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$$\mu(5,1910) = \mu(70,2019)?$$

Why do we care?

60s is the new 50s

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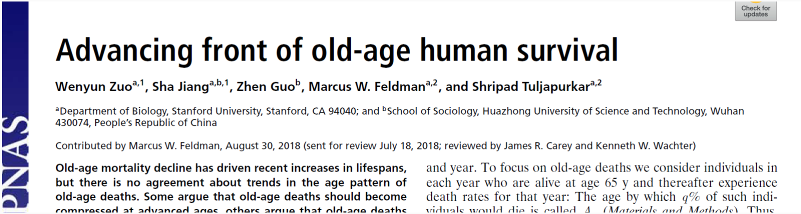
Survival is shifting from younger to older ages

(Myers and Manton 1984; Kannisto, 2001; Bongaarts 2005; Canudas-Romo 2008; Bergeron-Boucher et al. 2015)

Advancing front of old-age human survival



Check for updates

Advancing front of old-age human survival

Wenyun Zuo[a,1], Sha Jiang[a,b,1], Zhen Guo[b], Marcus W. Feldman[a,2], and Shripad Tuljapurkar[a,2]

[a]Department of Biology, Stanford University, Stanford, CA 94040; and [b]School of Sociology, Huazhong University of Science and Technology, Wuhan 430074, People's Republic of China

Contributed by Marcus W. Feldman, August 30, 2018 (sent for review July 18, 2018; reviewed by James R. Carey and Kenneth W. Wachter)

Old-age mortality decline has driven recent increases in lifespans, but there is no agreement about trends in the age pattern of old-age deaths. Some argue that old-age deaths should become

and year. To focus on old-age deaths we consider individuals in each year who are alive at age 65 y and thereafter experience death rates for that year: The age by which $q\%$ of such individuals would die is called

Zuo et. al (2018), *PNAS*

Advancing front of old-age human survival

Percentiles of deaths starting at **age 65**, from 1960 to 2010.

Advancing front of old-age human survival

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Advancing front of old-age human survival

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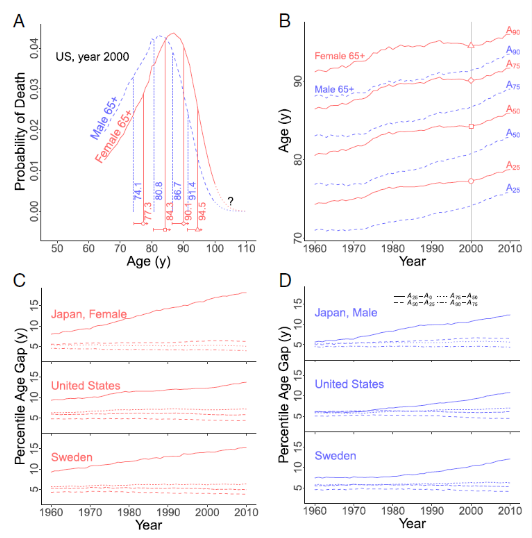
Overrule the long-lasting debate about **compression vs dispersion of deaths**

(Fries, 1980; Myers and Manton, 1985; Nusselder and Mackenback, 1996; Horuchi and Wilmoth, 1999; Kannisto, 2000; Tatcher et al, 2010; Bergeron-Boucher, 2015)

Advancing front of old-age human survival



Zuo et. al (2018), *PNAS*

Advancing front of old-age human survival

But...

But...

Why starting at age 65?

Advancing front of old-age human survival

But...

Why starting at age 65?

Young ages and old ages are part of the same continuous process of ageing.

We know that survival is shifting at a similar pace from older to even older ages...

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$s(x)$	$\overset{?}{\Longleftrightarrow}$	$\mu(x)$
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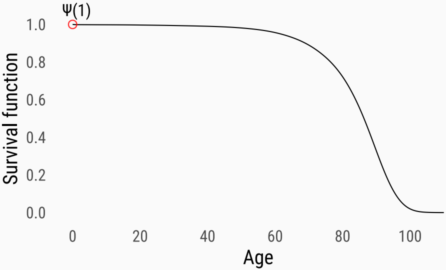
2. **Irregular relationship**
    - Mortality is an unstable and non-deterministic process,
    - Radical reshape of demographic thinking and redefine of theories of ageing (Wachter et al, 2014).

How?

s-percentiles

S-PERCENTILES

S-PERCENTILES



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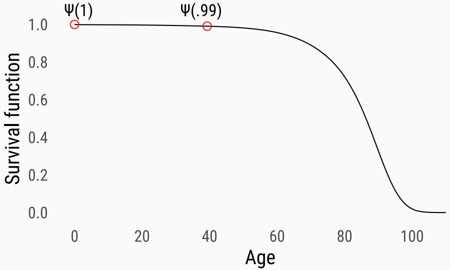

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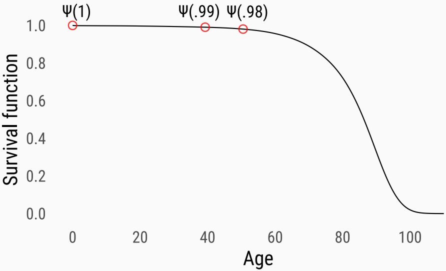

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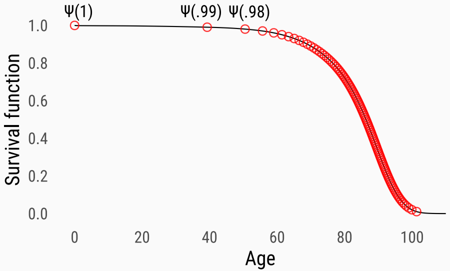

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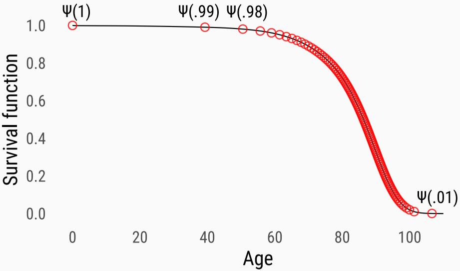

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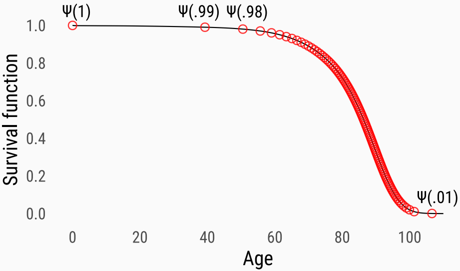

$\Psi(s,t)$: s-percentile

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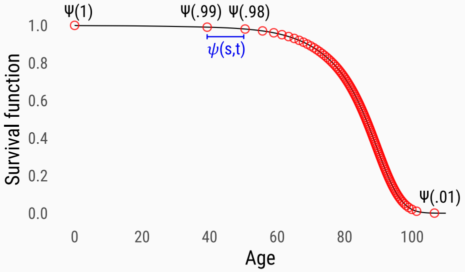

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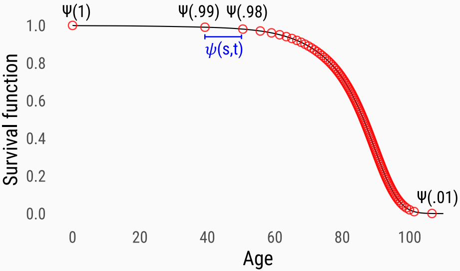

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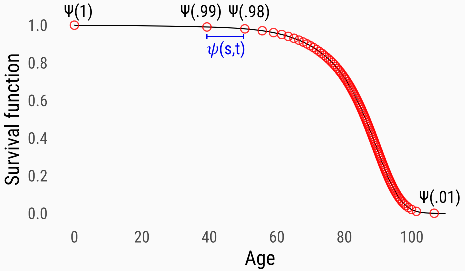

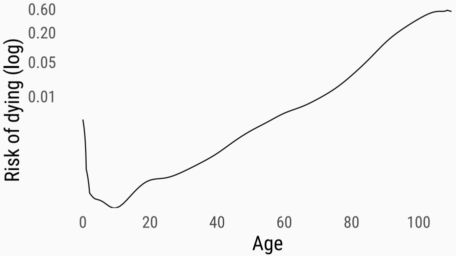

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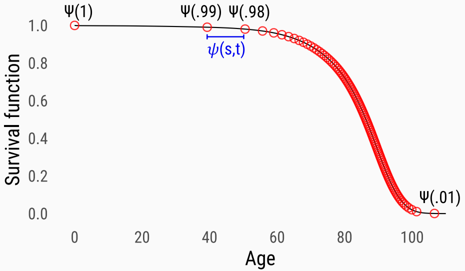

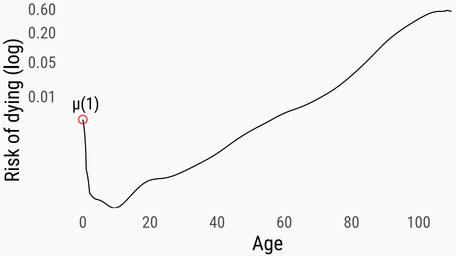

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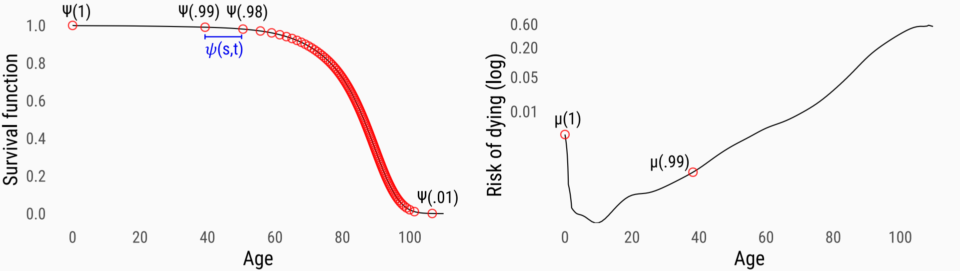

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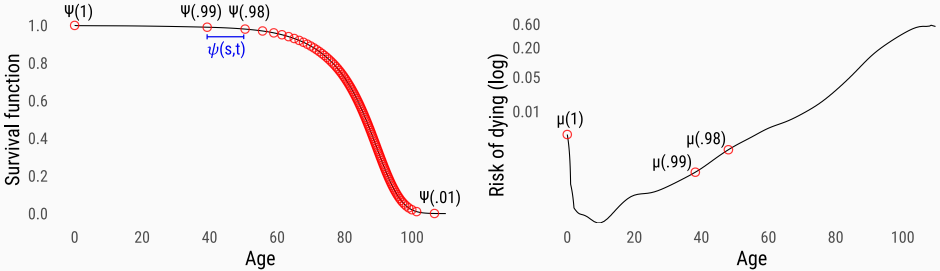

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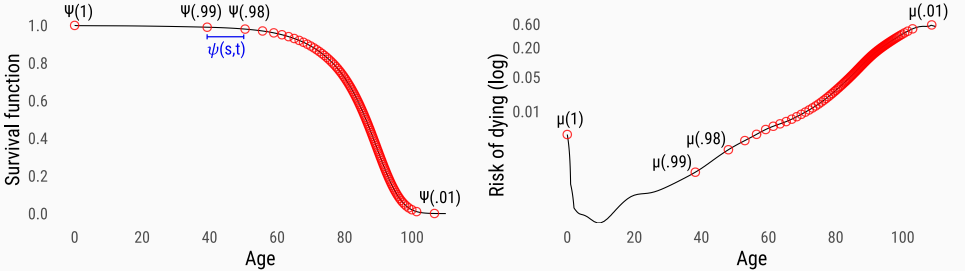

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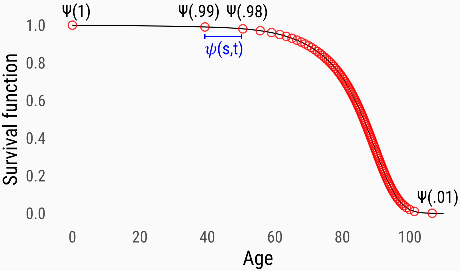

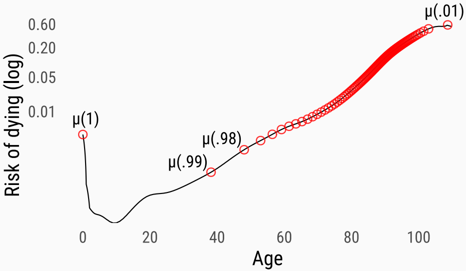

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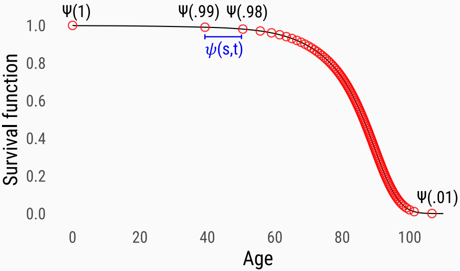

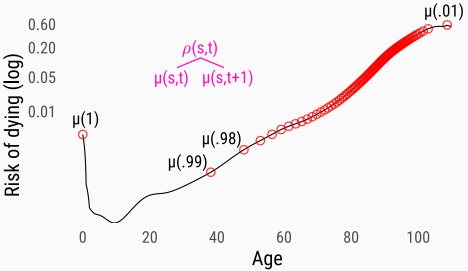

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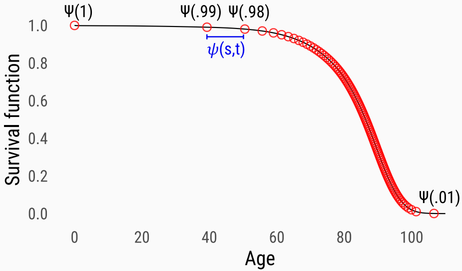

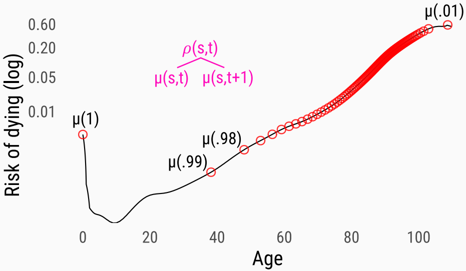

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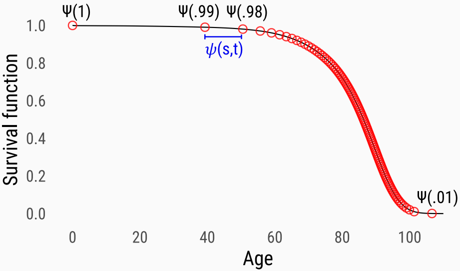

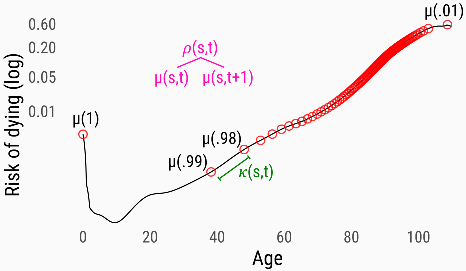

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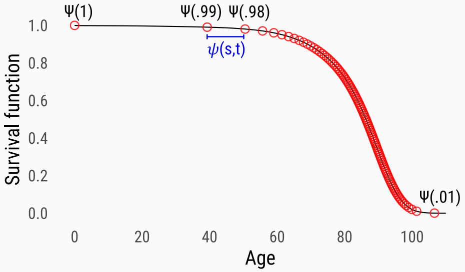

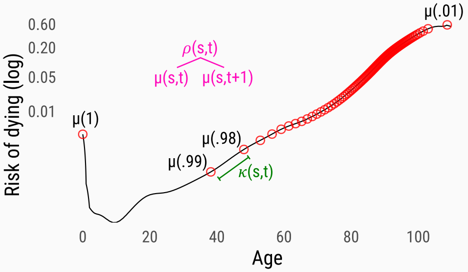

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$\kappa(s,t)$: change in $\mu(s,t)$ as s goes down

S-PERCENTILES

X: age at death with survival function $S(x)$.

$\exists$ age x_s such that $S(x_s) = s$

$$\Psi(s) = S^{-1}(x) = inf\,[x \mid S(x) \geq s]\,,\ 0 \leq s \leq 1.$$

$f(x_s) = f(\Psi(s))$ is the density function of x_s.

$\psi(s) = \frac{\partial \Psi(s)}{\partial s}$ is the density function of s.

Then, $f(x_s)$ and $\psi(s)$ are reciprocal functions

$$\frac{\partial S(\Psi(s))}{\partial s} = -f(x_s)\,\psi(s) = 1.$$

Therefore,

$$\mu(x_s) = \frac{f(x_s)}{S(x_s)} = -\frac{1}{s\ \psi(s)} = \mu(s).$$

Dynamics of the risk of dying in terms of s-percentiles

Change over time in $\mu(s,t)$

$$\rho(s,t) = -\frac{\frac{\partial \mu(s,t)}{\partial t}}{\mu(s,t)} = -\frac{\partial \ln(\mu(s,t))}{\partial t}$$

Change in $\mu(s,t)$ as s decreases

$$\kappa(s,t) = \frac{\frac{\partial \mu(s,t)}{\partial s}}{\mu(s,t)} = \frac{\partial\ ln(\mu(s,t))}{\partial s}$$

$\kappa(s,t)$ is not the rate of ageing $b(x,t) = \frac{\frac{\partial \mu(x,t)}{\partial x}}{\mu(x,t)}$

Data

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Raw death rates from the Human Mortality Database,

Data

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22 populations from 1900-2019,

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Non-parametric smoothing over time and over age: 2D P-splines (Camarda, 2012),

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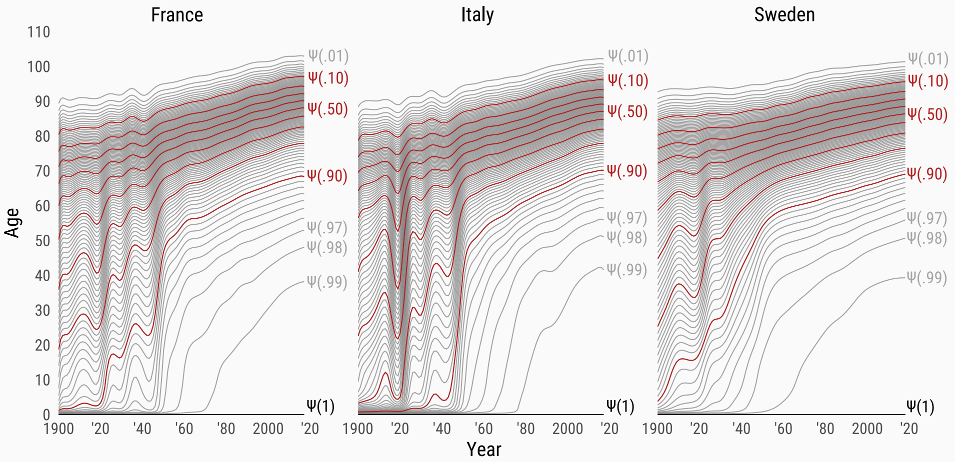
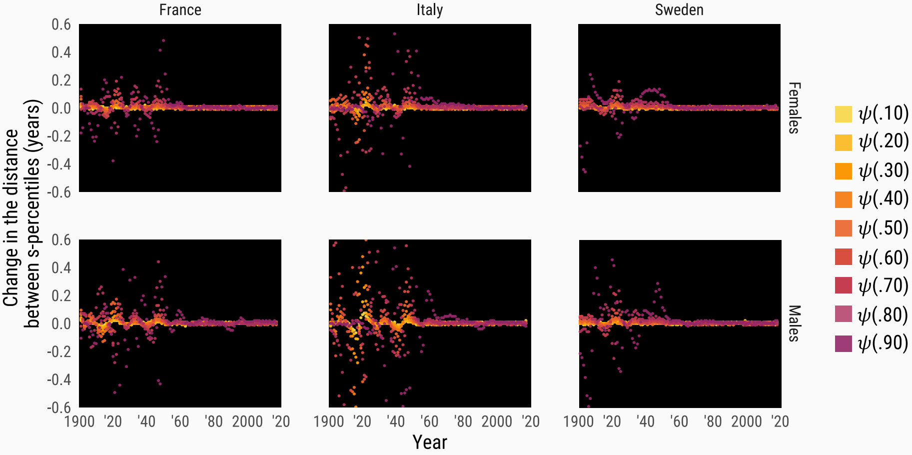
Start calculations at birth.

Dynamics of s-percentiles, $\Psi(s, t)$

Location of s-percentiles, females

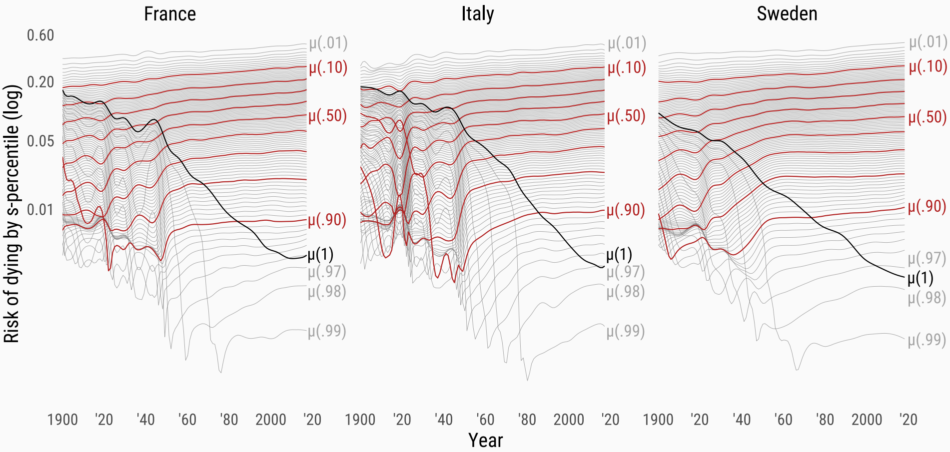


Distances between s-percentiles, both sexes



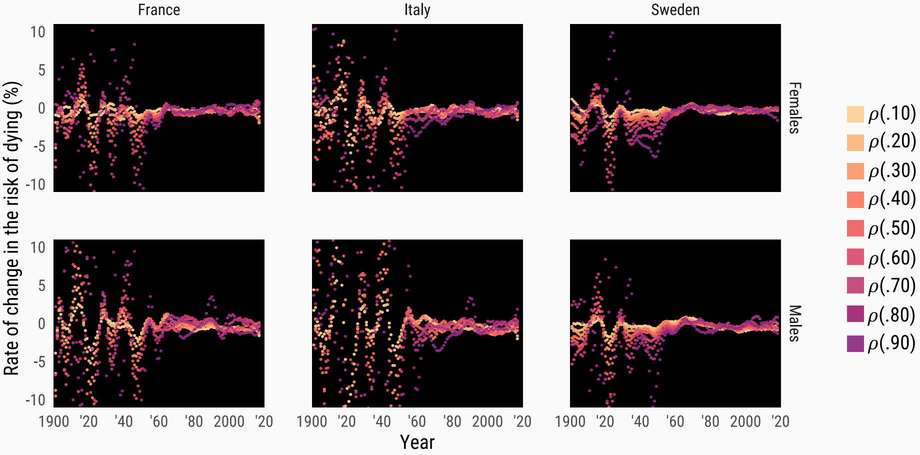
Dynamics of the risk of dying, $\mu(s, t)$

Risk of dying by survivorship level, females



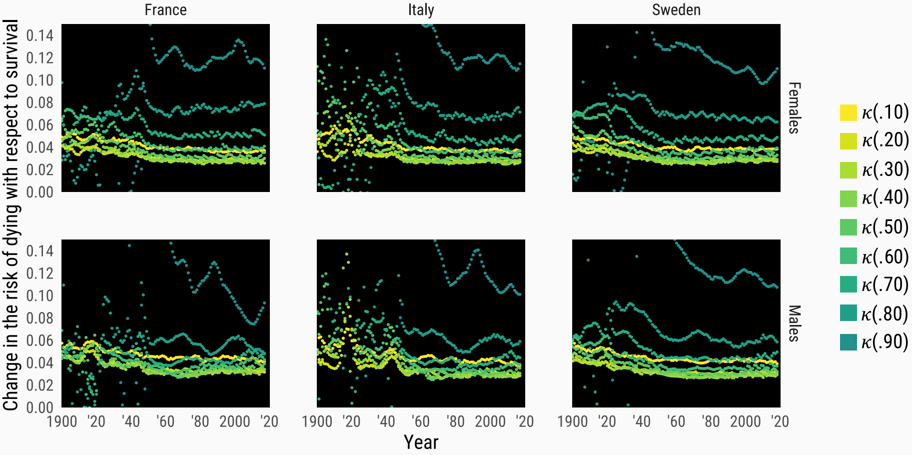
Change in the risk of dying with respect to time t

Changes over time in the risk of dying

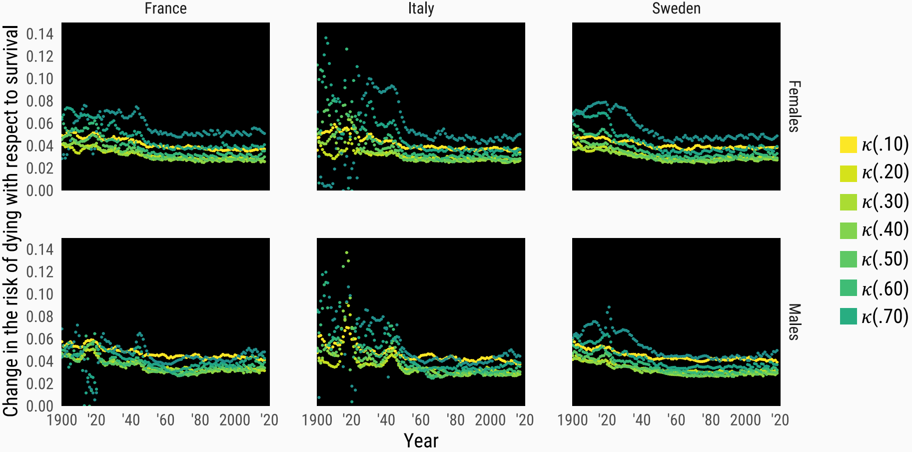


Change in the risk of dying with respect to decrease in survival s

Changes in the risk of dying as survival goes down



Changes in the risk of dying as survival goes down



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Populations experience similar risk of dying at specific levels of survival.

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Unsolved demographic questions

B-HYPOTHESIS

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Individual rate of ageing (by s-percentile) is constant?,
s-percentiles + heterogeneous populations (Vaupel and Missov, 2014)

Why?

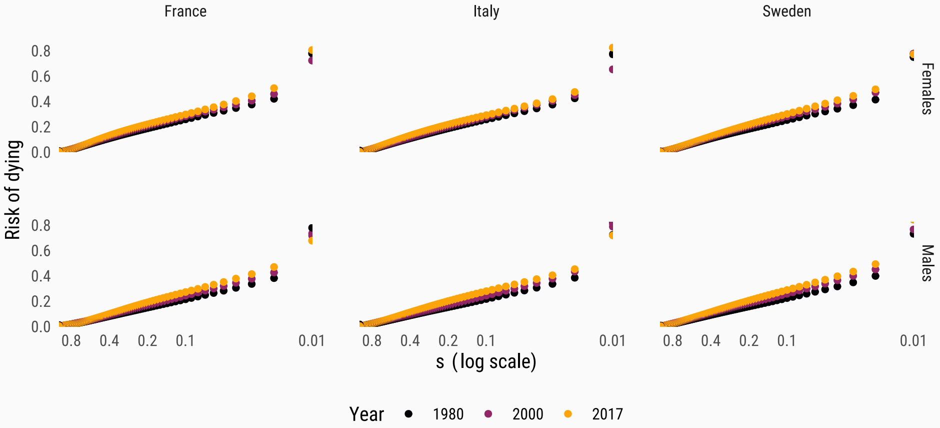
Steady shifts in human survival result in constant dynamics of the risk of dying

Jesús-Adrián Álvarez James W. Vaupel

SDU

Syddansk Universitet

Risk of dying by s - period



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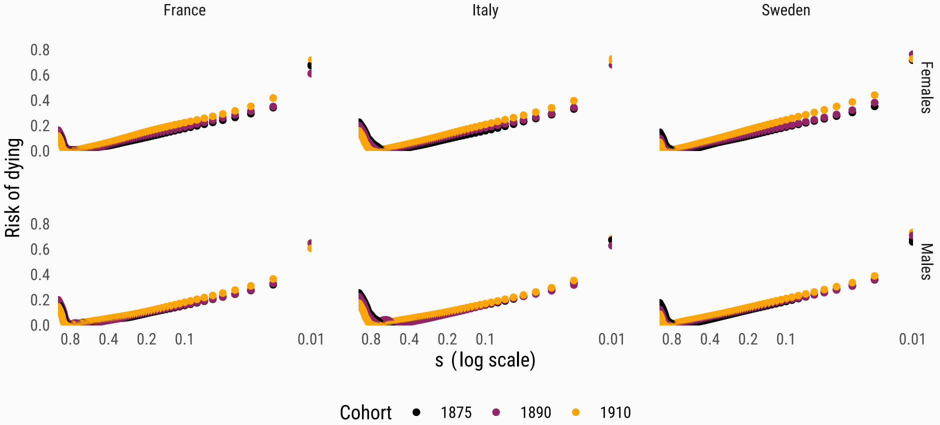

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S-PERCENTILES

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  - The distances between percentiles are constant before age 65 (starting at $\Psi(0.90)$)