

1. Regular Language:

* If A is a regular language then A has a pumping length P. Then any string $s \mid s \mid \geq P$ may be divided by 3 part (x, y, z) and so that true the following condition

$$xy^iz \in A \text{ for } i \in \mathbb{N}, i \geq 0$$

$$|y| > 0$$

$$|xy| \leq P.$$

2. Deterministic PDA

* PDA Define $(Q, \Sigma, T, \delta, q_0, z_0, F)$ is Deterministic.

For $q \in Q$, $z \in z_0$ and (q_0, Σ, z_0) not empty and (q_0, a, z_0) empty and all $a \in \Sigma$

For $q \in Q$, $z \in z_0$ and $\sum_{a \in \Sigma}$ does not (q_0, a, z_0) has a more than one element.

3. Technique

* The Technique for Turning Machine Construction.

1. ~~get~~ storage in infinite control
2. checking off symbol
3. shifting off
4. Multi track
5. subroutine.

4. Turning Machine:

* The basic mode of Turning Machine

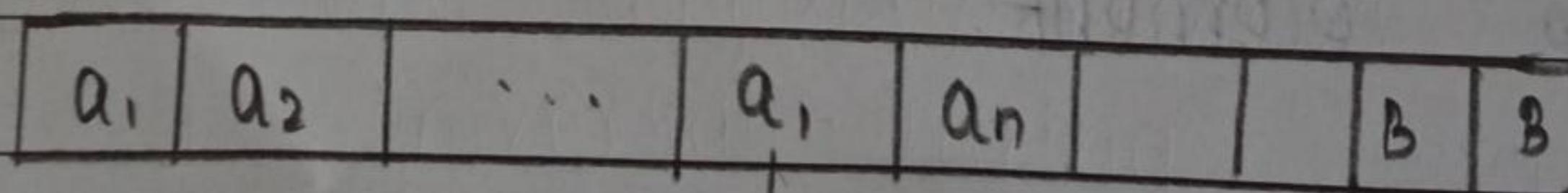
1. finite control
2. An Infite Tape
3. Tape Head.

* The input tape divided into a cell and the Tape infinite right.

* each cell must hold a only symbol

* Each Tape head move the both direction.

$$M = \{ Q, \Sigma, T, S, q_0, B, F \}$$



Finite
control

5. DEFINITION

* The instantaneous define has a $\alpha_1 Q \alpha_2$:

where

Q = is a current state.

α_1, α_2 = is a string input.

$(\alpha_1 Q \alpha_2)$ is a instantaneous description of Turning Machine.

6. Multitape:

* Multitape is a variation of the Turning Machine

* It utilizes several tape

* Each tape has a head for reading and writing.

* The first input appears in 1 and other has blank symbol.

7. Recursively enumerable

* The language accepted by Turning Machine is recursively enumerable language.

- * The recursively enumerable set include a regular and CFL.
- * Some language of recursively enumerable not accepted by the Turing machine.

Recursive Language:

- * The recursive set is subset of the recursively enumerable language.
- * All language of recursive set is accepted by the Turing machine.

8. Universal Turing machine.

- * The problem is undecidable.
- * The problem "Does turing machine M accept input w" Both M and w is a parameter of the problem.
- * In formulation problem is a language. The input restricted be over the $\{0,1\}^*$. And the tape alphabet $\{0,1,B\}$. Thus called a universal turing machine.

11 b)

$$\delta(q_0, 0, z_0) = \delta(q_0, xz_0)$$

$$\delta(q_0, 0, x) = \delta(q_0, xx)$$

$$\delta(q_0, 1, x) = \delta(q_1, e)$$

$$\delta(q_1, 1, x) = \delta(q_1, e)$$

$$\delta(q_1, e, x) = \delta(q_1, e)$$

$$\delta(q_1, e, z_0) = \delta(q_1, e).$$

$$\{ f q_0, q_1, y, f_0, 13, f_x, z_0^y, \delta, q_0, q_1 \}$$

$$V = \{ S, [q_0, x, q_0], [q_0, x, q_1], [q_1, x, q_0], \\ [q_1, x, q_1], [q_0, z_0, q_0], [q_0, z_0, q_1], \\ [q_1, z_0, q_0], [q_1, z_0, q_1] \}$$

$$T = \{0, 13\}$$

$$S \rightarrow [q_0, z_0, q_0]$$

$$S \rightarrow [q_0, z_0, q_1].$$

$$\delta(q_0, 0, z_0) = \delta(q_0, xz_0)$$

$$\times [q_0, z_0, q_0] = 0[q_0, x, q_0] [q_0, z_0, q_0]$$

$$\times [q_0, z_0, q_0] = 0[q_0, x, q_1] [q_1, z_0, q_0]$$

$$[q_0, z_0, q_1] = 0[q_0, x, q_0] [q_0, z_0, q_0]$$

$$[q_0, z_0, q_1] = 0[q_0, x, q_1] [q_1, z_0, q_1].$$

$$\delta(q_0, 0, x) = \delta(q_0, xx)$$

$$x [q_0, x, q_0] = 0 [q_0, x, q_0] [q_0, x, q_0]$$

$$x [q_0, x, q_0] = 0 [q_0, x, q_1] [q_1, x, q_0]$$

$$[q_0, x, q_1] = 0 [q_0, x, q_0] [q_0, x, q_0]$$

$$[q_0, x, q_1] = 0 [q_0, x, q_1] [q_1, x, q_1]$$

$$\delta(q_0, 1, x) = \delta(q_1, e)$$

$$[q_0, x, q_1] = 1$$

$$\delta(q_0, 1, x) = \delta(q_1, e)$$

$$[q_0, x, q_1] = 1$$

$$\delta(q_0, e, x) = \delta(q_1, e)$$

$$[q_1, x, q_1] = e$$

$$\delta((q_1, q_0), q_1)$$

$$\delta(q_1, e, q_0) = e(q_1, e)$$

$$[q_1, q_0, q_1] = e$$

- * above the NO production for the $[q_1, x, q_0]$, and $[q_1, y_0, q_0]$
- * Find the similar element then remove that production also.
- * then remove the $[q_0, x, q_0]$ and $[q_0, x, q_0]$ and $[q_0, z_0, q_0]$ and $[q_0, y_0, q_0]$.
- * then Final Resultant are:

$$S \rightarrow [q_0, z_0, q_0]$$

$$S \rightarrow [q_0, y_0, q_1].$$

Resultant:

$$[q_0, z_0, q_1] = 0 [q_0, x, q_0] [q_0, z_0, y_1]$$

$$[q_0, y_0, q_1] = 0 [q_0, x, q_1] [q_0, y_0, z_1]$$

$$[q_0, x, q_1] - 0 [q_0, x, q_0] [q_0, x, y_1]$$

$$[q_0, x, q_1] = 0 [q_0, x, q_1] [q_0, x, q_1].$$

$$[q_0, x, q_1] = 1$$

$$[q_1, x, q_1] = 1$$

$$[q_1, x, q_1] = \epsilon$$

$$[q_1, z_0, q_1] = \epsilon.$$

$$S \rightarrow [q_0, z_0, q_0]$$

$$S \rightarrow [q_0, z_0, q_1].$$

(2a) Turning Machine:

* The Max Basic Mode of the Turing Machine

1. finite control

2. An Infinite Tape

3. Tape Head.

* The input tape divided into a cell and the Tape infinite right.

* Each cell must contain a one symbol

* Each Tape head move the both direction.

$$M = \{ Q, \Sigma, T, \delta, q_0, B, F \}$$

Q - Set of State

Σ - Set of input

T - Set of Stack symbol

q_0 - initial state

δ - set of transition

B - Blank

F - Set of Final state.

$0^n 1^n$ $0^2 1^2$

0	0	1	1	B
---	---	---	---	---

* First replace the 0 to X then move to the right position then.

Move to the right position.

* then replace the 1 to Y then move to find the left most 0 then replace the next position

0 to X

* Then find the right most 1 then replace the next position 1 to Y

* Then read the Blank symbol.

$x \rightarrow$	X				
\emptyset	\emptyset	X	1	B	
↑	↑	↑	↑		
X	X	Y	Y		

	0	1	X	Y	B.
q_0	(q_1, X, R)			(q_3, Y, R)	
q_1	$(q_1, 0, R)$	(q_2, Y, L)		(q_1, Y, R)	
q_2	$(q_2, 0, L)$		(q_0, X, R)	(q_2, Y, L)	
q_3				(q_3, Y, R)	(q_4, B, R)
q_4	-	-	-	-	-.

check the Turing Machine

accept the 0011

$q_0 0011 \vdash x q_1 011 \vdash x q_1 11 \vdash$

$x q_2 x q_2 \quad x q_1 11 \vdash$

$x q_1 11 \vdash x y q_1 11 \vdash$

$x q_1 11 \vdash x x q_2 y y \vdash$

$x q_2 x y y \vdash x x q_0 y y$

$\vdash x x y q_3 y \quad \vdash x x y y q_3 B.$

$\vdash \text{xx YY B Q}_3.$

$M = \{q, \delta, T, S, q_0, B, F\}.$

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{0, 1\}$

$T = \{0, 1, X, Y, Z\}$

$q_0 = q_0$

$F = q_4$

δ

q_0	(q_1, X, R)	-	-	(q_3, Y, R)	-
-------	---------------	---	---	---------------	---

q_1	$(q_1, 0, R)$	(q_2, Y, L)	-	(q_1, Y, R)	-
-------	---------------	---------------	---	---------------	---

q_2	$(q_2, 0, L)$	$V - V, R$	(q_0, X, R)	(q_2, Y, L)	-
-------	---------------	------------	---------------	---------------	---

q_3	-	-	-	(q_3, Y, R)	(q_4, B, R)
-------	---	---	---	---------------	---------------

q_4	-	-	-	-	-
-------	---	---	---	---	---

13(b) Turing Machine:

* The Basic Mode of the Turing Machine :

1. Finite control

2. An Infinite tape

3. Tape Head.

* In Input Tape divided into a cell and the Tape infinite right.

* Each cell must contain a one symbol

* Each Tape head move the both direction.

$$M = \{Q, \Sigma, T, \delta, q_0, B, F\}.$$

Q - set of state

Σ - set of input symbol

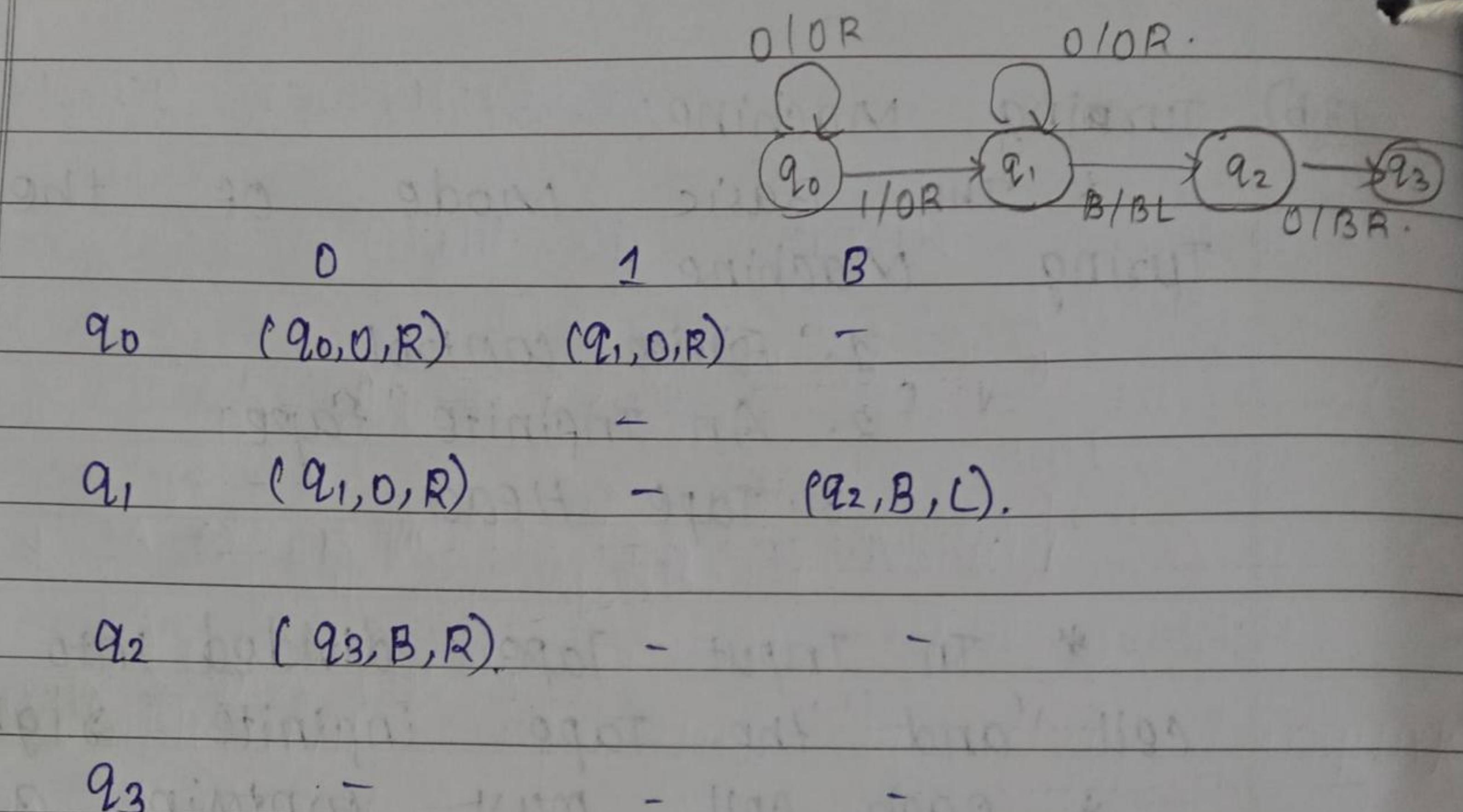
T - set of stack symbol

q_0 - initial state.

δ - set of transition.

B - blank symbol

F - set of final state.



$0^m 1 0^n$

~~00010~~
Now check = $0^3 1 0^1$

$q_0 00010 \leftarrow 0q_0 0010 \leftarrow$

$00q_0 010 \leftarrow 000 q_0 10 \leftarrow$

$0000 q_1 0 \leftarrow 00000 q_1 B.$

$\leftarrow 0000 q_2 BB. \leftarrow 0000 B q_3 B$

Result = $0000 B q_3 B.$

BD B13q₂ + BD Bq₄B + BDq₆BB.

Result = BDq₆BB.

M = { $\{Q\}$, Σ , T, S, q₀, B, F}.

Q = {q₀, q₁, q₂, q₃, q₄, q₅, q₆}

Σ = {0, 1}

T = {0, 1, B}

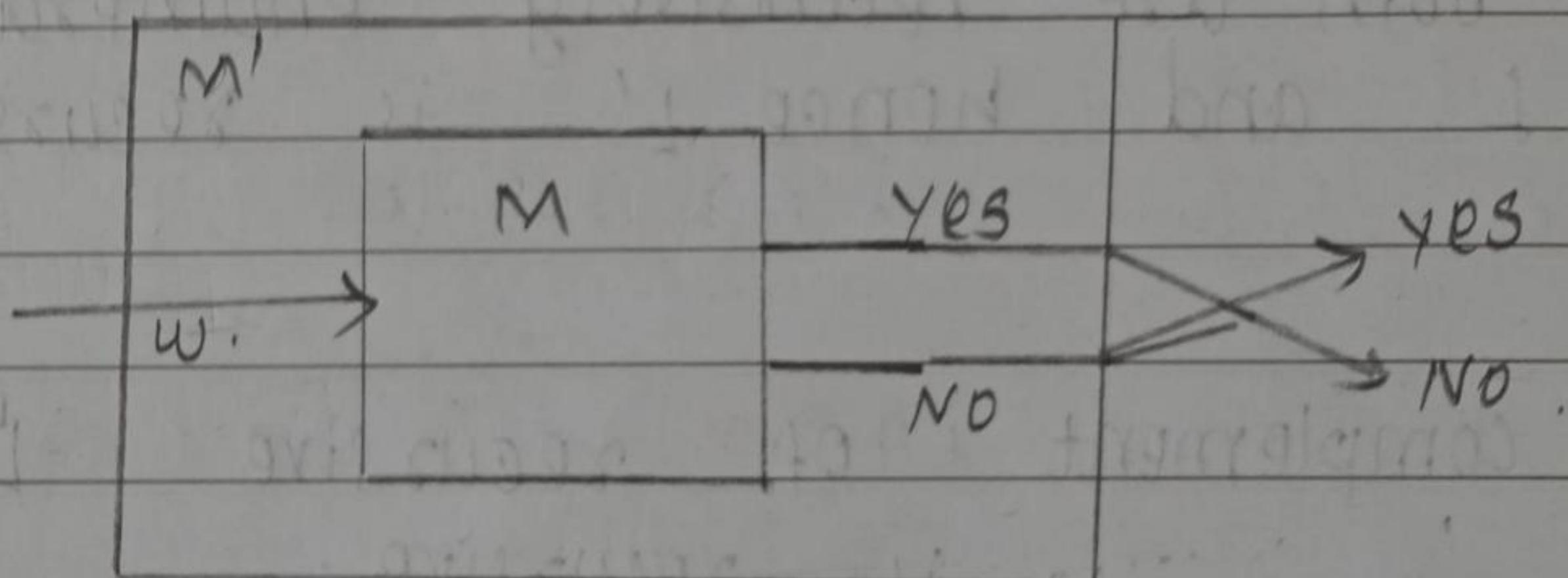
q₀ = q₀

F = {q₆}

14a) CLOSURE PROPERTIES.

1. complement of recursive language is recursive
 2. Union of two recursive language is recursive.
 3. union of two recursively enumerable language is recursively enumerable.
 4. If L and complement of L' both are recursively enumerable. Then L and hence L' is recursive.
1. complement of recursive Language is recursive.
- * Let L be a recursive language and M is a turing machine all input halts and accept L .
- * construct M' from M . M is enter a final state w . The M' halt without accepting.

- * M' is enter a Final state the M halt without accepted.
- * since one of those event is occur M is a Algorithm.
- * $L(M')$ is a L so, the complement of the recursive Language is recursive.
- * Hence proved complement of recursive language is recursive



2. union of the two recursive Language is recursive

union of the two recursively enumerable Language is recursively enumerable.

* L_1 and L_2 is a recursive Language accepted Algorithm M_1 and M_2 .



* Construct M from M_1 , simulate.

First M_1

* M_1 is accept M is accept and M_1 is reject M simulate the M and accept M accept.

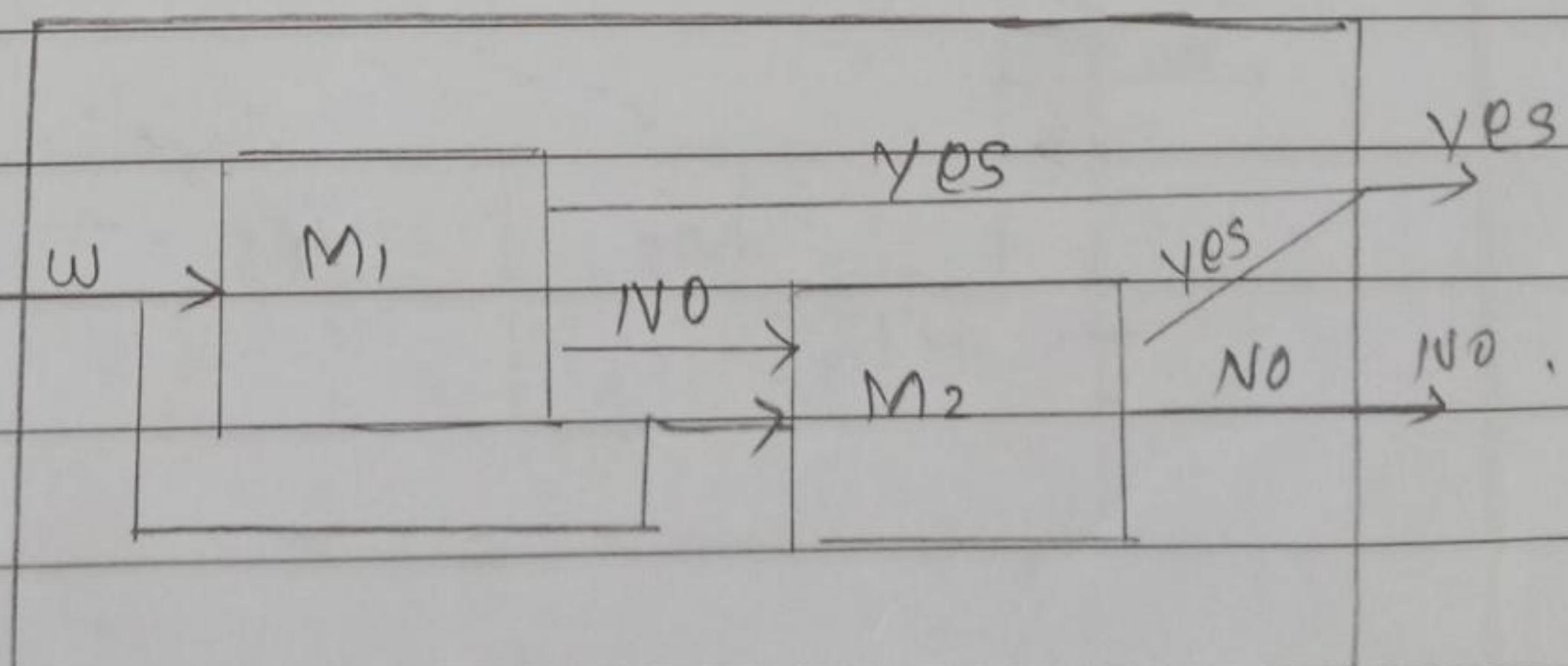
* Since M_1 and M_2 is a accepting algorithm so, M is $L_1 \cup L_2$.

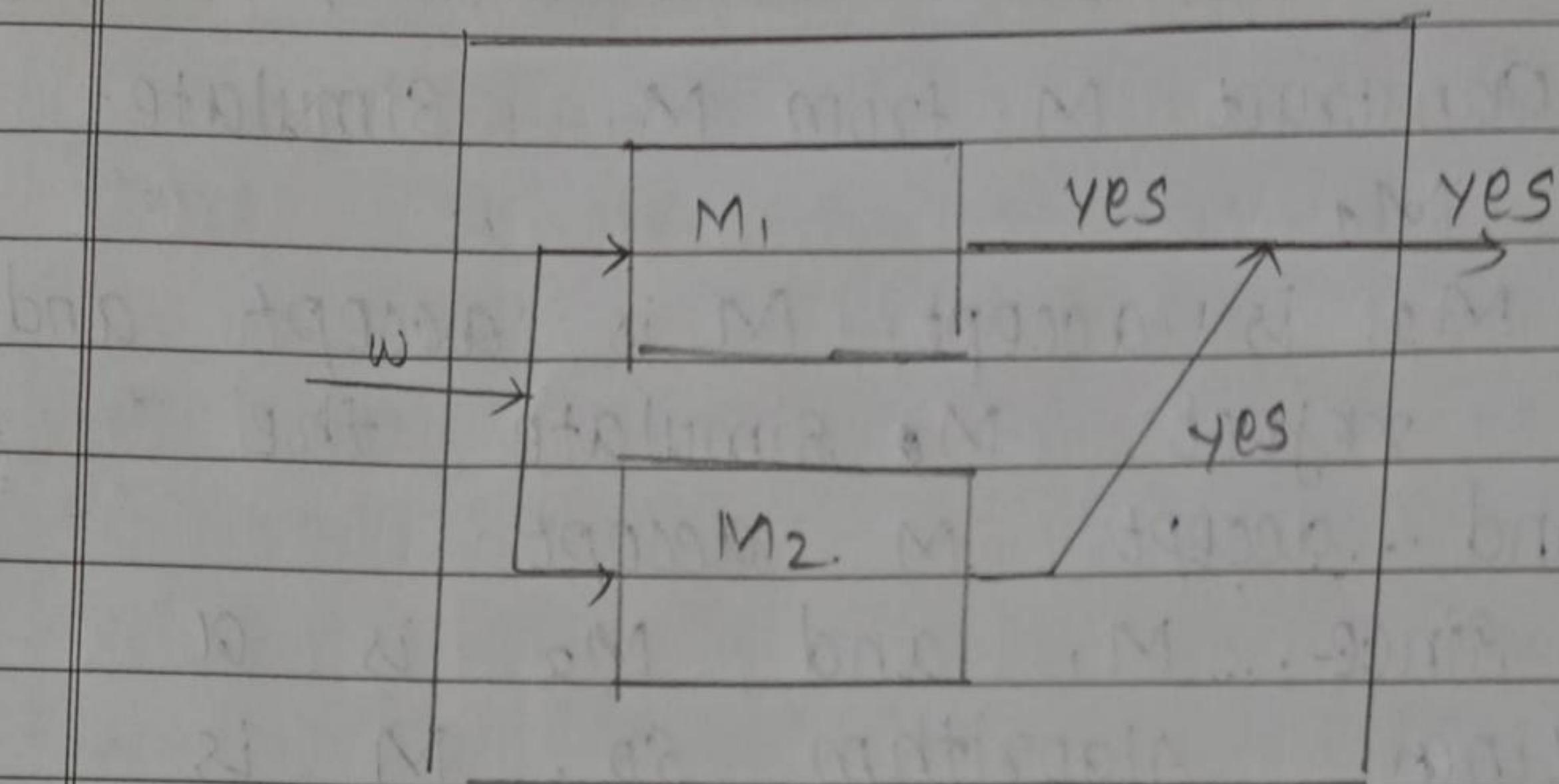
* since recursive Union of recursive language is recursive

* M_1 may not halt

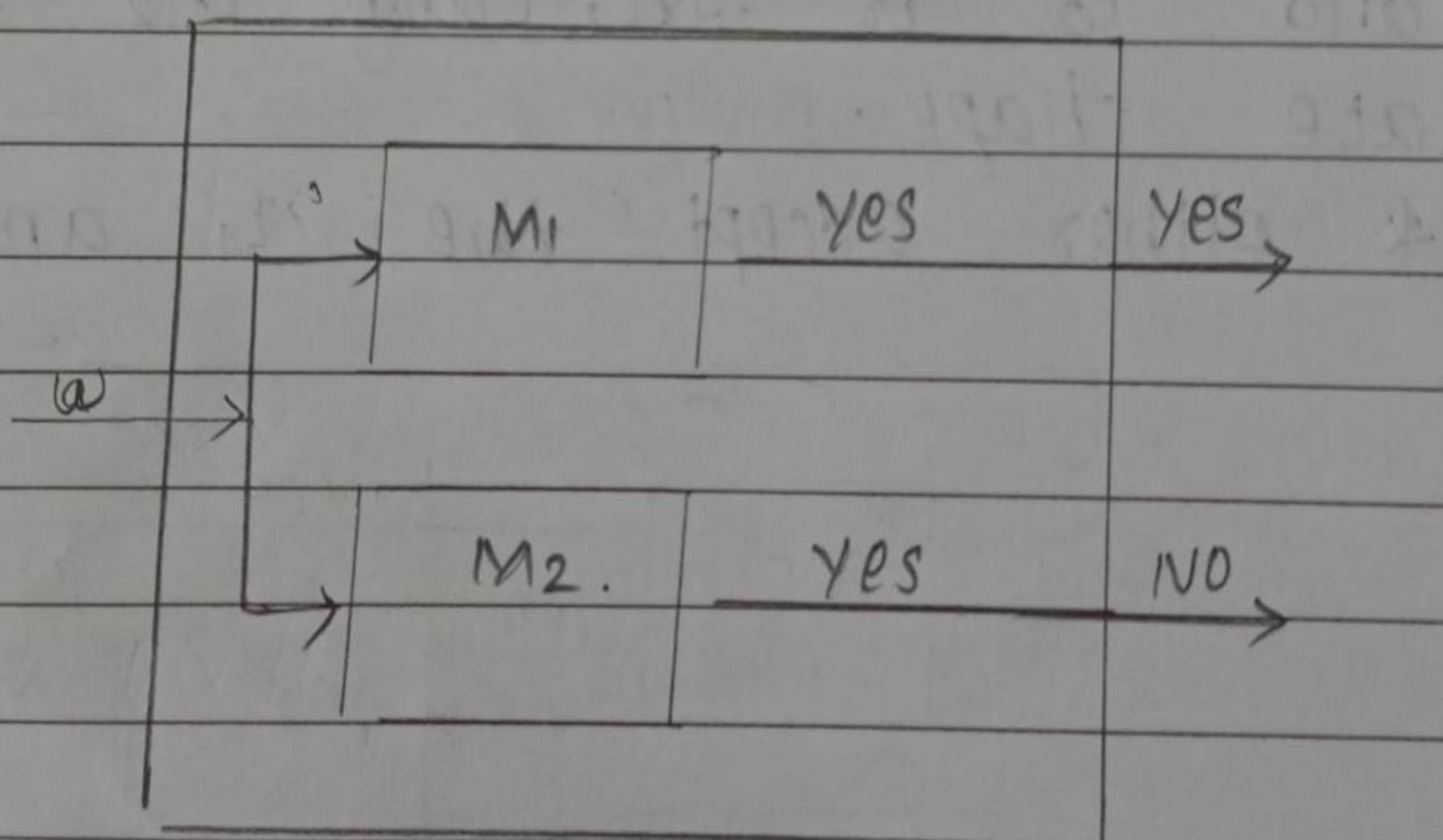
* Instead simulate simultaneously L_1 and L_2 or M_1 and M_2 in a separate tape.

* either accept the L_1 and L_2 .





It is a language L and complement of L' both are recursively enumerable. That L and hence L' is recursive.



- * Turing Machine M_1 and M_2 accepted language L and \bar{L}
- * construct M simulate the M_1 accept M accept reject w .
- * M simulate the M_1 and accept iff M accept.
- * we know that L and \bar{L} is accept.
- * The result is "yes" or "no" not a ~~both~~ say both.
- * So, LCM' is L so, L is recursive.
- * Hence proved the language L and \bar{L} is a recursively enumerable that L and complete \bar{L} is a recursive.

15(b) PCP.

First Pair

List A

#

List B.

H Q W

Group I

List A

X

#

List B.

X

X For in F

Group II

List A

q x

List B.

Y P S(q, x) = (P, Y, R)

Z Q X

P I Y S(q, x) = (P, Y, C)

Y Q #

P I Y S(q, B) = (P, Y, R)

Z Q #

P I Y S(q, B) = (P, Y, B)



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Additional Sheet

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Group III

List A

~~Q~~ x2y1

x2

y1

List B.

q

q

q

q For in F , x,y in F

Group IV

List A

q ##

List B

#

q For in F

M = {q, Σ, T, S, q0, B, q3}.

q = {q1, q2, q3, q}

T = {0, 1}.

Σ = {0, 1}

SOL:

Group I

First Pair.

List A

List B

#

$q_1, 01$

Group II.

List A

List B

1

1

0

0.

Group III

$$\delta(q_1, 0) = (q_2, 1, R).$$

$$q_1 0 = 1 q_2.$$

$$\delta(q_1, 1) = (q_2, 0, L)$$

$$0 q_1 1 = q_2 0 0.$$

$$1 q_1 1 = q_2 1 0.$$

$$\delta(q_1, B) = (q_2, 1, L).$$

$$0 q_1 B = q_2 0 1$$

$$1 q_{1,0} = q_{2,11}.$$

$$\delta(q_{2,0}) = (q_{3,0,L})$$

$$0 q_{2,0} = q_{3,0,0}$$

$$1 q_{2,0} = q_{3,2,0}.$$

$$\delta(q_{2,1}) = (q_{2,0,L})$$

$$q_{2,1} = 0 q_2.$$

$$\delta(q_{2,1,B}) = (q_{2,0,R})$$

$$q_{2,0} = 0 q_2.$$

GROUP $\overset{\leftarrow}{III}$

LIST A

LIST B

$| q_3 |$

q_3

$| q_3 0$

q_3

$0 q_3 |$

q_3

$| q_3 |$

q_3

$0 q_3$

q_3

$q_3 0$

q_3

GROUP IV.

LIST A

 $q_3 \# H$

LIST B

 $H \#$

$$\begin{aligned}
 q_1 01 &\vdash 1 q_2 1 \quad (\because q_1 0 = 1 q_2) & q_2 11 \\
 &\vdash 1 0 q_2 \quad (\because q_2 = 0 q_3) & q_2 0 11 \\
 &\vdash 1 q_2 0 \\
 &\vdash 0 q_2 \\
 &\vdash q_3.
 \end{aligned}$$

16(a) Pumping Lemma for regular Language.

* If A is a regular language then A has a pumping length p. then any string S. $|S| \geq p$ may be divided by 3 part (x,y,z) and so that true the following.

condition:

 $x y^i z \in A \text{ for } i \geq 0.$
 $|y| > 0$
 $|xy| \leq p.$



Application

- * Assume A is a regular language.
- * A has a pumping length (say p)
- * s is a string, a string long string pumped p ($|s| \geq p$)
- * s is divided into a 3 part (x, y, z)
- * $xy^iz \notin A$ for any $i \geq 0$.
- * Show that no one satisfy a three condition three condition at a same time.
- * s is not pumped = CONTRADICTION.

$$i) L = \{a^n b^n \mid n > 0\}$$

* Assume A is a regular language

* A has a pumping length p.

$$a^n b^n = a^p b^p$$

$$p = 7$$

$$S = a^7 b^7$$

$$S = \text{aaaaaaa } \underline{\text{bbbbb}}$$

S divided into a three part

$$S = xyz$$

Case 1

$$S = \underbrace{\text{aaaaaaa}}_x \underbrace{\text{bbbbb}}_y \underbrace{\text{bbb}}_z$$

$$S = xyz$$

$$i = 2$$

$$= xy^2 z$$

$$= aa \text{aaaaaaa} a bbbb bbb$$

$$= a^n b^7$$

$$= a^n b^7 \notin A$$

case 2

$$S = \underbrace{aaa \ a \ aaa}_{x} \ b \underbrace{bbb \ bbb}_{y \ z}$$

$$S = xy^iz \quad i=2.$$

$$= xy^2z$$

$$= aa \ a a \ a a a b \ b b b b b b \ b b b .$$

$$= a^7 b^{10} \notin A$$

case 3

$$S = \underbrace{aaa \ a}_{x} \ \underbrace{aaa \ bb}_{y} \ \underbrace{bbb \ bbb}_{z}$$

$$S = xy^iz \quad i=2.$$

$$= xy^2z$$

$$= aaa \ aaabb \ aaabb \ bbbbb$$

$$= a^{10} b^9 \notin A$$

L is not a regular.

$L = \{a^n b^n \mid n > 0\}$ not a regular.

pumping Lemma for CFL.

* If A is a context Free Language then A has a pumping length p. then any string s $|s| \geq p$ may be divided into 5 part (~~xxxzx~~) and so that true the following.

condition:

$$uv^i xy^i z \in A \text{ for } i \geq 0.$$

$$|vy| > 0$$

$$|vxy| \leq p.$$

Application:

* Assume A is a context Free language.

* A has a pumping length (say p)

* S is a string , a any long.

String pumped p ($|s| \geq p$)

* S is divided into a 5 part (u, v, x, y, z).

* Show that a no one satisfy a three condition at a same time.



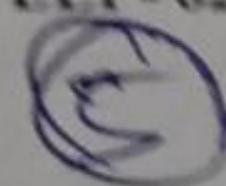
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S is not pumped = CONTRADICTION.

$$\cap L = \{ a^n b^n c^n \mid n > 0 \}$$

* Assume A is a context Free Language

* A has a pumping length P.

$$a^n b^n c^n = a^p b^p c^p$$

$$P = 5$$

$$S = a^5 b^5 c^5$$

$$S = aaaa a bbbb b cccc c$$

$$S = uvixyiz$$

case 1

$$S = \underbrace{aaaa}_{u} \underbrace{aa}_{v} \underbrace{bb}_{x} \underbrace{bb}_{y} \underbrace{ccccc}_{z}$$

$$= uv^i xy^i z \quad i=2$$

$$= aa aaaa aaaa bbbb bbbb cccc c$$

$$= a^8 b^7 c^5 \notin A$$

CASE 2.

$$S = \underbrace{aaaa}_{u} \underbrace{aa}_{v} \underbrace{bb}_{x} \underbrace{bb}_{y} \underbrace{ccccc}_{z}$$

$$= uv^i xy^i z \quad i=2$$

$= aa\ aaab\ aaab\ bbb\ bcc\ bcc\ ccc$
 $= a^8 b^7 b^7 \notin A$

case 3

aaaaaa bbbbbb cccccc
u x y
 uv^ixyiz $i = 2$

$= aaaa\ aaa\ bbb\ bbb\ bbb\ ccc\ ccc$
 $= a^6 b^7 c^5 \notin A$

so, L is not a context

Free Language.