Coloring SNR Graphs using Genetic Algorithm

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***Abstract*—In this paper, a novel approach has been proposed to find out the optimal broadcast scheduling in the SNR graphs.SNR graphs are basically Signal-to-Noise Ratio at the receiver side. A transmission will be heard by the receiver node only when this ratio will be above the threshold value. In this paper, we are first finding out the SNR of the transmission links, of the nodes deployed randomly in a fixed network region. Depending on this connectivity matrix, we try to optimize the number of channels required by using Graph Coloring in Genetic Algorithm. GA is a heuristic method based upon the theory of natural evolution. The Gas are quite efficient in directing the search towards relatively prospective regions of the search space. The propose method find out a valid transmission scheme for SNR graphs. It was also seen to outperform the UDG graphs for transmission scheduling.**

***IndexTerms*—Graph Coloring, SNR Graphs, UDG, Genetic Algorithm.**

I.INTRODUCTION

In Wireless Networks, the nodes are distributed over a wide geographical area and communicate with each other in a multi-hop fashion. These networks are hence often known as Packet Radio Networks (PRNETs) and are widely used foe communication over the area which do not have direct access to radio or cable connection.

There are different ways to allocate the different frequency bands to the nodes like frequency division [1][2], codes division, time division [3] and spatial reuse. In case of a single radio channel, the communication can be carried out either by broadcasting the messages or by activating a set of network links in some predefined sequence [4]-[6]. In a quest for a better scheduling algorithm there has been an intensive research for the assignment of channels based on the Signal to Noise Ratio of the links .

Whenever the nodes communicate using broadcast technique with the help of omnidirectional antennas, it becomes easier for the network to manage the traffic if all the nodes are tuned to the same radio frequency using time division and spatial reuse [7]-[9]. The spatial reuse refers to the simultaneous usage of the same channel at different non-interfering parts of the network. Because the nodes in a wireless network cannot directly communicate, hence the intermediate nodes act as store and forward repeaters. Here in this paper, we are assuming a PRNET which uses time division multiplexing with spatial reuse. TDMA avoids simultaneous transmissions while at the same time it also facilitates the sharing of the same frequency channel.

A PRNET is often represented using graphs having nodes which are connected by edges. Here the graph's vertices denote the network's nodes. A node can transmit successfully only when it does not face the following two collisions: a node must not transmit to two nodes simultaneously or it should not transmit and receive a packet simultaneously (known as primary conflict), and a node is not allowed to receive two transmissions simultaneously (known as secondary conflict). In order to meet the above requirements it must be ensured that the transmitting nodes must be at least two hops away. This way the time slots do not cause interference with the channels.

Since finding an optimal scheduling algorithm for a wireless network has been a difficult task, disk graph coloring is used as an appropriate method to solve the scheduling problem in quite a computationally affordable manner [10] [11] [12]. In this traditional method the transmitter and the receiver are represented by the vertices of the graph. Two vertices which are connected by the edge signify that they are within the communication range of the other, with the disk radius *d* centered on the other. In all such UDG models, the interference is depicted in a pairwise manner, which means that a contention between two links is always decided by the characteristics of the two links only. That is other links do not play in role in deciding an edge between any two vertices. This scheduling hence reduces the problem to coloring of the flow contention graph. Hence in UDG links within the distance d from each other are allocated different colors.

However, disk graph coloring suffers few critical problems. For example, it is not apparent as to what should be the appropriate value of disk radius *d* as there isn't any clear relation between *d* and the physical layer parameters [13]. If d is very small, then the coloring of the vertices (or resource management) may involve a lot of channels, much above then required. And if the value of *d* is very large, the scheduling may be too conservative, resulting in wastage of bandwidth. Since the graph coloring problem is NP-complete hence there is no polynomial time algorithm for finding out the chromatic number of the graph [10].

Another approach which can be used to for scheduling the frequency slots in the wireless networks is the SNR (Signal-to-Noise Ratio) which assigns the links to the nodes depending upon the quality of the link. If the value of SNR of some particular link is above the threshold level, only then will that link be assigned for transmission. It has been found that the scheduling based on the SNR value outperforms the traditional UDG scheduling for network spread out in the larger area.

Rest of the paper is organized as follows: Section II deals with the formulation of broadcast scheduling problem, Section III with the related work in this area, Section IV explains the Proposed Approach. Section V explains the simulation and the results and its analysis and Section VI gives the conclusion. Broadcast Scheduling problem is an NP-complete problem, hence in this paper first SNR technique is used to find the best links for transmission between any two nodes and then an evolutionary algorithm, the Genetic Algorithm, is used to optimize the solution.

II. Problem Definition

Let us assume a wireless multi-hop network as a graph $G=(V, E)$. Here $V = \{1,2,3 \ldots N\}$represents the nodes of the network and E the edges between them representing the set of transmission links. Two nodes i and j, where ($i,j \in V$) , are connected by an undirected edge $e_{ij} \in E$, if and only if they can receive each other's transmission. In this situation, the two nodes are said to be at a distance of one-hop or one hop away.A wireless multi-hop network basically suffers from two types of conflicts : a primary conflict which occurs when two nodes, which are at a distance of one hop, transmit in the same time slot and a secondary conflict which occurs when two nodes which are two hops away, transmit in the same time slot.

Based on this we find out the connectivity matrix $C = \{cij\}$, where $\{i, j = 1, \ldots., N\}$ for the wireless multi-hop network, which is defined as follows:

$$cij = \begin{cases} 1, if\ i\ and\ j\ are\ one\ hop\ away; \\ 0, \qquad otherwise. \end{cases}$$

Similarly, the compatibility matrix is also defined for nodes that are two hops away. Suppose, $e_{ij} \notin E$ and an intermediate node k exists, in a manner that $e_{ik} \in E$ and $e_{kj} \in E$. Hence the compatibility matrix is defined as follows:

$$dij = \begin{cases} 1, ifiandj\ are\ with\ in\ two\ hops; \\ 0, \qquad otherwise. \end{cases}$$

Therefore, the two main constraints in the wireless multi-hop network are: (1) Every node must be scheduled to transmit at least once in a TDMA cycle, and (2) A node cannot transmit and receive simultaneously, and that it cannot receive more than one transmission simultaneously. This implies that two nodes can transmit in the same time slot only if they are more than two hops away.

Considering these constraints, the optimal solution for BSP is a transmission schedule for M slots, for which $M\ X\ N$ binary matrix $T = \{t_{ij}\}$ is used to represent the transmission schedule, where

$$tij = \begin{cases} 1, if\ node\ i\ transmits\ in\ the\ slot\ j; \\ \quad 0, \quad otherwise. \end{cases}$$

Suppose, if we take the channel utilization factor as ρ, then for node j the channel utilization is defined by:

$$\rho j = \frac{number\ of\ time\ slots\ assigned\ to\ node\ j}{Total\ number\ of\ TDMA\ slots}$$

$$= \sum_{i=1}^{M} tij\ /M$$

Channel utilization total for the entire network is given by :

$$\rho = \frac{1}{N} \sum_{j=1}^{N} \rho j$$

Main objective of BSP is to find a transmission schedule having the shortest TDMA frame length (i.e. the smallest M) fulfilling all the above constraints, and maximizing the total transmissions.

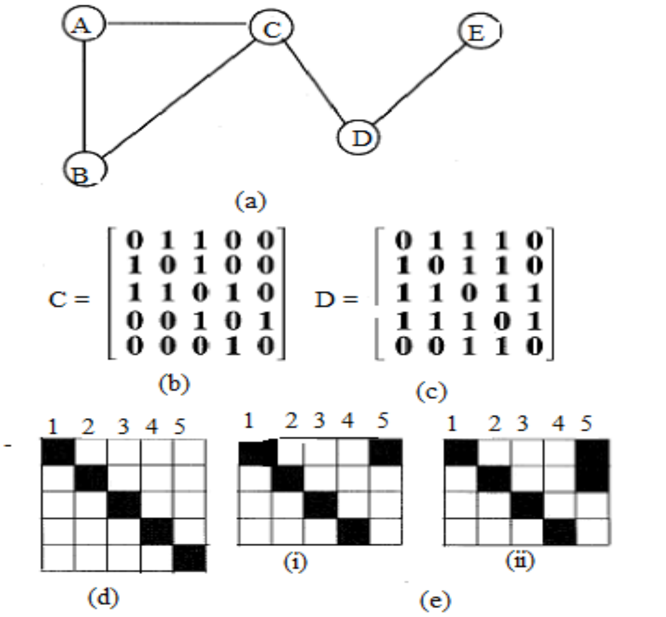

Fig. 1.(a) A Five Node Network (b)Corresponding C Matrix (c) Compatibility Matrix (d) Trivial TDMA schedule (e) Optimal TDMA Schedule.

Fig. 1 shows an example of five node PRNET. Fig. 1(a) depicts the association of nodes with each other. Like node-1 can communicate with node-2 and node-3 in a direct manner, but it cannot directly communicate with node-4 or node-5. Node -1 can communicate with node -5 via node – 3 and 4. Here nodes 3 and 4 are known as intermediate nodes. Fig. 1(b) denotes the connectivity matrix whereas fig. 1(c) represents the compatibility matrix.

III. RELATED WORK

An important research topic in multi-hop radio networks is to find a scheduling scheme, which allows nodes to communicate efficiently while avoiding message loss due to simultaneous transmissions from the neighboring nodes. A simple technique for coloring the nodes of the graph is Greedy Algorithm [14]. The greedy algorithm functions by coloring the nodes of a graph, initially defined in some particular order, with the minimum number of colors However, studies have shown that the greedy algorithm performs poorly.

Another approach used is known as the DSaturalgorithm[15]. It uses a heuristic to dynamically change the ordering of the nodes.The next vertex is chosen in a manner that the adjacent vertices have largest number of different colors assigned to them. Given a fixed integer k, considering an optimization problem, known as k-Graph coloring problem, which determines the minimum number of colors required to color the graph (k-colors), so that the number of conflicting edges is minimized. It then applies the greedy algorithm on the nodes to color it.

A number of algorithms have been proposed in [16, 17] finding out the optimum solution for scheduling problem. Further classification of these algorithms can be done into methods such as mean field annealing, neural networks, genetic algorithms, etc.These algorithms function on basically two points : one minimizes the frame length without taking into consideration the slot usage, while the other tries to maximize the slot utilization within the frame. Finding out an optimized result for these steps separately will not yield a good result. Hence, in order to get an improved result with the optimized solution, we need to incorporate both these criteria together.

A finite state machine is used in the work by Ahmad, Al-Kazemi, and Das [18].This defines the maximum compatibles as a maximal set of nodes which can be allocated the same time slots without any conflicts. It is known as maximal as any other node being scheduled in the same time slot would result in conflict.

Various techniques of Artificial Neural Networks have also been applied to solve the given problem. Ephremides and Truong, proposed a centralized and distributed algorithm for finding out maximal schedules, such that each node is assigned at a time slot diagonally; nodes having higher priority are allotted with the remaining time slots.

It was Hopfield and Tank, who first proposed neural network approach for combinatorial optimization problems. In this approach the neural network maps the Hopfield network with the k-coloring. This is done by reducing the network to a maximal independent set problem and then it maps it to the Hopfield network. If the algorithm does not gets an optimal solution in one phase, then the value of the proper coloring k is increased and the procedure is repeated.

Other successful methods to solve this broadcast scheduling problem are the genetic algorithms and the hybrid methods. While using GAs, the generation of a population represents the cycle forwhich the algorithm will run, the crossover operator helps in the cooperation step and then there is the mutation operator which helps in the self-evolution of the population. The Modified GA (MGA) approach defined by Chakraborty[19] defines a crossover operator which maintains only valid individuals in the population. It determines conflict free set of nodes as those nodes which can have the same timeslot schedules without any conflicts. However, as MGA uses fixed ordering, this leads to lesser diversity.

IV. THE PROPOSED APPROACH

Coloring SNR graphs is implemented in two steps. In the first step, calculation of SNR on the links is done. In the second step genetic algorithm is applied to the result from the first step for optimal coloring. Coloring can be simply explained as the problem of finding the color of the vertices of a graph in such a manner that no two adjacent vertices share the same color. The main objective of graph coloring is to minimize the total number of colors used in the assignment. It is an NP-hard problem.

Graph coloring finds wide application such as in register allocation, broadcast scheduling, etc. Finding out the optimum scheduling for a wireless multi-hop network is an example of graph coloring. In this work, we first find out the Signal-to-Noise ratio on each link. Then based on the adjacency matrix so obtained depicting the possible transmissions between the nodes having the connectivity, we optimize the solution. The optimization is done using a Genetic Algorithm. Genetic Algorithm is heuristic search technique that simulates the process of natural selection and evolution. Genetic Algorithms are effective, robust search procedure for NP-Complete problems.

A. Calculation of SNR on the links :

Let us denote the communication link as $l_i = (s_i, r_i)$, here s_i is the sender whereas r_i is the receiver node. Signal-to-Noise Ratio between any two particular nodes, which are within the transmission radius is calculated by using the following formula:

$$SNRi = Pi/N$$

In the above formula, the *SNRi*is the signal-to-noise ratio on the link $l_i$experienced at the receiver r_i,P_i is the received power atr_i of the signal which was transmitted by transmitter s_i and N represents the noise power experienced by the signal in the background.

In order to obtain the SNR on the links, in case of wireless networks, a classical model for radio signal propagation is adopted. This model is referred to as the log-distance path loss model. According to this model, the radio signal power or strength at a distance d from the transmitter is represented by P/d^{α}. Here α is the path loss coefficient. In this work, the value of α has been taken as 2 (actual value of α varies depending upon the environment). In the following work we have assumed that all the nodes transmit at the same constant power and that the background noise is also constant.

B. Genetic Algorithm for Optimization :

GA's provide an effective and robust search algorithm for NP-complete problems. Compared with other approaches, GA's are superior, because of wide applicability. They make few assumptions from the problem domain, and are not biased toward local minima. At the same time, GA's are very efficient in directing the search toward relatively prospective regions of the search space.

Chromosome Representation:

Here the chromosome is represented as the real-coded string which in turn represents a transmission schedule. Consider i^{th} position in the chromosome then it will represent a color for the i^{th} node in the graph. So here the chromosome will be of real coded. If graph is of N nodes, then the length of the chromosome is also taken as N. The next step in GA is to create a pool of solutions known as population. These solutions are generated randomly, without even considering how good they are.

Algorithm used for GA:

The basic steps followed to perform the GA is as given:

Pop : total population size
g : generation number;
Gen : maximum generation;
C(g) : set of chromosomes at generation ;
C"(g) : set of chromosomes after selection;
C'(g) : set of chromosomes after crossover.

Algorithm for GA (g, Gen, C(g), Pop)
begin
initially $g = 0$
  Creating Pop members of initial population $C(0)$;
  Calculate the fitness of all the members of $C(0)$;
while ($g <= Gen$)
    $g = g + 1$;
    C"(g) <— C(g-1); // k-tournament selection
$'(g)$ <— $C''(g)$; // crossover
C(g) <— C'(g);// mutation
end while
end

Whether a particular genetic search will be successful or not will depend upon balancing the two aspects of population diversity, one is exploring the different regions of the search space, and secondly the selection pressure to achieve the optimum point in a fast manner.

Fitness evaluation and Selection :

Here the fitness function considered is number of nodes that are assigned wrong colors i.e. two adjacent has been assigned the same color so it tends to be a wrong assignment, so in the fitness calculation we have to calculate the total number of wrong assignments. So if in fitness calculation if number of wrong assignments is zero then fitness value will be maximum and if it having some other value than zero then it is determined by

$$f = \frac{(n * n - count)}{n * n}$$

Here $count$ is the total number of wrong assignments. In this algorithm, k-tournament selection has been used. In k-tournament selection each time a random subset of k-solutions is selected from the population. Then the best solution out of these solutions is selected. This process is repeated until the number of populations in the newly formed population becomes equal to the total number of populations considered initially.

Crossover:

The process of crossover is done in order to recombine arbitrarily selected solutions in a pairwise manner, by substituting portions of them with each other, thereby producing a new divergent solution which would help in the exploration of the search space. A predetermined crossover rate defines the probability of performing crossover.

Here we have used a Neighborhood-Based Crossover Operators (NBCOs) which determine the genes of the offspring by extracting values from intervals defined in the neighborhood linked with the genes of the parents through probability distributions. NBCOs include a random component, i.e., they are basically non-deterministic. This process has used BLX-α. In this case two offspring are generated, $Hk = (h1k, h2k, \ldots..hik, \ldots..hnk), k = 1,2$ where hik is a randomly, i.e. it is a number chosen uniformly chosen from the interval$[Cmin - I\alpha, Cmax + I\alpha]$. Here,

$$Cmax = max\,\{ci1, ci2\}\ and$$
$$Cmin = min\,\{ci1, ci2\}\ and$$
$$I = Cmax - Cmin$$

Mutation:

In the mutation operation, a position in the chromosome is randomly mutated with a valid value which is in the range with some pre-defined mutation probability.

When applying GA to the wireless network, firstly the population is selected randomly by assigning the nodes some colors randomly and population size is fixed. The fitness of the population is then evaluated with the help of fitness function. The fitness function is checking the population with the help of the one-hop matrix C. After finding the no. of conflicts (fitness function) , and choosing the best population according to the k-tournament selection, now the population is selected for crossover and mutation operators.

V. Simulation And Results

The proposed method for finding out the optimum coloring for the SNR graphs was tested for a large number of times, using networks of different sizes and degrees of connectivity. As there are no benchmark sets available, we have performed simulation taking networks of different sizes. For simulation purposes, the graphs were tested for 100 meters, 150 meters and 250 meters of transmissions ranges.

Problem Setup:

For the experimental setup, the transmission power of each node was kept fixed at 100mW. The background noise was also fixed (N_0 = 5e-5). For calculating the SNR value, a code was written in MATLAB. This code returns the connectivity matrix of the nodes randomly deployed in an area, here it is taken as 500 x 500 units, depending on the criteria that the SNR value of the link must be above the threshold value.

The matrix so obtained is then used in the Genetic Algorithm so as to obtain the optimized result. The crossover probablility P_c= 0.8 and the mutation probability is taken as P_m= 0.008.The GA was run for several times for each particular set of values in order to get the best value.

Results :

We created a network of randomly connected nodes. Networks having nodes equal to 10, 20, 30, 40, 50 and 60 were used. Each value was run 30 times and the average value of the results so obtained is shown in Table 1. Each time up to 500 generations were executed for networks having less than 500 nodes, sufficient for the result to be stabilized. For networks having nodes greater than 500 nodes, the number of generations will have to be increased to 1000.

TABLE 1

Results for SNR graph having Txn Range as 250 mts.

No. of Nodes	No. of Colors	No. of Generations	Computational time (in secs)	Max. Degree
10	4	1	0.354	4
20	9	34	3.621	15
30	11	68	9.204	20
40	21	98	30.271	32
50	21	283	268.89	40
60	30	280	566.233	48

Graphical Analysis of SNR Graphs with UDG Graphs :

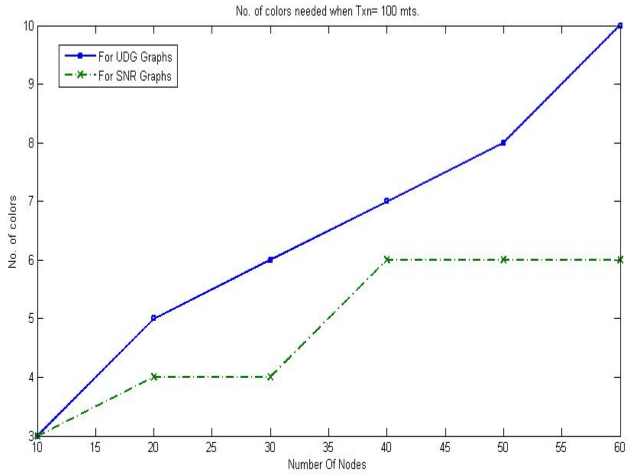

Fig. 1. No. of Colors required for SNR and UDG graphs when Txn Range is 100 mts.

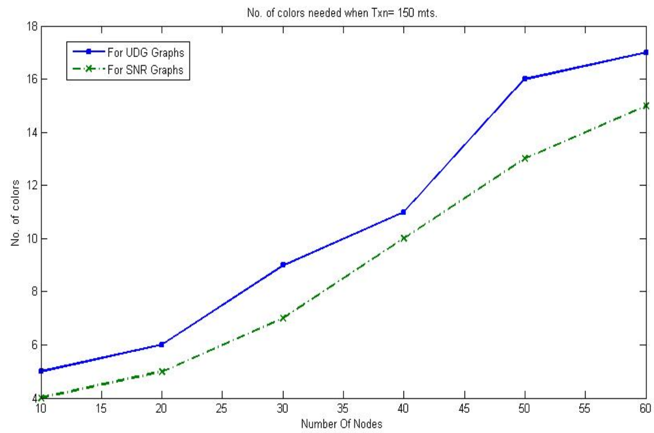

Fig. 2.No. of Colors required for SNR and UDG graphs when Txn Range is 150 mts.

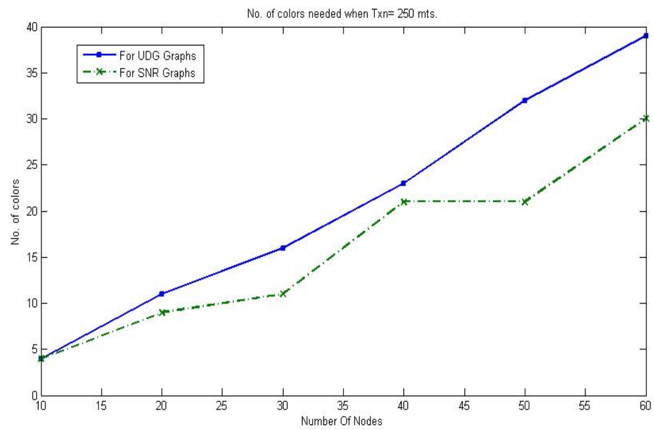

Fig. 3. No. of Colors required for SNR and UDG graphs when Txn Range is 250 mts.

Figure 1, 2 and 3 all depict the number of colors required for SNR and UDG graphs with the networks having 10, 20, 30, 40, 50 and 60 nodes. The axes of deployment have been taken as 500 x 500. In all the graphs so obtained, it has been found that for a given particular number of nodes in the network, the SNR gives better optimized result compared to UDG graphs.

Fig. 4 gives the analysis of number of generations required to reach the stable result by both the graphs. It was found that the SNR was able to reach the stabilized result quickly compared to UDG graph and hence was time efficient.

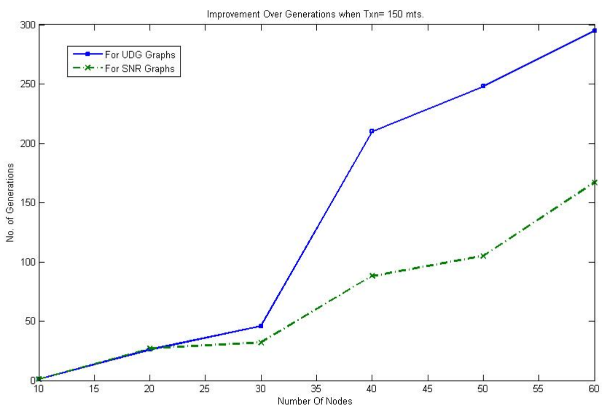

Fig. 4. Number of generations required to reach the result.

Genetic Algorithm is tested for many undirected graphs and the following figures will explain the results. The crossover probability chosen is 0.6 and mutation rate is 0.008.The number of generations chosen is 500. K-Tournament selection is used for selecting the best population among the population from the previous generations. Population size considered depends upon on the number of nodes in the graph. If there are less than 50 nodes in the graph the population size chosen is 50 and if population size is above 50 and less than 100, the population size chosen is 80.

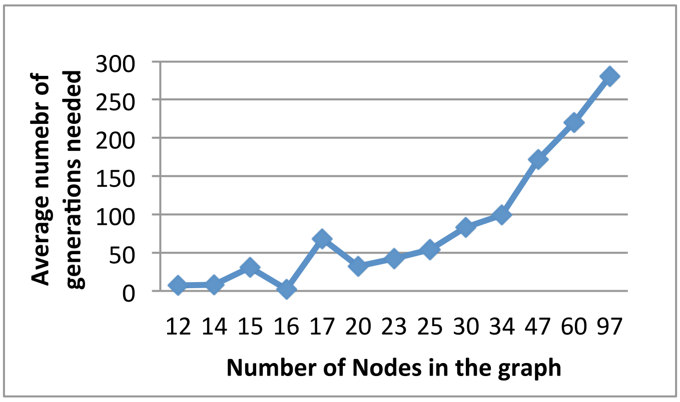

Fig.5 Average number of generations required to find out the minimum number of colors required for the graph.

So here by observing the fig.5, it gives the analysis of the number of generations required to reach the optimal solution by genetic algorithm for undirected graphs increases by increasing the number of nodes in the graph. Also if the number of edges in the graph increases then the time required to reach optimal solution i.e. finding minimum number of colors required also increases.Fig.5 illustrates it.

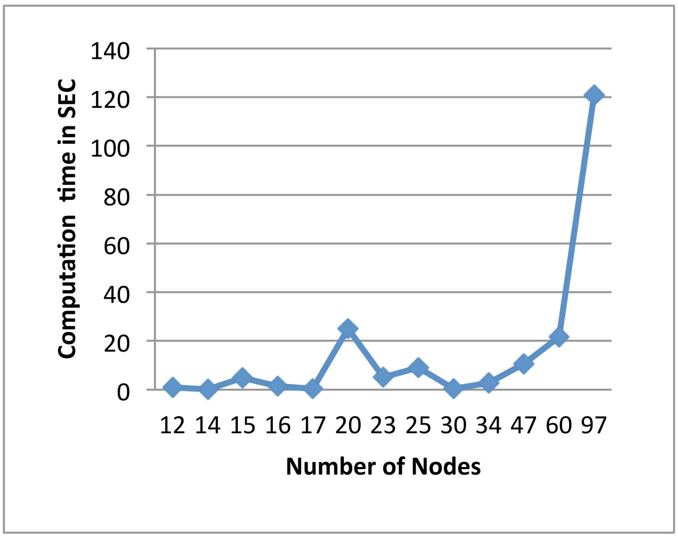

Fig.6 Comparison between computation time by genetic algorithm to reach the optimal coloring with number of nodes in the graph.

The computation time increases with increase in the number of nodes but may decrease with increase in number of nodes when number of edges in graph is less than compared with previous graphs where the number of nodes is less. So the computation time depends upon two factors namely the first one is number of nodes in the graph and the second one is number of edges present in the graph.

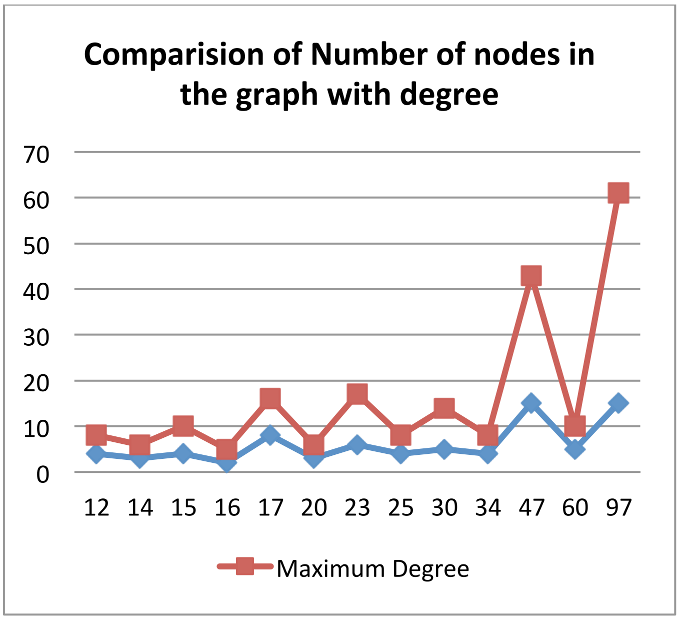

Fig.7 Comparison between degree and number of nodes in the graph.

Minimum number of colors required for coloring the graph depends on number of nodes, number of edges present in the graph, maximum degree of graph. So the fig.8, 9, 10 will show this relation. Number of colors required may increase if number of nodes increase with relatively same number of edges and may also decrease with increase in number of nodes where number of edges in the graph decrease.

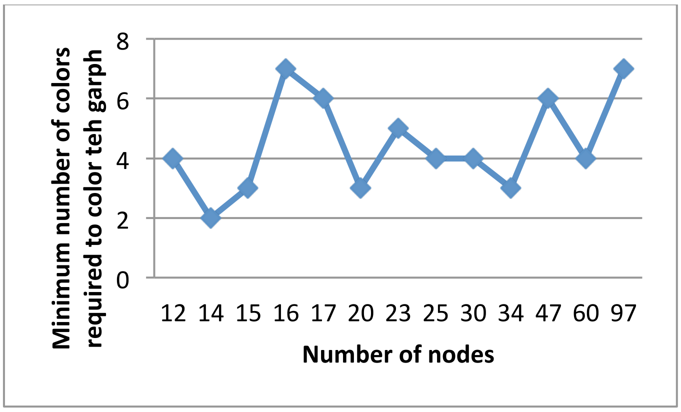

Fig.8 Comparison between number of nodes and minimum number of colors required for the graph.

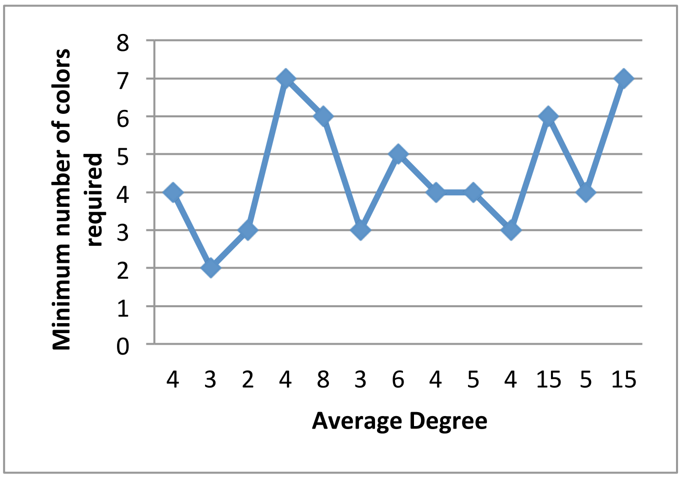

Fig.9 Comparison between Average degree of the graph and minimum number of colors required for the graph.

So from the above fig 8, 9 number of colors required to color the graph increase with increase with in number of nodes and number of edges. But also decrease with increase in number of nodes and decrease with number of edges.

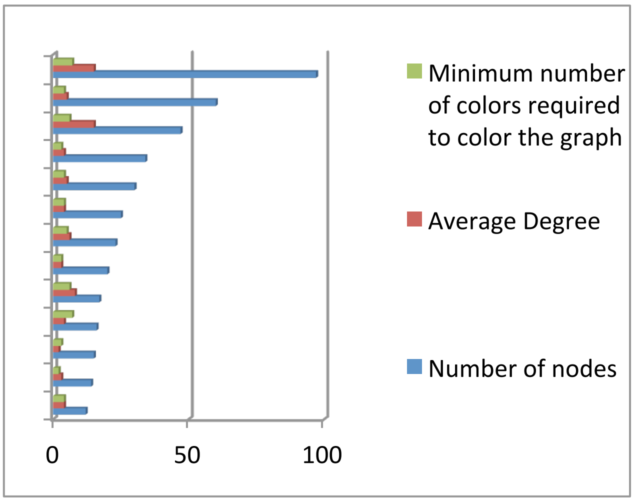

Fig.10 Comparison among number of nodes, average degree and minimum number of color required to color the graph.

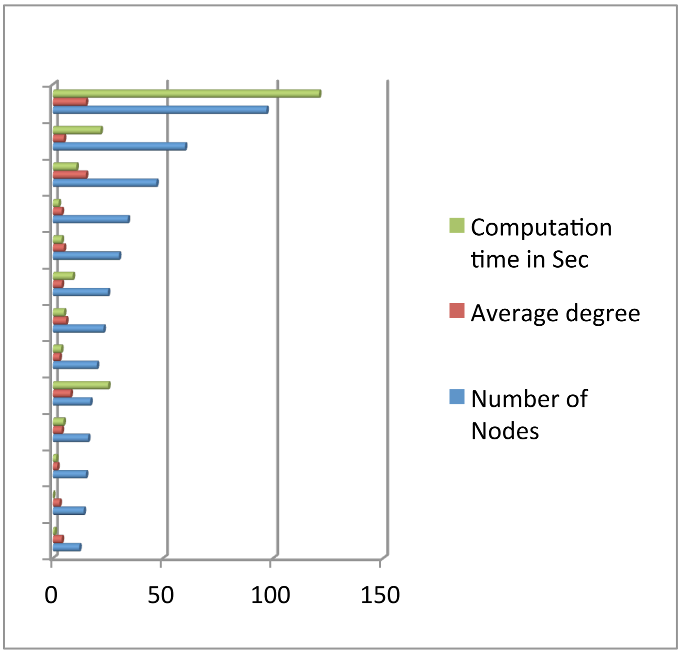

Fig.11 Comparison among Average degree, number of nodes in the graph and computation time (time required to find optimum number of colors).

VI. Conclusion

Broadcast scheduling problem is an NP-complete problem. Many randomized algorithms like neural networks and simulated annealing have been used to solve this problem. The approach using genetic algorithm is quite new to this problem. We have tried to use GA in optimizing the broadcast network transmission scheduling based on the SNR criterion. The results so obtained, upon comparison with the UDG graphs transmission scheduling, gave better results.

For smaller networks (like networks with 10 nodes) UDG and SNR function in a similar fashion. However, for larger networks, with about 30 and above nodes, SNR's performance was drastically improved. The proposed algorithm, however, is limited to only smaller number of nodes in the network because of the size of the available memory.

Genetic Algorithm developed can also be used for coloring of undirected graphs where it will find its uses in time table scheduling and register allocation. The number of colors needed to color the graph will increase with increase in number of nodes and number of edges present in the graph.

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