...tem and determines whe...
instance is "yes" or "no"

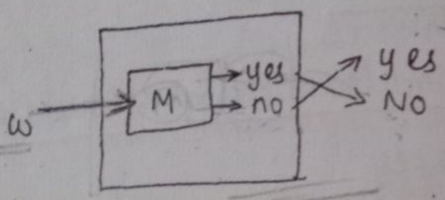
Closure Properties of recursive and Recursively Enumerable languages

Theorem 1

The complement of a recursive language is recursive

Proof

* Let L be a recursive language and M a turing machine that halts on all inputs and accepts L. (L enter into TM)
* Construct M' from M so that if M enters a final state on input w then M' halts without accepting.
* If M halts without accepting, M' enters a final state.
* Since one of these two events occurs, M' is an algorithm.
* Clearly $L(M')$ is the complement of L and thus the complement of L is a recursive language.



Theorem 2

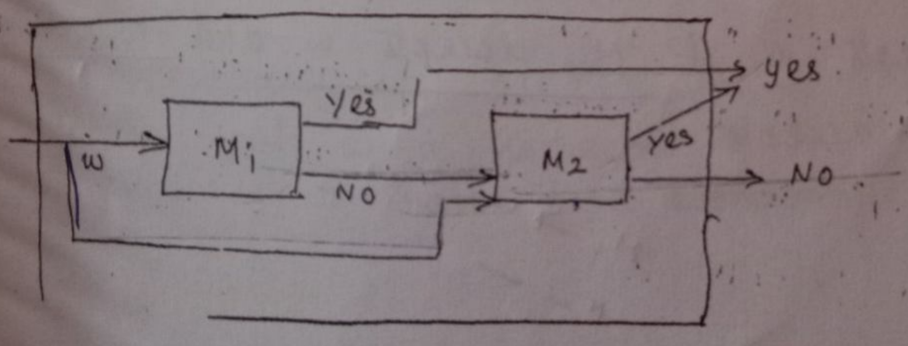
* The union of two recursive language is recursive.
* The union of two recursively enumerable language is recursively enumerable.

Proof:

* Let L_1 and L_2 be recursive languages accepted by algorithms M_1 and M_2.
* We construct M, which first simulates M_1.
* If M_1 accepts then M accepts
* If M_1 rejects then M simulates M_2 and accepts iff M_2 accepts.
* Since both M_1 and M_2 are algorithms, M is generated to halt.
* Clearly M accepts $L_1 \cup L_2$.

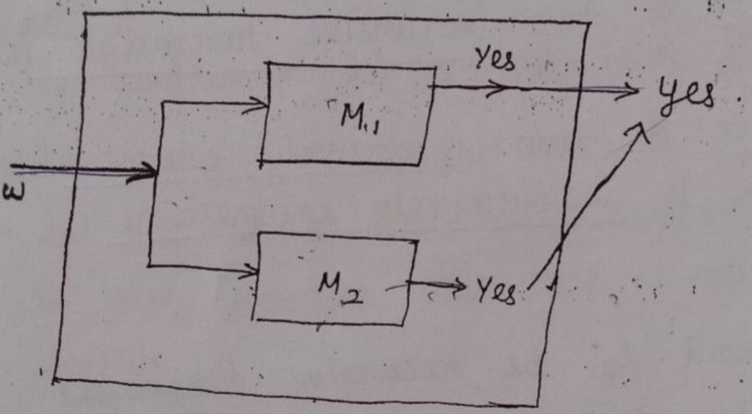
$M_1 \times \xrightarrow{4} M_2$

M_1 accept M_4TM accepted



... recursively enumerable ... construction does not work.

* Since M_1 may not halt.
* Instead, M can simultaneously simulate M_1 and M_2 on separate tapes.
* If either accepts, then M accepts.



M_1 ×

Theorem 3:

If a Language L and its complement $\bar{L}$ are both recursively Enumerable, then L (and hence $\bar{L}$) is recursive.

Proof

* Let M_1 and M_2 accept L and $\bar{L}$ respectively.
* Construct M to simulate simultaneously M_1 and M_2
* M accepts w if M_1 accepts w and rejects w if M_2 accepts w

$M_1 \to w$

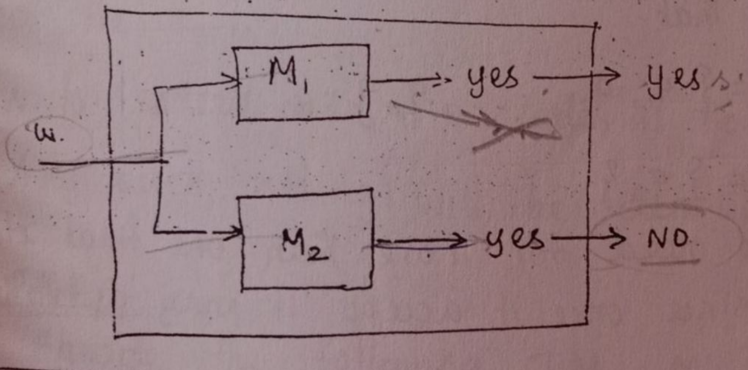
N ✓

* Since w is in either L or $\bar{L}$, we know that exactly one of M_1 or M_2 will accept.

* Thus M will always say either 'yes' or 'no' but will never say both.

accept

* Since M is an algorithm that accepts L, it follows that L is recursive.



M_1 accept pama
M_2 must reject

UNIVERSAL TURING MACHINE:

* Diagnolization is used to show the Problem