1. $A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_1 \mid b$

$A_3 \rightarrow A_1 A_2 \mid a$

Step 1:

$A_3 \rightarrow A_1 A_2 \mid a$

$A_3 \rightarrow A_2 A_3 A_2 \mid a$

$A_3 \rightarrow A_3 A_1 A_3 A_2 \mid b A_3 A_2 \mid a$

Resultant:

$A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_1 \mid b$

$A_3 \rightarrow A_3 A_1 A_3 A_2 \mid b A_3 A_2 \mid a.$

Step 2:

$A_3 \rightarrow A_3 A_1 A_3 A_2 \mid b A_3 A_2 \mid a$

$A_3 \rightarrow b A_3 A_2 \mid a$

$A_3 \rightarrow b A_3 A_2 B_3 \mid a B_3.$

$B_3 \rightarrow A_1 A_3 A_2$

$B_3 \rightarrow A_1 A_3 A_2 B_3.$

Resultant:

$A_1 \rightarrow A_2 A_3$

$A_2 \rightarrow A_3 A_1 \mid b$

$A_3 \rightarrow b A_3 A_2 \mid a \mid b A_3 A_2 B_3 \mid a B_3.$

$B_3 \rightarrow A_1 A_3 A_2 \mid A_1 A_3 A_2 B_3.$

NOW, A_3 is in the Griebach Normal Form.

Step 3:

$A_2 \rightarrow A_3 A_1 \mid b$

$A_2 \rightarrow b A_3 A_2 A_1 \mid a A_1 \mid b A_3 A_2 B_3 A_1 \mid a B_3 A_1 \mid b$

A_2 is in the Griebach Normal Form.

$A_1 \rightarrow A_2 A_3$

$A_1 \rightarrow b A_3 A_2 A_1 A_3 \mid a A_1 A_3 \mid b A_3 A_2 B_3 A_1 A_3 \mid a B_3 A_1 A_3 \mid b A_3$

A_1 is in the Griebach Normal Form.

$B_3 \rightarrow A_1 A_3 A_2$

$B_3 \rightarrow b A_3 A_2 A_1 A_3 A_3 A_2 \mid a A_1 A_3 A_3 A_2 \mid$

$bA_3A_2B_3A_1A_3 \; A_3A_2 \mid aB_3A_1A_3A_3A_2 \mid$
$bA_3A_3A_2.$

$B_3 \rightarrow A_1A_3A_2 \; B_3.$

$B_3 \rightarrow bA_3A_2A_1A_3 \; A_3A_2B_3 \mid aA_1A_3A_2A_2B_3 \mid$

$bA_3A_2B_3A_1A_3 \; A_3A_2B_3 \mid aB_3A_1A_3A_3A_2B_3 \mid$

$bA_3A_3A_2B_3.$

Resultant:

$A_1 \rightarrow bA_3A_2A_1A_3 \mid aA_1A_3 \mid bA_3A_2B_3A_1A_3 \mid$
$aB_3A_1A_3 \mid bA_3.$

$A_2 \rightarrow bA_3A_2A_1 \mid aA_1 \mid bA_3A_2B_3A_1 \mid aB_3A_1 \mid$
b

$A_3 \rightarrow bA_3A_2 \mid a \mid bA_3A_2B_3 \mid aB_3.$

$B_3 \rightarrow bA_3A_2A_1A_3A_3A_2 \mid aA_1A_3A_3A_2 \mid$

$bA_3A_2B_3A_1A_3A_3A_2 \mid aB_3A_1A_3A_3A_2 \mid$

$bA_3A_3A_2 \mid bA_3A_2A_1A_3A_3A_2B_3 \mid$

$aA_1A_3A_3A_2B_3 \mid bA_3A_2B_3A_1A_3A_3A_2B_3 \mid$

$aB_3A_1A_3A_3A_2B_3 \mid bA_3A_3A_2B_3.$

2. **Ambiguous grammar :**

A context free grammar G such that same word has two phase trees is said to be ambiguous.

An equivalent definition of ambiguity is that same word has more than one left most derivation or more than one ~~on~~ right most derivation.

$E \rightarrow E+E \mid E*E \mid (E) \mid id$ is ambiguous.

Deriving a string id + id * id

LMD	LMD
$E \Rightarrow E+E$	$E \Rightarrow E+E$
$E \Rightarrow id+E$	$E \Rightarrow E+E*E$
$E \Rightarrow id+E*E$	$E \Rightarrow id+E*E$
$E \Rightarrow id+id*E$	$E \Rightarrow id+id*E$
$E \Rightarrow id+id*id$	$E \Rightarrow id+id*id.$

For the word id + id * id. there exist two left derivations

<u>RMD 1</u>

$E \Rightarrow E + E$

$E \Rightarrow E + E * E$

$E \Rightarrow E + E * id$

$E \Rightarrow E + id * id$

$E \Rightarrow id + id * id$

<u>RMD 2</u>

$E \Rightarrow E * E$

$E \Rightarrow E * id$

$E \Rightarrow E + E * id$

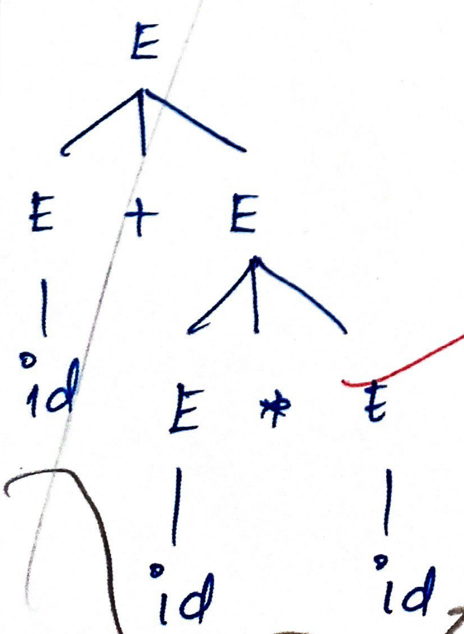
$E \Rightarrow E + id * id$

$E \Rightarrow id + id * id$

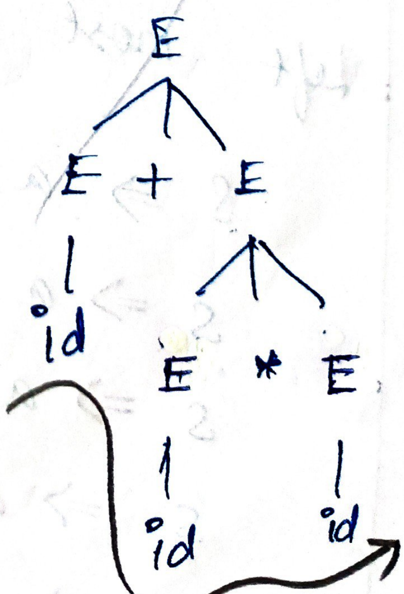
For the word id + id * id there exist two right derivations.

<u>Therefore</u> the given $E \rightarrow E+E \mid E*E \mid (E) \mid id$ is <u>ambiguous.</u>

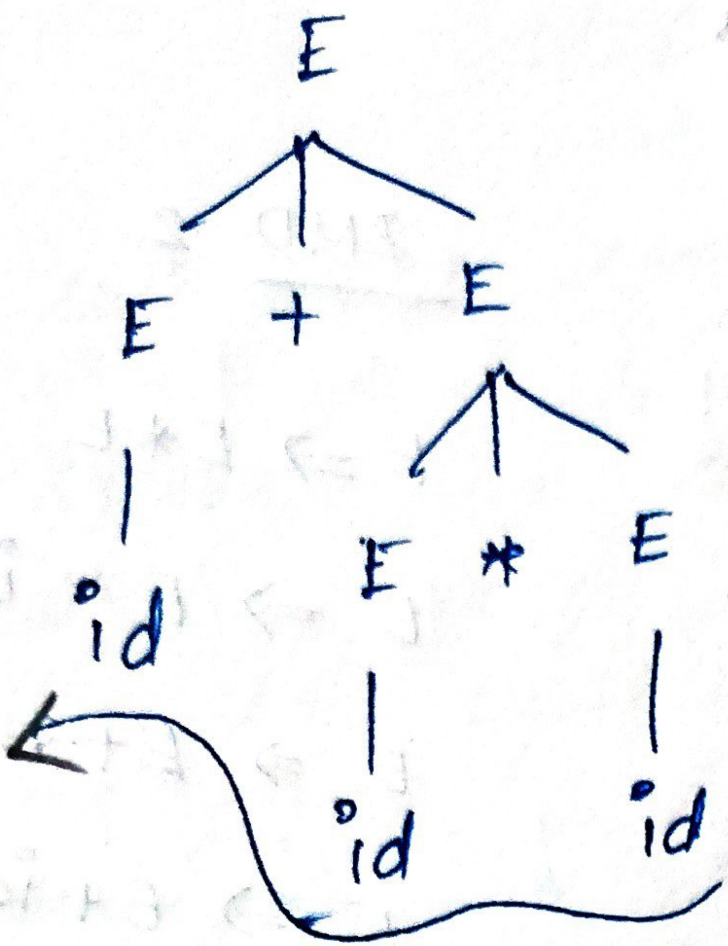
For <u>LMD</u>



For <u>RMD</u>



For RMD 1



For RMD2