2.3.1 Vector Generation/Digital Differential Analyzer (DDA) Algorithm

- The vector generation algorithms which step along the line to determine the pixel which should be turned on are sometimes called **Digital Differential Analyzer (DDA).**
- The slope of a straight line is given as,

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \qquad \ldots (2.3.1)$$

The above differential equation can be used to obtain a rasterized straight line. For any given x interval along a line, we can compute the corresponding y interval Δy from equation (2.3.1) as,

$$\Delta y = \frac{y_2 - y_1}{x_2 - x_1} \Delta x \qquad \ldots (2.3.2)$$

Similarly, we can obtain the x interval Δx corresponding to a specified Δy as

$$\Delta x = \frac{x_2 - x_1}{y_2 - y_1} \Delta y \qquad \ldots (2.3.3)$$

- Once the intervals are known the values for next x and next y on the straight line can be obtained as follows,

$$x_{i+1} = x_i + \Delta x$$

$$= x_i + \frac{x_2 - x_1}{y_2 - y_1} \Delta y \qquad \ldots (2.3.4)$$

and

$$y_{i+1} = y_i + \Delta y$$

$$= y_i + \frac{y_2 - y_1}{x_2 - x_1} \Delta x \qquad \ldots (2.3.5)$$

- The equations (2.3.4) and (2.3.5) represent a recursion relation for successive values of x and y along the required line. Such a way of rasterizing a line is called a **digital differential analyzer (DDA).** For simple DDA either Δx or Δy , whichever is larger, is chosen as one raster unit, i.e.

if $|\Delta x| \geq |\Delta y|$ then

else $\Delta x = 1$

$\Delta y = 1$

With this simplification, if $\Delta x = 1$ then

we have $y_{i+1} = y_i + \frac{y_2 - y_1}{x_2 - x_1}$ and

$x_{i+1} = x_i + 1$

If $\Delta y = 1$ then

we have $y_{i+1} = y_i + 1$ and

$$x_{i+1} = x_i + \frac{x_2 - x_1}{y_2 - y_1}$$

Let us see the vector generation/digital differential analyzer (DDA) routine for rasterizing a line.

Vector Generation/DDA Line Algorithm

```
1. Read the line end points $(x_1, y_1)$ and $(x_2, y_2)$ such that they are not equal.
        [if equal then plot that point and exit]
2. $\Delta x = |x_2 - x_1|$ and $\Delta y = |y_2 - y_1|$
3. If $(\Delta x \geq \Delta y)$ then
        length = $\Delta x$
     else
          length = $\Delta y$
     end if
4. $\Delta x = (x_2 - x_1)$ / length
     $\Delta y = (y_2 - y_1)$ / length
```

[This makes either Δx or Δy equal to 1 because length is either $|x_2 - x_1|$ or $|y_2 - y_1|$. Therefore, the incremental value for either x or y is one.]

```
5. $x = x_1 + 0.5 * \text{Sign}(\Delta x)$
     $y = y_1 + 0.5 * \text{Sign}(\Delta y)$
```

Here, Sign function makes the algorithm work in all quadrant. It returns – 1, 0, 1 depending on whether its argument is < 0, = 0, > 0 respectively. The factor 0.5 makes it possible to round the values in the integer function rather than truncating them.

```
        plot (Integer (x), Integer (y))
6. i = 1
     [Begins the loop, in this loop points are plotted]
     While ( $i \leq$ length)
     {
          x = x + $\Delta x$
          y = y + $\Delta y$
          plot (Integer (x), Integer (y))
          i = i + 1
     }
7. Stop
```

Let us see few examples to illustrate this algorithm.

11) a) consider the line from (0, 0) to (4, 6)
$x_1\ y_1$ $x_2\ y_2$
use the simple DDA Algorithm to rastrize this line.

Soln:-

$x_1\ y_1$ $x_2\ y_2$
$(0, 0)$ $(4, 6)$

$\Delta x = x_2 - x_1$ $\Delta y = y_2 - y_1$

$\Delta x = 4 - 0$ $\Delta y = 6 - 0$

$\boxed{\Delta x = 4}$ $\boxed{\Delta y = 6}$

$\Delta x < \underline{\Delta y}$

formula $\boxed{\text{Steps} = \text{abs}(\Delta y)}$

compare Δx & Δy ~~put~~ find the greatest and put it in the steps formula

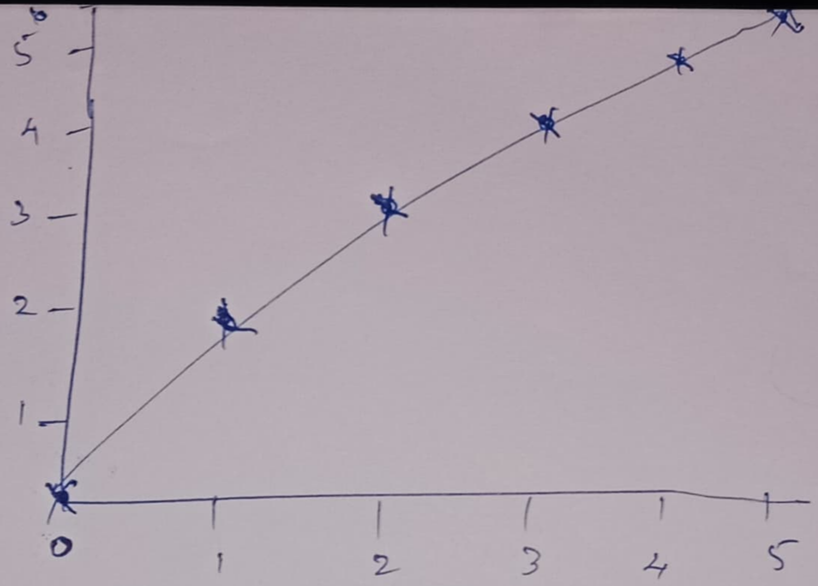
$3\overline{)20}$ 0.6, 18, 2; $\frac{2}{3}$

$$x \text{ increment} = \frac{\Delta x}{\text{Steps}} = \frac{4}{6} = 0.67$$

$$y \text{ increment} = \frac{\Delta y}{\text{Steps}} = \frac{6}{6} = 1$$

X	y	
0	0	→ initial value
0.67 (x increment)	1 (y increment)	
(0.67 + 0.67) 1.34	(1+1) 2	
(0.67 + 1.34) 2.01	(2+1) 3	
(0.67 + 2.01) 2.68	(3+1) 4	
(0.67 + 2.68) 3.35	(4+1) 5	
(0.67 + 3.35) 4.02	(5+1) 6	→ End value

1
0.67
1.34
2.01
0.67
2.68
0.67
3.35
6.67
4.02


6
5
4
3
2
1
0
1
2
3
4
5