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PART - A

1) Pumping lemma for regular language:

Pumping lemma is mostly used to prove that the lemma is said to see not - regular.

The condition to prove not regular is

1) $(xy^i z) = 0$

2) $|y| \geq 0$

3) $|xy| > 0$.

2) Deterministic Pushdown Automata.

A pushdown automata $M = \{Q, \Sigma, \delta, q_0, F\}$ is said to be deterministic if.

i) for each q in Q and Z in Γ, for all $\delta(q, \epsilon, z)$ for all non empty set to be in $\delta(q, a, z)$ then the a is set to be non empty.

ii) If a PDA is deterministic then the a must not be empty.

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3. Techniques for Turing machine:

* Multiple tracks
* Storage in finite control
* Checking off symbols
* Shifting over
* Subroutines.

4. Turing machine:

The basic three modes of the Turing machine are

* finite control
* infinite tape
* tape head

a_1	a_2	a_3	1	a_i	B	B	...

finite control.

* Each input tape head is divided into cells.
* Each cell will contain any symbol following with infinite input blank symbo
* Each turing machine will halts once it accept the input.

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5. Instanteous description:-

Instanteous description (ID) of a turing machine is defined as

$q_1 \alpha q_2$

where q_1 and q_2 is the current state of M.

$\alpha \to$ is the string in Γ^*.

6. Multitape Turing machines:-

* The multitape turing machine will consists of a finite control and k tape heads and k tapes.
* Each tape head will move in both the direction.
* One move of the tape will depend on the input scanned and the state of tape head.
* It changes the state
* Based on the scanned input it will rewrite
* Move in both direction.

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7. Recursively enumerable language and recursive set:

Recursively enumerable language set are the language which was accepted by the turing machine. Recursive enumerable set will includes regular languages and context free language (CFL's) some of the language in recursive enumerable will not be accepted by the turing machine.

Recursive set:

It is a subset of recursively enumerable set. The all language which was accepted by the turing machine is in the recursive set.

8. Universal Turing machine:-

If a problem is said to be undecidable the universal turing machine will be used. Does the turing M accept the input w? Here M and w are both the parameter of the problem and solving as a problem. M will over restrict the input {0,1} and w will set to {0,1,B}.

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9. Class NP problem:

The problem which was solved in the non-deterministic polynomial time by the non-deterministic turing machine is said to NP problem.

10. Rice Theorem:

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PART-C.

16)

a)

$L = \{\{a^n b^n \mid n > 0\}\}$.

The condition for a lemma to prove that is not regular is.

Conditions:

1) xy^iz
2) $|y| \geq 0$
3) $|xy| > 0$

Assume that the string

$s = a^n b^n$

Rewrite with p, $= a^p b^p$

consider a value for a $p = 2$

$= a^2 b^2$

$= \underbrace{a}_{x}\underbrace{ab}_{y}\underbrace{b}_{z}$

Based on the condition,

xy^iz

Case I:

Consider the value for $i = 3$.

aabababb

$= a^4 b^4$

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$a^2b^2 \neq a^4b^4$

Therefore the value of p is not equal therefore it is proved that as not regular.

Case II :

If we consider the value of $i = 4$

aababababb
a^5b^5
$a^2b^2 \neq a^5b^5$

Therefore in this case also it will be proved that it is not regular.

ii) $L = \{a^n b^n c^n \,||\, n > 0\}$ is not a CFL.

The language context free language will be described from the context free grammer.

The condition for a lemma to prove that it is not a context

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free grammer is.

Condition :

1) uv^ixy^iz
2) $|vy| \geq 0$
3) $|vxy| > 0$

Assume the given string as L as the string.

$S = \{a^n b^n c^n\}$

Replace the n with p,

$S = a^p b^p c^p$

consider for a value of $p = 2$.

$= a^2b^2c^2$

$= \underset{u}{a}\underset{v}{a}\underset{x}{b}\underset{y}{b}\underset{z}{cc}$

Based on the condition that

uv^ixy^iz

Case I :
Consider the value of $i = 4$

aabbbbbccccc
$a^2b^5c^5$

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Therefore

$$a^2b^2c^2 \neq a^2b^5c^5$$

From the case I it is clear that the value of p is not equal hence the given lemma is not context free language.

Case II:

Consider the value of $i=6$.

$$uv^6xy^6z$$

aabbbbbbbccccccc

$$a^2b^7c^7$$

Hence $a^2b^2c^2 \neq a^2b^7c^7$

It is infer from the case II also proved that it is not a context free language.

Good

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PART-B

11)b)

$$M = (V, T, P, S)$$

For converting the PDA to CFG we have to find the V, T, P, S.

$$V = \{S, [q_0, X, q_0], [q_0, X, q_1], [q_1, X q_0], [q_1, X q_1], [q_0, Z_0, q_0], [q_0, Z_0, q_1], [q_1, Z_0, q_0], [q_1, Z_0, q_1]\}.$$

$$T = \{0, 1\}.$$

The product of δ is

$\delta[q_0; X, q_1]$,

$\delta[q_0, Z_0, q_1]$. from rule (1).

i) $\delta(q_0, 0, Z_0) = \delta(q_0, XZ_0)$

$$\delta(q_0, Z_0, q_0) = 0[q_0, X, q_0][q_0, Z_0, q_0]$$
$$\delta(q_0, Z_0, q_0) = 0[q_0, X, q_1][q_1, Z_0, q_0]$$
$$\delta(q_0, Z_0, q_1) = 0[q_0, X, q_0][q_0, Z_0, q_1]$$
$$\delta(q_0, Z_0, q_1) = 0[q_0, X, q_1][q_1, Z_0, q_1]$$

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ii) $\delta(q_0, 0, x) = \delta(q_0, xx)$.

$\delta(q_0, x, q_0) = 0\,[q_0, x, q_0], [q_0, x, q_0]$

$\delta(q_0, x, q_0) = 0\,[q_0, x, q_1], [q_1, x, q_0]$

$\delta(q_0, x, q_1) = 0\,[q_0, x, q_0], [q_0, x, q_1]$

$\delta(q_0, x, q_1) = 0\,[q_0, x, q_1], [q_1, x, q_1]$

iii) $\delta(q_0, 1, x) = \delta(q_1, \epsilon)$

$\delta(q_0, x, q_1) = 1$

iv) $\delta(q_1, 1, x) = \delta(q_1, \epsilon)$

$\delta(q_1, x, q_1) = 1$

v) $\delta(q_1, \epsilon, x) = \delta(q_1, \epsilon)$

$\delta(q_1, x) \neq \epsilon$

vi) $\delta(q_1, \epsilon, z_0) = \delta(q_1, \epsilon)$

$(q_1, z_0) = \epsilon$.

* Hence therefore there is no productions is from the $[q_1, x, q_0]$, $[q_1, z_0, q_0]$.

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* The production $[q_0, x, q_0]$ and $[q_0, z_0, q_0]$ are have $[q_1, x, q_0], [q_1, z_0, q_0]$ on the string that is no other string is derived from the $[q_0, x, q_0]$, $[q_0, z_0, q_0]$.

* By deleting the string of these identifies will leads to the result of the production.

No. of productions available are.

x $[q_0, z_0, q_0] = 0\,[q_0, x, q_0][q_0, z_0, q_0]$

x $[q_0, z_0, q_0] = 0\,[q_0, x, q_1][q_1, z_0, q_0]$

x $[q_0, z_0, q_1] = 0\,[q_0, x, q_0][q_0, z_0, q_1]$

$[q_0, z_0, q_1] = 0\,[q_0, x, q_1][q_1, z_0, q_1]$

x $[q_0, x, q_0] = 0\,[q_0, x, q_0][q_0, x, q_0]$

x $[q_0, x, q_0] = 0\,[q_0, x, q_1][q_1, x, q_0]$

x $[q_0, x, q_1] = 0\,[q_0, x, q_0][q_0, x, q_1]$

$[q_0, x, q_1] = 0\,[q_0, x, q_1][q_1, x, q_1]$

$[q_0, x, q_1] = 1$

$[q_1, x, q_1] = 1$

$[q_1, x] = \epsilon$

$[q_1, z_0] = \epsilon$.

From the above production after cancelling the identity production the resultant product will be.

$S \rightarrow$

$[q_0, z_0, q_1] = 0[q_0, x, q_1][q_1, z_0, q_1]$

$[q_0, x, q_1] = 0[q_0, x, q_1][q_1, x, q_1]$

$[q_0, x, q_1] = 1$

$[q_1, x, q_1] = 1$

$[q_1, x] = \epsilon$

$[q_1, z_0] = \epsilon$.

12)

a) Turing machine:

There are three basic modes of a turing machine. They are:

* finite control
* infinite tape
* tape head.

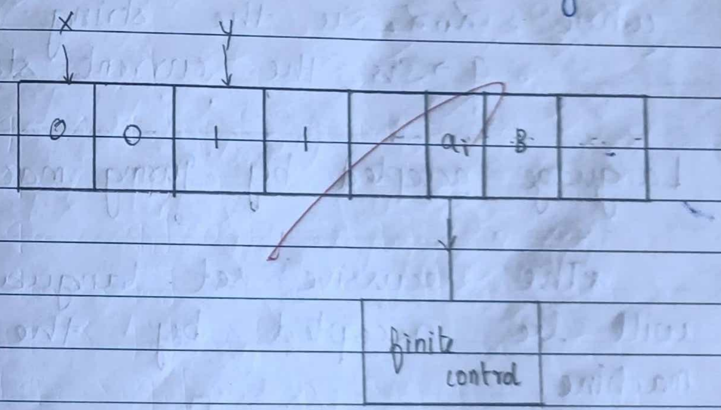
*Each head of the Input is divided into cells.

* Each cell with contain any symbol.

* The input tape is infinite in the right side and in left side it contains a tape symbol.

*Each move of the turing machine will depend on the scanned input head and the finite control.

* Turing machine will once halts when it accepts the string.



* ~~X will replaced for 0 continuously and~~

* 0 will be replaced by X on the left most and then in right the 1 will be replaced by Y on the leftmost and search in the right

to find the value of the righmost. and then it repeats the cycle until a B occur.

Instanteaus description of Turing machine.

The Instanteous description [ID] will be defined as.

$\alpha_1 q \alpha_2$

where, $\alpha_1 \alpha_2$ is the string in Γ^*.

$q \rightarrow$ is the current state of M.

Language accepted by Turing machine.

*The recursive set languages will be accepted by the turing machine.

* If a $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, B, F)$ is accepted by the machine then

$L(M) = \{ w | w$ for w in $\Gamma \}$

$M = \{Q, \Sigma, \Gamma, \delta, q_0, Z_0, B, F\}$.

where,

$Q \rightarrow$ is the set of finite states
$\Sigma \rightarrow$ is the set of input symbols.
$\Gamma \rightarrow$ stack symbol
$q_0 \rightarrow$ is the intial state
$B \rightarrow$ Blank symbol
$Z_0 \rightarrow$ set of
$F \rightarrow$ set of final state
$\delta \rightarrow$ is the transition,
$Q \times \delta \rightarrow Q \times \Sigma$

Ex:-
$M = \{0^n 1^n\}$

let $n = 1$, 01
$n = 2$, 0011
$n = 3$, 000111

$M = \{ 01, 0011, 000111 \ldots\ldots \}$.

0	0	1	1	...	...

M =

	0	1	X	Y	B
q_0	(q_1, X, R)	–	–	(q_3, Y, R)	–
q_1	$(q_1, 0, R)$	(q_2, X, L)	–	(q_1, Y, R)	–
q_2	$(q_2, 0, L)$	–	(q_0, X, R)	(q_3, Y, L)	–
q_3	–	–	–	(q_2, Y, R)	(q_4, B, L)
q_4	–	–	–	–	–

$M = \{\{q_0, q_1, q_2, q_3, q_4\}, \{0,1\}, \{0,1,B\}, \delta, q_0, B, \{q_4\}\}$.

Start with the intial state q_0 and w - 0011

$q_0 0011 \vdash Xq_1 011 \vdash X0q_1 11 \vdash Xq_2 0X1$

$\vdash q_2 X0X1$

$\vdash Xq_2 0Y1$

$\vdash q_2 X0Y1$

$\vdash Xq_0 0Y1$

$\vdash XXq_1 Y1$

$\vdash XXYq_1 1$

$\vdash XXYYq_4$

$\vdash XXYq_2 Y$

$\vdash XXYXq_0$

Therefore the Turing machine will accepts the string
$L = \{0^n 1^n \mid n > 0\}$.

14)

a) Closure properties of recursive and recursively enumerable language.

Theorem 1 :-

If L is recursive then the complement of L is also recursive.

Proof:-

* If a Turing machine M will accept the input w.

* M will creates a M' that will help the turing machine to enter into the final state for the input w.

* If M will accepts means it will

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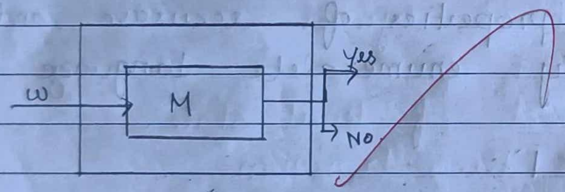
enter into final state and halts.

*If M' will accepts then it will enter into final state.

*There are one condition is to be done because in this M is an algorithm

*From that it clearly known that if L is recursive then it's complement L(M') is also recursive.



Theorem 2:

*The union of two recursive enumerable is recursively enumerable.

*The union of two recursive is recursive.

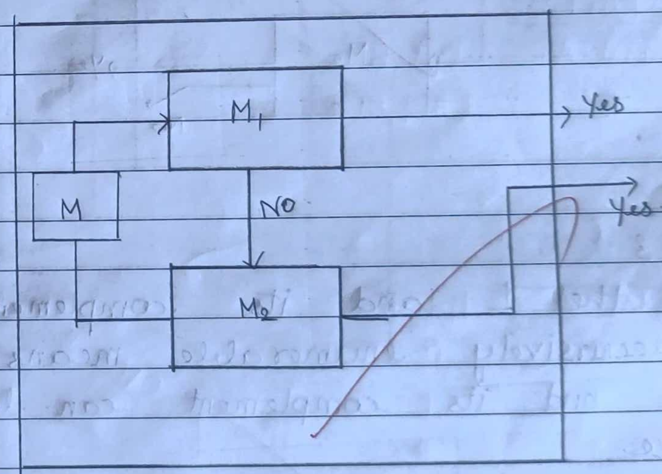
Proof:

*If the machine M_1 and M_2 will be in

*M will simulates M_1 if the M_1 will be accepts then M can also be accept.

*If M_1 will be reject then the M will stimulates M_2.

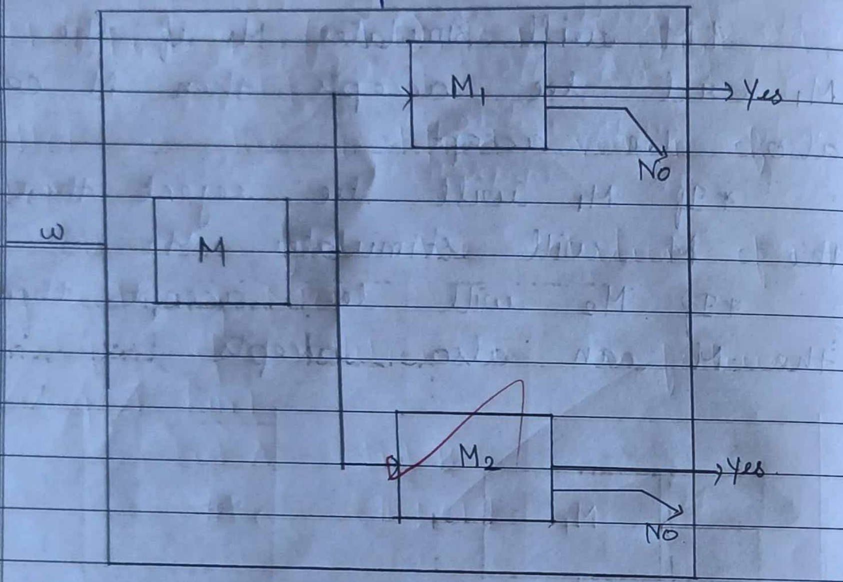
*If M_2 will be accept then the M can also accepts.



*The above theorem is not workout for the construction of the recursively enumerable.

*So that the M can simulate both the M_1 and M_2 simultaneously

If either one will accepts then the M will accept.



Theorem 3:-

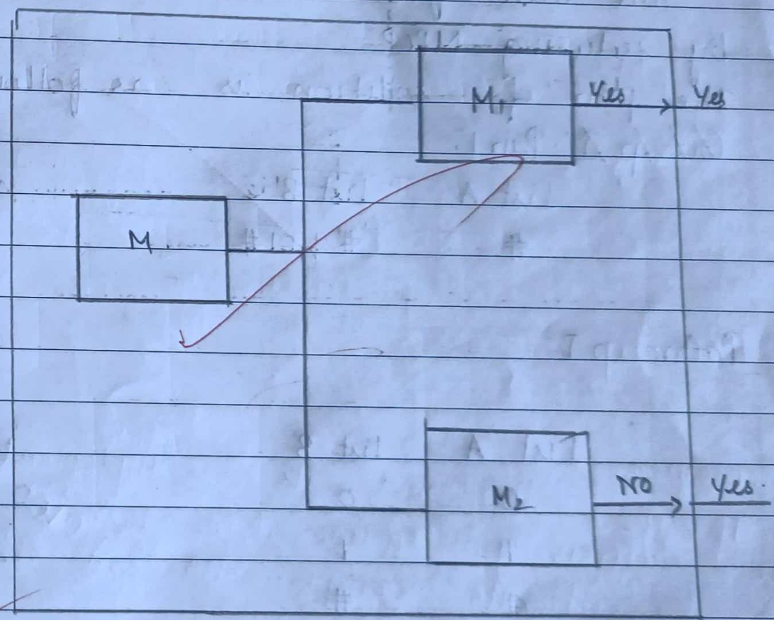
If the L and its complement are recursively enumerable means the L and its complement can be recursive.

Proof:

* In the M1 and M2 turing machine are in L and L̄ respectively.
* In this the M will accepts

when the M1 will accepts w.

* If M will rejects when the M2 will accepts w.
* L and its complement will recursively enumerable means the L and its complement must be recursive.



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15) b)

$M = (\{q_1, q_2, q_3\}, \{0, 1, B\}, \{0, 1\}, \delta, q_1, B, \{q_3\})$

MPCP is the modified version of PCP.

You can find the solution of PCP by reducing MPCP:

The pair of solution is as follows:

Group I: Pair 1:

List A	List B
#	$\#q_1 01\#$

Group II:

List A	List B
0	0
1	1
#	#

Group III:

List A	List B	
qx	yp	$\delta(q,x) = (p,y,R)$
zqx	yzp	$\delta(q,x) = (p,y,L)$
$\#qx$	$\#yp$	$\delta(q,B) = (p,y,R)$
$\#zqx$	$\#pzy$	$(q,B) = (p,y,L)$

List A	List B	
$q_1 0$	$1q_2$	$\delta(q_1,0) = \delta(q_2,1,R)$
$0q_1 1$	$00q_2$	$\delta(q_1,1) = \delta(q_2,0,L)$
$1q_1 1$	$01q_2$	

Group IV:

List A	List B	
$\#0q_1 B$	$\#q_2 01$	$\delta(q_1,B) = \delta(q_2,1,L)$
$\#1q_1 B$	$\#q_2 11$	
$0q_2 0$	$00q_3$	$\delta(q_2,0) = (q_3,0,L)$
$1q_2 0$	$01q_3$	
$q_2 1$	$0q_2$	$\delta(q_2,1) = (q_2,0,R)$

Group III:

List A	List B.
$0q_3 1$	q_3
$0q_3$	q_3
$q_3 1$	q_3

Group IV:

List A	List B
#	##

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$\{q_1 01\}, \{1q_2\}, \{0, 1, q_3\}, \{q_3 1\}, \{0q_2 0\},$
$\{1q_2 0\}, \{01q_3\}, \{0q_2\}, \{00q_3\}, \{q_3\},$

Solution of MPCP will exist if there is $x_{i1}x_{i2}x_{i3} \dots x_{ir} = w_{i1}w_{i2} \dots w_{ir}$.

---

13)
b) Turing machine subtraction:

Subtraction will be defined when

Subtraction of
$$m - n = \begin{cases} m-n & m \geq n \\ 0 & m<n \end{cases}$$

*The 0 will be replaced and the right most search will happens if there any 0 will be identified followed by an 1 then it will replaced.

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	0	1	B
q_0	(q_0, B, R)	(q_3, B, R)	-
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	-
q_2	$(q_3, 0, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, R)$	$(q_3, 1, L)$	(q_0, B, L)
q_4	$(q_4, 0, L)$	$(q_4, 1, L)$	$(q_0, 0, L)$
q_5	$(q_5, 0, R)$	$(q_5, 1, R)$	$(q_6, 0, L)$
q_6	-	-	-

$M = \{\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{0, 1\}, \{0, 1, B\}, \delta, q_0, z_0, F\}$

$-(1, 2) \to 1$

$q_0 0100 \vdash B q_0 100$
$\vdash BB q_3 00$

$\vdash BB0q_3 0$

$\vdash BB00q_3 B$

$\vdash BB0q_0 00$

$\vdash BB0Bq_0 0$

$\vdash BB0BBq_0$

$\vdash BBBB0q_0$

*The substraction will halt when there is no other move from that state.

*In this theorem the condition will be end because there is no other production with $q_0 B$.

*Subtraction will be done if there is the value which m will be greater to n.

*If the value of n will be greater than m means there is no subtraction is possible.