, XOR and XNOR gates are complex gates.**

Exclusive OR/ Exclusive NOR

- **Uses for the XOR and XNORs gate include:**
  - **Adders/subtractors/multipliers**
  - **Counters/incrementers/decrementers**
  - **Parity generators/checkers**
- **Definitions**
  - **The XOR function is:** $X \oplus Y = X\,\overline{Y} + \overline{X}\,Y$
  - **The eXclusive NOR (XNOR) function, otherwise known as *equivalence* is:** $\overline{X \oplus Y} = X\,Y + \overline{X}\,\overline{Y}$
- **Strictly speaking, XOR and XNOR gates do no exist for more that two inputs. Instead, they are replaced by odd and even functions.**

Truth Tables for XOR/XNOR

- **Operator Rules: XOR** **XNOR**

X	Y	$X \oplus Y$
0	0	0
0	1	1
1	0	1
1	1	0

X	Y	$\overline{(X \oplus Y)}$ or $X \equiv Y$
0	0	1
0	1	0
1	0	0
1	1	1

- **The XOR function means:**

  X OR Y, but NOT BOTH

- **Why is the XNOR function also known as the *equivalence* function, denoted by the operator $\equiv$?**

XOR/XNOR (Continued)

- **The XOR function can be extended to 3 or more variables. For more than 2 variables, it is called an *odd function* or *modulo 2 sum* (*Mod 2 sum*), not an XOR:**

$$X \oplus Y \oplus Z = \overline{X}\,\overline{Y}\,Z + \overline{X}\,Y\,\overline{Z} + X\,\overline{Y}\,\overline{Z} + X\,Y\,Z$$

- **The complement of the odd function is the even function.**
- **The XOR identities:**

$$X \oplus 0 = X \qquad X \oplus 1 = \overline{X}$$

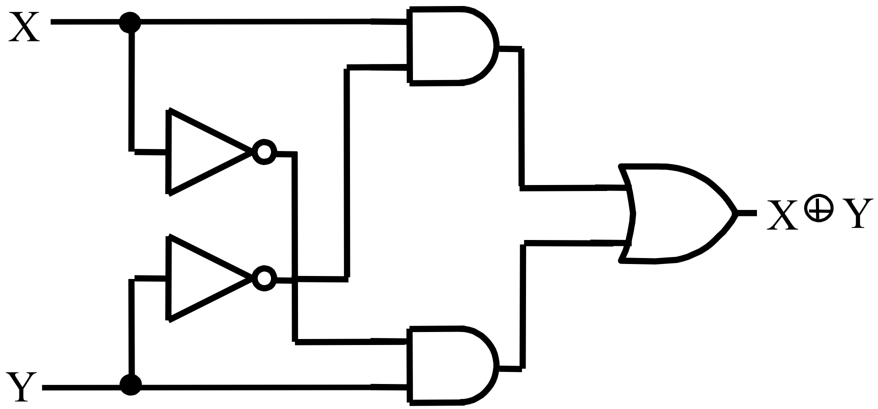
$$X \oplus X = 0 \qquad X \oplus \overline{X} = 1$$

$$X \oplus Y = Y \oplus X$$

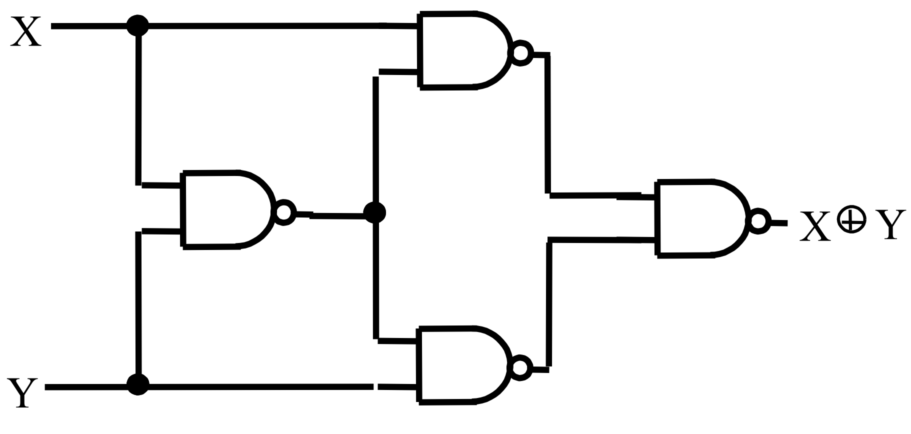
$$(X \oplus Y) \oplus Z = X \oplus (Y \oplus Z) = X \oplus Y \oplus Z$$

XOR Implementations

- **The simple SOP implementation uses the following structure:**



- **A NAND only implementation is:**

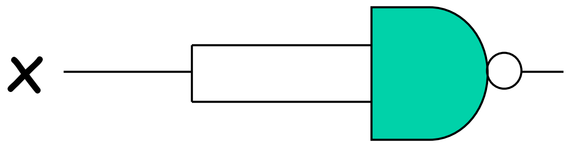


Universal Gate

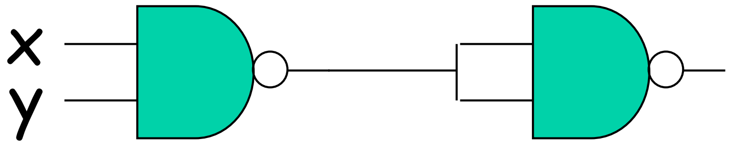
- NAND and NOR gates are universal.
- All Boolean functions can be implemented by AND, OR and NOT gates.
- These operations can be implemented by NAND and NOR gates.

x	y	(xy)′	x′	y′	(x′ y′)′
0	0	1	1	1	0
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	1

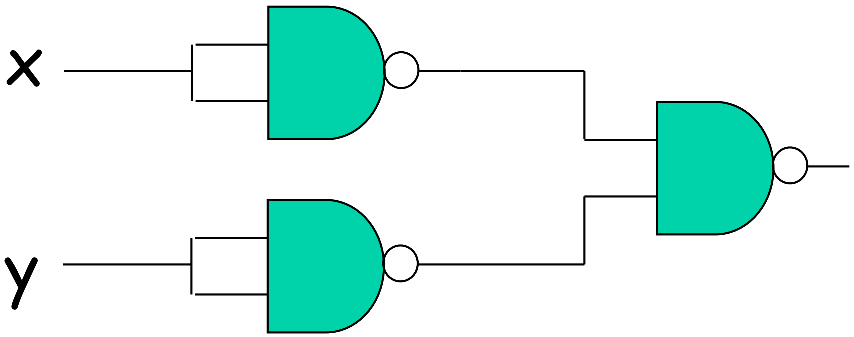
NAND Gates



$(x\,x)' = x' \rightarrow$ NOT

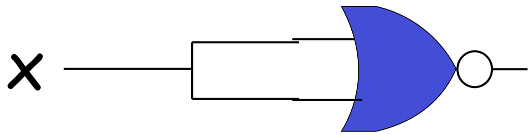


$[\,(x\,y)'\,]' = x\,y \rightarrow$ AND

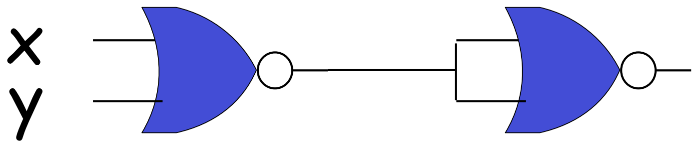


$(x'\,y'\,)' = x + y \rightarrow$ OR

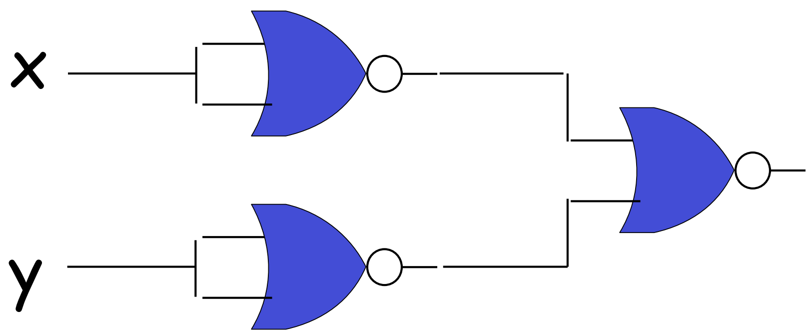
NOR Gate



$(x + x)' = x' \rightarrow$ NOT



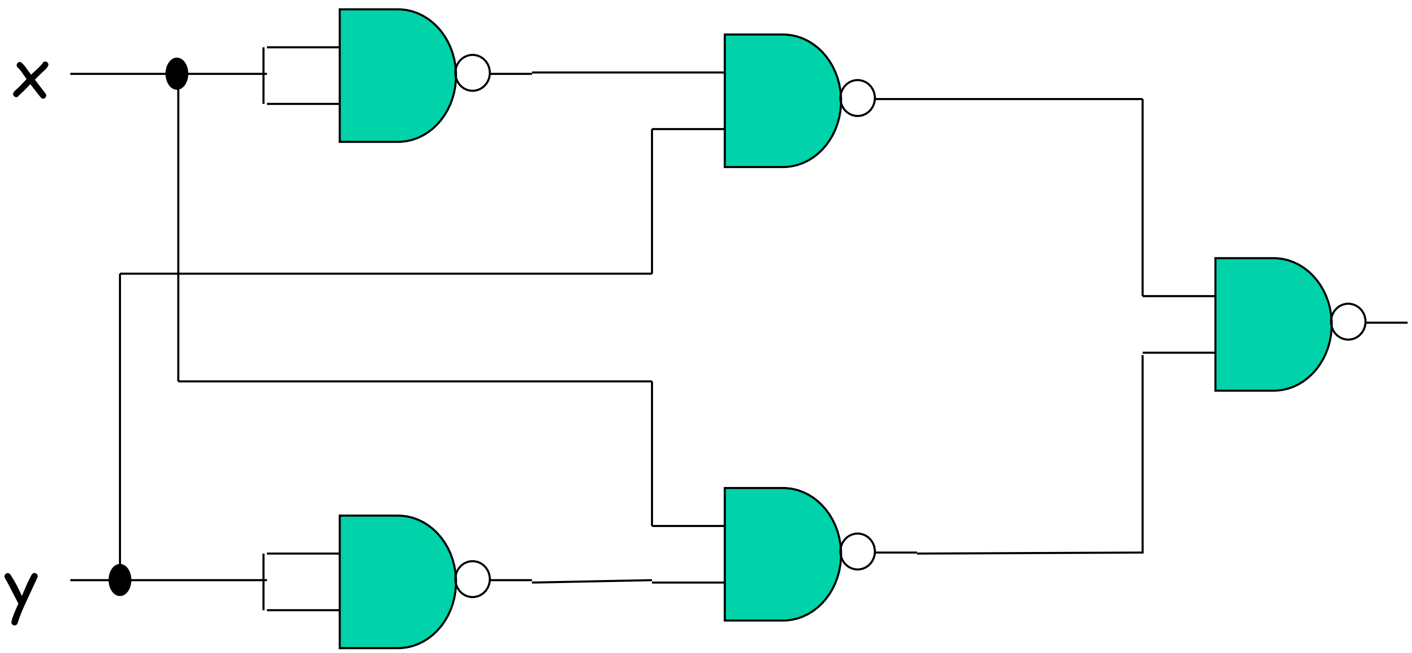
$[\,(x + y)'\,]' = x + y \rightarrow$ OR



$(x' + y'\,)' = x \cdot y \rightarrow$ AND

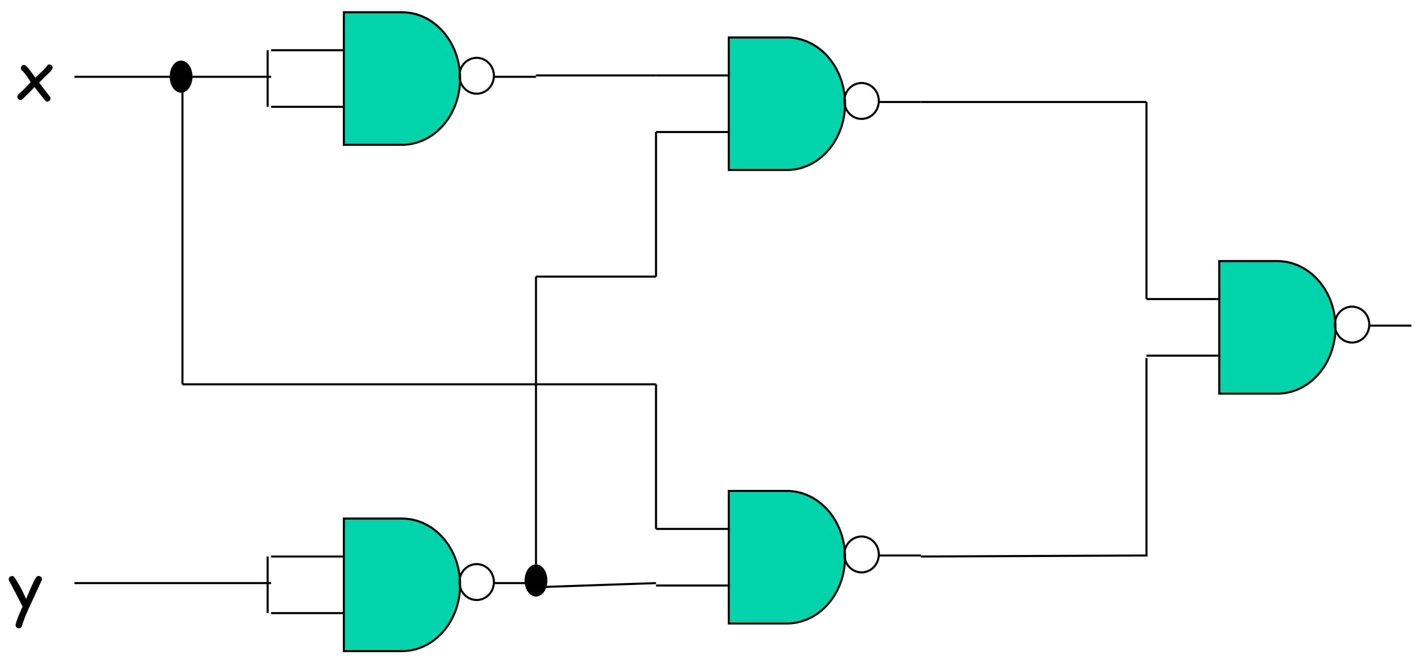
Example 1

- $F_1 = x' y + xy'$



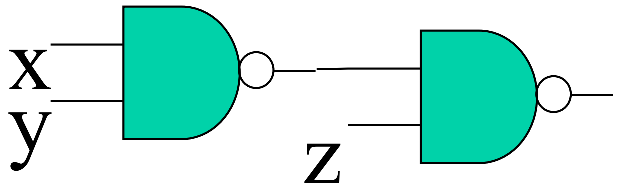
Example 2

- $F_2 = x' y' + xy'$



Multiple Input Gates

- AND and OR gates:
  - Commutative and associative properties exist.
  - There is no problem to increase the number of inputs.
- NAND and NOR gates:
  - They have commutative, but not associative property.
  - It is not easy to increase the number of inputs.
- <u>Example</u>: NAND gates
  - $((x\ y)'z)' \neq (x(yz)')'$
  - $((xy)'z)'$ $= ((x'+y')z)' = xy+z'$



  - $(x\ (yz)')' = x'+yz$