* In addition to language acceptor, the TM, may be viewed as a computer of functions from integers to integers.

0°

* The integer $i \geq 0$ is represented by the string 0^i
* If a function has k arguments $i_1, i_2, \dots i_k$ then these integers are initially placed on the tape separated by 1's as $0^{i_1} 1 0^{i_2} 1 \dots 1 0^{i_k}$

0^m

* If the TM halts with a tape consisting of 0^m for some m, then we say that $f(i_1, i_2, \dots i_k) = m$, where f is the function of k arguments computed by this Turing Machine

tuples

* If Turing machine M computes function f of k arguments then f need not have a value for all different k tuples.
* If $f(i_1, i_2, \dots i_k)$ is defined for all $i_1, i_2 \dots i_k$ then we say f is a total recursive function
* A function $f(i_1, \dots i_k)$ computed by a TM is called a partial recursive function.

$2^{3+2^{4}}$ 2^{3+1}

Constuct a Turing Machine to perform addition

<u>Solution:</u>

Suppose the input is $0^m 1 0^n$.

Finally the TM halts on tape containing 0^{m+n}

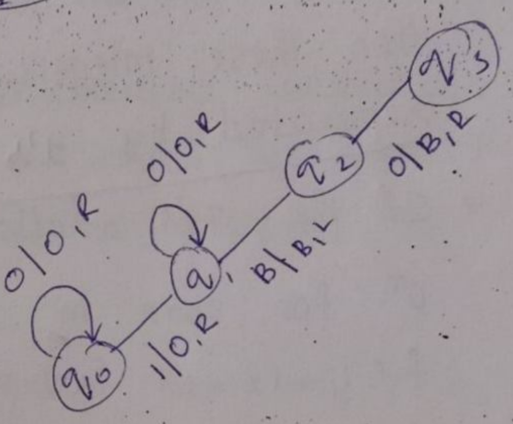
Suppose $w = 00010$. $0^3 1 0^1$

$3+1$ 0^i

0	0	0	1	0

$\uparrow$
q_0

4



0 0 0 0 0 B
q_1 q_2

0 0 0 0 B B
q_3

	0	1	B
q_0	$(q_0, 0, R)$	$(q_1, 0, R)$	
q_1	$(q_1, 0, R)$	~~$(q_2, 0, R)$~~	(q_2, B, L)
q_2	(q_3, B, R)	(	~~(q_3, B, R)~~
q_3	—	—	—

$M = (Q, \Sigma, \Gamma, \delta, q_0, B, \{q_3\})$

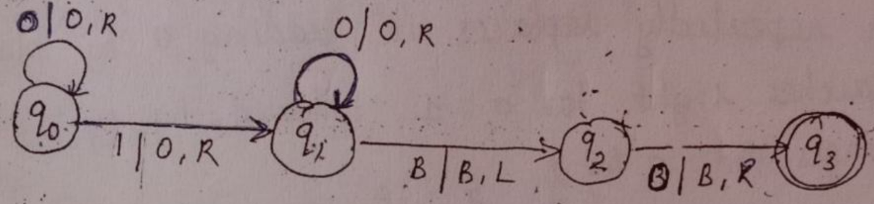
$Q = \{q_0, q_1, q_2, q_3\}$.

$\Sigma = \{0, 1\}$.

$\Gamma = \{0, 1, B\}$

$F = \{q_3\}$.

Transition Diagram



Simulating for $w = 00010$

$\underline{q_0}\,\underline{0}0010 \vdash 0\underline{q_0}\,\underline{0}010$ $(q_0, 0, R)$

$\vdash 00q_0\underline{0}10$ ↓

$\vdash 000q_0\underline{1}0$ $(q_1, 0, R)$

$\vdash 0000q_10$ $(q_1, 0, R)$

$\vdash 00000q_1B$ (q_2, B, L)

$\vdash 0000q_20B$ $q_2 \backslash B$

$\vdash 0000Bq_3B$. (q_3, B, R)

Hence Turing Machine performs addition.

proper subtraction.

<u>Solution:</u>

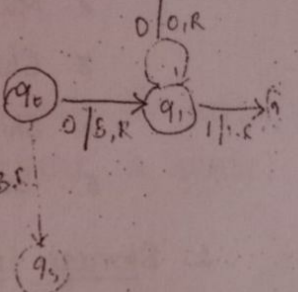
Proper subtraction $m \dot{-} n$ is defined as

$$m \dot{-} n = \begin{cases} m-n & \text{for } m \geq n \\ 0 & \text{for } m < n \end{cases}$$

m n
2-1
1-2 = 0

* The TM started with $0^m 1 0^n$ on its tape and halts on $0^{m \dot{-} n}$ on its tape.
* M repeatedly replaces its leading 0 by blank then searches right for a 1 followed by a 0 and changes the 0 to 1.
* Next M moves left until it encounters a blank and then repeats the cycle. The repetition ends if
  1) Searching right for a 0, M encounters a blank. Then the n 0's in $0^m 1 0^n$ have all been changed to 1's. and n+1 of the m ~~0's have been changed to B. M replaces the~~ n+1 1's by a 0 and n B's leaving m - n 0's on its tape.
  2) Beginning the cycle M cannot find a 0 to change to a blank, because the first m 0's already have been changed. Then $n \geq m$, so $m \dot{-} n = 0$. M replaces all remaining 1's and 0's by B

q_0	(q_1, B, R)	(q_5, B, R)	
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6	—	—	—



M is defined as

$M = \{Q, \Sigma, \Gamma, \delta, q_0, B, F\}$.

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$.

$\Sigma = \{0, 1\}$.

$\Gamma = \{0, 1, B\}$.

$F = \{q_6\}$.

2-1

A sample computation of M on input 0010 is

$q_0 0010 \vdash Bq_1 010 \vdash B0q_1 10 \vdash B01q_2 0 \vdash$

$B0q_3 11 \vdash Bq_3 011 \vdash q_3 B011 \vdash Bq_0 011 \vdash$

$BBq_1 11 \vdash BB1q_2 1 \vdash BB11q_2 \vdash$

$BB1q_4 1B \vdash BBq_4 1BB \vdash BBq_4 BBBB \vdash$

$B0q_6 BB\ldots$

$i < j$ True → more

$i \geq j$ − False → more ×