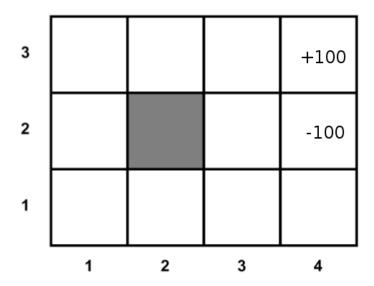
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TD and Q in Blockworld 1

Consider the following gridworld:



Suppose that we run two episodes that yield the following sequences of (state, action, reward) tuples:

${f S}$	\mathbf{A}	${f R}$	\mathbf{S}	\mathbf{A}	${f R}$
(1,1)	up	-1	(1,1)	up	-1
(2,1)	left	-1	(1,2)	up	-1
(1,1)	up	-1	(1,3)	right	-1
(1,2)	up	-1	(2,3)	right	-1
(1,3)	up	-1	(2,3)	right	-1
(2,3)	right	-1	(3,3)	right	-1
(3,3)	right	-1	(4,3)	exit	+100
(4,3)	exit	+100	(done)		
(done)					

1. According to direct estimation, what are the values for every state in the grid?

State	Calculation	Value
(1,1)	(93 + 95 + 94)/3	93.666
(1,2)	(96 + 95)/2	95.5
(1,3)	(97 + 96)/2	96.5
(2,1)	94/1	94
(2,2)	-	-
(2,3)	(98 + 97 + 98)/3	97.666
(3,1)	-	-
(3,2)	-	-
(3,3)	(99 + 99)/2	99
(4,1)	-	-
(4,2)	-	-
(4,3)	(100 + 100/2)	100

2. According to model-based learning, what are the transition probabilities for every (state, action, state) triple. Don't bother listing all the ones that we have no information about.

3. Suppose that we run Q-learning. However, instead of initializing all our Q values to zero, we initialize them to some large positive number ("large" with respect to the maximum reward possible in the world: say, 10 times the max reward). I claim that this will cause a Q-learning agent to initially explore a lot and then eventually start exploiting. Why should this be true? Justify your answer in a short paragraph.

2 Policy Gradient

In order to do policy gradient, we need to be able to compute the gradient of the value function J with respect to a parameter vector θ : $\nabla_{\theta} J(\theta)$. By our algebraic magic, we expressed this as:

$$\nabla_{\theta} J(\theta) = \sum_{a} \pi_{\theta}(s_0, a) R(a) \underbrace{\nabla_{\theta} \log (\pi_{\theta}(s_0, a))}_{g(s_0, a)} \tag{1}$$

If we us a linear function through a soft-max as our stochastic policy, we have:

$$\pi_{\theta}(s, a) = \frac{\exp\left(\sum_{i=1}^{n} \theta_{i} f_{i}(s, a)\right)}{\sum_{a'} \exp\left(\sum_{i=1}^{n} \theta_{i} f_{i}(s, a')\right)} \tag{2}$$

Compute a closed form solution for $g(s_0, a)$. Explain in a few sentences why this leads to a sensible update for gradience ascent (i.e., if we plug this in to Eq (1) and do gradient ascent, why is the derived form reasonable)?