HW6: Hidden Markov Models

CS6300: Artificial Intelligence, Spring 2018 **Jake Pitkin**
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1 Stationary Distributions of Markov Chains

1. **Suppose that in semester $t = 0$ Alice passes the class with probability 0.5. Compute the probability that she passes in semester $t = 1$ and semester $t = 2$.**

   We are given an initial distribution and transition matrix as follows:

$$\pi_0 = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}$$

   To compute the probabilities at $t = 1$ we apply matrix multiplication:

$$\pi_0 P = \pi_1 = \begin{bmatrix} 0.6 & 0.4 \end{bmatrix}$$

   To compute the probabilities at $t = 2$ we apply matrix multiplication again this time to π_1 and P:

$$\pi_0 P = \pi_1 = \begin{bmatrix} 0.64 & 0.36 \end{bmatrix}$$

$$\boxed{P(t_1 = pass) = 0.6, \quad P(t_2 = pass) = 0.64}$$

2. **Compute the stationary distribution of this chain. (Hint: the easiest way to do this is to start with some "guess" – say 50/50 – and then keep simulating the chain for a while until it seems to settle down. Once you think you've got a guess at what the stationary distribution might be, try transitioning from there to see if you get back to the stationary distribution.) (Not as helpful hint: if you like linear algebra, you can directly compute the stationary distribution from the transition matrix using an eigenvalue decomposition: take a look at Wikipedia for how!)**

   I'll use the technique of starting the distribution at $\pi_0 = [0.5\ 0.5]$, simulating the chain for a while and waiting for it to settle down. I simulated the chain for $t = 100$ using a program and arrived at $\pi = [0.6667\ 0.3333]$. We can verify this by transitioning and checking if we arrive at the stationary distribution:

$$\pi P = [0.6667\ 0.3333] \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix} = [0.6667\ 0.3333] = \pi$$

   Thus the stationary distribution is:

$$\boxed{\pi = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \end{bmatrix}}$$

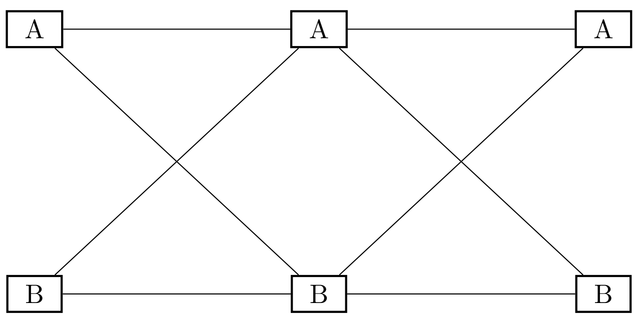
2 Alice and the Crazy Coke Machine

Alice is up late studying for her algorithms final exam and needs to stay hydrated (and caffienated!). Unfortunately, Clarence was the one who bought the soda machine in the lab. He went for the cheapest model. All you can do with this soda machine is put money in and hope it gives you the type of drink you want. It carries three types of soda: Coke (C), Diet Coke (D) and Sprite (S).

Alice has been monitoring the soda machine for a while and has figured out that it behaves as an HMM. It has two internal states (call them A and B). When asked for a soda from state A, it gives a coke with probability 1/2, and a diet coke and a sprite each with probability 1/4. On the other hand, when asked for a soda in state B, it gives a diet coke with probability 1/2, a sprite with probability 1/3 and a coke with probability 1/6. Furthermore, it transitions from state A to A with probability 4/5 and from state B to B with probability 3/5.

The machine formally works as follows. It is in some state s. Someone puts in money and it dispenses a soda according to the probability rules set out above, being in state s. It then (randomly) transitions to a new state s' according to the transition probabilities above.

1. Draw a state space lattice for this soda machine (as on the final slide of day 20) for three time steps.



2. Suppose that Alice doesn't know what state the machine is in currently (specifically, she believes it's equally likely to be in either state), but puts money in and gets a Sprite out. What is the probability distribution over states that it was in when Alice put her money in? What is the probability distribution over states that it is in now?

3. Suppose Alice comes back the next day (so again she doesn't know what state the machine is in) and really wants a diet coke. Unfortunately, the machine isn't being particularly nice to her and it produces the following series of emissions upon taking money from Alice: C S S D. What is the most likely sequence of states the machine went through in this process?