... (4.4.3)

Example 4.4.1 *Translate a polygon with co-ordinates A (2, 5), B (7, 10) and C (10, 2) by 3 units in x direction and 4 units in y direction.*

Solution :

Translate:

A (2, 5) B (7, 10) C (10, 2)

$x' = x + tx$

$y' = y + ty$

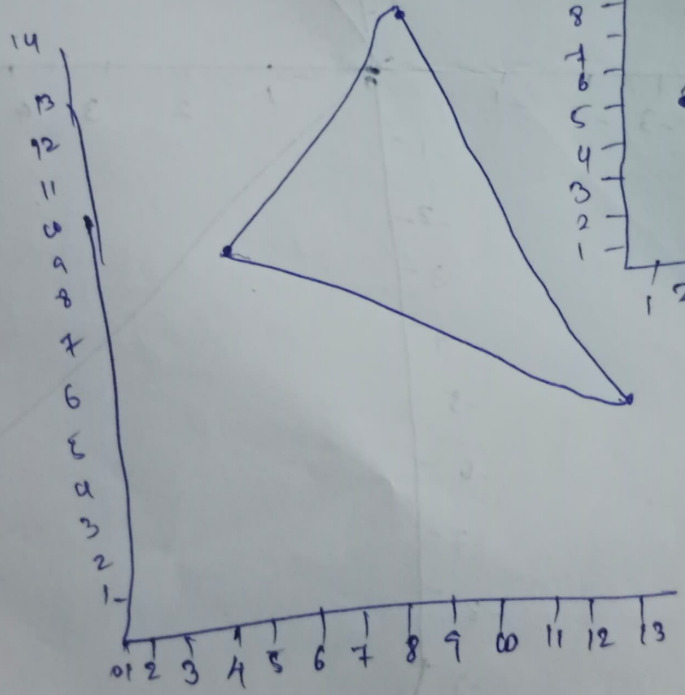
$A' = (5, 9)$

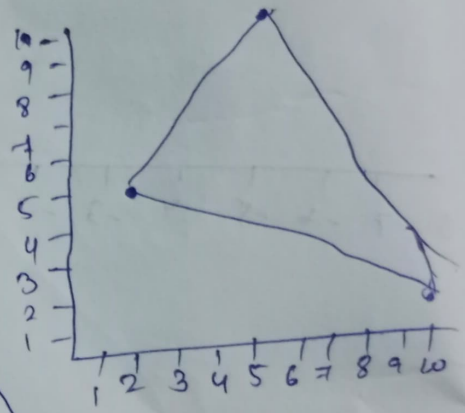
$B' = (10, 14)$

$C' = (13, 6)$

tx ty

(3, 4)





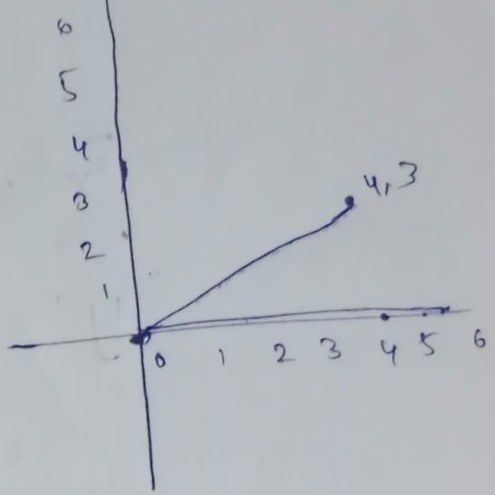
Example 4.4.2 *A point (4, 3) is rotated counterclockwise by an angle of Find the rotation matrix and the resultant point.*

Solution :

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$\therefore \quad P^1 = \begin{bmatrix} 4 & 3 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 4/\sqrt{2} - 3/\sqrt{2} & 4/\sqrt{2} + 3/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 7/\sqrt{2} \end{bmatrix}$$

$P_2' = (6.28, 10)$



Rotation $(4, 3)$, $45°$

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$

$$x' = 4\cos 45° - 3\sin 45°$$

$$= 4\left(\frac{1}{\sqrt{2}}\right) - 3\left(\frac{1}{\sqrt{2}}\right)$$

$$= \frac{4}{\sqrt{2}} - \frac{3}{\sqrt{2}} = \frac{4\sqrt{2} - 3\sqrt{2}}{2} = \frac{\sqrt{2}}{2\sqrt{2}} = \boxed{\frac{1}{\sqrt{2}}}$$

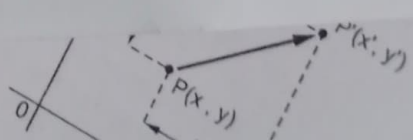
$$= 4\left(\frac{1}{\sqrt{2}}\right) + 3\frac{1}{\sqrt{2}}$$

$$= \frac{4}{\sqrt{2}} + \frac{3}{\sqrt{2}}$$

$$\boxed{= \frac{7}{\sqrt{2}}}$$

$\sqrt{2} = 1.414$

$$\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$$



Example 4.4.3 *Scale the polygon with co-ordinates A (2, 5), B (7, 10) and C (10, 2) by two units in x direction and two units in y direction.*

Solution : Here $S_x = 2$ and $S_y = 2$. Therefore, transformation matrix is given as

16) Scaling:- 2D

a (2, 5) b (7, 10) c (10, 2)

x dir = 2 y dir = 2

scaling factor (2, 2)
S_x S_y

fixed point- (7, 10)
x_f y_f

Formula:-

$$x' = x_f + (x - x_f) S_x$$

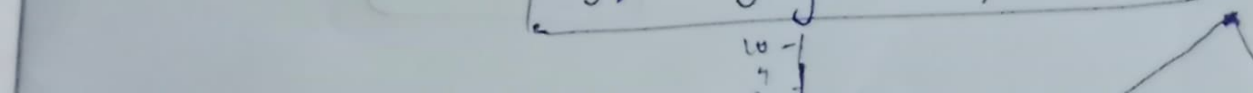
$$x' = x_f + S_x x - S_x x_f$$

$$\boxed{x' = x S_x + x_f (1 - S_x)}$$

$$y' = y_f + (y - y_f) S_y$$

$$= y_f + S_y y - S_y y_f$$

$$\boxed{y' = y S_y + y_f (1 - S_y)}$$



$(\underset{x}{2}, \underset{y}{5})$

$x' = xS_x + x_f(1 - S_x)$

$= 2(2) + 7(1-2)$

$= 4 + 7(-1)$

$= 4 - 7$

$\boxed{x' = -3}$

$y' = yS_y + y_f(1 - S_y)$

$= 5(2) + 10(1-2)$

$= 10 + 10(-1)$

$= 10 - 10$

$\boxed{y' = 0}$

$(10, 2)$

$x \quad y$

$x' = xS_x + x_f(1 - S_x)$

$= 10(2) + 7(1-2)$

$= 20 - 7$

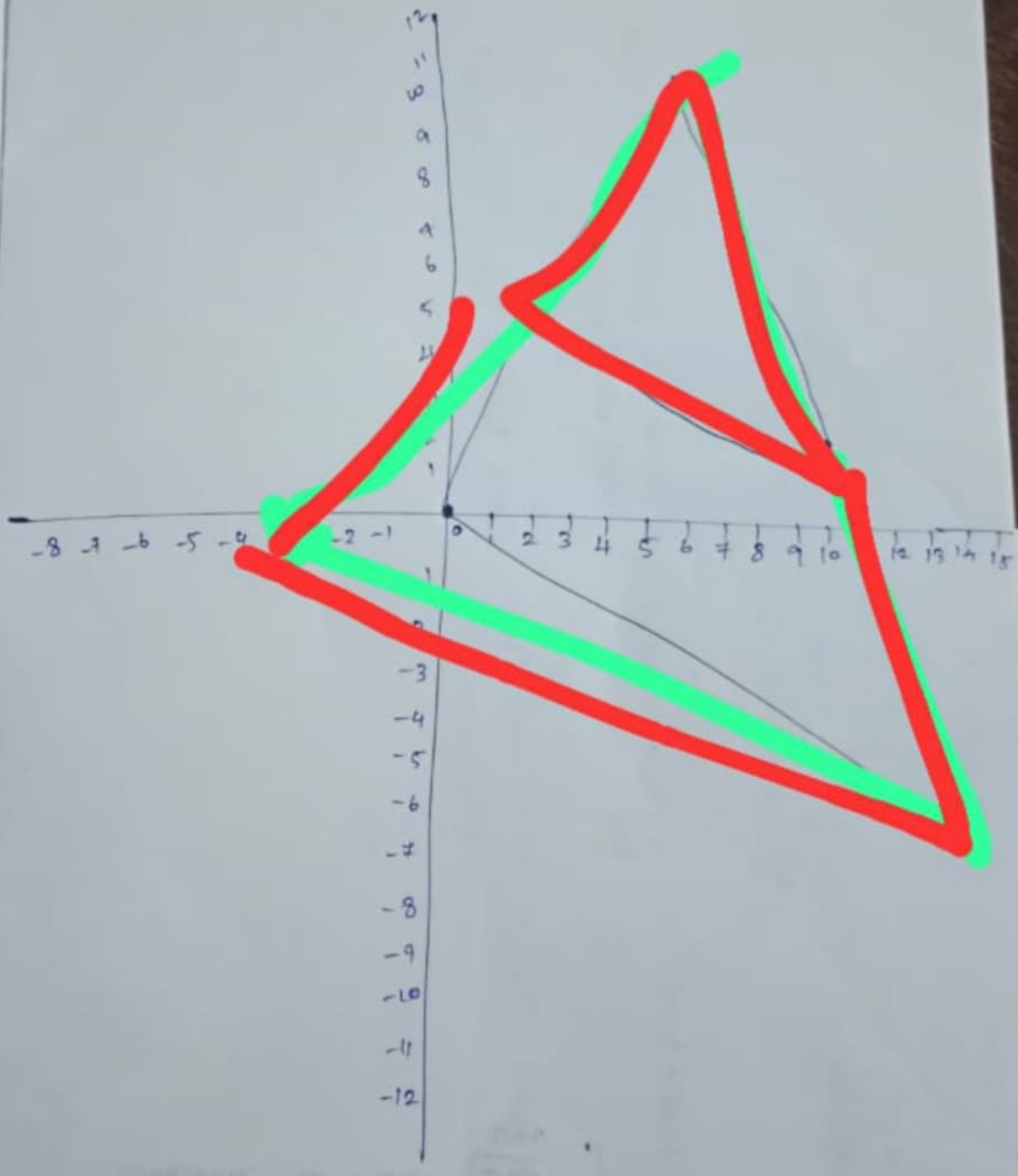
$\boxed{x' = 14}$

$y' = yS_y + y_f(1 - S_y)$

$= 2(2) + 10(1-2)$

$= 4 - 10$

$\boxed{y' = -6}$


12
11
10
9
8
7
6
5
4
3
2
1
-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 12 13 14 15
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12