* A problem is ~~decidable~~ if there is an
algorithm that takes as input an instance of the
problem and determines whether the answer to
that instance is "yes" or "no"

Closure Properties of recursive and Recursively
Enumerable languages.

Theorem 1

The complement of a recursive language is recursive

Proof

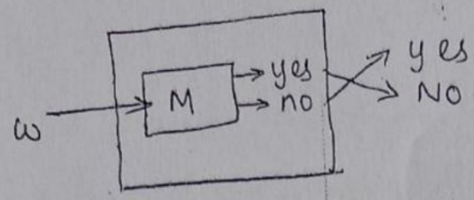
* Let L be a recursive language and M
a turing machine that halts on all inputs and
accepts L.

* Construct M' from M so that if M enters a
final state on input w then M' halts without
accepting.

* If M halts without accepting, M' enters a
final state.

* Since one of these two events occurs, M' is
an algorithm.

* Clearly $L(M')$ is the complement of L and
thus the complement of L is a recursive
language.

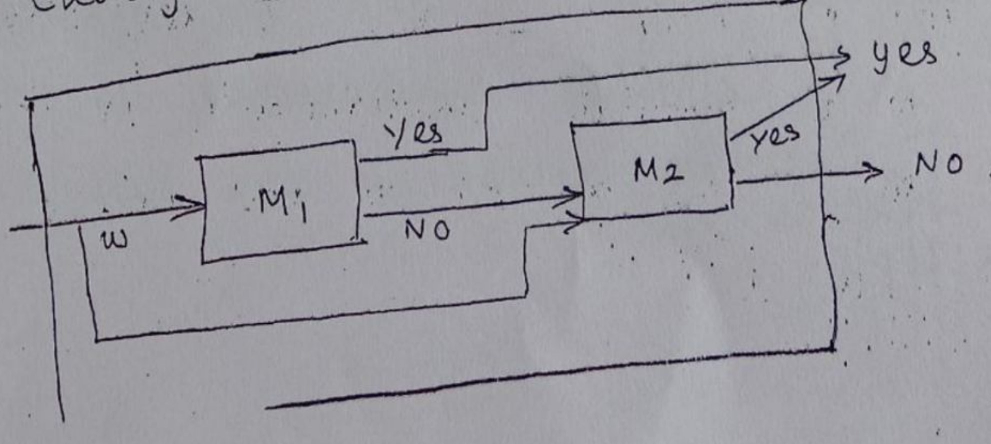


Theorem 2

* The union of two recursive language is recursive.
* The union of two recursively enumerable language is recursively enumerable.

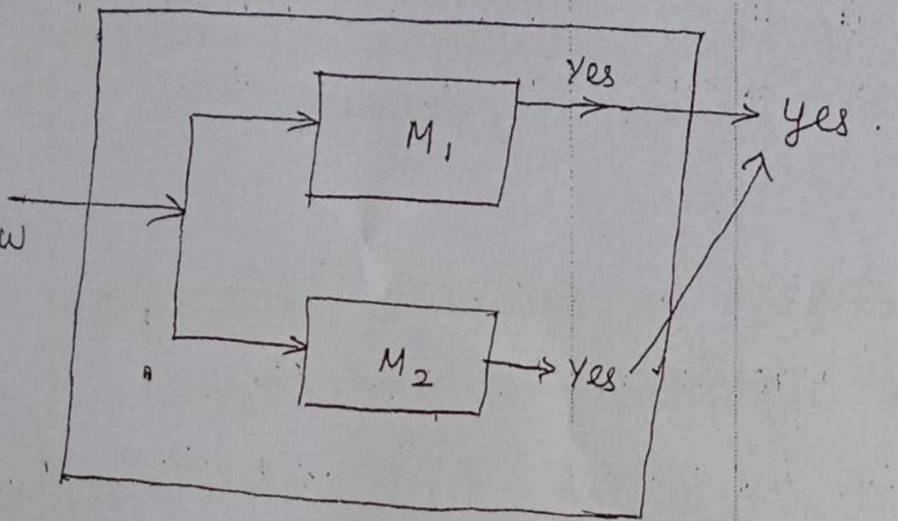
Proof:

* Let L_1 and L_2 be recursive languages accepted by algorithms M_1 and M_2.
* We construct M, which first simulates M_1.
* If M_1 accepts then M accepts.
* If M_1 rejects then M simulates M_2 and accepts iff M_2 accepts.
* Since both M_1 and M_2 are algorithms, M is generated to halt.
* Clearly M accepts $L_1 \cup L_2$.



construction does not work.

* Since M_1 may not halt.
* Instead, M can simultaneously simulate M_1 and M_2 on separate tapes.
* If either accepts, then M accepts.



Theorem 3:

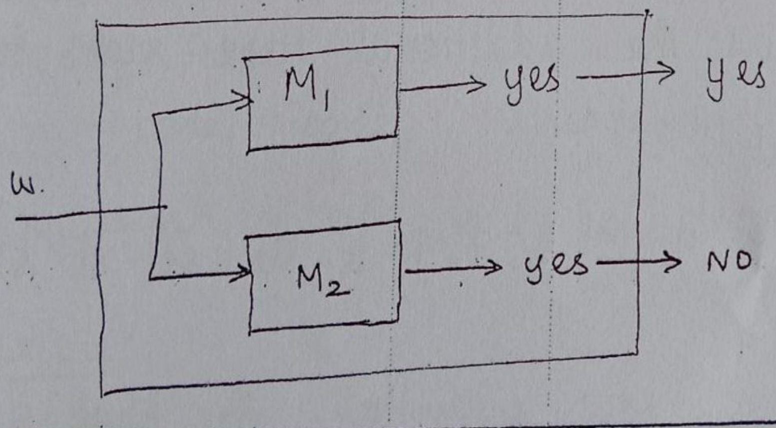
If a Language L and its complement $\bar{L}$ are both recursively Enumerable, then L (and hence $\bar{L}$) is recursive.

Proof

* Let M_1 and M_2 accept L and $\bar{L}$ respectively.
* Construct M to simulate simultaneously M and M_2
* M accepts w if M_1 accepts w and rejects w if M_2 accepts w

Since w is in either L or $\bar{L}$, we know that exactly one of M_1 or M_2 will accept.

* Thus M will always say either 'yes' or 'no' but will never say both.
* Since M is an algorithm that accepts L, it follows that L is recursive.



UNIVERSAL TURING MACHINE:

* Diagnolization is used to show the Problem