# How long is *long* in *long*evity?

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**between ages 60 and 70**

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*"Last scene of all,*
*That ends this strange, eventful history,*
*Is second childishness and mere oblivion;*
*Sans teeth, sans eyes, sans taste, sans everything."*

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***Thane, P. (2020). Old Age in European Cultures: A Significant Presence from Antiquity to the Present. The American Historical Review, 125(2), 385-395.***

# What about demography?

# Demographic thresholds of old age

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Issue:

$$\lim_{x \to \infty} H(x) = \infty$$

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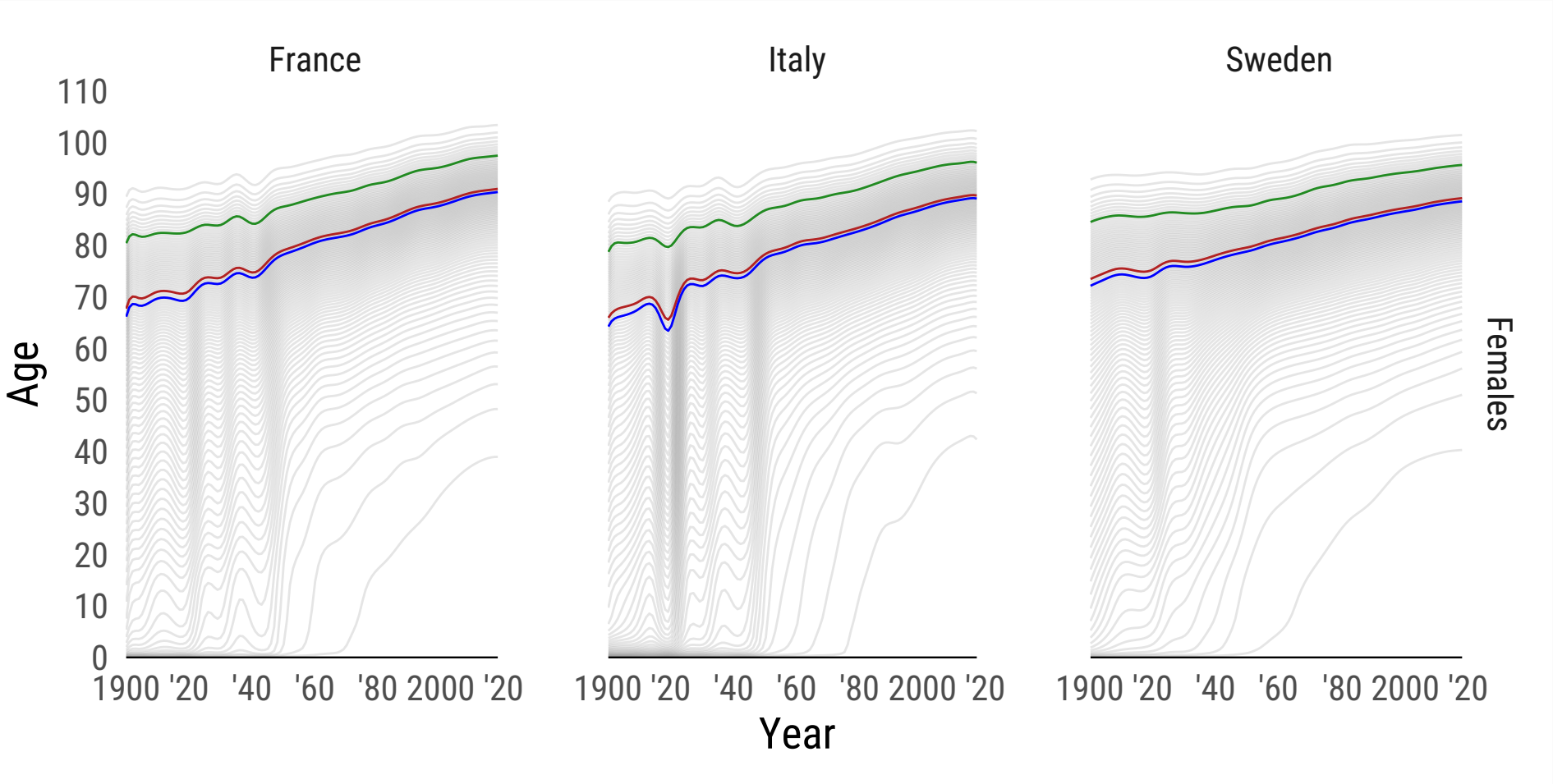
$$\bar{H} = 1$$

# Demographic thresholds of old age - Hazard potential

$\bar{H} = 1$ is reached when $s(x) = e^{-\bar{H}} = e^{-1} = 0.37$

$\bar{H}$ is the **sufficient hazard to kill the average person** and it is located at the s-percentile $\Psi(.37)$

# Hazard potentian in s-percentiles



# Post-Darwinian Longevity

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Longevity: no longer any age-specific pressure from natural selection. However, health and vitality of some species are determined by evolutionary forces at young ages (Carey and Gruendfeler, 1997)

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$H(x)$ captures the "force of failure" and $\Psi(0.37)$ is when failure governs the length of life.

# Let’s zoom out

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## Fixed chronological ages

- ages 70, 65, 62, 73.283, etc.

# Defining longevity

## Fertility

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A person is born $\rightarrow$ population $\rightarrow$ fertility

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An **individual** lives **long**???

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**Longevity**

An **individual** lives **long**???

How do we know that something is **long**?????

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*x lives beyond age 67.234*

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1. To know that something is longer than something else:

$$x > y$$

2. Candidates of *y*:

**Characteristics of the population**: life expectancy, mode, s-percentiles, etc.

*x lives beyond life expectancy*

**Points in time**: age 60, 65, 67.234, 75+2$\pi$, etc.

*x lives beyond age 67.234*

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Population

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**Longevity**

Population $\rightarrow$ characteristic

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An individual re-locates $\rightarrow$ population $\rightarrow$ migration

**Longevity**

Population $\rightarrow$ characteristic $\rightarrow$ *individual live long or not*

# Questions?