The Mathematics of Fused Filament Fabrication

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1 Drive Mechanisms

1.1 Core XY

Equations of motion for a Core XY system are:

1.2 Delta

1.3 Drive Belts

For an endless belt, what is the circumference required to fit two pullies, radii r_1, r_2 and separation, d?

$$l = \pi \left(r_1 + r_2\right) + 2\sqrt{s^2 + \left(r_2 - r_1\right)^2} \tag{1}$$

!!! This assumes $r_1 \approx r_2$!!! TODO

1.4 Steps per Unit Distance

Steps per millimeter of travel in the X direction:

2 Polymer Extrusion

2.1 Extruder Calibration

2.2 Volumetric Extrusion

3 Temperature

3.1 Thermistor Calibration

Thermistors, both NTC and PTC, and thermocouples are all used to sense temperatures at various critical locations on and around a typical FFF machine. See the Wikipedia page relating to thermistors: `https://en.wikipedia.org/wiki/Thermistor`. A widely used third-order model developed for semiconductor resistance is the Steinhart-Hart equation.

$$\frac{1}{T} = a + b\ln(R) + c(\ln(R))^3 \gg k(\ln(R))^2 \tag{2}$$

Coefficients a, b and c can be determined from empirically acquired data using a fitting method, in this case we choose least squares approximation. Starting from acquired data 1:

Summarising the least squares process and relating it to the numpy/scipy implementation referenced here: `https://github.com/jsr38/savagecorexy/blob/cuboid/scripts/thermistor_calibration.py`.

The general idea is to minimise the following norm:

$$\|\mathbf{b} - \mathbf{Ax}\|^2$$

Suppose we took n measurements where:

$$\mathbf{x} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \mathbf{b} = \begin{bmatrix} \frac{1}{T_1} \\ \vdots \\ \frac{1}{T_n} \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ \ln(R_1) & \ln(R_2) & \ln(R_3) & \dots & \ln(R_n) \\ \ln(R_1)^3 & \ln(R_2)^3 & \ln(R_3)^3 & \dots & \ln(R_n)^3 \end{bmatrix}^\intercal$$

This can be achieved by inversion of the matrix $\mathbf{A}$. The following concerns the detail of generating a suitable lookup table given a circuit configuration and ADC of known resolution, that achieves the best resolution over our region of interest.

Resistance R (k ohms)	Temperature T (degrees celsius)
18.0	70.0
18.4	68.0
20.0	66.5
23.5	60.0
28.7	55.5
38.1	48.8
68.0	34.0
74.1	32.0
79.8	30.3
84.3	29.0
92.2	27.3
96.4	26.3
100.3	25.3
107.0	23.9
112.6	22.8
114.2	22.4
115.7	22.1
117.3	21.8
120.0	22.0
130.0	20.0

Table 1: Thermistor measurements peformed in lab.

The function used to generate lookup tables within the Repetier firmware assumes a different model, the β model which discounts the third order effects:

$$\frac{1}{T} = \frac{1}{T_0} + \frac{1}{\beta} \ln\left(\frac{R}{R_0}\right) \tag{3}$$

We could derive β and supply a pair of points to this function in order to generate a lookup table but since we have the third-order approximation we may as well use this to generate the lookup table. The accompanying script assumes a pull-up or pull-down resistor which linearises the thermistor response and hence reduces error introduced by finite resolution ADC, quantisation error, over our region of interest.

$$N = 2^B, B = 12, \Delta b = 2^{B-L} = 2^4 = 16$$

$$\mathbf{n} = \left[\begin{array}{ccccc} 0 & 2^1 & 2^2 & \dots & 2^L \end{array}\right]$$

$$\alpha_n = \left(n\Delta b + \frac{\Delta b - 1}{2} + \frac{1}{2} \right)$$

$$\alpha = \left[\begin{array}{ccccc} \alpha_0 & \alpha_1 & \alpha_2 & \dots & \alpha_n \end{array}\right] = \frac{V_o}{V_i}$$

Results of running the process on our data: