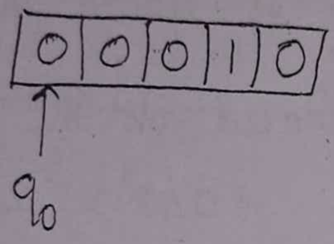
Constuct a Turing Machine to perform addition.

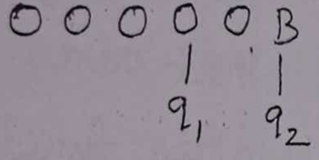
Solution:

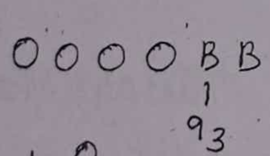
Suppose the input is $0^m 1 0^n$.

Finally the TM halts on tape containing 0^{m+n}

Suppose $w = 00010$.







	0	1	B
q_0	$(q_0, 0, R)$	$(q_1, 0, R)$	
q_1	$(q_1, 0, R)$	~~$(q_2, 0, R)$~~	(q_2, B, L)
q_2	(q_3, B, R)	(–	~~(q_3, B, R)~~
q_3	—	—	—

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, \{q_3\})$

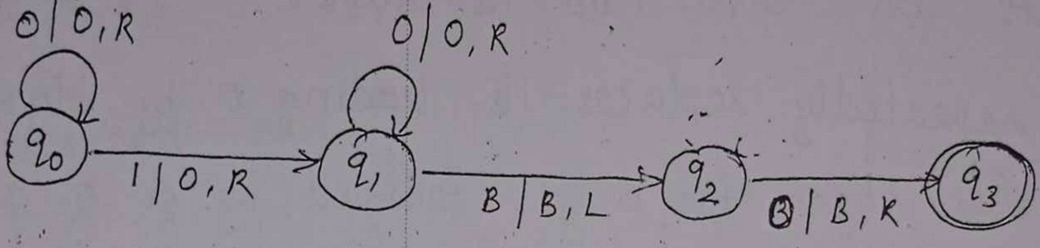
$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0, 1\}$.

$\Gamma = \{0, 1, B\}$.

$F = \{q_3\}$.

Transition Diagram



Simulating for w = 00010

$q_0 00010 \vdash 0 q_0 0010$

$\vdash 00 q_0 010$

$\vdash 000 q_0 10$

$\vdash 0000 q_1 0$

$\vdash 00000 q_1 B$

$\vdash 0000 q_2 BB$

$\vdash 0000 B q_3$.

Hence Turing Machine performs addition

proper subtraction.

Solution:

Proper subtraction $m \dot{-} n$ is defined as.

$$m \dot{-} n = \begin{cases} m-n & \text{for } m \geq n \\ 0 & \text{for } m < n \end{cases}$$

* The TM started with $0^m 1 0^n$ on its tape and halts on $0^{m \dot{-} n}$ on its tape.

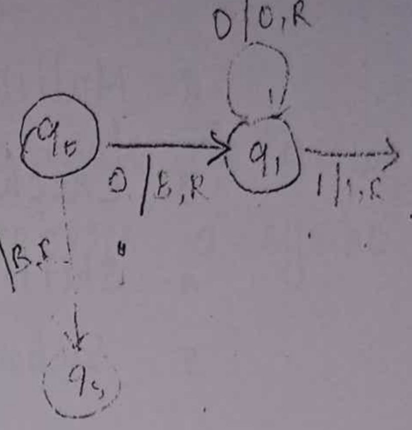
* M repeatedly replaces its leading 0 by blank then searches right for a 1 followed by a 0 and changes the 0 to 1.

* Next M moves left until it encounters a blank and then repeats the cycle. The repetition ends if

1) Searching right for a 0, M encounters a blank. Then the n 0's in $0^m 1 0^n$ have all been changed to 1's. and n+1 of the m 0's have been changed to B. M replaces the n+1 1's by a 0 and n B's leaving m-n 0's on its tape.

2) Beginning the cycle M cannot find a 0 to change to a blank, because the first m 0's already have been changed. Then $n \geq m$, so $m \dot{-} n = 0$. M replaces all remaining 1's and 0's by B

q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6	—	—	—



M is defined as

$M = \{Q, \Sigma, \Gamma, \delta, q_0, B, F\}$.

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$.

$\Sigma = \{0, 1\}$.

$\Gamma = \{0, 1, B\}$.

$F = \{q_6\}$.

A sample computation of M on input 0010 is

$q_0 0010 \vdash Bq_1 010 \vdash B0q_1 10 \vdash B01q_2 0 \vdash$

$B0q_3 11 \vdash Bq_3 011 \vdash q_3 B011 \vdash Bq_0 011 \vdash$

$BBq_1 11 \vdash BB1q_2 1 \vdash BB11q_2 . \vdash$

$BB1q_4 1B \vdash BBq_4 1BB \vdash Bq_4 BBBB . \vdash$

$B0q_6 BB \ldots$