Example:

Prove $0^2+1^2+2^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

by inductive proof.

Basis : $n=0$

Let $P(n)$ be

$$0^2+1^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

LHS:	RHS:
$\sum_{i=0}^{n} i^2 = 0$	$\frac{n(n+1)(2n+1)}{6} = 0$

LHS = RHS. proved.

Induction step:

$P(n-1)$ implies $P(n)$

$$P(n-1) = \sum_{i=0}^{n-1} i^2 = \frac{(n-1)(n-1+1)(2(n-1)+1)}{6}$$

$$= \frac{(n-1)(n)(2n-2+1)}{6}$$

$$= \frac{(n-1)(n)(2n-1)}{6}$$

$$P(n-1) = \frac{n(n-1)(2n-1)}{6}$$

$$\sum_{i=0}^{n} i^2 = \sum_{i=0}^{n-1} i^2 + n^2$$

$$= \frac{n(n-1)(2n-1)}{6} + n^2$$

$$= \frac{n(n-1)(2n-1) + 6n^2}{6}$$

$$= \frac{(n^2-n)(2n-1) + 6n^2}{6}$$

$$= \frac{2n^3 - n^2 - 2n^2 + n + 6n^2}{6}$$

$$= \frac{n(2n^2 - n - 2n + 1 + 6n)}{6}$$

$$= \frac{n(2n^2 + 6n - 3n + 1)}{6}$$

$$= \frac{n(2n^2 + 3n + 1)}{6}$$

$$= \frac{n(n+1)(2n+1)}{6}$$

Part B

EQUIVALENCE OF NFA and DFA.

Construct an Equivalent DFA for the following NFA.

$M = (\{q_0, q_1\}, \{0,1\}, \delta, q_0, \{q_1\})$

δ:

	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$* q_1$	ϕ	$\{q_0, q_1\}$

$\rightarrow$ ⇒ resembles initial state

$*$ ⇒ Final state.

$\delta(q_1, 0) = \{\}$

{ } → set of states

$\delta(q_1, 1) = (q_0$

[ ] → single state.

$M = (Q, \Sigma, \delta, q_0, F)$

$\delta(q_0, 0) = \{q_0, q_1\}$

$\delta(q_0, 01) = \{q_1\}$

Soln:

We construct an equivalent DFA.

	0	1
$[q_0]$	$[q_0, q_1]$	$[q_1]$
$[q_1]$	$[\phi]$	$[q_0, q_1]$
$[q_0, q_1]$	$[q_0, q_1]$	$[q_0, q_1]$
$[\phi]$	$[\phi]$	$[\phi]$

1)a $\delta([q_0, q_1], 0) = ?$

$$\delta(\{q_0, q_1\}, 0) = \delta(q_0, 0) \cup \delta(q_1, 0)$$

$$= \{q_0, q_1\} \cup \phi$$

$$= \{q_0, q_1\}$$

$$\delta([q_0, q_1], 0) = [q_0, q_1]$$

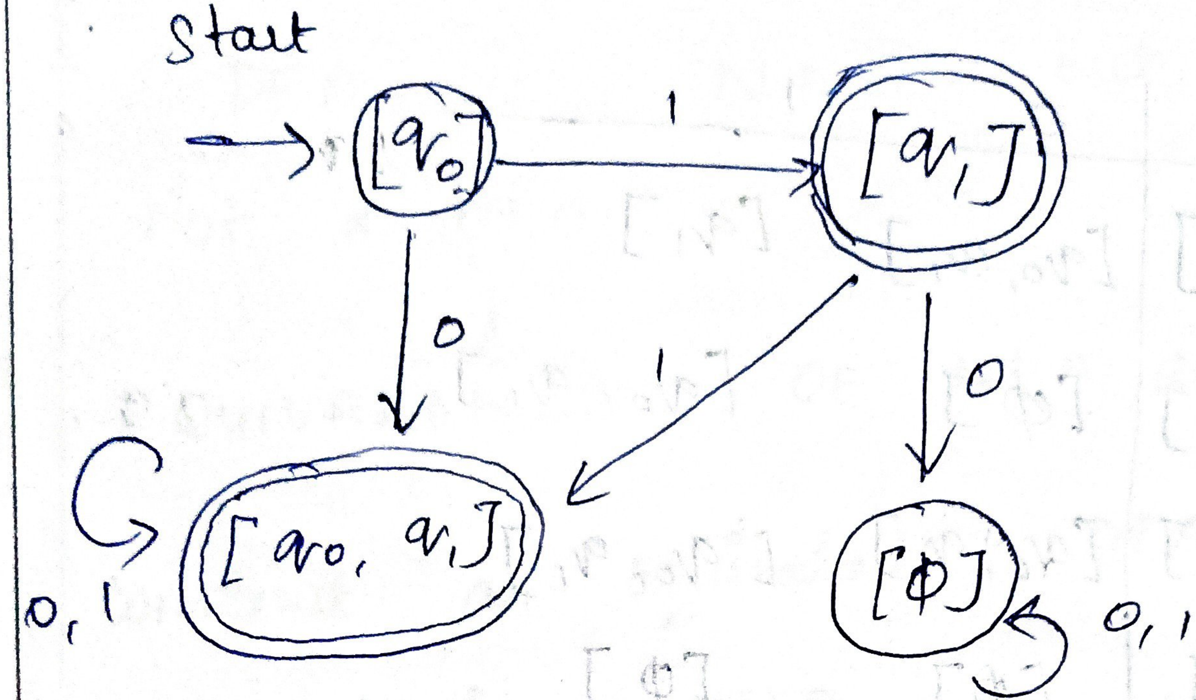
b) $\delta([q_0, q_1], 1) = ?$

$$\delta[\{q_0, q_1\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1)$$

$$= \{q_1\} \cup \{q_0, q_1\}$$

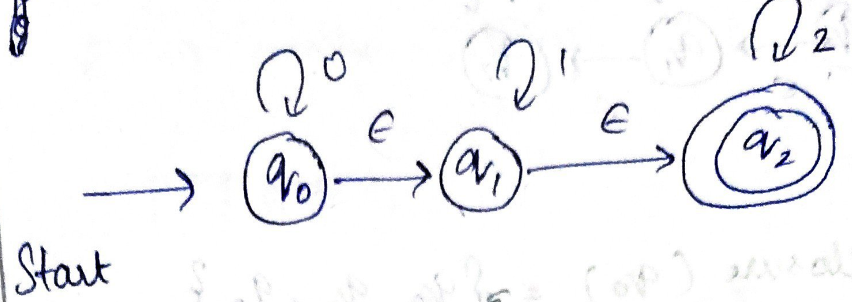
$$= \{q_0, q_1\}$$

$$\delta([q_0, q_1], 1) = [q_0, q_1]$$



When the transition leads from initial to final state.

Construct NFA for the following.



Solution:

Given NFA with ϵ transition.

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1, 2\}$ → Here ϵ is not added bcoz we want to construct NFA without ϵ.

For to find δ' we want to find e-closure for each state.

ϵ. closure $(q_0) = \{q_0, q_1, q_2\}$

ϵ. closure $(q_1) = \{q_1, q_2\}$

ϵ. closure $(q_2) = \{q_2\}$

Here $q_0 \rightarrow q_0$

For Finding Final state F, if we have the final state in the inital state, so, we should add the initial state (ie) q_0 in F. so the F becomes

$F \to \{q_0, q_2\}$

δ:	0	1	2
q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
q_2	ϕ	ϕ	$\{q_2\}$

$\delta(q_0, 0) = \epsilon.\text{closure}(\delta(\delta(q_0, \epsilon), 0)$

$= \epsilon.\text{closure}(\delta(\epsilon.\text{closure}(q_0), 0)$

$= \epsilon.\text{closure}(\delta(q_0, q_1, q_2), 0)$

$= \epsilon.\text{closure}(\delta(q_0, 0) \cup (q_1, 0) \cup (q_2, 0))$

$= \epsilon.\text{closure}(q_0 \cup \phi \cup \phi)$

$= \epsilon.\text{closure}(q_0)$

$= \{q_0, q_1, q_2\}$

$\delta(q_0, 1) = \epsilon.\text{closure}(\delta(\delta(q_0, \epsilon), 1))$

$= \epsilon.\text{closure}(\delta(\epsilon.\text{closure}(q_0), 1))$

$= \epsilon.\text{closure}(\delta(q_0, q_1, q_2), 1))$

$= \epsilon.\text{closure}(\delta(q_0, 1) \cup (q_1, 1) \cup (q_2, 1))$

$= \epsilon.\text{closure}(\phi \cup q_1 \cup \phi)$

$= \epsilon.\text{closure}(q_1)$

$= \{q_1, q_2\}$

$\delta(q_0, 2) = \epsilon.\text{closure}(\delta(\delta(q_0, \epsilon), 2))$

$= \epsilon.\text{closure}(\delta(\epsilon.\text{closure}(q_0), 2))$

$= \epsilon.\text{closure}(\delta(q_0, q_1, q_2), 2)$

$= \epsilon.\text{closure}(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2))$

$= \epsilon.\text{closure}(\phi \cup \phi \cup q_2)$

$= \epsilon.\text{closure}(q_2)$

$= \{q_2\}$

$\delta(q_1, 0) = \epsilon.\text{closure}(\delta(\delta(q_1, \epsilon), 0))$

$= \epsilon.\text{closure}(\delta(\epsilon.\text{closure}(q_1), 0))$

$= \epsilon.\text{closure}(\delta(q_1, q_2), 0))$

$= \epsilon.\text{closure}(\delta(q_1, 0) \cup \delta(q_2, 0))$

$= \epsilon\text{. closure}(\phi \cup \phi)$

$= \epsilon\text{. closure}(\phi)$

$= \phi$

$\delta(q_1, 1) = \epsilon\text{. closure}(\delta(\delta(q_1, \epsilon), 1))$

$= \epsilon\text{. closure}(\delta(\epsilon\text{. closure}(q_1), 1))$

$= \epsilon\text{. closure}(\delta(q_1, q_2), 1))$

$= \epsilon\text{. closure}(\delta(q_1, 1) \cup \delta(q_2, 1))$

$= \epsilon\text{. closure}(q_1 \cup \phi)$

$= \cancel{\{q_2\}}\ \{q_1, q_2\}$

$\delta(q_2, 2) = \epsilon\text{. closure}(\delta(\delta(q_2, \epsilon), 0))$

$= \epsilon\text{. closure}(\delta(\epsilon\text{. closure}(q_2), 0))$

$= \epsilon\text{. closure}(\delta(q_2), 0))$

$= \epsilon\text{. closure}(\delta(q_2, 0))$

$= \epsilon\text{. closure}(\cancel{q_2}\ \phi)$

$$\delta(q_1, 2) = \epsilon\text{-closure}(\delta(\delta(q_1, \epsilon), 2))$$

$$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_1, \cancel{\epsilon}), 2))$$

$$= \epsilon\text{-closure}(\delta(\cancel{\epsilon\text{-closure}}(q_1, q_2), 2))$$

$$= \epsilon\text{-closure}(\delta(q_1, 2) \cup \delta(q_2, 2))$$

$$= \epsilon\text{-closure}(\phi \cup q_2)$$

$$= \epsilon\text{-closure}(q_2)$$

$$= \{q_2\}$$

$$\delta(q_2, 1) = \epsilon\text{-closure}(\delta(\delta(q_2, \epsilon), 1))$$

$$= \epsilon\text{-closure}(\delta(\epsilon\text{-closure}(q_2), 1))$$

$$= \epsilon\text{-closure}(\delta(q_2), 1))$$

$$= \epsilon\text{-closure}(\delta(q_2, 1))$$

$$= \epsilon\text{-closure}(\phi)$$

$$= \phi$$

$$\delta(q_2, 2) = \epsilon.\text{closure}(\delta(\delta(q_2, \epsilon), 2))$$

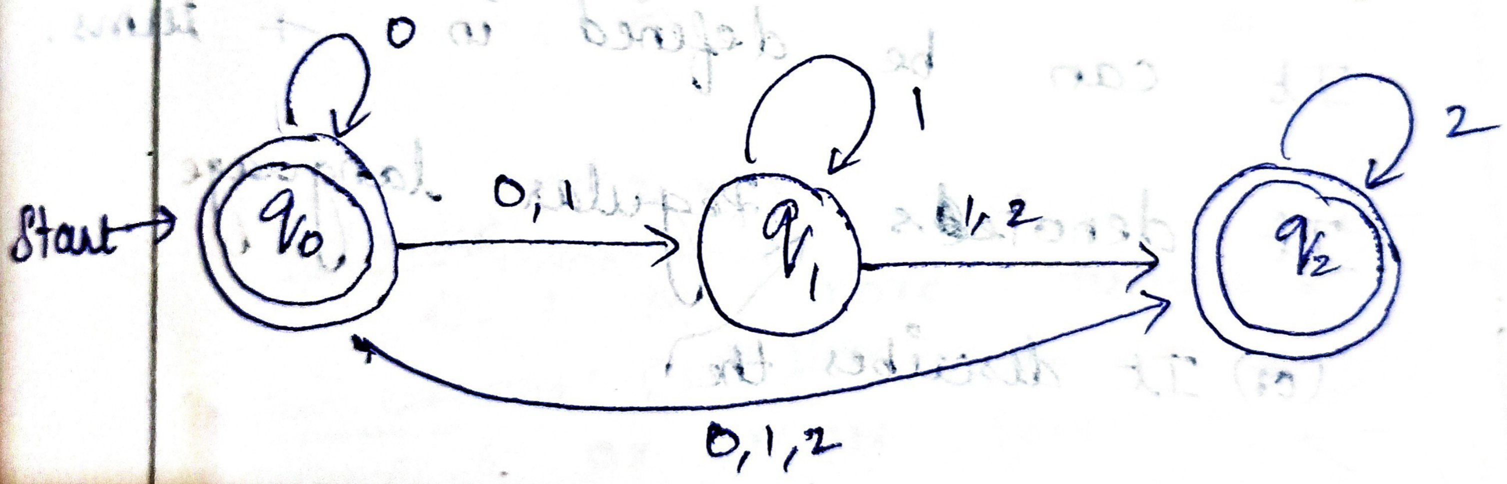
$$= \epsilon.\text{closure}(\delta(\epsilon.\text{closure}(q_2), 2))$$

$$= \epsilon.\text{closure}(\delta(q_2), 2))$$

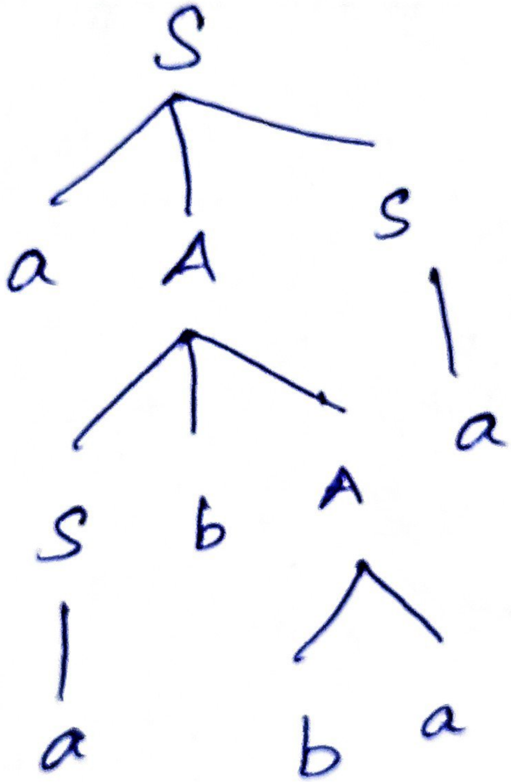
$$= \epsilon.\text{closure}(\delta(q_2, 2))$$

$$= \epsilon.\text{closure}(q_2)$$

$$= \{q_2\}$$



5) Find the left most and Right most derivation for



Ans :

1) What is derivation ?

2) defn for left most derivation

3) defn for right most derivation

LMD

Left most non terminal should be derived first

$S \Rightarrow aAS$

root node

$\Rightarrow aSbAS$

$\Rightarrow aabAS$

$\Rightarrow aabbaS$

$\Rightarrow aabbaa$

RMD

$S \Rightarrow aAS$

$\Rightarrow aAa$

$\Rightarrow aSbAa$

$\Rightarrow aSbbaa$

$\Rightarrow aabbaa$