$\langle pf \rangle \quad \vec{\nabla} \cdot B^{n+1} = \vec{\nabla} \cdot B^{n}$ in 3D

$$\cdot\ (\vec{\nabla} \cdot B^{n})_{i,j,k} = \Big[\frac{1}{\Delta x} \left(B^{n}_{x,i+1/2,j,k} - B^{n}_{x,i-1/2,j,k} \right) + \frac{1}{\Delta y} \left(B^{n}_{y,i,j+1/2,k} - B^{n}_{y,i,j-1/2,k} \right) + \frac{1}{\Delta z} \left(B^{n}_{z,i,j,k+1/2} - B^{n}_{z,i,j,k-1/2} \right) \Big] = 0$$

$$\cdot\ B^{n+1}_{x,i-1/2,j,k} = B^{n}_{x,i-1/2,j,k} - \frac{\Delta t}{\Delta y} \left(\mathcal{E}^{n+1/2}_{z,i-1/2,j+1/2,k} - \mathcal{E}^{n+1/2}_{z,i-1/2,j-1/2,k} \right) + \frac{\Delta t}{\Delta z} \left(\mathcal{E}^{n+1/2}_{y,i-1/2,j,k+1/2} - \mathcal{E}^{n+1/2}_{y,i-1/2,j,k-1/2} \right)$$

$$\begin{aligned}
\vec{\nabla} \cdot B^{n+1} - \vec{\nabla} \cdot B^{n} &= \frac{1}{\Delta x} \left(B^{n+1}_{x,i+1/2} + B^{n}_{x,i+1/2} - B^{n+1}_{x,i-1/2} - B^{n}_{x,i-1/2} \right) \\
&\quad + \frac{1}{\Delta y} \left(B^{n+1}_{y,j+1/2} + B^{n}_{y,j+1/2} - B^{n+1}_{y,j-1/2} - B^{n}_{y,j-1/2} \right) \\
&\quad + \frac{1}{\Delta z} \left(B^{n+1}_{z,k+1/2} + B^{n}_{z,k+1/2} - B^{n+1}_{z,k-1/2} - B^{n}_{z,k-1/2} \right) \\
&= \frac{1}{\Delta x} \Big[2B^{n}_{x,i+1/2} - 2B^{n}_{x,i-1/2} + \frac{\Delta t}{\Delta y} \left(\mathcal{E}^{n+1/2}_{z,i+1/2,j-1/2} - \mathcal{E}^{n+1/2}_{z,i+1/2,j+1/2} + \mathcal{E}^{n+1/2}_{z,i-1/2,j-1/2} - \mathcal{E}^{n+1/2}_{z,i+1/2,j+1/2} \right) \\
&\quad + \frac{\Delta t}{\Delta z} \left(\mathcal{E}^{n+1/2}_{y,i+1/2,k+1/2} - \mathcal{E}^{n+1/2}_{y,i+1/2,k-1/2} + \mathcal{E}^{n+1/2}_{y,i-1/2,k+1/2} - \mathcal{E}^{n+1/2}_{y,i-1/2,k-1/2} \right) \\
&\quad + \frac{1}{\Delta y} [\ \vdots\ \ \vdots\ ] + \frac{1}{\Delta z} [\ \vdots\ \ \vdots\ ] \longrightarrow \text{similar expressions like } \frac{1}{\Delta x}[\ \vdots\ \ \vdots\ ] \\
&= 0
\end{aligned}$$