HW5: Bayesian Networks & Independence

CS6300: Artificial Intelligence, Spring 2018 **Jake Pitkin**
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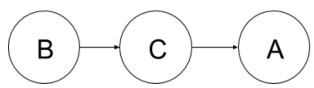
1 Independences from Probability Tables

A	**B**	**C**	p
T	T	T	1/16
T	T	F	1/3
T	F	T	1/32
T	F	F	1/12
F	T	T	3/16
F	T	F	1/6
F	F	T	3/32
F	F	F	1/24

Table 1: $P(A, B, C)$

I considered a few factorizations and found one that satisfied the above table.



This factorizes the joint probability $P(A, B, C)$ as a product of the conditional probabilities $P(B)$, $P(C|B)$, and $P(A|C)$.

B	p
T	3/4
F	1/4

Table 2: $P(B)$

C	**B**	p
T	T	1/3
T	F	1/2
F	T	2/3
F	F	1/2

Table 3: $P(C|B)$

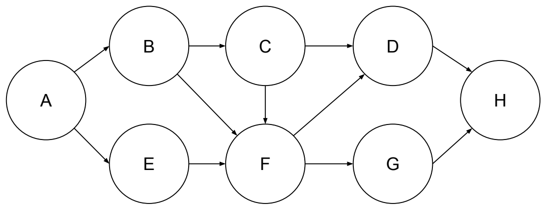
A	**C**	p
T	T	1/4
T	F	2/3
F	T	3/4
F	F	1/3

Table 4: $P(A|C)$

With 10 total entries, this is the smallest you can get the conditional probability tables (keeping the graph connected). These were derived by marginalizing out variables from $P(A, B, C)$ and using the product rule for dependent variables.

2 Independence in Graphical Models

Consider the graphical model shown below:



Please answer the following conditional independence questions from this model:

1. $A \perp H$ - **No**: There is an active path between A and H by causal chain.
2. $A \perp H|C$ - **No**: There is an active path between A and H by causal chain.
3. $A \perp H|C, F$ - **Yes**: There is no active path between A and H.
4. $E \perp B|A$ - **Yes**: There is no active path between E and B.
5. $E \perp B|C, F$ - **No**: There is an active path between E and B by common cause.
6. $E \perp B|A, C, F$ - **No**: There is an active path between E and B by common cause.

3 Inference by Enumeration and Variable Elimination

The following queries are answered using the probability tables found in the lecture slides.

1. $p(b, \neg e|a, j, m)$

$$p(b, \neg e|a, j, m) = \frac{p(b, \neg e, a, j, m)}{p(a, j, m)}$$

$$= \frac{p(b)\ p(\neg e)\ p(j|a)\ p(m|a)\ p(a|b, \neg e)}{p(j|a)\ p(m|a)\ \sum_b \sum_e p(B)\ p(E)\ p(a|B, E)}$$

$$= \frac{0.001 * 0.998 * 0.9 * 0.7 * 0.94}{\begin{array}{r}0.9 * 0.7 * (0.001 * 0.002 * 0.95 + 0.999 * 0.002 * 0.29 + \\ 0.001 * 0.998 * 0.94 + 0.999 * 0.998 * 0.001)\end{array}}$$

$$= \frac{0.0005910156}{0.00158535846}$$

$$= \mathbf{0.372796}$$

2. $p(b|a)$

$$p(b|a) = \frac{\sum_e \sum_j \sum_m p(b)\ p(E)\ p(a|b, E)\ p(J|a)\ p(M|a)}{\sum_b \sum_e \sum_j \sum_m p(E)\ p(E)\ p(a|B, E)p(J|a)\ p(J|a)}$$

$$= \frac{p(b) \sum_e p(E)\ p(a|b, E)}{\sum_b \sum_e p(B)\ p(E)\ p(a|B, E)}$$

$$= \frac{0.001 * (0.002 * 0.95 + 0.998 * 0.94)}{0.001 * 0.002 * 0.95 + 0.999 * 0.002 * 0.29 + 0.001 * 0.998 * 0.94 + 0.999 * 0.998 * 0.001}$$

$$= \frac{0.00094002}{0.002516442}$$

$$= \mathbf{0.3735512}$$

3. $p(b|e,a)$ (how does this compare to the previous one?)

$$p(b|e,a) = \frac{\sum_j \sum_m p(b)\ p(e)\ p(a|b,e)\ p(J|a)\ p(M|a)}{\sum_b \sum_j \sum_m p(B)\ p(e)\ p(a|B,e) p(J|a)\ p(M|a)}$$

$$= \frac{p(b)\ p(e)\ p(a|b,e)}{p(e) \sum_b p(B)\ p(a|B,e)}$$

$$= \frac{0.001 * 0.002 * 0.95}{0.002 * (0.001 * 0.95 + 0.999 * 0.29)}$$

$$= \frac{0.0000019}{0.00058132}$$

$$= \mathbf{0.0032684}$$

This makes sense. If there is an earthquake and the alarm is going off, it's unlikely that there is a burglary occurring at the same moment. But in question 2, we know only the alarm is going off and there is a much larger chance there is a burglary occurring (causing the alarm to go off).

4. $p(a|j,\neg m)$

Note: in the numerator I take the result of marginalizing b and e when a is true from question 1 rather than showing the work again. For denominator I solved it using Excel.

$$p(a|j,\neg m) = \frac{p(j|a)\ p(\neg m|a)\ \sum_b \sum_e p(B)\ p(E)\ p(a|B,E)}{\sum_b \sum_e \sum_a p(B)\ p(E)\ p(A|B,E) p(j|A)\ p(\neg m|A)}$$

$$= \frac{0.9 * 0.01 * 0.002516442}{\sum_b \sum_e p(B)\ p(E)\ f(B,E,j,\neg m)}$$

$$= \frac{0.000226479}{0.0166848}$$

$$= \mathbf{0.013574}$$

Now, repeat items (2) and (4) using variable elimination. When you have to choose a variable to eliminate, choose alphabetically.

2. $p(b|a)$

Variable elimination order: E, J, then M.

First we will observe a is true to reduce the size of the resulting factors.

$$p(b|a) = \alpha\ p(B)\ \sum_e p(E)\ p(a|B,E) \sum_j p(J|a) \sum_m p(M|a)$$

Step 1: Sum out E and join:

$$f_1(a, B) = \sum_e p(E)\ p(a|B, E)$$

A	**B**	p
T	T	0.940020
T	F	0.001578

Table 5: $f_1(a, B)$

Giving the new equation:

$$p(b|a) = \alpha\ p(B)\ f_1(a, B) \sum_j p(J|a) \sum_m p(M|a)$$

Step 2: Sum out J and join:

$$f_2(a) = \sum_j p(J|a) = 1$$

Giving the new equation:

$$p(b|a) = \alpha\ p(B)\ f_1(a, B)\ f_2(a) \sum_m p(M|a)$$

Step 3: Sum out M and join:

$$f_3(a) = \sum_m p(M|a) = 1$$

Giving the new equation:

$$p(b|a) = \alpha\ p(B)\ f_1(a, B)\ f_2(a)\ f_3(a)$$
$$p(b|a) = \alpha\ p(B)\ f_1(a, B)$$

Joining $p(B)$ and $f_1(a, B)$:

a	**B**	p
T	T	0.00094
T	F	0.001576

Alpha can be found by taking the inverse of the sum of the probability table:

$$\alpha = \frac{1}{0.002516}$$

Finally, we finish the inference:

$$p(b|a) = \frac{0.00094}{0.002516} = \mathbf{0.3736}$$

4. $p(a|j, \neg m)$

   Variable elimination order: B then E.

   We will observe j to be true and m to be false.

$$p(a|j, \neg m) = \alpha\ p(j|A)\ p(\neg m|A) \sum_e p(E) \sum_b p(B)p(A|B, E)$$

   Step 1: Sum out B and join:

$$f_1(A, E) = \sum_b p(B)p(A|B, E)$$

A	**E**	p
T	T	0.29066
T	F	0.001939
F	T	0.70934
F	F	0.998061

Table 6: $f_1(A, E)$

   Giving the new equation:

$$p(a|j, \neg m) = \alpha\ p(j|A)\ p(\neg m|A) \sum_e p(E)\ f_1(A, E)$$

   Step 2: Sum out E and join:

$$f_2(A) = \sum_e p(E)\ f_1(A, E)$$

A	p
T	0.002516
F	0.997483

Table 7: $f_2(A)$

   Giving the new equation:

$$p(a|j, \neg m) = \alpha\ p(j|A)\ p(\neg m|A)\ f_2(A)$$

   Joining $p(j|A)$, $p(\neg m|A)$, and $f_2(A)$ with $\neg m$ and j observed:

A	p
T	0.000679
F	0.049375

Alpha can b e found by taking the inverse of the sum of the probability table:

$$\alpha = \frac{1}{0.056165}$$

Then we can finish the inference:

$$p(a|j, \neg m) = \frac{0.000679}{0.056165} = \mathbf{0.013574}$$