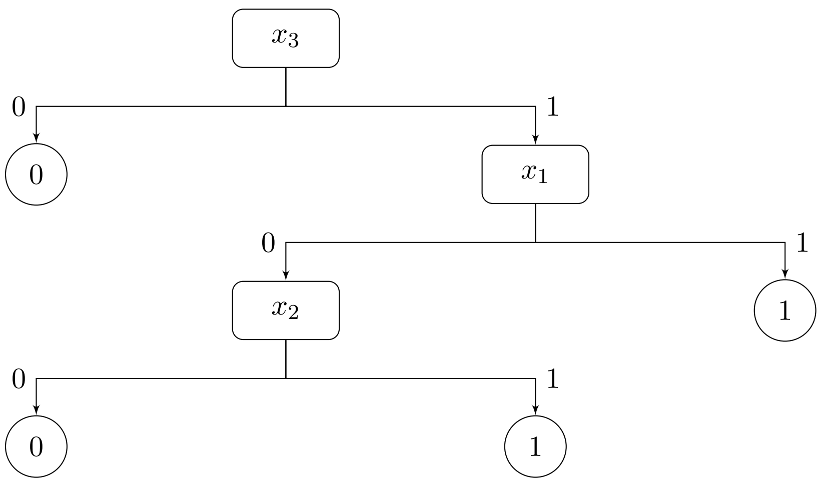
1: Decision trees

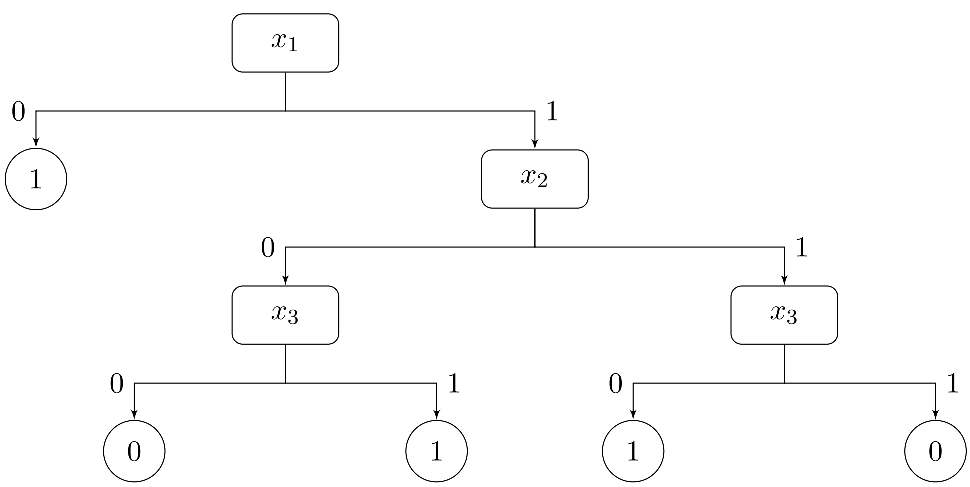
Note: Square nodes test for feature values and round leaf nodes specify the class labels.

(1) Representing Boolean functions as decision trees.

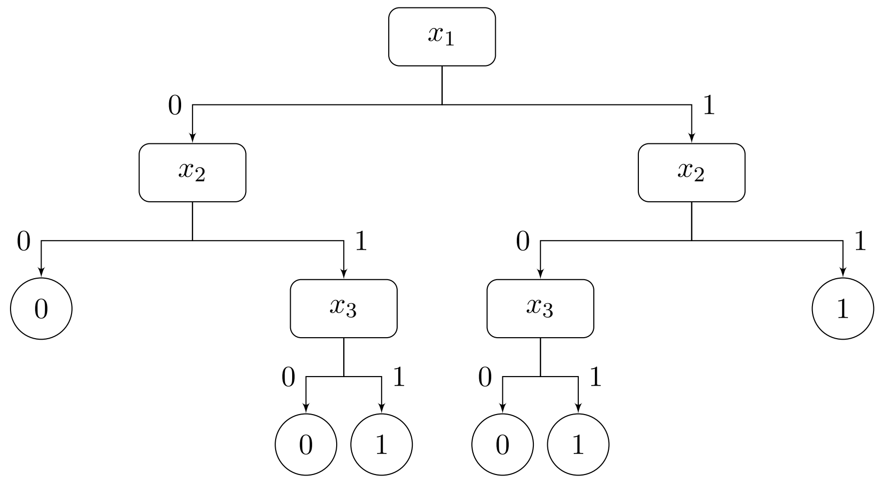
(a) $(x_1 \vee x_2) \wedge x_3$



(b) $(x_1 \wedge x_2)$ *xor* $(\neg x_1 \vee x_3)$



(c) The 2-of-3 function defined as follows: at least 2 of $\{x_1, x_2, x_3\}$ should be true for the output to be true.



(2) Pokémon Go decision tree to determine whether a Pokémon can be caught.

(a) There are 2 choices for Berry, 3 choices for Ball, 3 choices for Color, and 4 choices for type. This gives $2 * 3 * 3 * 4 = 72$ possible outputs. We are making a Boolean decision, so there are two ways to fill each of those 72 outputs giving 2^{72} possible functions.

$\boxed{2^{72} \text{ possible functions}}$

(b) Entropy is given by:

$$H(S) = -p_+ \log(p_+) - p_- \log(p_-)$$

The training set contains 16 examples, 8 of which result in a catch while the other 8 do not.

$$H(Caught) = -\frac{8}{16}\log(\frac{8}{16}) - \frac{8}{16}\log(\frac{8}{16}) = 1$$

$\boxed{H(Caught) = 1}$

(c) Information gain is given by:

$$Gain(S, A) = H(S) \ - \sum_{v \in Values(A)} \frac{|S_v|}{|S|} H(S_v)$$

Berry = Yes. 7 out of 16 examples. Caught = 6/7 - Not Caught = 1/7. $H(berry_{yes}) = 0.592$.
Berry = No. 9 out of 16 examples. Caught = 2/9 - Not Caught = 7/9. $H(berry_{no}) = 0.764$.

$$Gain(S, Berry) = 1 - ((\frac{7}{16})(0.592) + (\frac{9}{16})(0.764)) = 0.689$$

$\boxed{Gain(S, Berry) = 0.311}$

Ball = Poké. 6 out of 16 examples. Caught = 1/6 - Not Caught = 5/6. $H(ball_{poke}) = 0.65$.

Ball = Great. 7 out of 16 examples. Caught = 4/7 - Not Caught = 3/7. $H(ball_{great}) = 0.985$.

Ball = Ultra. 3 out of 16 examples. Caught = 3/3 - Not Caught = 0/3. $H(ball_{ultra}) = 0$.

$$Gain(S, Ball) = 1 - ((\frac{6}{16})(0.65) + (\frac{7}{16})(0.985) + (\frac{3}{16})(0)) = 0.325$$

$\boxed{Gain(S, Ball) = 0.325}$

Color = Green. 3 out of 16 examples. Caught = 2/3 - Not Caught = 1/3. $H(color_{green} = 0.918$.

Color = Yellow. 7 out of 16 examples. Caught = 3/7 - Not Caught = 4/7. $H(color_{yellow}) = 0.985$.

Color = Red. 6 out of 16 examples. Caught = 3/6 - Not Caught = 3/6. $H(color_{red}) = 1$.

$$Gain(S, Color) = 1 - ((\frac{3}{16})(0.918) + (\frac{7}{16})(0.985) + (\frac{6}{16})(1)) = 0.022$$

$\boxed{Gain(S, Color) = 0.022}$

Type = Normal. 6 out of 16 examples. Caught = 3/6 - Not Caught = 3/6. $H(type_{normal}) = 1$.

Type = Water. 4 out of 16 examples. Caught = 2/4 - Not Caught = 2/4. $H(type_{water}) = 1$.

Type = flying. 4 out of 16 examples. Caught = 3/4 - Not Caught = 1/4. $H(type_{flying}) = 0.811$.

Type = psychic. 2 out of 16 examples. Caught = 0/2 - Not Caught = 2/2. $H(type_{psychic}) = 0$.

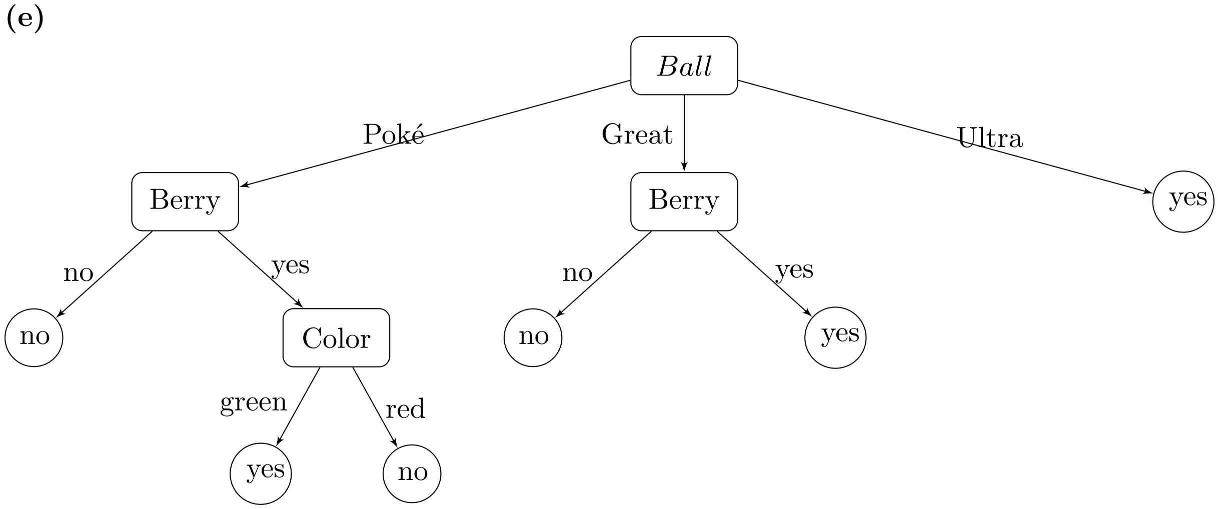
$$Gain(S, Type) = 1 - ((\frac{6}{16})(1) + (\frac{4}{16})(1) + (\frac{4}{16})(0.811) + (\frac{2}{16})(0)) = 0.172$$

$\boxed{Gain(S, Type) = 0.172}$

(d) When using the ID3 algorithm to create a decision tree, I would select the attribute Ball as the root of the tree.

$\boxed{\text{Ball}}$

(e)



(f)

Berry	Ball	Color	Type	Caught	Correct Prediction?
Yes	Great	Yellow	Psychic	Yes	Yes
Yes	Poké	Green	Flying	No	No
No	Ultra	Red	Water	No	No

Table 1: Predictions for test set

Out of the three examples in the test set, my decision tree predicted only one correct.

$\boxed{\text{Accuracy} = 33\%}$

(g) I think using a decision tree to classify if a Pokémon will be caught or not is a good idea. The training set provided is very sparse and I would expect a better rate of accuracy given more training data.

(3) Using the Gini measure with the ID3 algorithm.

(a)

(b)

2: Linear Classifiers

(1)

(2)

(3)