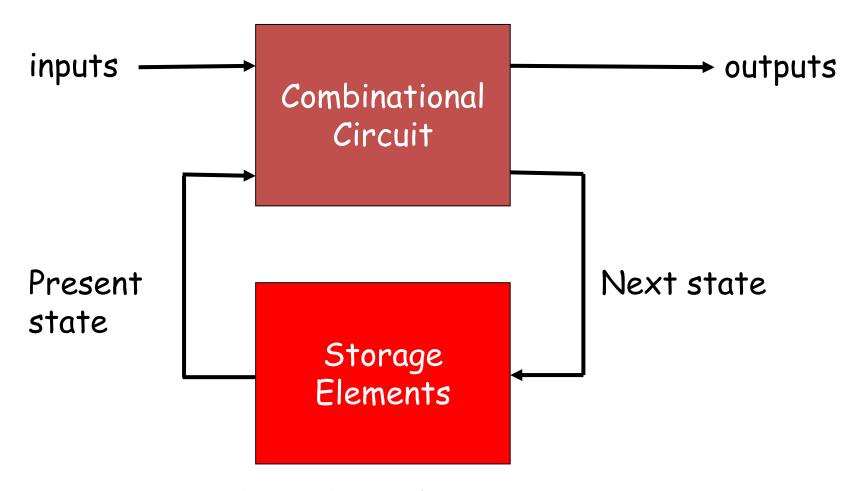
Sequential Circuit Model



Present state depends on the previous inputs

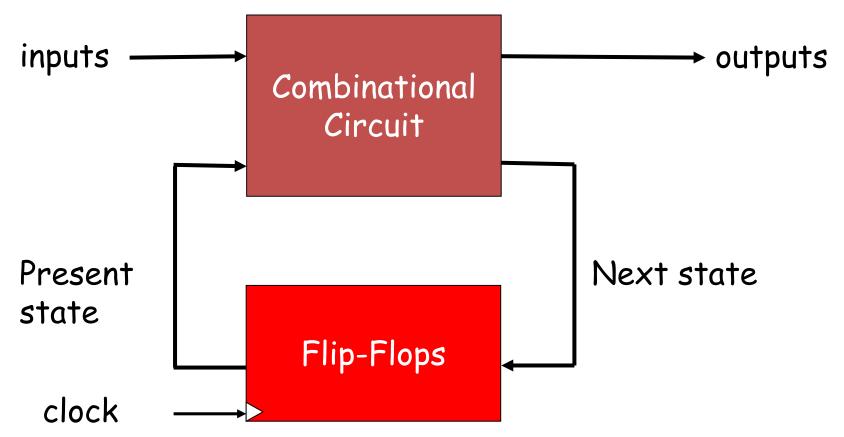
Synchronous Sequential Circuits

- Behavior defined from knowledge of its signals at discrete instances of time.
- Discrete instances of time need synchronization.
- Synchronization is done by a common clock signal.
- Clock signal is a periodic square signal.
- Storage elements observe inputs and can change state only in relation to a timing signal (clock pulses from a clock)



Synchronous Sequential Circuits

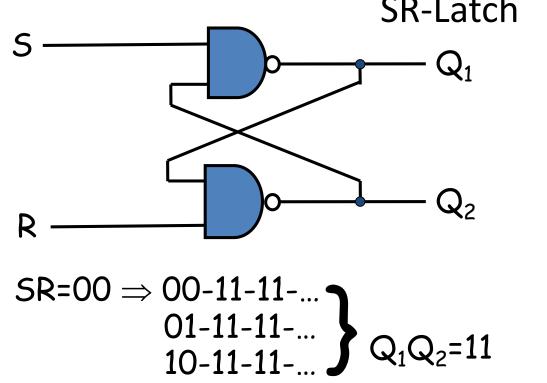
 The storage elements are flip-flops that can store one bit of information.



Latch

- Basic storage element
- A latch is a storage element that can store its content forever.
- Latches are asynchronous circuits and do not need a clock signal to operate.
- Hence they can not be used in synchronous circuits directly.
- They are used to construct flip-flops.

SR-Latch



$$\begin{array}{c}
10-11-11-... \quad J \quad Q_1Q_2-11 \\
SR=01 \Rightarrow 00-11-10-10-... \quad Q_1Q_2=10
\end{array}$$

10-10-...

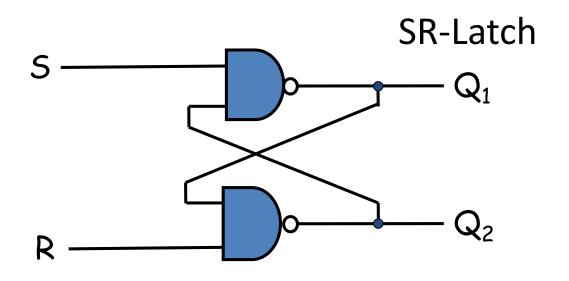
$$\begin{array}{c} \text{SR=10} \Rightarrow \text{00-11-01-01-...} \\ \text{10-11-01-01-...} \end{array} \hspace{-0.5cm} Q_1 Q_2 \text{=01}$$

$$SR=11 \Rightarrow 00-11-00-11-...$$
 Q₁Q₂=osilates 01-01-...

$$Q_1 = (S Q_2)' = S' + q_2'$$

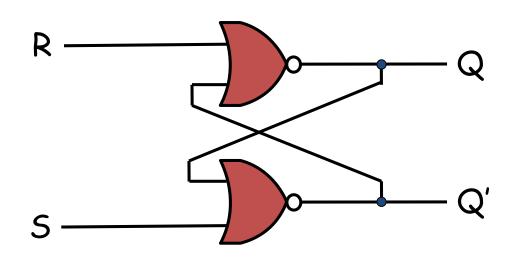
 $Q_2 = (R Q_1)' = R' + q_1'$

S	R	q_1	q_2	Q_1	Q_2	
0	0	0	0	1	1	
0	0	0	1	1	1	
0	0	1	0	1	1	
0	0	1	1	1	1	2
0	1	0	0	1	1	
0	1	0	1	1	1	
0	1	1	0	1	0	7
0	1	1	1	1	0	
1	0	0	0	1	1	
1	0	0	1	0	1	7
1	0	1	0	1	1	N
1	0	1	1	0	1	4
1	1	0	0	1	1	3
1	1	0	1	0	1	
1	1	1	0	1	0	
1	1	1	1	0	0	4



 S	R	Q_1	Q_2	
0	0	×	×	Undefined
 0	1	1	0	
1	0	0	1	Q ₁ = Q ₂ '
 1	1	q_1	q_2	
				- Q ₁ = Q Q ₂ = Q '

SR-Latch



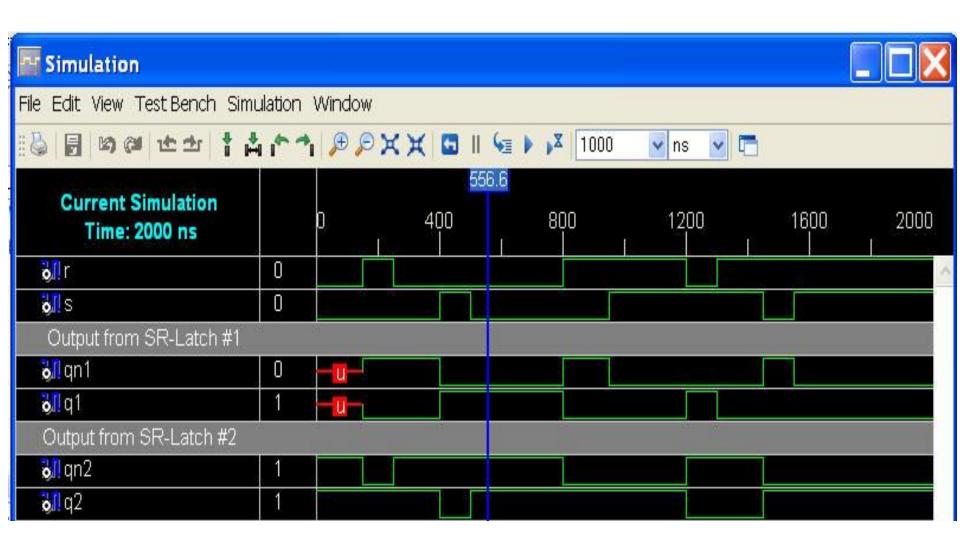
$$Q = (R + Q')' = R' q$$

 $Q' = (S + Q)' = S' q'$

5	R	Q	Q'	_
0	0	q	q'	_
0	1	0	1	
1	0	1	0	
1	1	X	X	_ (

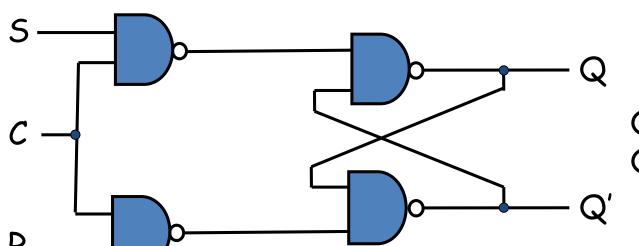
Undefined

Simulation of SR-Latch



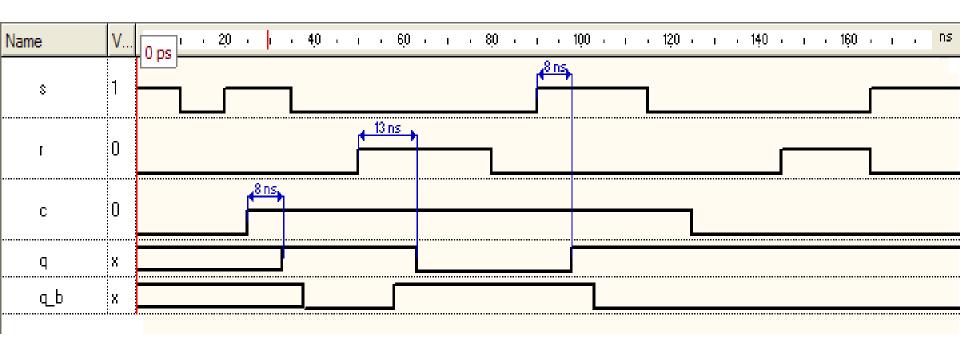
SR-Latch with Control Input





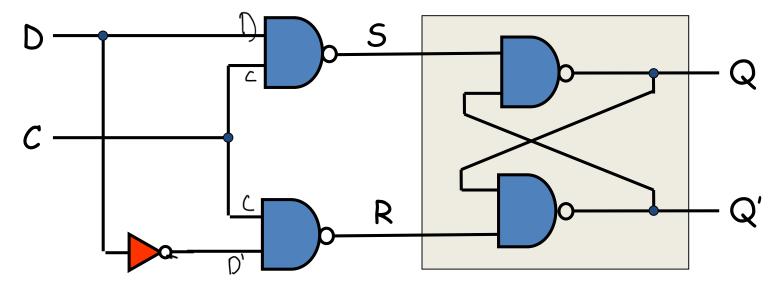
C	5	R	Q &			
0	X	X	No change			
1	0	0	No change No change			
1	0	1	Q = 0 Reset state			
1	1	0	Q = 1 Set state			
1	1	1	Undefined			

Simulation of SR-Latch with Control Input



D-Latch

- Because the undefined situation can cause stability problems, SR latches are not used offen.
- Solution: D-latch

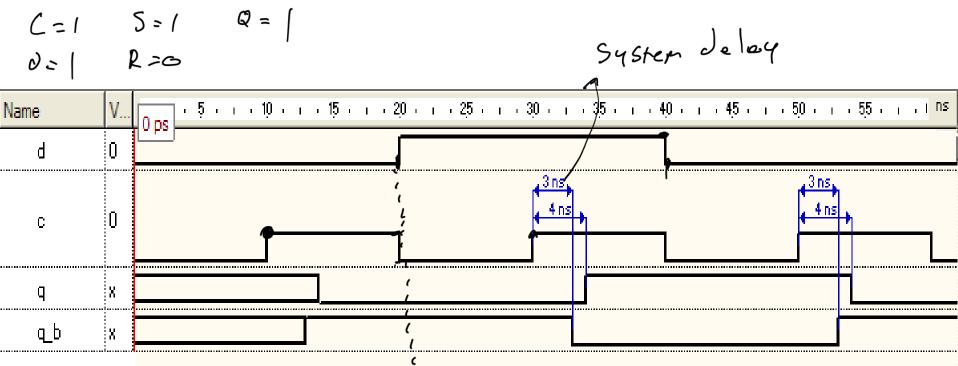


This circuit garanties that S and R inputs are always each other's complement.

Simulation of D-Latch

$$0 = 0 \longrightarrow 5 = 0$$

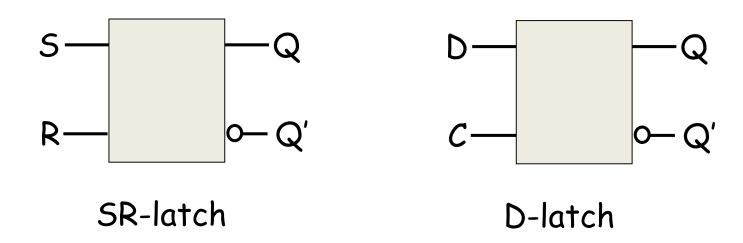
$$R = 1$$



D-Latch

C	D	Next state of Q
0	X	No change
1	0	Q = 0; reset state
1	1	Q = 1; set state

• D input is sampled when C=1.

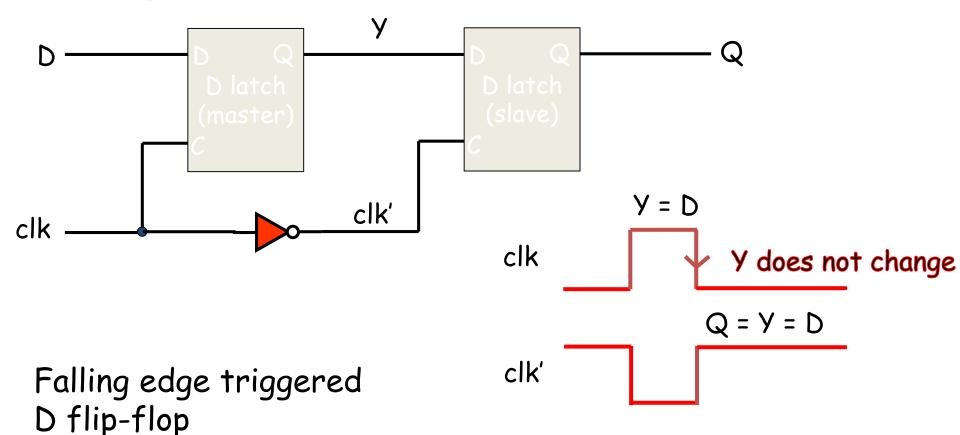


D Latch as a Storage Element

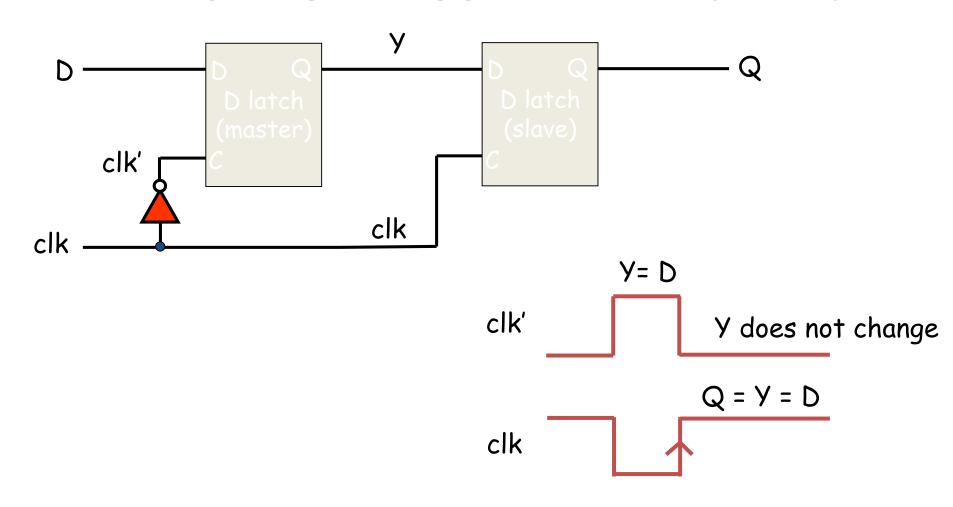
- When C = 1 D latch copies the input to the output.
- When C = 0 the information is kept unchanged.
- Latches are called level triggered.
 - While C is in logic-1 level, the changes at the input cause changes at the output.
- The states of the storage elements should change synchronously.
- We need a storage element which changes the state in a very short time spot.
- These storage elements are called edge triggered and specially flip-flops.

Edge Triggered D Flip-Flop

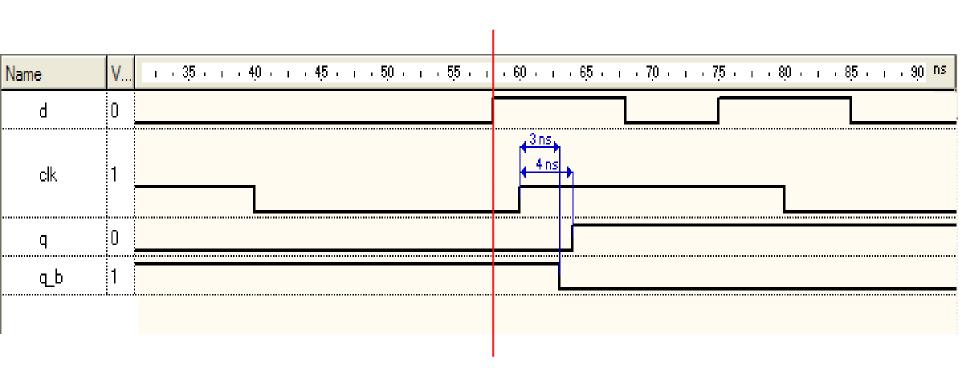
 Edge triggered D flip-flop can be constructed by using two D latches.



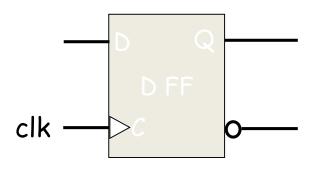
Rising Edge Triggered D Flip-Flop



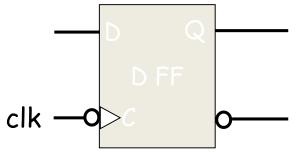
Simulation of Rising Edge Triggered D $\lim_{\beta \neq 0} \Rightarrow \sup_{\alpha \neq 1} Flip-Flop$



D Flip-Flop Symbols



Rising edge triggered D Flip-Flop

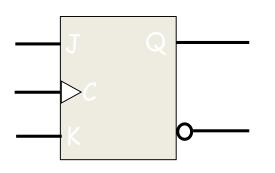


Falling edge triggered D Flip-Flop

- Characteristic Equation
 - Q(t+1) = D

JK Flip-Flop

K



J	K	Q(†+1)	Next State
0	0	Q(†)	No change
0	1	0	Reset
1	0	1	Set
1	1	Q'(†)	Complement

• Characteristic Equation ⁰

$$-Q(t+1) = JQ'(t) + K'Q(t)$$

- Y = Jy' + K'y

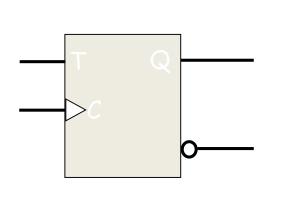
		•	J
0	1	0	Reset
1	0	1	Set
1	1	y'	Complement

Characteristic Table

Next State

No change

T (Toggle) Flip-Flop



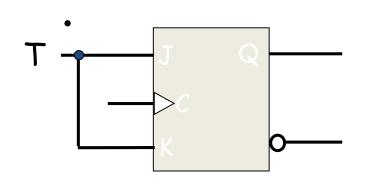
Τ	Q(†+1)	next state
0	Q(†)	no change
1	Q'(†)	Complement
T	У	next state
0	У	no change
1	у'	Complement

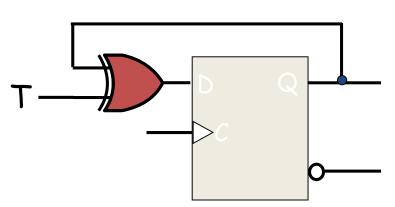
Characteristic Equation

Characteristic Table

•
$$Q(t+1) = T \oplus Q(t) = TQ'(t) + T'Q(t)$$

•
$$Y = T \oplus y = Ty' + T'y$$





Analysis of Synchronous Sequential Circuits

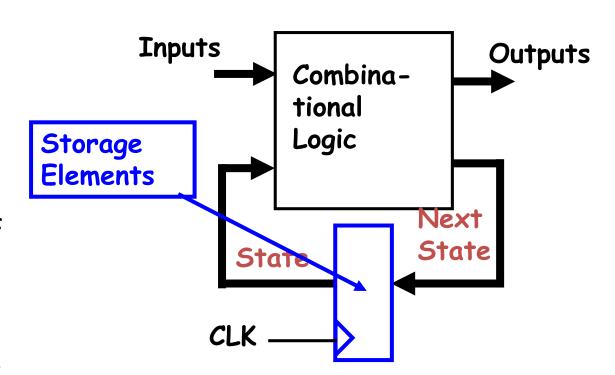
• Aim:

- Finding the behaviour of the synchronous sequential circuits
- "Behaviour"
 - Inputs
 - Outputs
 - States of the flip-flops
- Finding the Boolean functions of the outputs and the inputs of the flip-flops
 - Output and state equations
 - state table
 - state diagram

Analysis of Synchronous Sequential

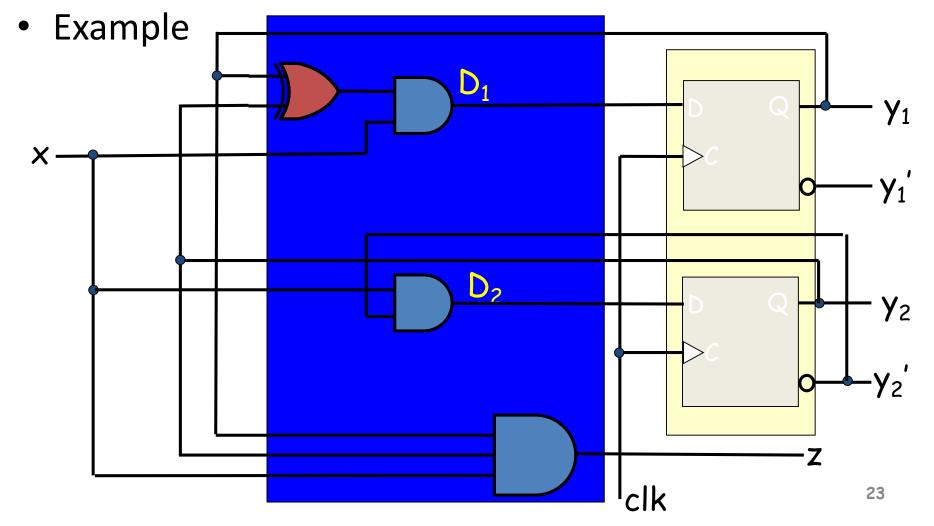
- Current State at time t is stored in an array of flipflops.
- Next State at time t+1 is a Boolean function of Current State and Inputs.
- Outputs at time t are a Boolean function of Current State and sometimes Inputs.

Circuits



State and Output Equations

- They are also called transition equations.
 - They show the next state as a function of the present state and the inputs.

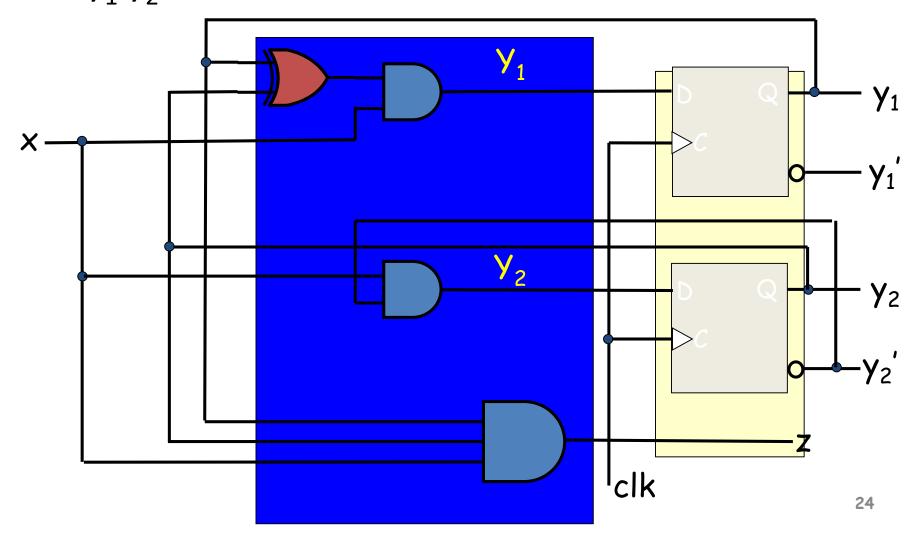


State and Output Equations

•
$$D_1 = (y_1 \oplus y_2) x = Y_1$$

•
$$D_2 = x y_2' = Y_2$$

•
$$z = y_1 y_2 x$$



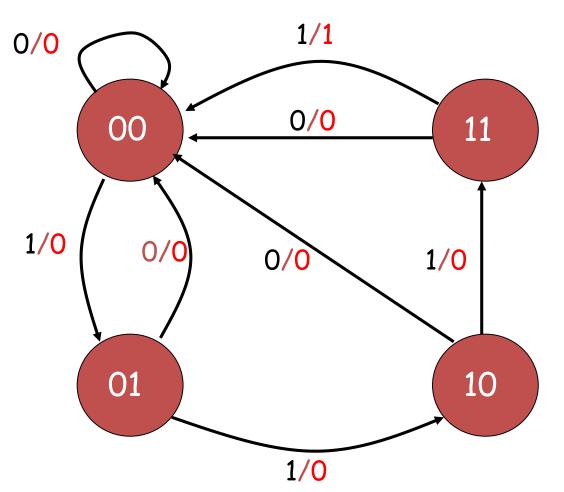
Example: State (Transition) Table

$$z = ?$$

Presen	t State	Input	Next	State	Output
y ₁	y ₂	×	Y_1	y_2	z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	1	1	0
1	1	0	0	0	0
1	1	1	0	0	1

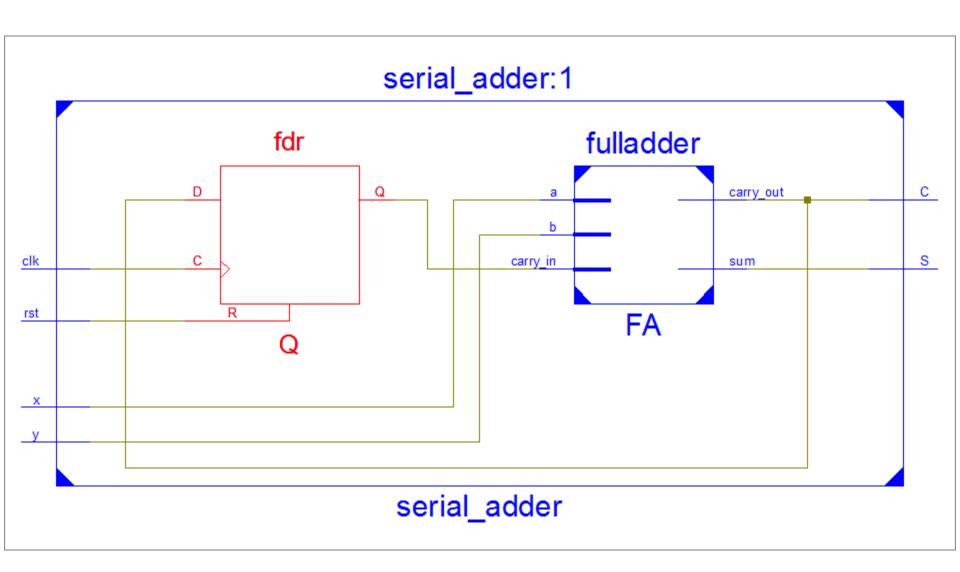
There are 2^{m+n} rows in the state table of a synchronous sequential circuit with m FFs and n inputs.

Example: State (Transition) Diagram



Currei State	nt	Input	Next State		Output
y ₁	y ₂	Х	Y ₁	Y ₂	Z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	1	1	0
1	1	0	0	0	0
1	1	1	0	0	1

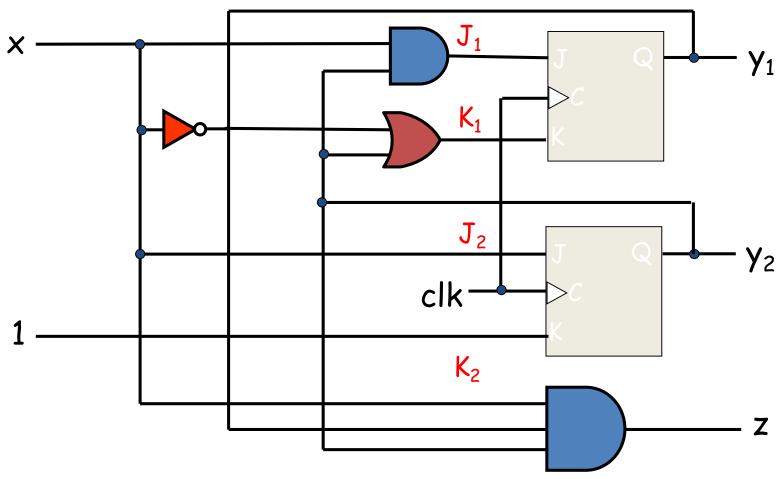
The state diagram and table give the same information



Analysis of a Synchronous Sequential Circuit with JK Flip-Flops

- For a D flip-flop, the state equation is the same as the flip-flop input equation
 - Q(t+1) = D
- For JK flip-flops, situation is different
 - Goal is to find state equations
 - Method
 - 1. determine flip-flop input equations
 - 2. List the binary values of each input equation
 - 3. Use the corresponding flip-flop characteristic table to determine the next state values in the state table

Example: Analysis with JK FFs



Flip-flop input equations

$$-J_1 = xy_2$$

and
$$K_1 = x' + y_2$$

$$-J_2 = x$$

and
$$K_2 = 1$$

Example: Analysis with JK FFs

$$- J_1 = xy_2$$
 and $K_1 = x' + y_2$
 $- J_2 = x$ and $K_2 = 1$

presen	t State	input	next	state		FF ir	puts	
y ₁	y ₂	X	y ₁	y_2	J_1	K_1	J_2	K_2
0	0	0	0	0	0	1	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	0	0	1	0	1
0	1	1	1	0	1	1	1	1
1	0	0	0	0	0	1	0	1
1	0	1	1	1	0	0	1	1
1	1	0	0	0	0	1	0	1
1	1	1	0	0	1	1	1	1

Example: Analysis with JK FFs

$$Q(t+1) = JQ'(t) + L'Q(t)$$

Characteristic equations

$$- Y_1 = J_1 y_1' + K_1' y_1$$

 $- Y_2 = J_2 y_2' + K_2' y_2$

Flip-flop Input equations

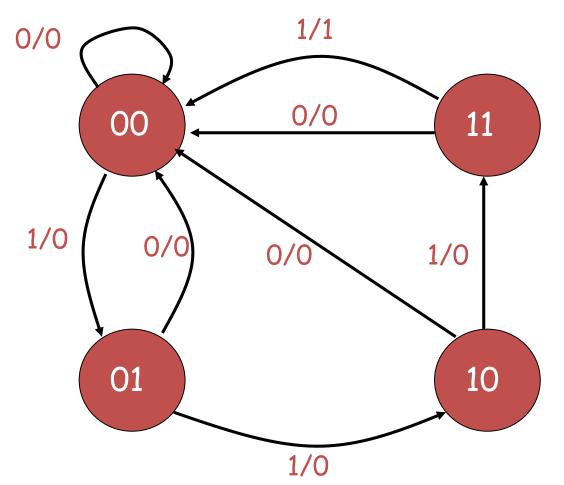
$$-J_1 = xy_2$$
 ve $K_1 = x' + y_2$
 $-J_2 = x$ ve $K_2 = 1$

State equations

$$- Y_1 = xy_2y_1' + (x' + y_2)'y_1 = xy_2y_1' + xy_2'y_1 = x(y_2 \oplus y_1)$$

$$- Y_2 = xy_2' + 1'y_2 = xy_2'$$

State Diagram



Prese state		Input	Next State		Output
y ₁	y ₂	X	Y ₁	Y ₂	Z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	1	1	0
1	1	0	0	0	0
1	1	1	0	0	1

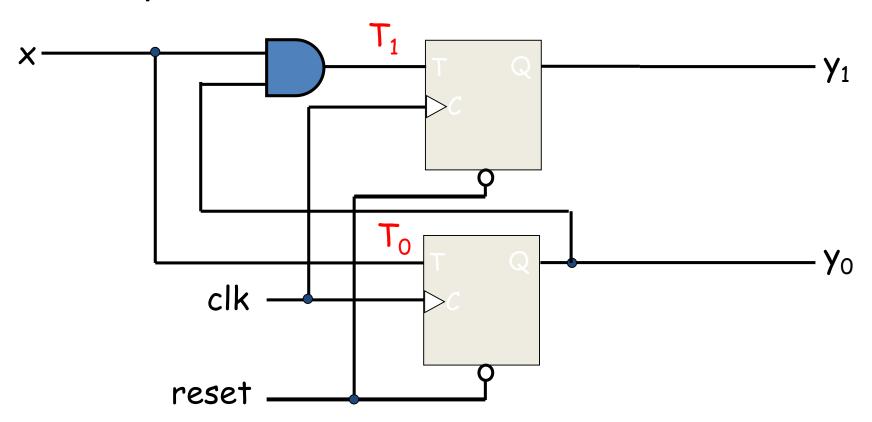
What is the circuit doing?

Analysis with T Flip-Flops

- Method is the same
- Example

$$T_1 = xy_0$$

 $T_0 = x$



Example: Analysis with T Flip-Flops

Characteristic equation

$$- Y_0 = T_0 \oplus y_0$$
$$- Y_1 = T_1 \oplus y_1$$

Flip-flop Input equations

$$-T_1 = x y_0$$
$$-T_0 = x$$

State equations

$$- Y_0 = x \oplus y_0$$
$$- Y_1 = x y_0 \oplus y_1$$

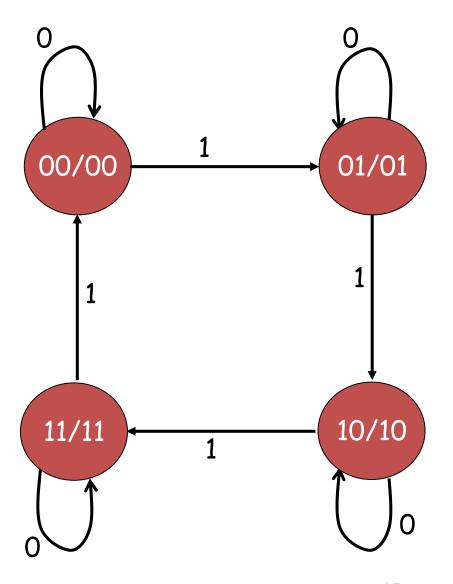
State Table & Diagram

•
$$Y_0 = x \oplus y_0$$

• $Y_1 = x y_0 \oplus y_1$

•
$$\mathbf{Y}_1 = \mathbf{x} \mathbf{y}_0 \oplus \mathbf{y}_1$$

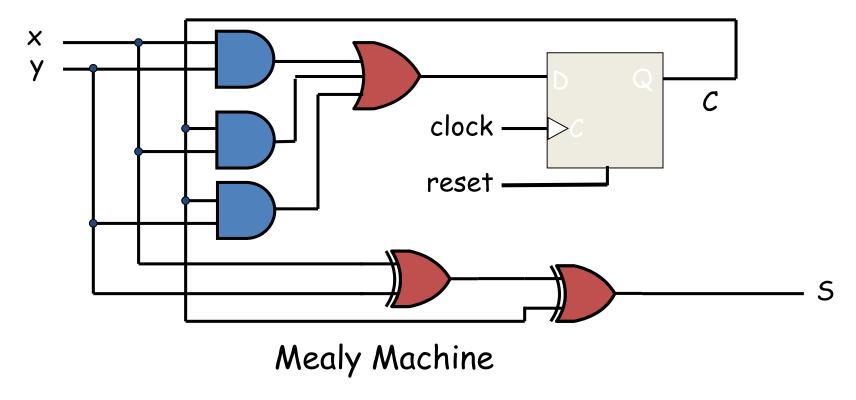
Present State		Input	Next State		Output	
y ₁	y 0	×	y_1	Y ₀	y ₁	y 0
0	0	0	0	0	0	0
0	0	1	0	1	0	0
0	1	0	0	1	0	1
0	1	1	1	0	0	1
1	0	0	1	0	1	0
1	0	1	1	1	1	0
1	1	0	1	1	1	1
1	1	1	0	0	1	1



Mealy and Moore Models

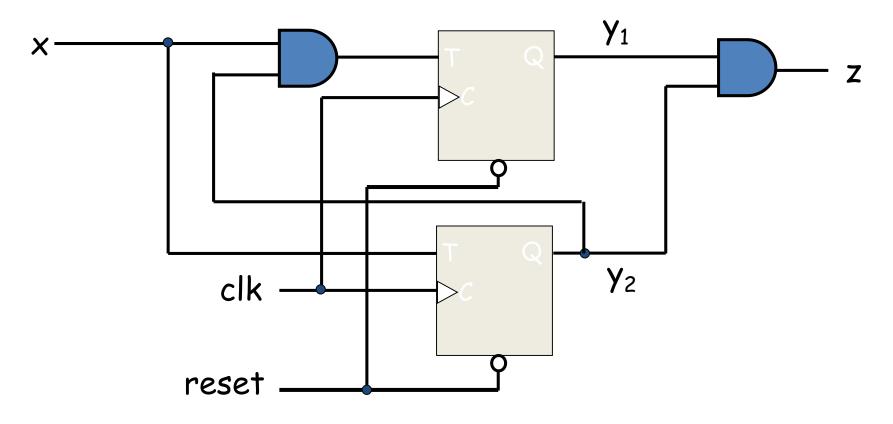
- There are two models for sequential circuits
 - Mealy
 - Moore
- They differ in the way the outputs are generated
 - Mealy:
 - output is a function of both present states and inputs
 - Moore
 - output is a function of present state only

Example: Mealy and Moore Machines



- External inputs, x and y, are asynchronous
- · Thus, outputs may have momentary (incorrect) values
- · Inputs must be synchronized with clocks
- Outputs must be sampled only during clock edges.

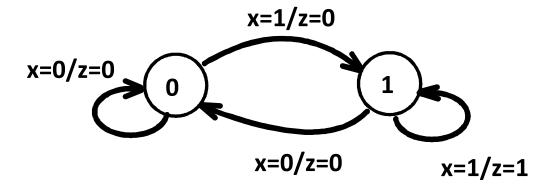
Example: Moore Machines



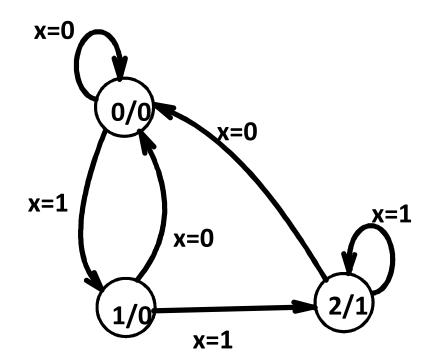
- Outputs are already synchronized with clock.
- They change synchronously with the clock edge.

Example State Diagrams for Moore and Mealy Machines

State Diagram for Mealy Model



• State Diagram for Moore Model



Design Process

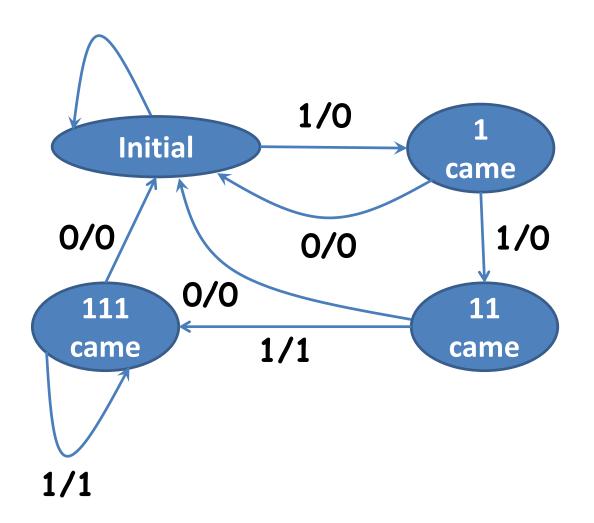
- 1. Verbal description of desired operation
- 2. Draw the state diagram
- Reduce the number of states if necessary and possible: s = number of states
- 4. Determine the number of flip-flops: $n = |\log_2 s|$
- 5. State assignment: $\underbrace{00...0}_{n-bits},\underbrace{00...1}_{n-bits},\underbrace{00...1}_{n-bits},\underbrace{00...1}_{n-bits},\underbrace{00...1}_{n-bits}$
- 6. Obtaine the encoded state table
- 7. Choose the type of the flip-flops
- 8. Derive the simplified flip-flop input equations
- 9. Derive the simplified output equations
- 10. Draw the logic diagram

Example: Design of a Synchronous Sequential Circuit

- Verbal description
 - 1st Step: we want a circuit that detects three or more consecutive 1's in a string of bits.
 - <u>Input</u>: string of bits of any length
 - Output:
 - "1" if the circuit detects such a pattern in the string
 - "0" otherwise

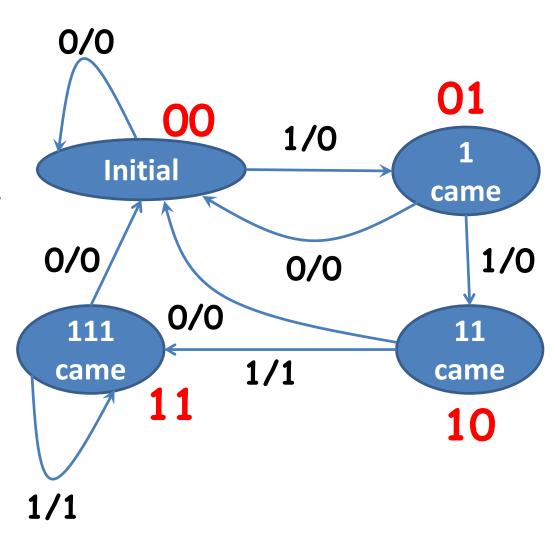
Example: State Diagram

2nd Step: Draw the state diagram 0/0



Synthesis with D Flip-Flops 1/5

- 3rd Step: State reduction
 - Not possible
- 4th Step: Number of flip-flops
 - 4 states
 - -? flip-flop
- 5th Step: State assignment



Synthesis with D Flip-Flops 2/5

• 6th Step: Obtain the state table

Presen	Present State		Next	State	Output
y ₁	y ₂	×	У ₁	y_2	Z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	1	1	øj
1	1	0	0	0	10
1	1	1	1	1	1

Synthesis with D Flip-Flops 3/5

- 7th Step: Choose the type of the flip-flops
 - D type flip-flops
- 8th Step: : Derive the simplified flip-flop input equations
 - Boolean expressions for D₁ and D₂

$\chi_2 x$				
y_1	00	01	11	10
0	0	0	1	0
1	0	1	1	0

$$D_1 = y_1 x + y_2 x$$

Y ₂ X				
y_1	00	01	11	10
0	0	1	0	0
1	0	1	1	0

$$D_2 = y_1 x + y_2' x$$

Synthesis with D Flip-Flops 4/5

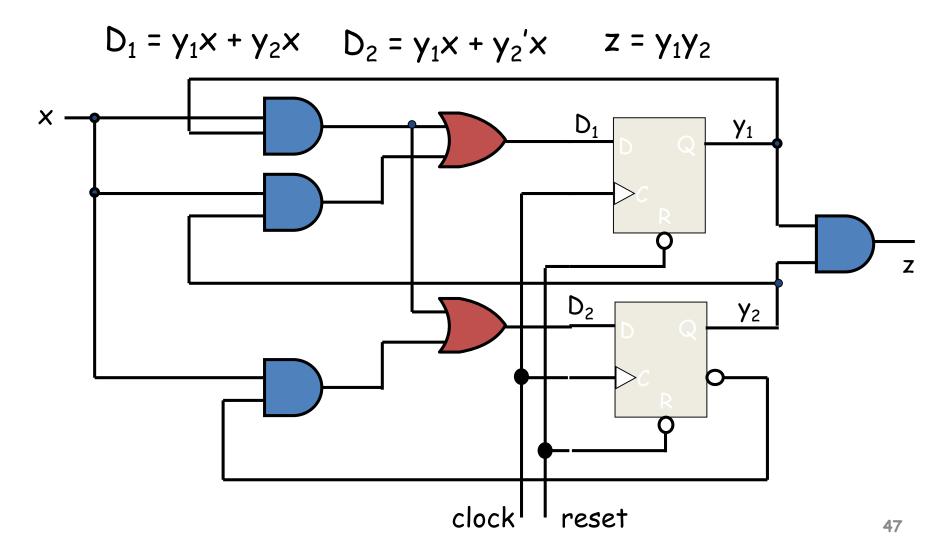
- 9th Step: : Derive the simplified output equations
 - Boolean expressions for z

$\chi_2 X$				
y ₁	00	01	11	10
0	0	0	0	0
1	0	0	1	1

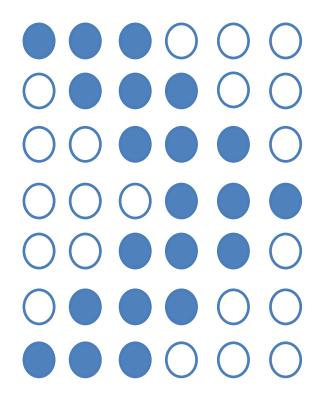
$$z = y_1y_2$$

Synthesis with D Flip-Flops 5/5

• 10th Step: Draw the logic diagram



Synthesis with JK Flip-Flops and MUXs



Number of states = 6

Number of state variables= 3

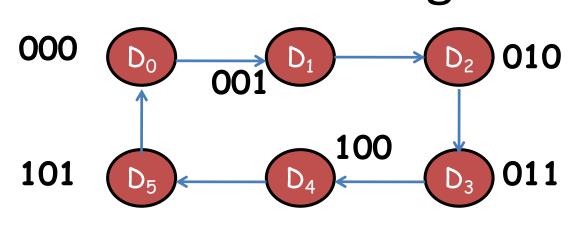
Number of flip-flops= 3

Number of Inputs = 0

Number of Outputs= 6

- •6 shifting lights
- ••= lojik-1
- •O= lojik-0

State Diagram & Table

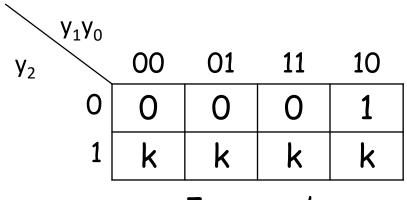


$$Y = Jy' + K'y$$

_		
J	K	У
0	0	У
0	1	0
1	0	1
1	1	Q'

Present State	Next State	Flip-flop inputs Outp	outs
Y ₂ Y ₁ Y ₀	$y_2 y_1 y_0$	$J_2 K_2 J_1 K_1 J_0 K_0 z_5 z_4 z_3$	$\mathbf{z}_2 \ \mathbf{z}_1 \ \mathbf{z}_0$
0 0 0	0 0 1	0 k 0 k 1 k 1 1 1	0 0 0
0 0 1	0 1 0	0 k 1 k k 1 0 1 1	100
0 1 0	0 1 1	0 k k 0 1 k 0 0 1	1 1 0
0 1 1	1 0 0	1 k k 1 k 1 0 0 0	1 1 1
1 0 0	1 0 1	k 0 0 k 1 k 0 0 1	1 1 0
1 0 1	0 0 0	k 1 0 k k 1 0 1 1	1 0 0

Inplementation of Flip-Flop Input Equations

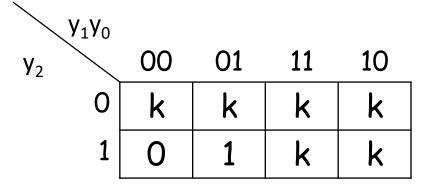


$y_1 y_0$	$J_2 = y_1 y_0'$							
	00	01	11	10				
0	0	1	k	k				
1	0	0	k	k				

 y_2

y_1y_0		$J_1 = \gamma$	/2 Y 0	
y ₂	00	01	11	10
0	1	k	k	1
1	1	k	k	k

$$J_0 = 1$$

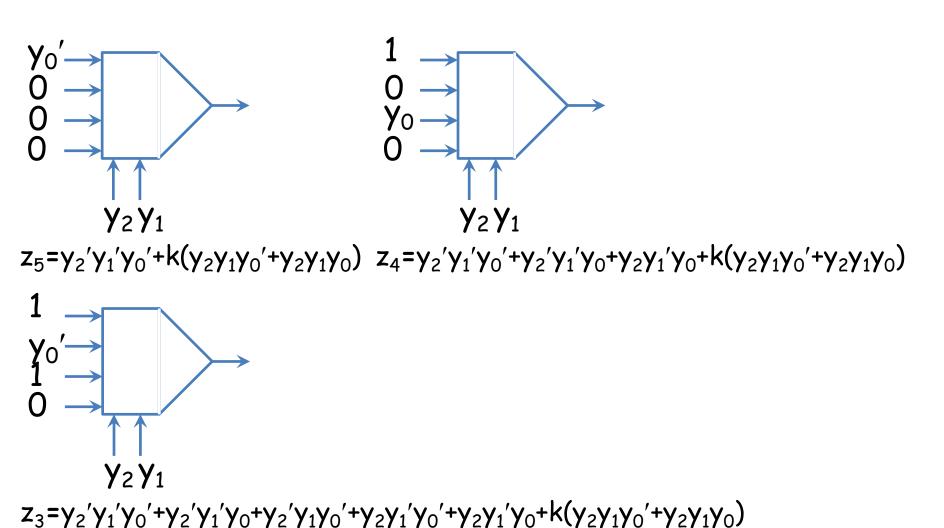


y_1y_0		K ₂ =	y 0	
y ₂	00	01	11	10
0	k	k	1	0
1	k	k	k	k

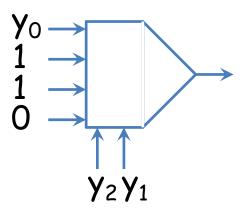
y_1y_0		$K_1 =$	y ₀	
y ₂	00	01	11	10
Ο	k	1	1	k
1	k	1	k	k

$$K_1 = 1$$

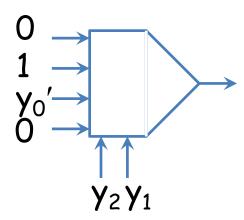
Inplementation of Output Equations



Inplementation of Output Equations

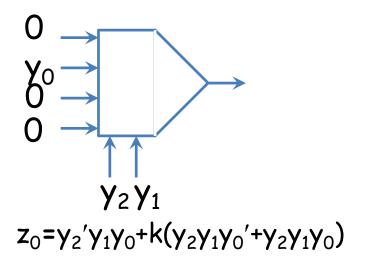


 $z_2 = y_2' y_1' y_0 + y_2' y_1 y_0' + y_2' y_1 y_0 + y_2 y_1' y_0' + y_2 y_1' y_0 + k(y_2 y_1 y_0' + y_2 y_1 y_0)$



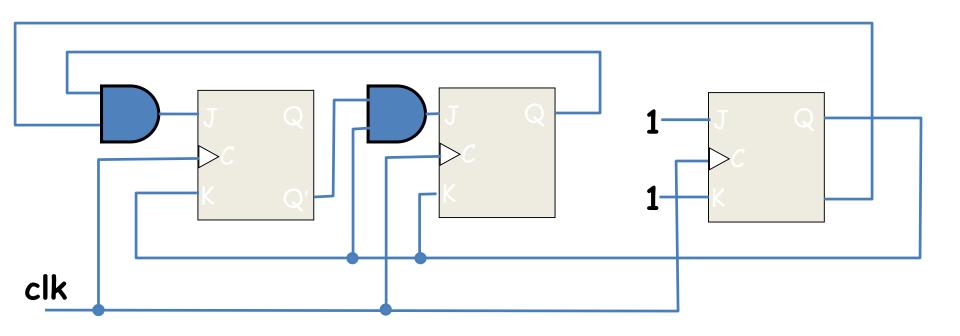
 $z_1 = y_2' y_1 y_0' + y_2' y_1 y_0 + y_2 y_1' y_0' + k(y_2 y_1 y_0' + y_2 y_1 y_0)$

Inplementation of Output Equations



Logic Diagram

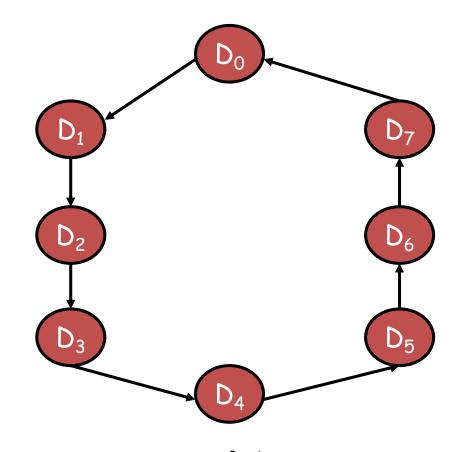
$$J_2 = y_1 y_0' K_2 = y_0 J_1 = y_2' y_0 K_1 = y_0 J_0 = 1 K_1 = 1$$



Synthesis with T Flip-Flops 1/4

• Example: 3-bit binary counter

$$0 \rightarrow 1 \rightarrow 2 \rightarrow ... \rightarrow 7 \rightarrow 0 \rightarrow 1 \rightarrow 2$$



State Diagram

How many flip-flops?

State assignments

- $D_0 \to 000$
- $D_1 \rightarrow 001$
- $D_2 \rightarrow 010$
- . . .
- $D_7 \rightarrow 111$

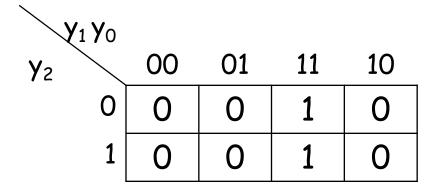
Synthesis with T Flip-Flops 2/4

• State Table

pre	sent st	tate	ne	xt sta	te	F	F input	ts
y ₂	y ₁	Yo	y ₂	\mathbf{y}_{1}	\mathbf{y}_{0}	T_2	T_1	T_0
0	0	0	0	0	1	0	0	1
0	0	1	0	1	0	0	1	1
0	1	0	0	1	1	0	0	1
0	1	1	1	0	0	1	1	1
1	0	0	1	0	1	0	0	1
1	0	1	1	1	0	0	1	1
1	1	0	1	1	1	0	0	1
1	1	1	0	0	0	1	1	1 5

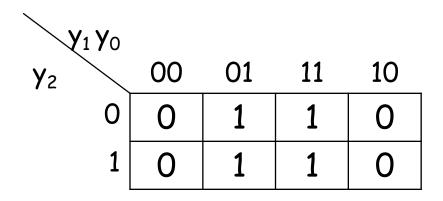
Synthesis with T Flip-Flops 3/4

Flip-Flop input equations



$$T_2 = y_1 y_0$$

$$T_0 = 1$$

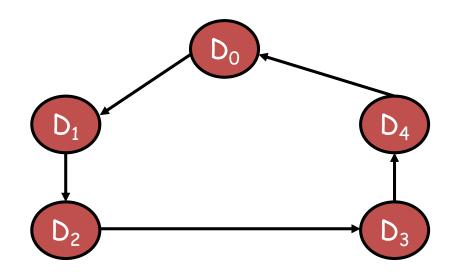


$$T_1 = y_0$$

Synthesis with T Flip-Flops 4/4

• Circuit lojik-1 T_0 **y**₀ $T_2 = y_1 y_0$ $T_1 = y_0$ $T_0 = 1$ **y**₂ clock · reset

Unused States



Modulo-5 counter

Present State			Next State		
y ₂	y_1	y ₀	Y ₂	Y_1	Y_0
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	0	0	0

Example: Unused States 1/4

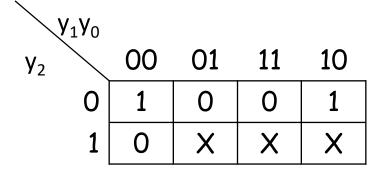
Present State			Next State		
y ₂	У ₁	y_0	Y ₂	Y_1	Y_0
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	0	0	0

y_1y_0	00	01	11	10
0	0	0	1	0
1	0	X	X	X

$$Y_2 = y_1 y_0$$

y_1y_0				
y ₂	00	01	11	10
0	0	1	0	1
1	0	X	X	X

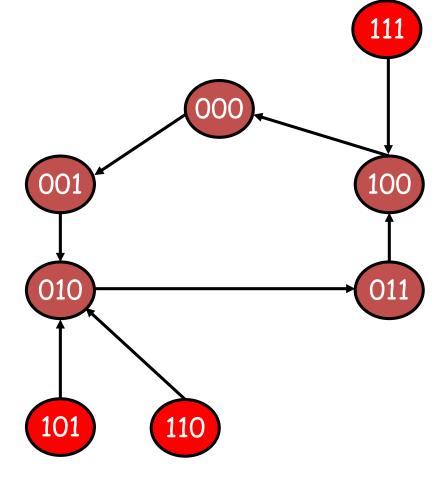
$$y_1 = y_1' y_0 + y_1 y_0'$$
$$= y_1 \oplus y_0$$



$$y_0 = y_2' y_0'$$

Example: Unused States 2/4

Present State			Next State		
у ₂	У ₁	y_0	Y ₂	Y ₁	Y ₀
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	0	0	0
1	0	1	0	1	0
1	1	0	0	1	0
1	1	1	1	0	0



$$Y_2 = y_1 y_0$$

$$Y_1 = y_1 \oplus y_0$$

$$Y_0 = y_2' y_0'$$

The circuit is not locked type.

Example: Unused States 3/4

Not using don't care conditions

Present State			Next State		
Α	В	С	Α	В	С
0	0	0	0	0	1
0	0	1	0	1	0
0	1	0	0	1	1
0	1	1	1	0	0
1	0	0	0	0	0

BC				
A	00	01	11	10
0	0	0	1	0
1	0	0	0	0

$$A(t+1) = A'BC$$

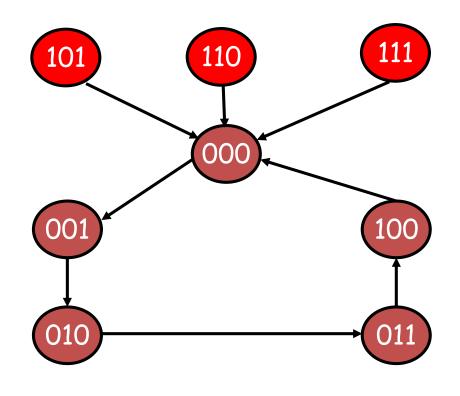
BC				
A	00	01	11	10
0	0	1	0	1
1	0	0	0	0

$$B(t+1) = A'B'C + A'BC'$$
$$= A'(B \oplus C)$$

$$C(\dagger + 1) = A'C'$$

Example: Unused States 4/4

Pres	Present State			Next State		
Α	В	С	Α	В	С	
0	0	0	0	0	1	
0	0	1	0	1	0	
0	1	0	0	1	1	
0	1	1	1	0	0	
1	0	0	0	0	0	



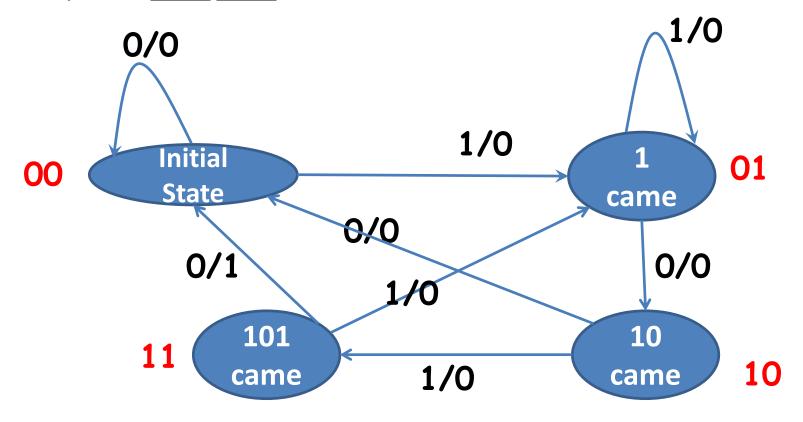
$$A(t+1) = A'BC$$

$$B(t+1) = A'(B \oplus C)$$

$$C(\dagger+1) = A'C'$$

Design Example

- Design the synchronous sequential circuit which gives "1" as output when the last 4 values from the 1-bit input are 1010.
- Eaxmple: x= <u>1010</u> <u>1011</u> ise z= 0001 0000



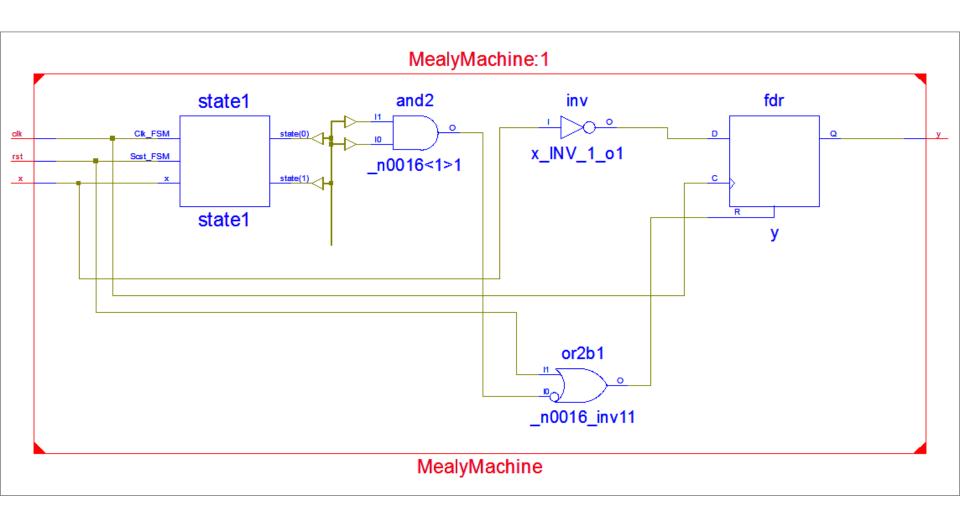
State Table

Presen	t State	Input	Next	State	Output
y ₁	y ₂	×	У ₁	y_2	z
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	0	1	0
1	0	0	0	0	0
1	0	1	1	1	0
1	1	0	0	0	1 1
1	1	1	0	1	0

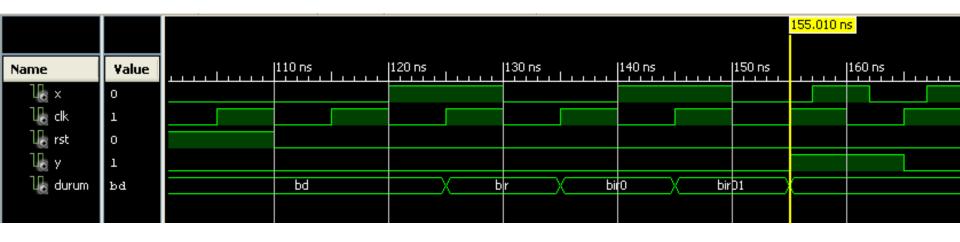
Verilog Code

```
module
                                                            S_10: begin
                           always @(posedge clk)
   MealyMachine(x,y,clk begin
                                                                    v = 0:
   ,rst);
                             if (rst) begin state = IS; y = 0;
                                                                    if(x) state = S 101;
                             end else
                                                                    else state = IS;
input x,clk,rst;
                               case (state)
                                                                 end
                                 IS: begin
output y;
                                                                  S 101: begin
                                  y = 0;
                                                                    if(x) begin
                                  if(x) state = S_1;
parameter IS=0, S 1=1,
                                                                      state = S = 1; y = 0;
                                  else state = IS;
   S 10=2, S 101=3;
                                                                    end else begin
                                 end
                                                                      state = IS; v = 1;
                                S 1: begin
reg [1:0] state;
                                  y = 0;
                                                                    end
reg y;
                                  if(x) state = S_1;
                                                                  end
                                  else state = S 10;
                                                                endcase
                               end
                                                              end
                                                            endmodule
```

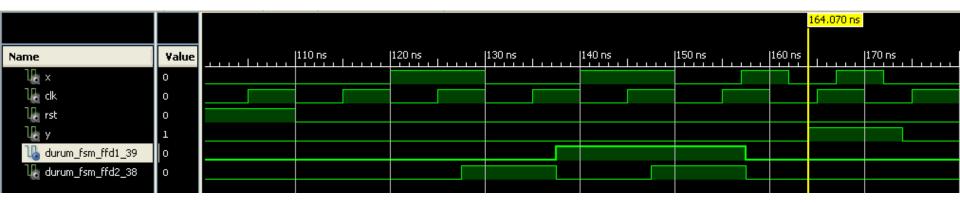
RTL Schematic



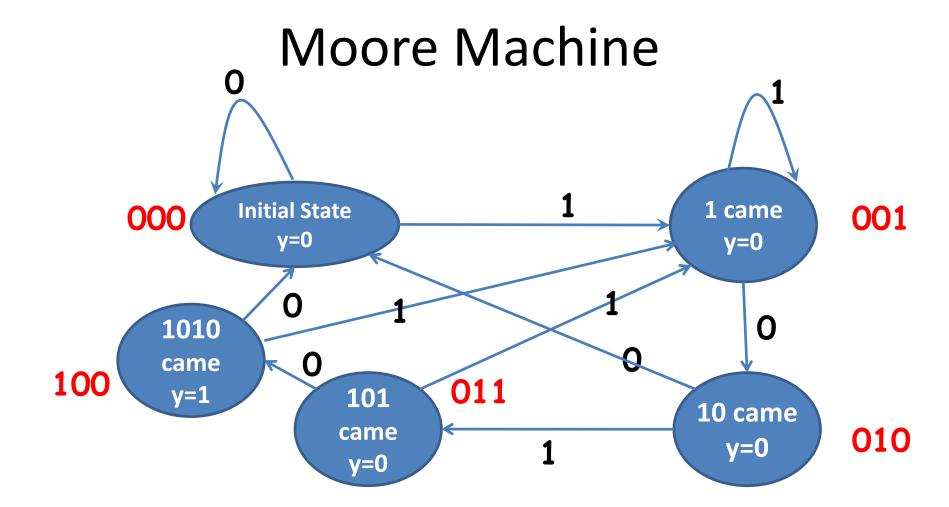
Timing Diagram



Ideal Case: Delay = 0



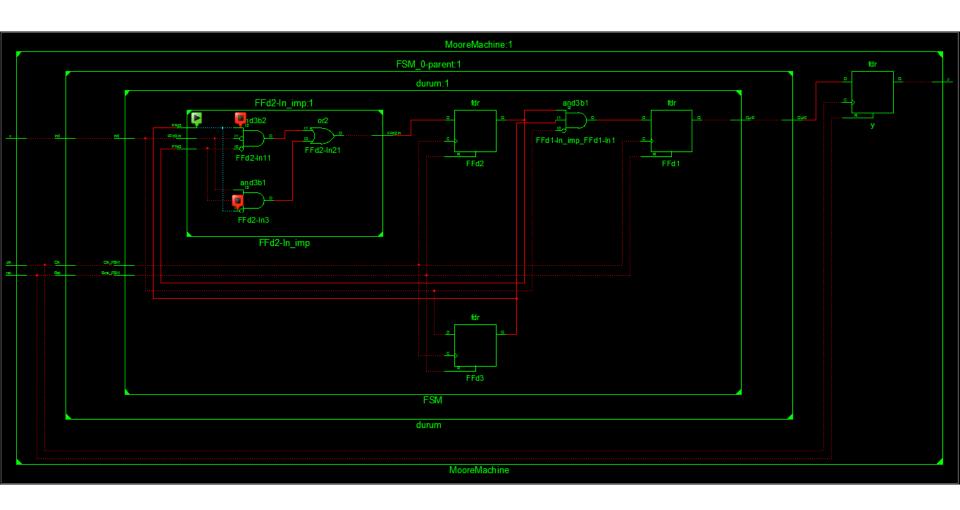
Not Ideal Case: Delay ≠ 0



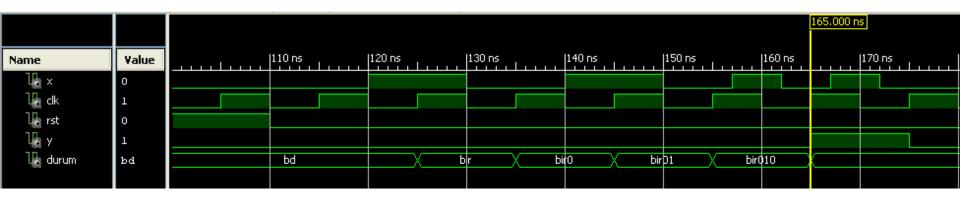
Verilog Code

```
S 10:
                                                                  y = 0;
                          always @(posedge clk)
                                                                  if(x) state = S_101;
module
                            begin
   MealyMachine(x,y,clk
                                                                  else state = IS;
                             if (rst) state = IS; y = 0;
   ,rst);
                                                                S 101:
                             else
                                                                  v = 0;
                               case (state)
Input x,clk,rst;
                                                                  if(x) state = S_1;
                                 IS:
output y;
                                                                  else state = S 1010;
                                  y = 0;
                                                                S_1010:
                                  if(x) state = S_1;
Parameter IS=0, S 1=1,
                                                                  y = 1;
                                  else state = IS;
   S 10=2, S 101=3,
                                                                  if(x) state = S = 1;
                                 S 1:
   S 1010=4;
                                                                  else state = S_IS;
                                  v = 0;
                                                              endcase
                                  if(x) state = S 1;
reg [2:0] state;
                                                             end
                                  else state = S_10;
reg y;
                                                          endmodule
```

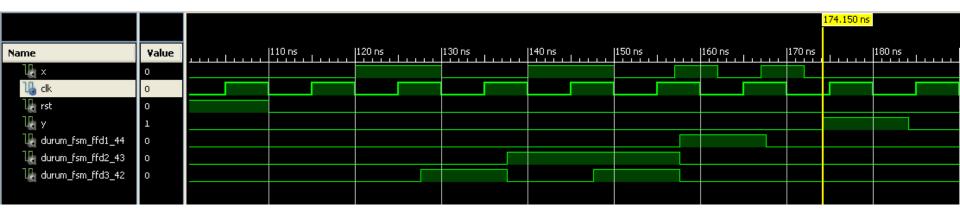
RTL Schematic



Timing Diagram



Ideal Case: Delay = 0



Not Ideal Case: Delay ≠ 0