Q Prove by mathematical induction.

$$0^2 + 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Solution:

Let $P(n) = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

Basis step:

For $n = 0$

L.H.S $\sum_{i=0}^{n} i^2 = 0$.

R.H.S $\frac{n(n+1)(2n+1)}{6} = 0$.

Inductive step:

for $n = n-1$

$$\sum_{i=0}^{n-1} i^2 = \frac{n(n)(n-1)(2n-1)}{6}$$ implies $\sum_{i=0}^{n} i^2 = \frac{n(n+1)}{6}$

Since

$$\sum_{i=0}^{n} i^2 = \sum_{i=0}^{n-1} i^2 + n^2$$

$$= \frac{n(n-1)(2n-1)}{6} + n^2$$

$$= \frac{(n^2-n)(2n-1) + 6n^2}{6}$$

$$= \frac{2n^3 - 2n^2 - n^2 + n + 6n^2}{6}$$

$$= \frac{2n^3 + 3n^2 + n}{6}$$

$$= \frac{n(2n^2 + 3n + 1)}{6}$$

$$= \frac{n(n+1)(2n+1)}{6}$$

L.H.S = R.H.S

$\therefore$ Thus by induction it is true for all n

1. Let $M = (\{q_0, q_1\}, \{0, 1\}, \delta, q_0, \{q_1\})$ be an NFA, where

$\delta(q_0, 0) = \{q_0, q_1\}$.
$\delta(q_0, 1) = \{q_1\}$.
$\delta(q_1, 0) = \phi$.
$\delta(q_1, 1) = \{q_0, q_1\}$.

	0	1
q_0	$\{q_0, q_1\}$	$\{q_1\}$
q_1	ϕ	$\{q_0, q_1\}$

Solution:

We can construct a DFA

$M' = (Q', \{0, 1\}, \delta', [q_0], F')$.

Q' = all subsets of $\{q_0, q_1\}$.

$Q' = \{[q_0], [q_1], [q_0, q_1], \phi\}$.

Tecan F' = Set of states of Q' containing a state in F

$F' = \{[q_1], [q_0, q_1]\}$

Transition Table: δ'

	0	1
$[q_0]$	$[q_0, q_1]$	$[q_1]$
$[q_1]$	ϕ.	$[q_0, q_1]$
$[q_0, q_1]$	?	?
ϕ	ϕ	ϕ

To find $\delta'([q_0, q_1], 0)$

$$\delta(\{q_0, q_1\}, 0) = \delta(q_0, 0) \cup \delta(q_1, 0)$$
$$= \{q_0, q_1\} \cup \phi$$
$$= \{q_0, q_1\}$$

$$\delta'([q_0, q_1], 0) = [q_0, q_1]$$

To find $\delta'([q_0, q_1], 1)$

$$\delta(\{q_0, q_1\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1)$$
$$= \{q_1\} \cup \{q_0, q_1\}$$
$$= \{q_0, q_1\}$$

$$\therefore \delta'([q_0, q_1], 1) = [q_0, q_1]$$

$$\therefore M' = (Q', \Sigma', \delta', q_0', F')$$

$$Q' = \{[q_0], [q_1], [q_0, q_1], \phi\}$$

$$\Sigma = \{0, 1\}$$

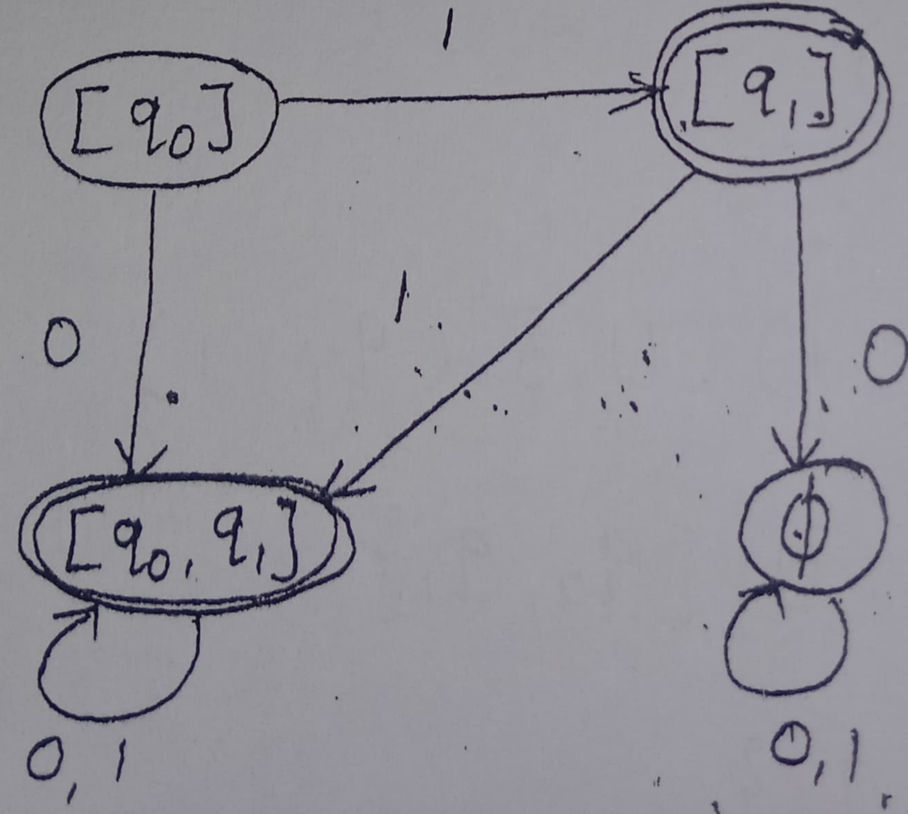
$$q_0' = [q_0]$$

$$F' = \{[q_1], [q_0, q_1]\}$$

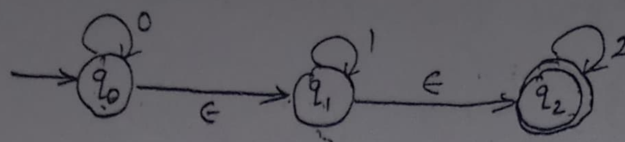
δ :

	0	1
$[q_0]$	$[q_0, q_1]$	$[q_1]$
$[q_1]$	ϕ	$[q_0, q_1]$
$[q_0, q_1]$	$[q_0, q_1]$	$[q_0, q_1]$
ϕ	ϕ	ϕ

Transition Diagram:



Consider the NFA with ϵ-moves.



Find an equivalent NFA without ϵ-moves.

Solution:

Given NFA with ϵ-move.

$$M = (\{q_0, q_1, q_2\}, \{0, 1, 2, \epsilon\}, \delta, q_0, \{q_2\})$$

Now we have to define NFA without ϵ-move

$$M' = (Q', \Sigma, \delta', q_0, F')$$

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0, 1, 2\}$

$F' = \{q_0, q_2\}$ $\quad \because$ ϵ-closure $(q_0) = \{q_0, q_1, q_2\}$ contains a state of F.

δ':

	0	1	2
q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
q_1	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
q_2	ϕ	ϕ	$\{q_2\}$

ϵ-closure $(q_1) = \{q_1, q_2\}$

ϵ-closure $(q_2) = \{q_2\}$

$\delta(q_0, \epsilon) = \epsilon$-closure (q_0)
$= \{q_0, q_1, q_2\}$

$\delta(q_0, 0) = \epsilon\text{-closure}(\delta(q_0, 0)).$

$= \epsilon\text{-closure}(\delta(\delta(q_0, \epsilon), 0).$

$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 0).$

$= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$

$= \epsilon\text{-closure}(\{q_0\} \cup \phi \cup \phi).$

$= \{q_0, q_1, q_2\}.$

$\delta'(q_0, 1) = \epsilon\text{-closure}(\delta(q_0, 1)).$

$= \epsilon\text{-closure}(\delta(\delta(q_0, \epsilon), 1).$

$= \epsilon\text{-closure}(\delta\{q_0, q_1, q_2\}, 1).$

$= \epsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1))$

$= \epsilon\text{-closure}(\phi \cup \{q_1\} \cup \phi)$

$= \{q_1, q_2\}.$

$\delta'(q_0, 2) = \epsilon\text{-closure}(\delta(q_0, 2)).$

$= \epsilon\text{-closure}(\delta(\delta(q_0, \epsilon), 2).$

$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 2).$

$= \epsilon\text{-closure}(\delta(q_0, 2) \cup \delta(q_1, 2) \cup \delta(q_2, 2))$

$= \epsilon\text{-closure}(\phi \cup \phi \cup \{q_2\}).$

$= \{q_2\}.$

$\delta'(q_1, 0) = \epsilon\text{-closure}(\delta(q_1, 0)).$

$= \epsilon\text{-closure}(\delta(\delta(q_1, \epsilon), 0).$

$= \epsilon\text{-closure}(\delta\{q_1, q_2\}, 0)$

$= \epsilon\text{-closure}(\delta(q$

$= \phi.$

$$\delta'(q_1, 1) = \epsilon\text{-closure}(\delta(q_1, 1))$$
$$= \epsilon\text{-closure}(\delta(\delta(q_1, \epsilon), 1))$$
$$= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, 1))$$
$$= \epsilon\text{-closure}(\delta(q_1, 1) \cup \delta(q_2, 1))$$
$$= \epsilon\text{-closure}(\{q_1\} \cup \phi)$$
$$= \{q_1, q_2\}.$$

$$\delta'(q_1, 2) = \epsilon\text{-closure}(\delta(q_1, 2))$$
$$= \epsilon\text{-closure}(\delta(\delta(q_1, \epsilon), 2))$$
$$= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, 2))$$
$$= \epsilon\text{-closure}(\delta(q_1, 2) \cup \delta(q_2, 2))$$
$$= \epsilon\text{-closure}(\phi \cup \{q_2\}).$$
$$= \{q_2\}.$$

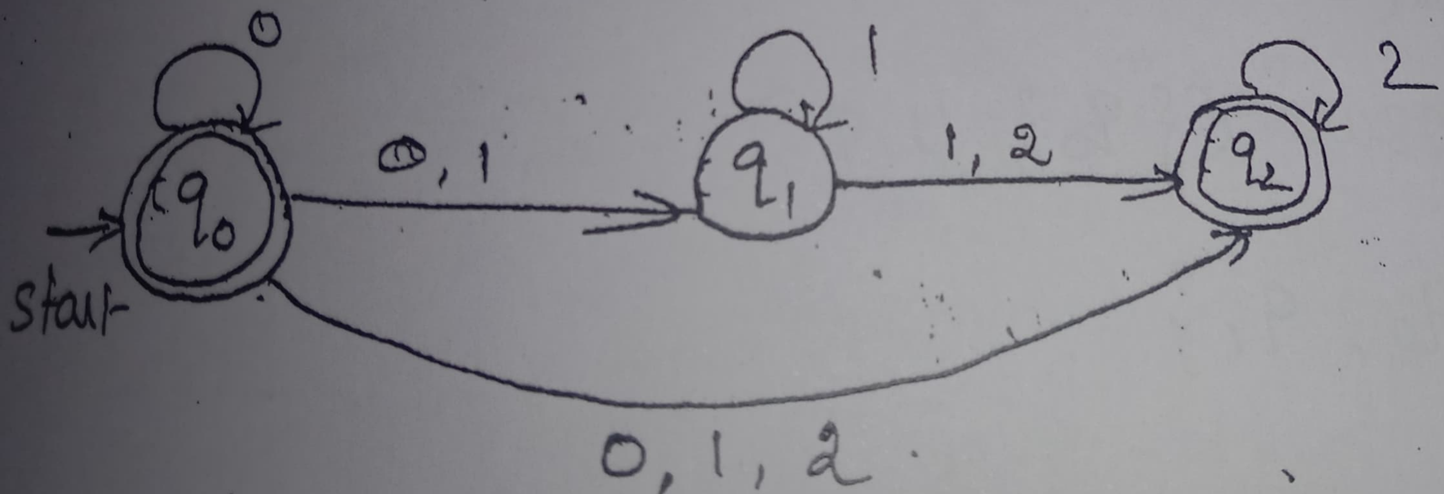
$$\delta'(q_2, 0) = \epsilon\text{-closure}(\delta(\delta(q_2, \epsilon), 0))$$
$$= \epsilon\text{-closure}(\delta(\{q_2\}, 0))$$
$$= \epsilon\text{-closure}(\phi)$$
$$= \phi.$$

$$\delta'(q_2, 1) = \epsilon\text{-closure}(\delta(\delta(q_2, \epsilon), 1))$$
$$= \epsilon\text{-closure}(\delta(\{q_2\}, 1))$$
$$= \epsilon\text{-closure}(\phi)$$
$$= \phi.$$

$= \epsilon\text{-closure}(\delta(\{q_2\}, 2))$

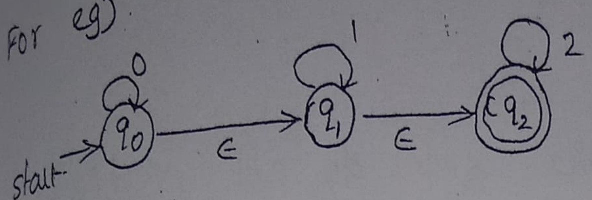
$= \epsilon\text{-closure}(q_2)$

$= \{q_2\}$



* The NFA may be extended to include transitions on the empty input ϵ.

* For eg)



* We say an NFA accepts a string w if there is some path labeled w from the initial state to a final state, of course edges labeled ϵ may be included in the path, although the ϵ's do not appear explicitly in w.

* For eg) the word 002 is accepted by the NFA by the path $q_0 - q_0 - q_0 - q_1 - q_2 - q_2$ with arcs labeled $0, 0, \epsilon, \epsilon, 2$.

Formal definition:

* A NFA with ϵ-moves to be a 5-tuple,

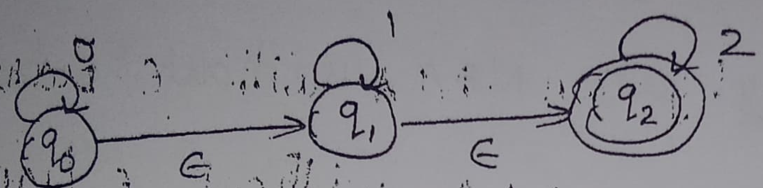
$$M = (Q, \Sigma, \delta, q_0, F)$$

δ is the transition function maps $Q \times (\Sigma \cup \{\epsilon\})$ to 2^Q.

* $\delta(q, a)$ will consist of all states p such that there is a transition labeled a from q to p, where a is either ϵ or a symbol in

* **Example:**

Consider the NFA,



Find $\hat{\delta}(q_0, 01)$.

$\hat{\delta}(q_0, 01) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, 0), 1))$.

$\hat{\delta}(q_0, 0) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), 0))$.

$\hat{\delta}(q_0, \epsilon) = \epsilon\text{-closure}(q_0)$.

$= \{q_0, q_1, q_2\}$.

$\hat{\delta}(q_0, 0) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_0, \epsilon), 0))$

$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 0))$.

$= \epsilon\text{-closure}(\delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0))$

$= \epsilon\text{-closure}(\{q_0\} \cup \phi \cup \phi)$.

$= \epsilon\text{-closure}(\delta(\{q_0, q_1, q_2\}, 1))$

$= \epsilon\text{-closure}(\delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1))$

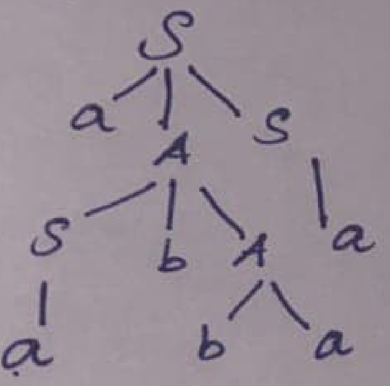
$= \epsilon\text{-closure}(\phi \cup \{q_1\} \cup \phi)$

$= \epsilon\text{-closure}(q_1)$

$= \{q_1, q_2\}$

on Fin. Automata with ϵ Transition.

5. Find the Left most & Right most derivation for:



What is Derivation?

defn. for L.m. "

R.m. ".

defn. for Deriv. Tree.

LMD: (Left most non-Terminal)

$S \overset{lmd}{\Rightarrow} a\underline{A}S$

$\overset{lmd}{\Rightarrow} a\underline{S}bAS$

$\Rightarrow aabAS$

$\Rightarrow aabbaS$

$\Rightarrow aabbaa$

RMD: (Right most non-terminal)

$S \Rightarrow aA\underline{S}$

$\Rightarrow aAa$

$\Rightarrow aSb\underline{A}a$

$\Rightarrow a\underline{S}bbaa$

$\Rightarrow aabbaa.$

id is ambiguous.

III CSE B

* Example.

Show that the Grammar G

$E \rightarrow E+E \mid E*E \mid (E) \mid id$ is ambiguous

Solution:

* Deriving a string id + id * id

LMD 1:

$E \underset{lm}{\Longrightarrow} E+E$

$\underset{lm}{\Longrightarrow} id+E$

$\underset{lm}{\Longrightarrow} id+E*E$

$\underset{lm}{\Longrightarrow} id+id*E$

$\underset{lm}{\Longrightarrow} id+id*id$

LMD 2:

$E \underset{lm}{\Longrightarrow} E*E$

$\underset{lm}{\Longrightarrow} E+E*E$

$\underset{lm}{\Longrightarrow} id+E*E$

$\underset{lm}{\Longrightarrow} id+id*E$

$\underset{lm}{\Longrightarrow} id+id*id$.

* For the word id + id * id, there exists two left most derivation.

RMD 1:

$E \underset{lm}{\Longrightarrow} E+E$

$\underset{rm}{\Longrightarrow} E+E*E$

$\underset{lm}{\Longrightarrow} E+E*id$

$\underset{rm}{\Longrightarrow} E+id*id$

$\underset{rm}{\Longrightarrow} id+id*id$

RMD 2:

$E \underset{rm}{\Longrightarrow} E*E$

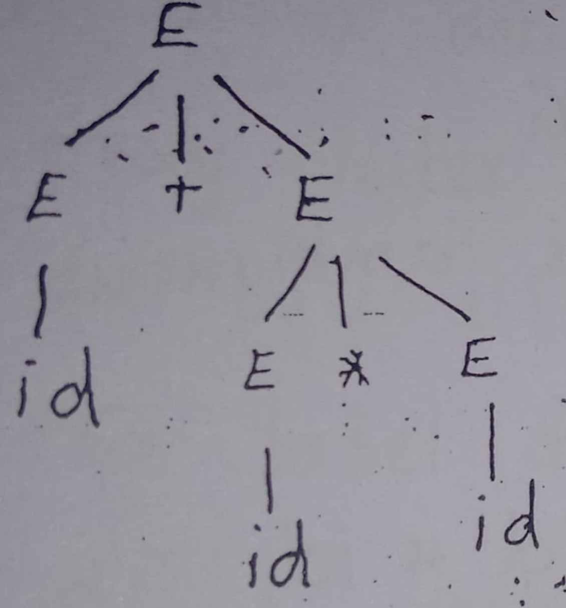
$\underset{rm}{\Longrightarrow} E*id$

$\underset{rm}{\Longrightarrow} E+E*id$.

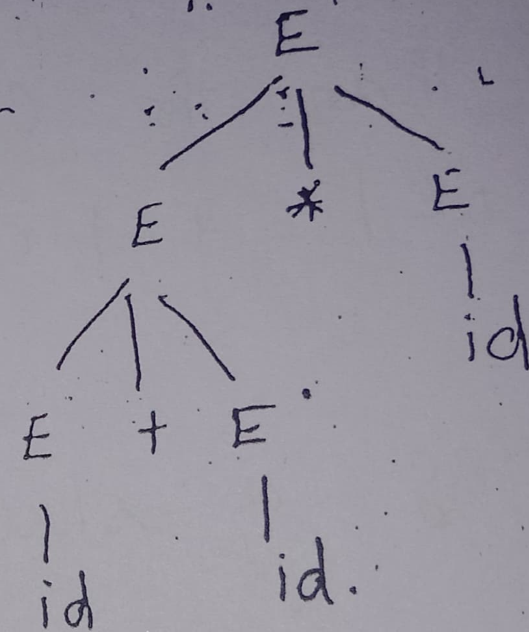
$\underset{rm}{\Longrightarrow} E+id*id$

$\underset{rm}{\Longrightarrow} id+id*id$.

* For the word id + id * id, there exists two right most derivation.



Parse tree 2



* For the word id + id * id has two parse

i.e. Grammar is ambiguous.

Example:

Convert into GNF from the grammar

$$G = (\{A_1, A_2, A_3\}, \{a, b\}, P, A_1$$

where P consists of

$$A_1 \rightarrow A_2 A_3$$
$$A_2 \rightarrow A_3 A_1 \mid b$$
$$A_3 \rightarrow A_1 A_2 \mid a$$

Since RHS of the productions for A_1 and A_2 start with terminal or higher-numbered variables. We begin with the production:

$$A_3 \to \underline{A_1} A_2$$

$$A_3 \to \underline{A_2 A_3} A_2 \quad (\because A_1 \to A_2 A_3);$$

$$A_3 \to A_3 A_1 A_3 A_2 \mid b A_3 A_2 \quad (\because A_2 \to A_3 A_1 \mid b$$

* The new resultant set of productions

$$A_1 \to A_2 A_3$$

$$A_2 \to A_3 A_1 \mid b.$$

$$A_3 \to A_3 A_1 A_3 A_2 \mid b A_3 A_2 \mid a$$

* We now apply lemma 2 to the production

$$A_3 \to A_3 A_1 A_3 A_2 \mid b A_3 A_2 \mid a$$

Symbol B_3 is introduced and the production

$A_3 \to A_3 A_1 A_3 A_2$ is replaced by

$$A_3 \to b A_3 A_2 \mid a$$

$$A_3 \to b A_3 A_2 B_3 \mid a B_3$$

$$B_3 \to A_1 A_3 A_2$$

$$B_3 \to A_1 A_3 A_2 B_3$$

$A_1 \to A_2 A_3$

$A_2 \to A_3 A_1 \mid b$

$A_3 \to b A_3 A_2 B_3 \mid a B_3 \mid b A_3 A_2 \mid a$

$B_3 \to A_1 A_3 A_2 \mid A_1 A_3 A_2 B_3$

* Now all the productions with A_3 on the left RHS that start with terminals.

* These are used to replace A_3 in the prod $A_2 \to A_3 A_1$ and then the production with A_2 the left are used to replace A_2 in the produ $A_1 \to A_2 A_3$

* The new set of productions.

$A_3 \to b A_3 A_2 B_3 \mid a B_3 \mid b A_3 A_2 \mid a$

$A_2 \to b A_3 A_2 B_3 A_1 \mid a B_3 A_1 \mid b A_3 A_2 A_1 \mid a A_1 \mid$

$A_1 \to b A_3 A_2 B_3 A_1 A_3 \mid a B_3 A_1 A_3 \mid b A_3 A_2 A_1 A_3 \mid a A_1 A_3 \mid b A_3$

$B_3 \to b A_3 A_2 B_3 A_1 A_3 A_3 A_2 \mid a B_3 A_1 A_3 A_3 A_2 \mid b A_3 A_2 A_1 A_3 A_3 A_2 \mid a A_1 A_3 A_3 A_2 \mid b A_3 A_3 A_2$

$B_3 \to b A_3 A_2 B_3 A_1 A_3 A_3 A_2 B_3 \mid a B_3 A_1 A_3 A_3 A_2 B_3 \mid b A_3 A_2 A_1 A_3 A_3 A_2 B_3 \mid a A_1 A_3 A_3 A_2 B_3 \mid b A_3 A_3 A_2$