1 Graph Search

For the following problems, S will denote Start and G will denote Goal. When choosing an arbitrary order of state expansions, alphabetical ordering will be used. Once visited, a state will not be expanded again.

1. Greedy search

Step	Priority Queue	Expand
1	(S, 0)	S
2	(S-A, 3), (S-B, 2)	В
3	(S-A, 3), (S-B-D, 1), (S-B-C, 2), (S-B-G, 4)	D
4	(S-A, 3), (S-B-C, 2), (S-B-G, 4), (S-B-D-G, 5)	С
5	(S-A, 3), (S-B-G, 4), (S-B-D-G, 5), (S-B-C-G, 1)	G

Table 1: Greedy search

Greedy search final path: $S \to B \to C \to G$

2. Depth first search

Note: a LIFO queue is used and sibling nodes are enqueued in alphabetical order.

Step	LIFO Queue	Expand
1	S	S
2	В, А	В
3	G, D, C, A	G

Table 2: Depth first search

Depth first search final path: $S \rightarrow B \rightarrow G$

3. Breadth first search

Note: a FIFO queue is used and sibling nodes are enqueued in alphabetical order. If a node is already in the queue, it is not re-added.

Step	FIFO Queue	Expand
1	S	S
2	A, B	A
3	B, G	В
4	G, C, D	G

Table 3: Breadth first search

Breadth first search final path: $S \to A \to G$

4. Uniform cost search

Step	Priority Queue	Expand
1	(S, 0)	S
2	(S-A, 3), (S-B, 2)	В
3	(S-A, 3), (S-B-C, 4), (S-B-D, 3), (S-B-G, 6)	A
4	(S-B-C, 4), (S-B-D, 3), (S-B-G, 6), (S-A-G, 6)	D
5	(S-B-C, 4), (S-B-G, 6), (S-A-G, 6), (S-B-D-G, 8)	С
6	(S-B-G, 6), (S-A-G, 6), (S-B-D-G, 8), (S-B-C-G, 5)	G

Table 4: Uniform cost search

Uniform cost search final path: $S \to B \to C \to G$

5. Greedy search with the heuristic values listed at each state

Step	Priority Queue	Expand
1	(S, 0)	S
2	(S-A, 2), (S-B, 3)	A
3	(S-B, 3), (S-A-G, 0)	G

Table 5: Greedy search with heuristic values

Greedy search with heuristic final path: $S \to A \to G$

6. A^* search with the heuristic values listed at each state

Step	Priority Queue	Expand
1	(S, 0)	S
2	(S-A, 5), (S-B, 5)	A
3	(S-B, 5), (S-A-G, 6)	В
4	(S-A-G, 6), (S-B-C, 5), (S-B-D, 4), (S-B-G, 6)	D
5	(S-A-G, 6), (S-B-C, 5), (S-B-G, 6), (S-B-D-G, 8)	С
6	(S-A-G, 6), (S-B-G, 6), (S-B-D-G, 8), (S-B-C-G, 5)	G

Table 6: A* search

A* search final path: $S \to B \to C \to G$

2 Downhill Skiing

1. If the mountain is N units tall (eg., it is N=6 units tall in the figure), what is the size of the state space? Justify your answer. (You may ignore "unreachable" states.) What are the start/goal states?

Let's consider the state space S of the example where N=6. A state $s_i \in S$ is defined by a position and a current velocity as such: $s_i = (position, velocity)$. First we will define the start state s_0 and goal state g.

$$s_0 = (1,0), g = (6,0)$$

For small N, we can simply enumerate all the possible velocities at each position on the hill to determine the reachable states. Her action adjusts her velocity *before* she moves squares, which is how a velocity of 1 at square 1 is possible. Also, Alice makes a final action at the goal state.

(1, 0)	(1, 1)	-	-
(2, 0)	(2, 1)	(2, 2)	-
(3, 0)	(3, 1)	(3, 2)	-
(4, 0)	(4, 1)	(4, 2)	(4, 3)
(5, 0)	(5, 1)	(5, 2)	(5, 3)
(6, 0)	(6, 1)	(6, 2)	(6, 3)

Table 7: State space for Alice's ski run.

To justify this table, we will consider it one column at a time.

Column 1: Alice can be present in each square with a velocity of 0 by alternating accelerate and decelerate actions all the way down the hill.

Column 2: Alice can accelerate at square 1 and coast the rest of the way down the hill.

Column 3: From square n-1 with a velocity of 0 before performing an action, accelerate twice. The state will be arrived at after the second accelerate but before moving.

Column 4: From square n-2 with a velocity of 1 before performing an action, accelerate twice. The state will be arrived at after the second accelerate but before moving.

There is no way to have a velocity greater than 3 when N=6. The best chance at the largest velocity is to accelerate for every action (as the higher on the hill you are the more space you have to accelerate before hitting the parking lot). Consider we accelerate 3 times. This will take us to space (4,3) after performing the action but before moving. Once we move, we will be overshooting the goal into the parking lot.

Thus, |S| = 20.

2. Give an example of a state that is not reachable. Suppose that Alice cannot coast (she must either accelerate or decelerate): does this yield

more unreachable states? If so, give an example of one and justify your answer either way.

An example of a state that is not reachable is (1,2) (being at square 1 with a velocity of 2). Her velocity at square 1 is either 0 (she coasts as her first move) or 1 (she accelerates as her first move). There is no way for her to move back up the hill either.

When we remove the action of coast, the number of unreachable states does not change. In question one, I used coast to justify being at each square with a velocity of 1. Another way to achieve this without coasting is to alternate between accelerate and decelerate. This will place Alice in all squares with a velocity of 1.

3. Is Alice's current elevation (i.e., distance from the chair lift) an admissible heuristic? Why or why not?

A heuristic h(n) is admissible if $h(n) \le h * (n)$ where h * (n) is the true cost to the nearest goal.

Alice's current elevation is an admissible heuristic, we can prove this by comparing the heuristic with the best true cost for each square on the hill.

h(n)	h*(n)	Possible Best Path
-	-	-
-	-	-
-	-	-
-	-	-
_	-	-
-	-	-

Table 8: Comparison of h(n) and h*(n) for $N \in [0, 5]$

4. State and justify a non-trivial, admissible heuristic for this problem which is *not* current elevation.