1: Unique Minimum Spanning Trees

(a) Proof by contradiction: assume there are two minimum spanning trees T1 and T2.

take the smallest edge that exists in T1 or T2 and add it to T2 or T1. This forms a cycle.

Cycle property: the largest weighted edge in the cycle doesn't belong in the MST.

We won't remove e1, as it's the smallest weighted edge that exists. Therefore T2 wasn't a minimum spanning tree as we will remove an edge from T2 to form the new MST.

(b)

Wikipedia: If the minimum cost edge e of a graph is unique, then this edge is included in any MST.

2: Max Flow Basics

(a)

(b)

(c)

3: More Reductions to Flow

(a)

(b)

(c)

(d)

4: Unexpected Reductions to Flow/Matchings

(a)

(b)