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PART-A

1) Pumping lemma for Regular Languages :-

Pumping lemma is used for pumping (generating) multiple substring from the input given string.

In other words, pumping lemma is a break of large input string into ~~m~~ several substrings.

2) Deterministic Push Down Automata.

$M = \{Q, \Sigma, \Gamma, \delta, q_0, Z, F\}$

* For every q in Q and Z in Γ ~~whatev~~ whenever (q, ϵ, Z) is non empty, (q, a, Z) is empty for all a in Σ

* For no q in Q and Z in Γ, a in $\Sigma \cup \{\epsilon\}$ does (q, a, Z) atleast have more element.

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3) Different ~~Techni~~ Techniques for Turning Machine Construction :-

There are five techniques for Turning Machine construction

* Storage of finite control
* Multiple track
* Checking off symbol
* State change.

4) Turning Machine :-

Turning Machine contain

* Finite control
* Infinite tape
* Tape head.

Specification of Turning Machine :-

Each tape is divided into cells.

The tape has infinite to the right.

Each cell of the tape contain only one symbol.

The tape head can move in both direction in the cell.

→ finite control

a_1	a_2	...	a_i	B	B

5) Instantaneous description of turning Machine:

Instantaneous description of turning machine is defined as

$\alpha_1 q \alpha_2$

where q is the current state of M

α_1 & α_2 is the string

6) Multitape Turning Machine:-

* Multitape turning Machine contain finite control with K tapes and K tape heads
* The type of tape has infinite in both side.
* The tape head can move independently on both side.

Turning Machine follows

* Changes the state
* Change the symbol scanned by the tape head
* Either move in left or right Direction.

B	B	a_1	a_2	... a_n	B	B

7) Recursively Enumerable language:

The language accepted by turning machine is called Recursively ~~Enumer~~ Enumerable language.

Recursively Enumerable set: It consists of regular languages and context-free languages. But not all the languages are accepted by Turning Machine.

Recursive set: Recursive set is the subset of Recursively Enumerable set. ~~The~~ Any language is accepted by turning Machine.

8) Universal turning Machine:

It shows the problem in undesirable.

The problem is "Does the turning Machine M accept the input w?"

Both M and w are parameters. Formalizing the problem, w should be required to have (0,1) where M have the alphabet (0,1,B). This is called Universal turning Machine.

9) Class NP problem:
The problem can be solved non-deterministic polynomial language by non deterministic Turning Machine.
Eg:- Undesirable problem.

10) RIce theorem:
If the POST correspondance is desirable, then MPCP is also desirable

PART-C.

16) a) Prove that:-

i) $L = \{a^n b^n \mid n>0\}$ is not regular.

Proof:-

Given $L = \{a^n b^n \mid n>0\}$

Let us consider L is regular.

Pumping length = P.

The string of the L

~~$s = a^p b^p$.~~

$s = a^p b^p$.

s is going to separate into 3 parts

$s = xyz$.

Let us put $p=7$

$s = a^7 b^7$

= aaaaaaabbbbbbb.

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Case 1:

y be in 'a' part

$$\underbrace{aa}_{x}\underbrace{a\,a\,a\,a\,a}_{y}\underbrace{b\,b\,b\,b\,b\,b\,b}_{z}$$

Case 2:

y be in both 'a' and 'b' part

$$\underbrace{a\,a\,a\,a}_{x}\underbrace{a\,a\,a\,b\,b}_{y}\underbrace{b\,b\,b\,b\,b}_{z}$$

Case 3:

y be in 'b' part

$$\underbrace{a\,a\,a\,a\,a\,a\,a}_{x}\underbrace{b\,b}_{y}\underbrace{b\,b\,b\,b\,b}_{z}$$

Now we write

xy^iz.

We have to prove $xy^iz \notin L$ for any value of i.

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Put $i=2$.

Case 1:

$xy^iz = xy^2z$

$= aaaaaaaaaaaa\,bbbbbbb$

Case 2:

$xy^iz = xy^2z$

$= aaaa\,aaaaaa\,bbbb\,bbbbb$

Case 3:

$xy^iz = xy^2z$

$= aaaaaaa\,bbbb\,bbbbb$.

Check under the given condition.

$|xy| \le p$.

Case 1: $|xy| \le p$

$12 \le 7$ not satisfied.

Case 2: $|xy| \le p$

$13 \le 7$ not satisfied

Case 2: $|xy| \le p$

$11 \le p$ not satisfied.

Put $i = 3$.

Case 1:

$xy^iz = xy^3z$

$= aaaaaaaaaaaaaaaa\ bbbbbbb.$

Case 2:

$xy^iz = xy^3z$

$= aaaa\ aaaaaaaaaa\ bbbbbb\ bbbbb$

Case 3:

$xy^iz = xy^3z$

$= aaaaaaa\ bbbbbb\ bbbbb.$

Check under the given condition.

$|xy| \leq p$

Case 1: $|xy| \leq p$

$17 \leq 7$ not satisfied

Case 2: $|xy| \leq p$

$19 \leq 7$ not satisfied

Case 3: $|xy| \leq p$

$13 \leq 7$ not satisfied.

It is concluded that of is not pumped.

Our assumption is wrong.

$\therefore$ L is not regular

Hence proved.

ii) $L = \{a^n b^n c^n \mid n > 0\}$ is not a CFL.

Proof:

Given $L = \{a^n b^n c^n \mid n > 0\}$

According to pumping lemma we write $L = uvwxy$

L is going to separate into 5 parts.

$n = 1$ $L = \{ab\}$

$n = 2$ $L = \{aabb\}$

$n = 3$ $L = \{aaabbb\}$

$L = \{ab, aabb, aaabbb, \ldots\}$

Consider $L = aaaabbbb$.

$$\underbrace{a}_{u}\;\underbrace{aaa}_{v}\;\underbrace{b}_{w}\;\underbrace{bb}_{x}\;\underbrace{b}_{y}$$

$u = a$, $v = aaa$, $w = ab$, $x = bb$, $y = b$.

Checking conditions :-

1) $|vx| \neq \epsilon$

v has 3 value

x has 2 value.

$\therefore 5 \neq \epsilon \rightarrow$ satisfied.

2) $|vwx| \leq m$

$6 \leq 8 \rightarrow$ satisfied.

It satisfy both condition.
then we write
$L = uv^k wx^k y$.

$k = 1$

$uv^k wx^k y = uvwxy$

$= aaaabbbb \in L$.

$= a^4 b^4$

$k = 2$

$uv^k wx^k y = uv^2 wx^2 y$

$= aaaaaaabbbbbb$

$= a^7 b^6 \notin L$.

$k = 3$.

$uv^k wx^k y = uv^3 wx^3 y$

$= aaaaaaaaaabbbbbbbb \notin L$.

$= a^{10} b^8$

$k=2$ is not belongs to L.

$\therefore L$ is not a context free language.

Hence proved.

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PART - A

11) b) PDA to CFG.

Construct a CFG G

$G = \{V, T, P, S\}$

where V → set of non terminal
T → set of terminal
P → set of production in the form of $A \rightarrow \alpha$.
where A → set of Terminals
α → string of both terminal and non terminal
S → starting symbol.

Gn :- $M = (\{q_0, q_1\}, \{0, 1\} \{x, z_0\}, \delta, q_0, z_0, \phi)$

$\boxed{\begin{array}{l} V = \{S, \{q_0, x, q_0\} \{q_0, x, q_1\} \{q_1, x, q_0\} \\ \{q_1, x, q_1\} \{q_0, z_0, q_0\} \{q_0, z_0, q_1\} \\ \{q_1, z_0, q_0\} \{q_1, z_0, q_1\} \}. \end{array}}$

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T = same as the ~~sim~~ input given in PDA

$\boxed{T = \{0, 1\}}$

S → [initial state, initial stack symbol, state]

state → $\{q_0, q_1\}$

initial state → q_0

initial stack symbol → z_0

$\boxed{\begin{array}{l} S \rightarrow [q_0, z_0, q_0] \\ S \rightarrow [q_0, z_0, q_1] \end{array}}$

Set of Productions:-

i) $\delta(q_0, 0, z_0) = \delta(q_0, xz_0)$

eliminate ← $[q_0, z_0, q_0] \rightarrow 0 [q_0, x, q_0] [q_0, z_0, q_0]$

eliminate ← $[q_0, z_0, q_0] \rightarrow 0 [q_0, x, q_1] [q_1, z_0, q_0]$

eliminate ← $[q_0, z_0, q_1] \rightarrow 0 [q_0, x, q_0] [q_0, z_0, q_1]$

$[q_0, z_0, q_1] \rightarrow 0 [q_0, x, q_1] [q_1, z_0, q_1]$

2) $\delta(q_0, 0, x) = \delta(q_0, xx)$

eliminate $\leftarrow [q_0, x, q_0] \rightarrow 0\,[q_0, x, q_0]\,[q_0, x, q_0]$

eliminate $\leftarrow [q_0, x, q_0] \rightarrow 0\,[q_0, x, q_1]\,[q_1, x, q_0]$

eliminate $\leftarrow [q_0, x, q_1] \rightarrow 0\,[q_0, x, q_0]\,[q_0, x, q_1]$

$[q_0, x, q_1] \rightarrow 0\,[q_0, x, q_1]\,[q_1, x, q_1]$

3) $\delta(q_0, 1, x) = \delta(q_1, \epsilon)$

$[q_0, x, q_1] \rightarrow 1$

4) $\delta(q_1, 1, x) = \delta(q_1, \epsilon)$

$[q_1, x, q_1] \rightarrow 1$

5) $\delta(q_1, \epsilon, x) = \delta(q_1, \epsilon)$

$[q_1, x, q_1] \rightarrow \epsilon$

6) $\delta(q_1, \epsilon, z_0) = \delta(q_1, \epsilon)$

$[q_1, z_0, q_1] \rightarrow \epsilon$

Check the non terminal of V is present in the left side of production.

$\{q_1, x, q_0\}$ & $\{q_1, z_0, q_0\}$ is not present in the set of production ~~so, eliminate the production.~~

Check these two in the right side of the production and eliminate it.

Similar terms in both left and right of the production is eliminate.

Set of production P =

$[q_0, x, q_1] \rightarrow 0\,[q_0, x, q_1]\,[q_1, x, q_1]$

$[q_0, z_0, q_1] \rightarrow 0\,[q_0, x, q_1]\,[q_1, z_0, q_1]$

$[q_0, x, q_1] \rightarrow 1$

$[q_1, x, q_1] \rightarrow 1$

$[q_1, x, q_1] \rightarrow \epsilon$

$[q_1, z_0, q_1] \rightarrow \epsilon$

14) a) Closure properties of recursive and recursively enumerable languages:-

Theorem 1 :-

The complement of L is recursive, if L is recursive.

Proof :-

Let L be the language accepted by turning Machine M.

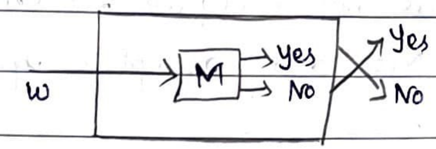
Construct M' from M, if M is entered into the final state of input w, then M' is entered without accepting.

Likewise if M' is entered into the final state, then M is entered without accepting.

Since both are accepted, M' is an ~~ta~~ algorithm.

Therefore L is recursive, where complement of L is also recursive.

Hence proved.



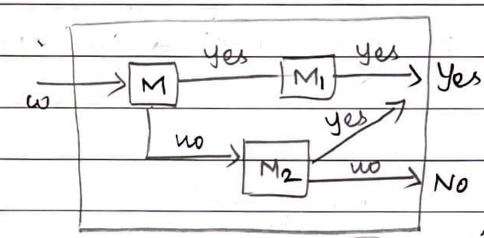
Theorem of 2 :-

i) The union of ~~recs~~ recursive is recursive.

ii) The union of recursively Enumerable is recursively Enumerable.

Proof :-

i)



Let L1 and L2 be the languages accepted by M.

Construct M, first M stimulates M_1

If M_1 is accepted, then M accepted

If M_1 is not accepted, then M stimulates M_2 and accepted iff M_2 is accepted.

M accept any of the language $L_1 \cup L_2$.

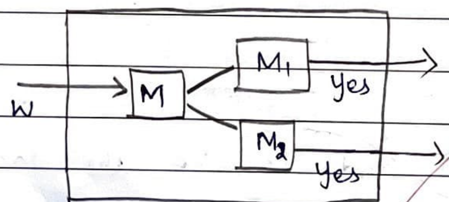
∴ The union of recursive is recursive.

Hence proved.

ii) The above construction is not applicable for recursively Enumerable

M_1 may not accepted.

So, M is simultaneously stimulate M_1 and M_2 into two ~~differ~~ separate tapes.



∴ The union of recursively enumerable is recursively ~~one~~ enumerable.

Hence proved.

Theorem 3:

If the language L and $\bar{L}$ is recursively enumerable then L is recursive.

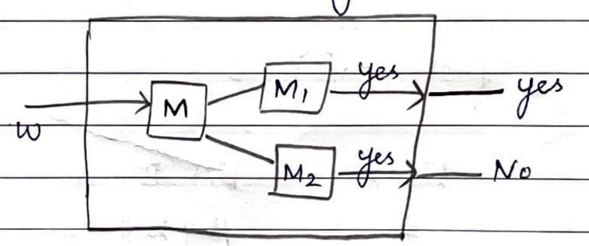
Proof:

Let M_1 and M_2 accept the language L and $\bar{L}$ respectively.

If M accepts the input w if M_1 accepts w and reject w if M_2 accepts w.

Since w is in either on L and $\bar{L}$, we know that any of the M_1 or M_2 is going to accepted.

M says only whether it is yes or no and not say both.



∴ If the language L and its complement $\bar{L}$ is recursively enumerable, then L is recursive.

Hence Proved.

15) b) Gn:- $M = \{\{q_1, q_2, q_3\}, \{0, 1, B\}, \{0, 1\}, \delta, q_1, B, \{q_3\}\}$

	0	1	B
q_1	$(q_2, 1, R)$	$(q_2, 0, L)$	$(q_2, 1, L)$
q_2	$(q_3, 0, L)$	$(q_1, 0, R)$	$(q_2, 0, R)$
q_3	—	—	—

Making first pair:

List A	List B
#	$\# q w \#$

$q = q_1$ $\quad$ $w = 01$

List A	List B
#	$\# q_1 01 \#$

Making remaining pairs:-

Group 1:-

List A	List B
X	X
#	#

$x \in$ ~~Q~~ Γ

$x \in (0, 1)$

List A	List B
0	0
1	1
#	#

Group 2:-

List A	List B	
qX	Yp	from $(q, x) \to (P, Y, R)$
$Zq X$	pZY	from $(q, x) \to (P, Y, L)$
$\# q \#$	$YP\#$	from $(q, B) \to (P, Y, R)$
$Zq\#$	$PZY\#$	from $(q, B) \to (P, Y, L)$

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$q \in Q - F$

$q \in \{q_1, q_2\} - \{q_3\}$ $x, y \in \{0, 1\}$

$q \in \{q_1, q_2\}$

List A	List B	
$q_1 0$	$1 q_2$	$(q_1, 0) \rightarrow (q_2, 1, R)$
$0 q_1 1$	$q_2 0 0$	$(q_1, 1) \rightarrow (q_2, 0, L)$
$1 q_1 1$	$q_2 1 0$	
$0 q_1 \#$	$q_2 0 1 \#$	$(q_1, B) \rightarrow (q_2, 1, L)$
$1 q_1 \#$	$q_2 1 1 \#$	
$0 q_2 0$	$q_3 0 0$	$(q_2, 0) \rightarrow (q_3, 0, L)$
$1 q_2 0$	$q_3 1 0$	
$q_2 1$	$0 q_1$	$(q_2, 1) \rightarrow (q_1, 0, R)$
$q_2 B \#$	$0 q_2 \#$	$(q_2, B) \rightarrow (q_2, 0, R)$

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~~List~~ Group 3 :-

List A	List B
$x q y$	q
$x q$	q
$q y$	q

$x \& y \in \{0, 1\}$

$q \in F$

$q \in q_3$

List A	List B
$0 q_3 0$	q_3
$0 q_3 1$	q_3
$1 q_3 0$	q_3
$1 q_3 1$	q_3
$0 q_3$	q_3
$1 q_3$	q_3
$q_3 0$	q_3
$q_3 1$	q_3

Group 4:-

List A	List B
#q	#

$q \in F$

$q \in q_3$

~~List~~ List A	List B
$\#q_3$	#

Let us consider $w = \cancel{0011}\ 01$

$q_1 01 \vdash 1 q_2 1$

$\vdash 1 0 q_1$

$\vdash 1 q_2 0 1$

$\vdash q_3 \cancel{1} 0 1$

$\vdash q_3 0 1$

$\vdash q_3 1$

$\vdash q_3$

Hence, the instance of MPCP of list A and list B is constructed.

12) a) Given :- $L = \{0^n 1^n \mid n \geq 0\}$

Turning Machine:-

Turning Machine contain

* Finite control
* Infinite Tape
* Tape head.

Specification of Turning Machine:-

* Each tape is divided into cells. The tape has infinite to the right.
* Each cell of the tape contain only one symbol.
* The tape head can move in both direction in the cell.

a_1	a_2	. . .	a_n	B	B

↓

~~[illegible]~~ finite control.

$L = \{0^n 1^n \mid n \geq 0\}$

when $n = 1$ $L = \{01\}$

$n = 2$ $L = \{0011\}$

$n = 3$ $L = \{000111\}$

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$L = \{01, 0011, 000111, \dots\}$

Let us take 0011

0	0	1	1	B	B	...

Turning Machine follows.

* Changes the state
* Change the symbol scanned by the tape head.
* Either move in left or right direction.

Initially it start with state q_0

The readed 0's are marked as x, and the readed 1's are marked as y.

We read the leftmost 0' and move to right and read the ~~rig~~ leftmost 1 and then move left to read 0 and so on.

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	a	ψ	x	y	B
q_0	(q_1, x, R)	-	-	(q_3, y, R)	-
q_1	$(q_1, 0, R)$	(q_2, y, L)	-	(q_1, y, R)	-
q_2	$(q_2, 0, L)$	-	(q_0, x, R)	(q_2, y, L)	-
q_3	-	-	-	(q_3, y, R)	(q_4, B, R)
q_4	-	-	-	-	-

Let us consider string $w = 0011$

$q_0 0011 \vdash x q_1 011$

$\vdash x 0 q_1 11$ $\quad x \delta q_2 y$

$\vdash x q_2 0 y 1$

$\vdash q_2 x 0 y 1$ $\quad q_0 x 0$

$\vdash x q_0 0 y 1$

$\vdash XX q_1 y1$

$\vdash XXy q_1 1$

$\vdash XX q_2 yy$

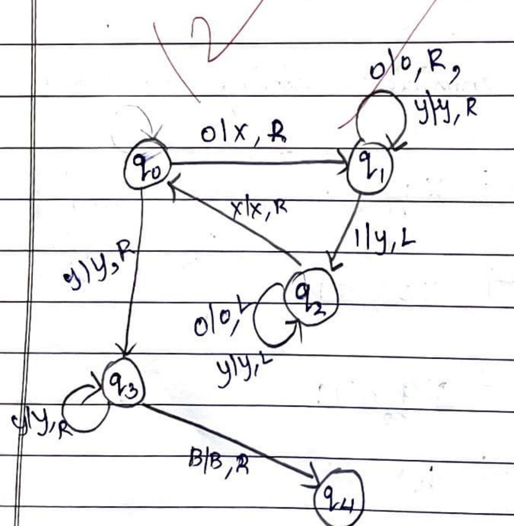
$\vdash XXy q_2 y$

$\vdash XXyy q_3$

$\vdash XXyy q_4$

Hence Turning Machine accepts the language $0^n 1^n$

Transition diagram:-



13) b) Turning Machine to perform $f(x, y) = x - y$ for $x > y$.

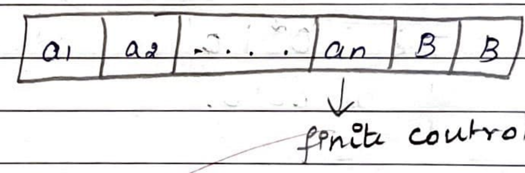
Turning Machine:-

Turning Machine contain

* Finite control
* Infinite control
* Tape head.

Specification of turning Machine:-

* Each tape is divided into cells. The tape has finite to the right
* Each cell of the tape contain only one symbol
* The tape head can move in both direction in the cell.



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Turning Machine follows:

* Changes the state
* Changes the symbol scanned by the tape head.
* Either move in left or right direction.

Example : $-(0,0) = 1$

Here the values are considered as 0 and the minus (−) sign is considered as 1.

$m = 2 \qquad n = 1. \qquad 0^m \quad 0^n$

$0^m 1 0^n = 0^{m-n}$ $\quad 0^1$

$0^2 1 0^1$

$w = 0010.$

Additional Sheet

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	0	1	B
q_0	(q_1, B, R)	–	–
q_1	$(q_1, 0, R)$	$(q_2, 10, R)$	(q_4, B, L)
q_2	$(q_3, 1, R)$	–	–
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	~~(q_3, B, L)~~ (q_0, B, L)
q_4	–	(q_4, B, L)	$(q_6, 0, R)$
q_5	~~(q_5, B, R)~~	(q_5, B, R)	(q_6, B, R)
q_6	–	–	–

Subtract $-(2,1) = 1$. $w = 0010$.

$q_0 0010 \vdash B q_0 010$

$\vdash B 0 q_1 1 0$

$\vdash B 0 1 q_2 0$

$\vdash B 0 1 1 q_3$

$\vdash B 0 1 q_3 1$

$\vdash B 0 q_3 1 1$

$\vdash B q_3 0 1 1$

$\vdash q_3 B 0 1 1$

$\vdash B q_0 0 1 1$

$\vdash B B q_1 1 1$

$\vdash B B 1 q_2 1$

$\vdash B B 1 1 q_2$

$\vdash B B 1 q_4 1 B$

$\vdash B B q_4 1 B B$

$\vdash B q_4 B B B$

$\vdash B 0 q_6 B B \ldots$

$-(2,1) = 1$

The subtraction is performed successfully.

12

$0^m 1 0^n = 0^{m-n}$

$0^2 1 0^1 = 0^1$

In the table the first 0 is replaced by Blank (B)

Then the moves to the right 0 of 1.

Then change that 0 as 1.

Then the state move forward.

Then the steps are repeated.

Transition Diagram