Population dynamics

with emphasis on mortality research

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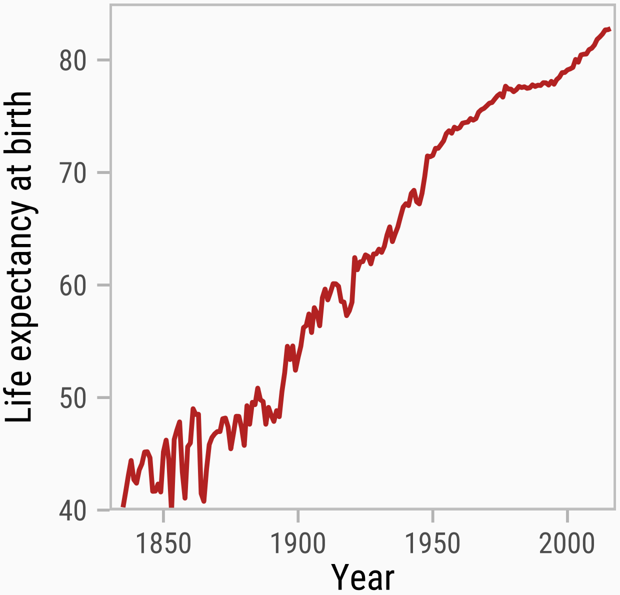
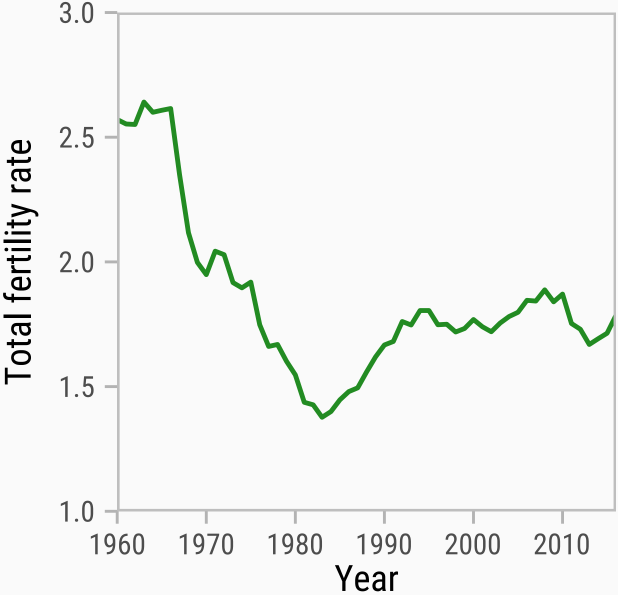
Life expectancy at birth, Danish females. 1835-2017



Total fertility rate, Danish females. 1960-2018



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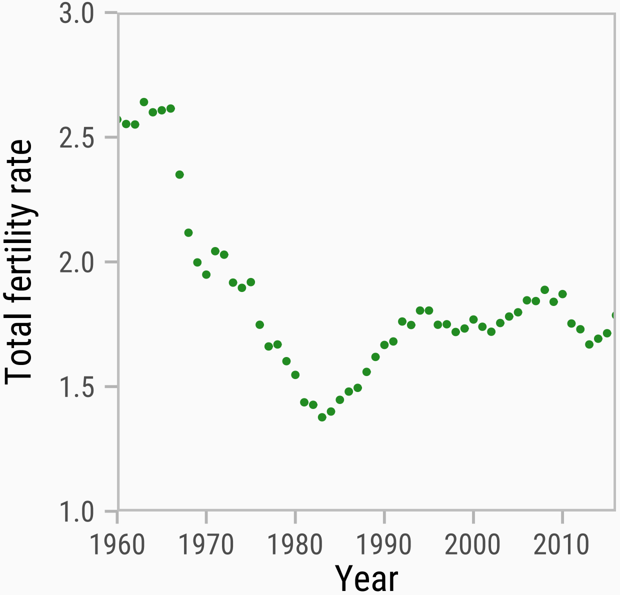
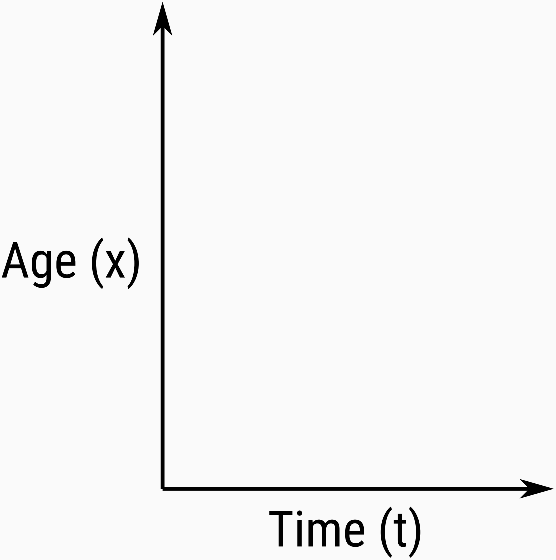
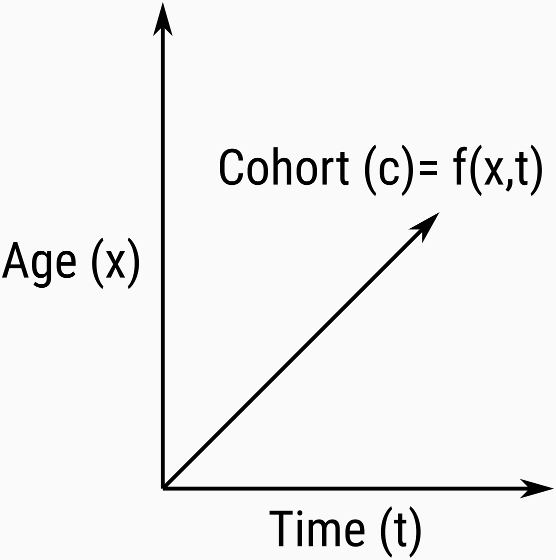


Changes over time and age


Age (x)
Time (t)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Changes over time and age

Mortality

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Family demography

- Expected years ever married (Mogi and Canudas-Romo, 2018)

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- the modal age at death?

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Limitations:

Bad approximations from discrete to continuous measures

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Discrete approach

$$e(0) = L_0 + L_1 + L_2 + ...L_{100+}$$

How to calculate changes over time in demographic measures?

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How to apply neat continuous formulas to messy discrete data?

Data

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   - Sources: HMD, UN, WHO, LAMBdA (Latin America), etc.

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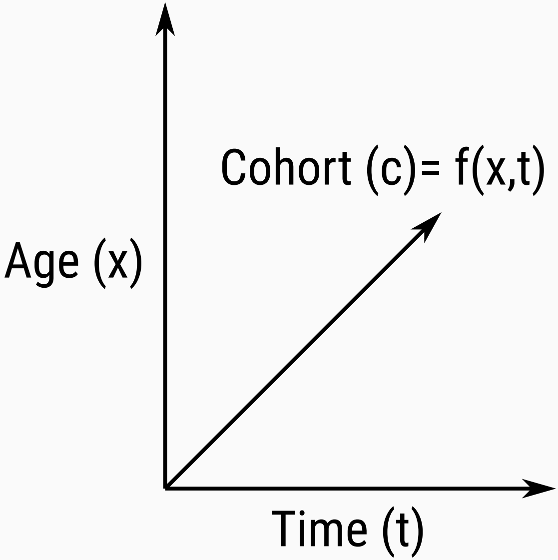
$$\frac{df(x)}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

In what direction should we derive?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Rate of ageing (in the cohort direction)

$$\beta(x,t) = \frac{\frac{\partial \mu(x,t)}{\partial t \partial x}}{\mu(x,t)}$$

$$\beta(x,t) = b(x,t) - \rho(x,t)$$

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The approximation depends on the size of h.

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$$\int_a^b f(x,t)dx \cong \frac{b-a}{8}[f(a)+3f(\frac{2a+b}{3})+3f(\frac{a+2b}{3})+f(b)]$$

Producing continuous estimates

Some approaches to produce continuous estimates

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    - Ungrouping (Rizzi, 2015)

Discrete approximation from life tables

Vaupel and Canudas-Romo (2003) approximation

$$\rho(x, t + h/2) \cong \frac{\ln\left(\frac{\mu(x, t+h)}{\mu(x, t)}\right)}{h}$$

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- Easy to estimate from life tables,
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Disadvantages

- Not very precise.

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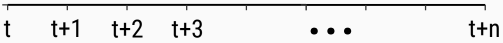

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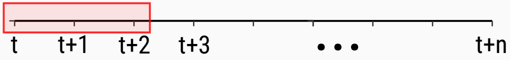

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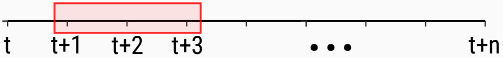

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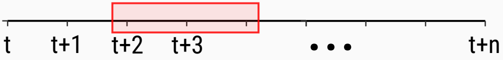

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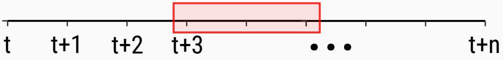

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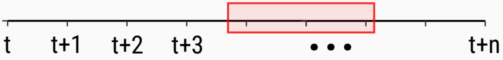

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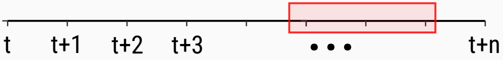

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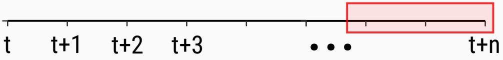

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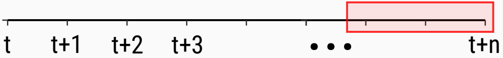

Advantages

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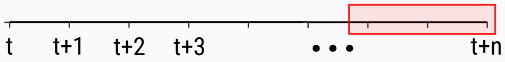

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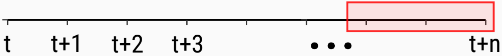

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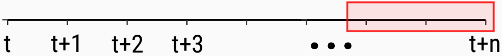

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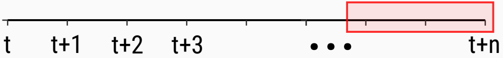

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- It requires many years/ages to provide meaningful results.

Simulating lifespans using exponential distribution with piecewise constant rate

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- Can derive and integrate any function (with `trapz()` from `pracma`).

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- It requires a lot of RAM (and time).

Smoothing

Poisson counts with P-splines (Camarda, 2012)

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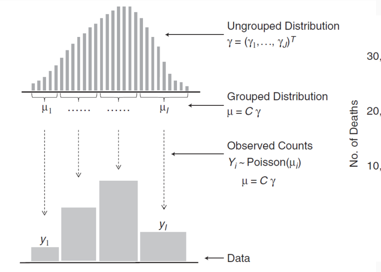

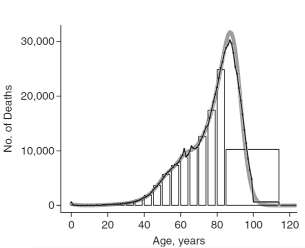

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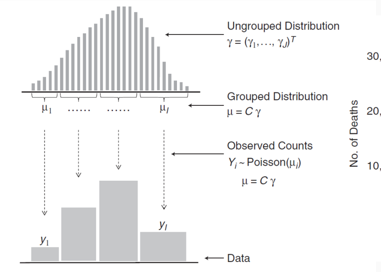

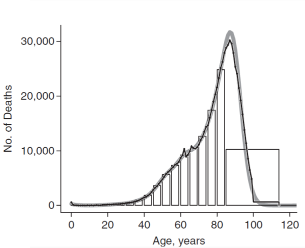

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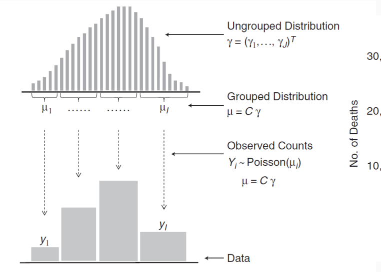

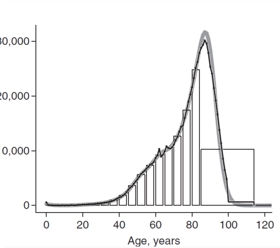

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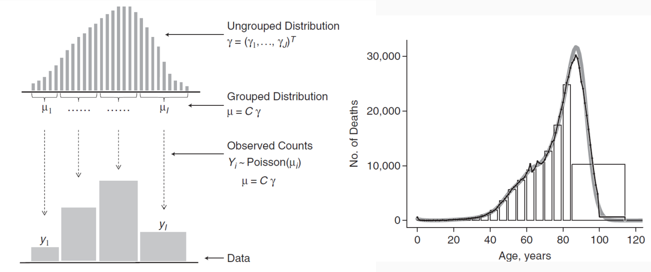

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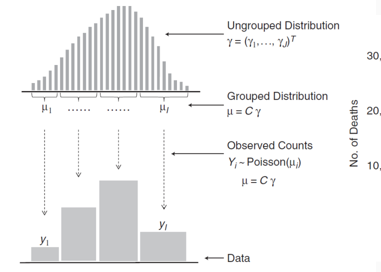

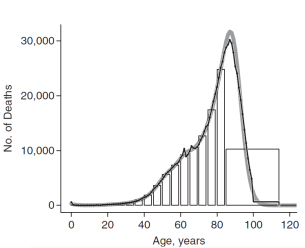

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- Not able to produce confidence intervals (so far).

Does Demography need differential equations?

by Thomas K. Burch

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Burch, Thomas K. Model-based demography: Essays on integrating data, technique and theory. Springer Nature, 2018.

Questions?