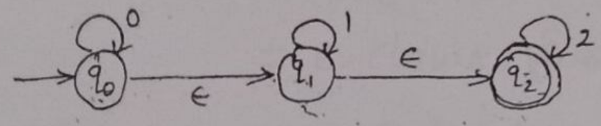
Example

Consider the NFA with ϵ-moves.



Find an equivalent NFA without ϵ-moves.

Solution:

$Q \quad \Sigma \quad \delta \quad q_0 \quad F$ — set of final states

Given NFA with ϵ-move

states — input symbols

$$M = (\{q_0, q_1, q_2\}, \{0, 1, 2, \epsilon\}, \delta, q_0, \{q_2\})$$

Now we have to define NFA without ϵ-move

$$M' = (Q', \Sigma, \delta', q_0, F')$$

$$Q = \{q_0, q_1, q_2\}$$

ϵ-closure $(q_0, \epsilon) = \{q_0\}$

$$\Sigma = \{0, 1, 2\}$$

$$F' = \{q_0, q_2\}$$

$\because$ [ϵ-closure $(q_0) = \{q_0, q_1, q_2\}$ contains a state of F.]

δ':

	0	1	2
q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
q_2	ϕ	ϕ	$\{q_2\}$

ϵ-closure $(q_1) = \{q_1, q_2\}$

ϵ-closure $(q_2) = \{q_2\}$

$$\delta(q_0, \epsilon) = \epsilon\text{-closure}(q_0)$$
$$= \cancel{\{q_0, q_1, q_2\}}$$

$$\delta(q_0, \epsilon) = \{q_0, q_1, q_2\}$$

$$\delta(q_1, \epsilon) = \{q_1, q_2\}$$

$$\delta(q_3, \epsilon) = \{q_2\}$$

$F = \{q_0, q_2\}$

$\delta(q_0, 0) = \epsilon\text{-closure}(\delta(q_0, 0))$

$= \epsilon\text{-closure}(\delta(\delta(q_0, \epsilon), 0)$

$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 0)$.

$= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$

$= \epsilon\text{-closure}(\{q_0\} \cup \phi \cup \phi)$.

$= \{q_0, q_1, q_2\}$.

$\delta'(q_0, 1) = \epsilon\text{-closure}(\delta(q_0, 1))$

$= \epsilon\text{-closure}(\delta(\delta(q_0, \epsilon), 1)$.

$= \epsilon\text{-closure}(\delta\{q_0, q_1, q_2\}, 1)$.

$= \epsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1))$

$= \epsilon\text{-closure}(\phi \cup \{q_1\} \cup \phi)$

$= \{q_1, q_2\}$.

$\delta'(q_0, 2) = \epsilon\text{-closure}(\delta(q_0, 2))$.

$= \epsilon\text{-closure}(\delta(\delta(q_0, \epsilon), 2)$.

$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 2)$.

$= \epsilon\text{-closure}(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2))$

$= \epsilon\text{-closure}(\phi \cup \phi \cup \{q_2\})$.

$= \{q_2\}$.

$\delta'(q_1, 0) = \epsilon\text{-closure}(\delta(q_1, 0))$.

$= \epsilon\text{-closure}(\delta(\delta(q_1, \epsilon), 0)$.

$= \epsilon\text{-closure}(\delta.\{q_1, q_2\}, 0)$

$= \epsilon\text{-closure}(\delta(q_1, 0) \cup \delta(q_2, 0))$

$= \phi.$

$\delta'(q_1, 1) = \epsilon\text{-closure}(\delta(q_1, 1))$

$= \epsilon\text{-closure}(\delta(\delta(q_1, \epsilon), 1)$.

$= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, 1)$.

$= \epsilon\text{-closure}(\delta(q_1, 1) \cup \delta(q_2, 1))$.

$= \epsilon\text{-closure}(\{q_1\} \cup \phi)$

$= \{q_1, q_2\}$.

$\delta'(q_1, 2) = \epsilon\text{-closure}(\delta(q_1, 2))$.

$= \epsilon\text{-closure}(\delta(\delta(q_1, \epsilon), 2)$

$= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, 2)$.

$= \epsilon\text{-closure}(\delta(q_1, 2) \cup \delta(q_2, 2))$

$= \epsilon\text{-closure}(\phi \cup \{q_2\})$.

$= \{q_2\}$.

$\delta'(q_2, 0) = \epsilon\text{-closure}(\delta(\delta(q_2, \epsilon), 0)$.

$= \epsilon\text{-closure}(\delta(\{q_2\}, 0))$

$= \epsilon\text{-closure}(\phi)$

$= \phi$.

$\delta'(q_2, 1) = \epsilon\text{-closure}(\delta(\delta(q_2, \epsilon), 1)$

$= \epsilon\text{-closure}(\delta(\{q_2\}, 1)$.

$= \epsilon\text{-closure}(\phi)$.

$= \phi$.

$$= \in\text{-closure}\,(\delta(\{q_2\}, 2))$$

$$= \in\text{-closure}\,(q_2).$$

$$= \{q_2\}.$$