Population dynamics

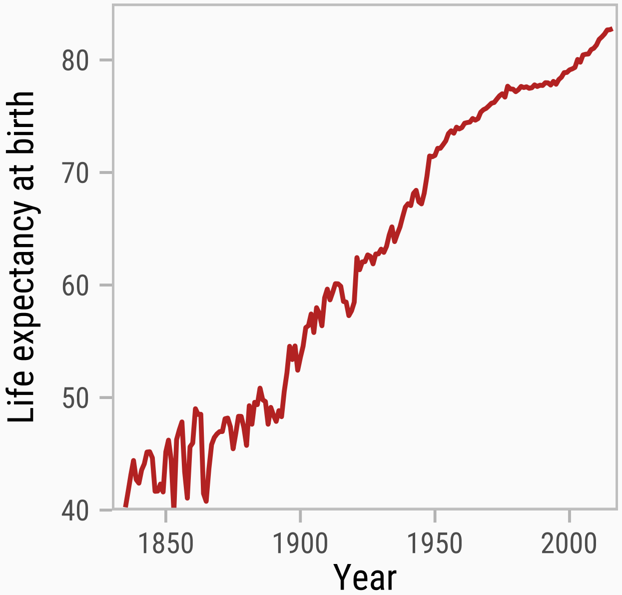
with emphasis on mortality research

Jesús-Adrián Álvarez

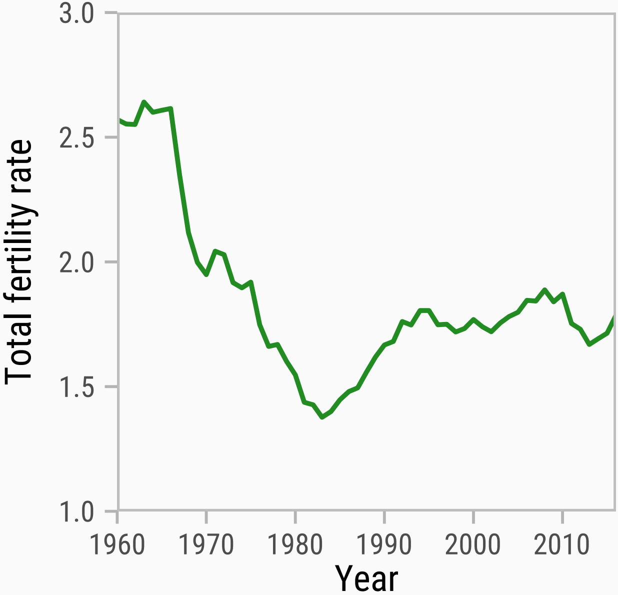
alvarez@sdu.dk

Interdisciplinary Centre on Population Dynamics
University of Southern Denmark

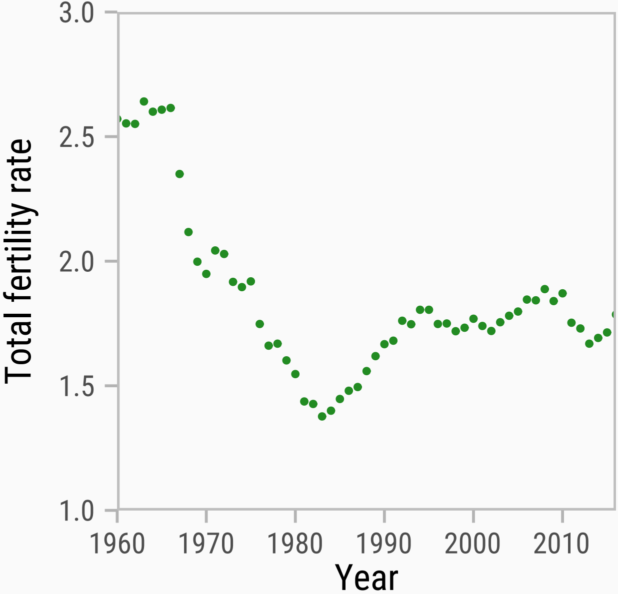
Life expectancy at birth, Danish females. 1835-2017



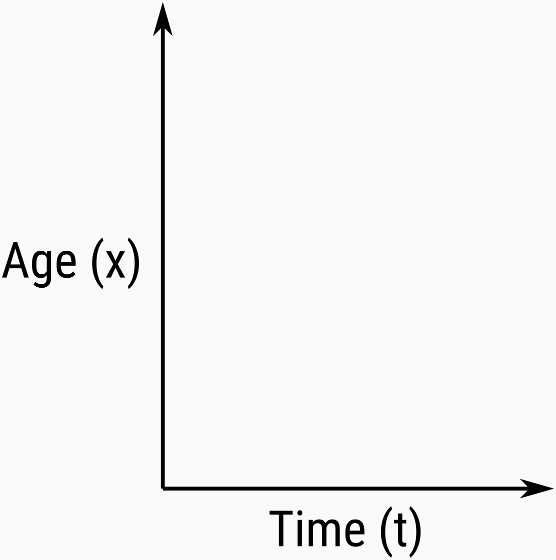
Total fertility rate, Danish females. 1960-2018

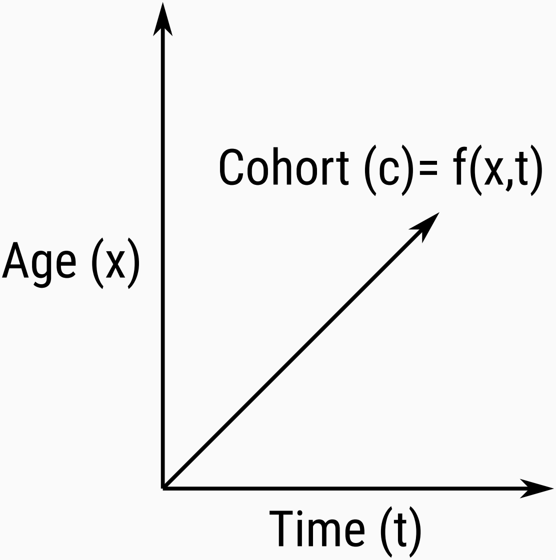


Total fertility rate, Danish females. 1960-2018



Changes over time and age


Age (x)
Time (t)



Cohort (c)= f(x,t)
Age (x)
Time (t)

Mortality

Changes over time and age

Mortality

- Mortality hazard,

Changes over time and age

Mortality

- Mortality hazard,
- Life expectancy,

Mortality

- Mortality hazard,
- Life expectancy,
- Life table entropy.

Changes over time and age

Mortality

- Mortality hazard,
- Life expectancy,
- Life table entropy.

Fertility

Changes over time and age

Mortality

- Mortality hazard,
- Life expectancy,
- Life table entropy.

Fertility

- Fertility rates,

Mortality

- Mortality hazard,
- Life expectancy,
- Life table entropy.

Fertility

- Fertility rates,
- Mean age at birth

Mortality

- Mortality hazard,
- Life expectancy,
- Life table entropy.

Fertility

- Fertility rates,
- Mean age at birth

Family demography

- Expected years ever married (Mogi and Canudas-Romo, 2018)

Precise calculations

How can I calculate changes over time in...

- remaining life expectancy at the 80th percentile?

Precise calculations

How can I calculate changes over time in...

- remaining life expectancy at the 80th percentile?
- the age at which remaining life expectancy is 14.5 years (i.e. $e(x) = 14.5$)

Precise calculations

How can I calculate changes over time in...

- remaining life expectancy at the 80th percentile?
- the age at which remaining life expectancy is 14.5 years (i.e. $e(x) = 14.5$)
- the probability of dying between percentiles 70th and 90th of the death distribution?

Precise calculations

How can I calculate changes over time in...

- remaining life expectancy at the 80th percentile?
- the age at which remaining life expectancy is 14.5 years (i.e. $e(x) = 14.5$)
- the probability of dying between percentiles 70th and 90th of the death distribution?
- the difference in standard deviation between females and males at age 100?

Precise calculations

How can I calculate changes over time in...

- remaining life expectancy at the 80th percentile?
- the age at which remaining life expectancy is 14.5 years (i.e. $e(x) = 14.5$)
- the probability of dying between percentiles 70th and 90th of the death distribution?
- the difference in standard deviation between females and males at age 100?
- the modal age at death?

The struggle

The struggle

We develop neat and **elegant equations** in continuous mathematics but....

The struggle

We develop neat and **elegant equations** in continuous mathematics but....
when we deal with **real data**, everything gets **messy**.

The struggle

We develop neat and **elegant equations** in continuous mathematics but....
when we deal with **real data**, everything gets **messy**.

In particular, when **data is scarce** (e.g. above age 80)

The struggle

We develop neat and **elegant equations** in continuous mathematics but....
when we deal with **real data**, everything gets **messy**.

In particular, when **data is scarce** (e.g. above age 80)

Advantages of working in continuous time:

The struggle

We develop neat and **elegant equations** in continuous mathematics but....
when we deal with **real data**, everything gets **messy**.

In particular, when **data is scarce** (e.g. above age 80)

Advantages of working in continuous time:

- We can use **calculus**,

The struggle

We develop neat and **elegant equations** in continuous mathematics but....
when we deal with **real data**, everything gets **messy**.

In particular, when **data is scarce** (e.g. above age 80)

Advantages of working in continuous time:

- We can use **calculus**,
- More intuitive when we think about the dynamics and processes of populations.

The struggle

We develop neat and **elegant equations** in continuous mathematics but.... when we deal with **real data**, everything gets **messy**.

In particular, when **data is scarce** (e.g. above age 80)

Advantages of working in continuous time:

- We can use **calculus**,
- More intuitive when we think about the dynamics and processes of populations.

Limitations:

Bad approximations from discrete to continuous measures

The limitations of life tables

Everything we do (or most of it) is based on life tables.

Life tables are powerful but imply many assumptions behind:

The limitations of life tables

Everything we do (or most of it) is based on life tables.

Life tables are powerful but imply many assumptions behind:

- How deaths are distributed between age groups (i.e. $ax = 0.5$)

The limitations of life tables

Everything we do (or most of it) is based on life tables.

Life tables are powerful but imply many assumptions behind:

- How deaths are distributed between age groups (i.e. $ax = 0.5$)
- The open age interval (i.e. age 110+)

The limitations of life tables

Everything we do (or most of it) is based on life tables.

Life tables are powerful but imply many assumptions behind:

- How deaths are distributed between age groups (i.e. $ax = 0.5$)
- The open age interval (i.e. age 110+)

Continuous representation

$$e(0) = \int_0^{\infty} \ell(x)dx$$

The limitations of life tables

Everything we do (or most of it) is based on life tables.

Life tables are powerful but imply many assumptions behind:

- How deaths are distributed between age groups (i.e. $ax = 0.5$)
- The open age interval (i.e. age 110+)

Continuous representation

$$e(0) = \int_0^{\infty} \ell(x)dx$$

Discrete approach

$$e(0) = L_0 + L_1 + L_2 + ...L_{100+}$$

How to calculate changes over time in demographic measures?

How to calculate changes over time in demographic measures?

How to apply neat continuous formulas to messy discrete data?

Data

Type of data

1. Individual data

Type of data

1. Individual data
    - Date of birth, date of death, date of migration, individual characteristics, etc.

Type of data

1. Individual data
    - Date of birth, date of death, date of migration, individual characteristics, etc.
    - Sources: Statistics Denmark, Statistics Netherlands, International Database on Longevity, Surveys, etc.

Type of data

1. Individual data
    - Date of birth, date of death, date of migration, individual characteristics, etc.
    - Sources: Statistics Denmark, Statistics Netherlands, International Database on Longevity, Surveys, etc.
2. Aggregated data

Type of data

1. Individual data
   - Date of birth, date of death, date of migration, individual characteristics, etc.
   - Sources: Statistics Denmark, Statistics Netherlands, International Database on Longevity, Surveys, etc.
2. Aggregated data
   - Death counts and exposures by age-group and year,

Type of data

1. Individual data
    - Date of birth, date of death, date of migration, individual characteristics, etc.
    - Sources: Statistics Denmark, Statistics Netherlands, International Database on Longevity, Surveys, etc.
2. Aggregated data
    - Death counts and exposures by age-group and year,
    - Life tables,

Type of data

1. Individual data
    - Date of birth, date of death, date of migration, individual characteristics, etc.
    - Sources: Statistics Denmark, Statistics Netherlands, International Database on Longevity, Surveys, etc.
2. Aggregated data
    - Death counts and exposures by age-group and year,
    - Life tables,
    - Sources: HMD, UN, WHO, LAMBdA (Latin America), etc.

Some approaches to measure changes over time

Some approaches to measure changes over time

1. Approximations based on life table functions

Some approaches to measure changes over time

1. Approximations based on life table functions
    - Appendix of every new method in continuous time

Some approaches to measure changes over time

1. Approximations based on life table functions
   - Appendix of every new method in continuous time
   - Wilmoth and Horuchi (2009),

Some approaches to measure changes over time

1. Approximations based on life table functions
    - Appendix of every new method in continuous time
    - Wilmoth and Horuchi (2009),
    - Beltran-Sanchez and Fernandez (2015),

Some approaches to measure changes over time

1. Approximations based on life table functions
    - Appendix of every new method in continuous time
    - Wilmoth and Horuchi (2009),
    - Beltran-Sanchez and Fernandez (2015),
    - Vaupel and Canudas-Romo (2003).

Some approaches to measure changes over time

1. Approximations based on life table functions
   - Appendix of every new method in continuous time
   - Wilmoth and Horuchi (2009),
   - Beltran-Sanchez and Fernandez (2015),
   - Vaupel and Canudas-Romo (2003).
2. Rolling window

Some approaches to measure changes over time

1. Approximations based on life table functions
    - Appendix of every new method in continuous time
    - Wilmoth and Horuchi (2009),
    - Beltran-Sanchez and Fernandez (2015),
    - Vaupel and Canudas-Romo (2003).
2. Rolling window
3. Simulation of lifespans using exponential distribution with piecewise constant rate (Willikens, 2009)

Some approaches to measure changes over time

1. Approximations based on life table functions
    - Appendix of every new method in continuous time
    - Wilmoth and Horuchi (2009),
    - Beltran-Sanchez and Fernandez (2015),
    - Vaupel and Canudas-Romo (2003).
2. Rolling window
3. Simulation of lifespans using exponential distribution with piecewise constant rate (Willikens, 2009)
4. Smoothing

Some approaches to measure changes over time

1. Approximations based on life table functions
   - Appendix of every new method in continuous time
   - Wilmoth and Horuchi (2009),
   - Beltran-Sanchez and Fernandez (2015),
   - Vaupel and Canudas-Romo (2003).
2. Rolling window
3. Simulation of lifespans using exponential distribution with piecewise constant rate (Willikens, 2009)
4. Smoothing
   - Parametric
     - Gompertz

Some approaches to measure changes over time

1. Approximations based on life table functions
   - Appendix of every new method in continuous time
   - Wilmoth and Horuchi (2009),
   - Beltran-Sanchez and Fernandez (2015),
   - Vaupel and Canudas-Romo (2003).
2. Rolling window
3. Simulation of lifespans using exponential distribution with piecewise constant rate (Willikens, 2009)
4. Smoothing
   - Parametric
     - Gompertz
     - Kannisto (HMD)

Some approaches to measure changes over time

1. Approximations based on life table functions
    - Appendix of every new method in continuous time
    - Wilmoth and Horuchi (2009),
    - Beltran-Sanchez and Fernandez (2015),
    - Vaupel and Canudas-Romo (2003).
2. Rolling window
3. Simulation of lifespans using exponential distribution with piecewise constant rate (Willikens, 2009)
4. Smoothing
    - Parametric
        - Gompertz
        - Kannisto (HMD)
        - Gamma-Gompertz

Some approaches to measure changes over time

1. Approximations based on life table functions
    - Appendix of every new method in continuous time
    - Wilmoth and Horuchi (2009),
    - Beltran-Sanchez and Fernandez (2015),
    - Vaupel and Canudas-Romo (2003).
2. Rolling window
3. Simulation of lifespans using exponential distribution with piecewise constant rate (Willikens, 2009)
4. Smoothing
    - Parametric
        - Gompertz
        - Kannisto (HMD)
        - Gamma-Gompertz
5. Non-Parametric
    - Splines, P-Splines (Camarda, 2012)

Some approaches to measure changes over time

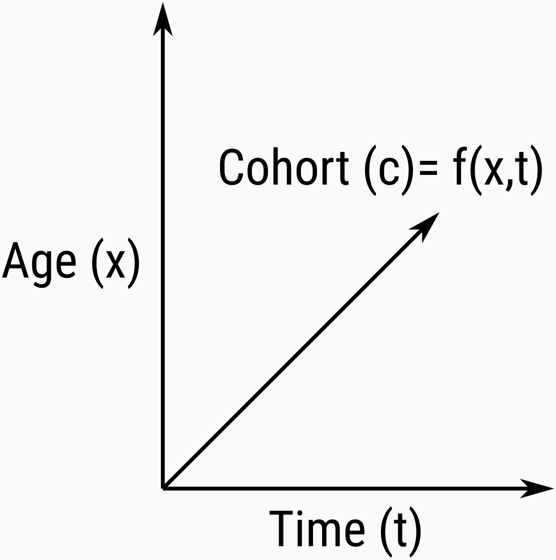
1. Approximations based on life table functions
    - Appendix of every new method in continuous time
    - Wilmoth and Horuchi (2009),
    - Beltran-Sanchez and Fernandez (2015),
    - Vaupel and Canudas-Romo (2003).
2. Rolling window
3. Simulation of lifespans using exponential distribution with piecewise constant rate (Willikens, 2009)
4. Smoothing
    - Parametric
        - Gompertz
        - Kannisto (HMD)
        - Gamma-Gompertz
5. Non-Parametric
    - Splines, P-Splines (Camarda, 2012)
    - Ungrouping (Rizzi, 2015)

How do we measure change?

How do we measure change?

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

In what direction should we derive?


Cohort (c)= f(x,t)
Age (x)
Time (t)

In what direction should we derive?

Rate of ageing (in the age direction)

$$b(x,t) = \frac{\frac{\partial \mu(x,t)}{\partial x}}{\mu(x,t)}$$

In what direction should we derive?

Rate of ageing (in the age direction)

$$b(x,t) = \frac{\frac{\partial \mu(x,t)}{\partial x}}{\mu(x,t)}$$

Rate of mortality improvement

$$\rho(x,t) = -\frac{\frac{\partial \mu(x,t)}{\partial t}}{\mu(x,t)}$$

In what direction should we derive?

Rate of ageing (in the age direction)

$$b(x,t) = \frac{\frac{\partial \mu(x,t)}{\partial x}}{\mu(x,t)}$$

Rate of mortality improvement

$$\rho(x,t) = -\frac{\frac{\partial \mu(x,t)}{\partial t}}{\mu(x,t)}$$

Rate of ageing (in the cohort direction)

$$\beta(x,t) = \frac{\frac{\partial \mu(x,t)}{\partial t \partial x}}{\mu(x,t)}$$

$$\beta(x,t) = b(x,t) - \rho(x,t)$$

Discrete approximation from life tables

Vaupel and Canudas-Romo (2003) approximation

$$\rho(x, t + h/2) \cong \frac{\ln\left(\frac{\mu(x, t+h)}{\mu(x, t)}\right)}{h}$$

and

$$\mu(x, t + h/2) \cong \frac{\mu(x, t+h) + \mu(x, t)}{2}$$

Vaupel and Canudas-Romo (2003) approximation

$$\rho(x, t + h/2) \cong \frac{\ln\left(\frac{\mu(x, t+h)}{\mu(x, t)}\right)}{h}$$

and

$$\mu(x, t + h/2) \cong \frac{\mu(x, t+h) + \mu(x, t)}{2}$$

Advantages

- Easy to estimate from life tables,
- It can be applied to any function (changes over time and age).

Vaupel and Canudas-Romo (2003) approximation

$$\rho(x, t + h/2) \cong \frac{\ln\left(\frac{\mu(x,t+h)}{\mu(x,t)}\right)}{h}$$

and

$$\mu(x, t + h/2) \cong \frac{\mu(x,t+h) + \mu(x,t)}{2}$$

Advantages

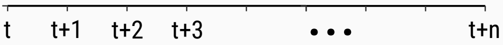
- Easy to estimate from life tables,
- It can be applied to any function (changes over time and age).

Disadvantages

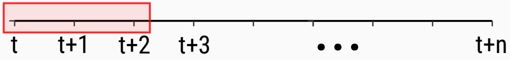
- Not very precise.

Rolling window

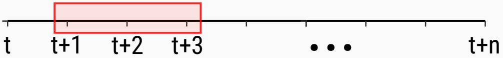
Rolling window



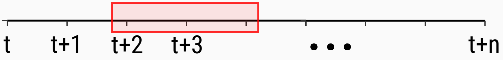
Rolling window



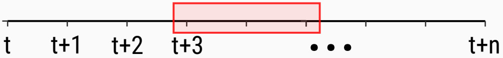
Rolling window



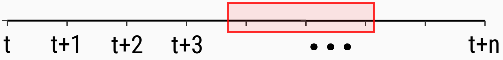
Rolling window



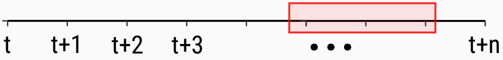
Rolling window



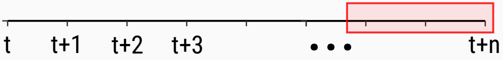
Rolling window



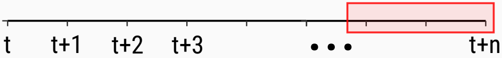
Rolling window



Rolling window

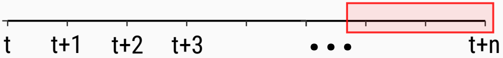


Rolling window



Advantages

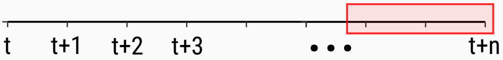
- In R, it can be computed easily with the `zoo` package.



Advantages

- In R, it can be computed easily with the `zoo` package.
- It can be combined with the Vaupel and Canudas-Romo approximation to achieve better results.

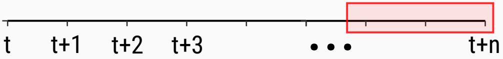
Rolling window



Advantages

- In R, it can be computed easily with the `zoo` package.
- It can be combined with the Vaupel and Canudas-Romo approximation to achieve better results.
- It can be applied to any function.

Rolling window

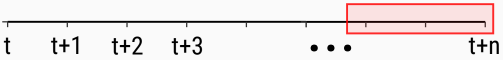


Advantages

- In R, it can be computed easily with the `zoo` package.
- It can be combined with the Vaupel and Canudas-Romo approximation to achieve better results.
- It can be applied to any function.

Disadvantages

Rolling window



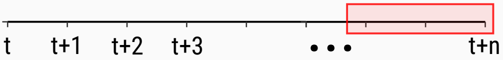
Advantages

- In R, it can be computed easily with the `zoo` package.
- It can be combined with the Vaupel and Canudas-Romo approximation to achieve better results.
- It can be applied to any function.

Disadvantages

- The greater the window, the more information we lose.

Rolling window



Advantages

- In R, it can be computed easily with the `zoo` package.
- It can be combined with the Vaupel and Canudas-Romo approximation to achieve better results.
- It can be applied to any function.

Disadvantages

- The greater the window, the more information we lose.
- It requires many years/ages to provide meaningful results.

Simulating lifespans using exponential distribution with piecewise constant rate

Simulating lifespans

Simulating lifespans

- We start with a vector of death rates $M(x, t)$,

Simulating lifespans

- We start with a vector of death rates $M(x, t)$,
- Simulate a great number of lifespans (>100,000) using $M(x, t)$ as the vector of rates in the function `rpexp()` of the package `msm`.

$$s(x, t) = e^{-nM(x,t)} \;\; and \;\; \mu(x, t) = \frac{f(x, t)}{s(x, t)}$$

Simulating lifespans

- We start with a vector of death rates $M(x,t)$,
- Simulate a great number of lifespans (>100,000) using $M(x,t)$ as the vector of rates in the function `rpexp()` of the package `msm`.

$$s(x,t) = e^{-nM(x,t)} \ \ and \ \ \mu(x,t) = \frac{f(x,t)}{s(x,t)}$$

Advantages

- Can calculate hazard, density, survival function, life expectancy, entropy, quantiles and associated confidence intervals in a continuous setting.

SIMULATING LIFESPANS

- We start with a vector of death rates $M(x,t)$,
- Simulate a great number of lifespans (>100,000) using $M(x,t)$ as the vector of rates in the function `rpexp()` of the package `msm`.

$$s(x,t) = e^{-nM(x,t)} \ \ and \ \ \mu(x,t) = \frac{f(x,t)}{s(x,t)}$$

Advantages

- Can calculate hazard, density, survival function, life expectancy, entropy, quantiles and associated confidence intervals in a continuous setting.
- It is equivalent to calculating life tables in a continuous setting.

Simulating lifespans

- We start with a vector of death rates $M(x,t)$,
- Simulate a great number of lifespans (>100,000) using $M(x,t)$ as the vector of rates in the function `rpexp()` of the package `msm`.

$$s(x,t) = e^{-nM(x,t)} \text{ and } \mu(x,t) = \frac{f(x,t)}{s(x,t)}$$

Advantages

- Can calculate hazard, density, survival function, life expectancy, entropy, quantiles and associated confidence intervals in a continuous setting.
- It is equivalent to calculating life tables in a continuous setting.
- Can derive and integrate any function (with `trapz()` from `pracma`).

Simulating lifespans

- We start with a vector of death rates $M(x,t)$,
- Simulate a great number of lifespans (>100,000) using $M(x,t)$ as the vector of rates in the function `rpexp()` of the package `msm`.

$$s(x,t) = e^{-nM(x,t)} \ \ and \ \ \mu(x,t) = \frac{f(x,t)}{s(x,t)}$$

Advantages

- Can calculate hazard, density, survival function, life expectancy, entropy, quantiles and associated confidence intervals in a continuous setting.
- It is equivalent to calculating life tables in a continuous setting.
- Can derive and integrate any function (with `trapz()` from `pracma`).

Disadvantages

- It only helps when you are looking at changes over age

Simulating lifespans

- We start with a vector of death rates $M(x,t)$,
- Simulate a great number of lifespans (>100,000) using $M(x,t)$ as the vector of rates in the function `rpexp()` of the package `msm`.

$$s(x,t) = e^{-nM(x,t)} \;\; and \;\; \mu(x,t) = \frac{f(x,t)}{s(x,t)}$$

Advantages

- Can calculate hazard, density, survival function, life expectancy, entropy, quantiles and associated confidence intervals in a continuous setting.
- It is equivalent to calculating life tables in a continuous setting.
- Can derive and integrate any function (with `trapz()` from `pracma`).

Disadvantages

- It only helps when you are looking at changes over age
- It requires a lot of RAM (and time).

Smoothing

Poisson counts with P-splines (Camarda, 2012)

Age and year specific death counts are assumed to follow a Poisson distribution. P-splines and GLM are employed.

Poisson counts with P-splines (Camarda, 2012)

Age and year specific death counts are assumed to follow a Poisson distribution. P-splines and GLM are employed.

1. Fit model.

Poisson counts with P-splines (Camarda, 2012)

Age and year specific death counts are assumed to follow a Poisson distribution. P-splines and GLM are employed.

1. Fit model.
2. Predict log-hazard for very small intervals $x = 0, 0.01, 0.02...$ and $t = 0, 0.01, 0.02...$

Poisson counts with P-splines (Camarda, 2012)

Age and year specific death counts are assumed to follow a Poisson distribution. P-splines and GLM are employed.

1. Fit model.
2. Predict log-hazard for very small intervals $x = 0, 0.01, 0.02...$ and $t = 0, 0.01, 0.02...$

Advantages

- In R, `MortalitySmooth` package.

Poisson counts with P-splines (Camarda, 2012)

Age and year specific death counts are assumed to follow a Poisson distribution. P-splines and GLM are employed.

1. Fit model.
2. Predict log-hazard for very small intervals $x = 0, 0.01, 0.02...$ and $t = 0, 0.01, 0.02...$

Advantages

- In R, `MortalitySmooth` package.
- It can smooth in two dimensions (age and time).

Poisson counts with P-splines (Camarda, 2012)

Age and year specific death counts are assumed to follow a Poisson distribution. P-splines and GLM are employed.

1. Fit model.
2. Predict log-hazard for very small intervals $x = 0, 0.01, 0.02...$ and $t = 0, 0.01, 0.02...$

Advantages

- In R, `MortalitySmooth` package.
- It can smooth in two dimensions (age and time).
- Can derive and integrate any function (with `trapz()` from `pracma`).

Poisson counts with P-splines (Camarda, 2012)

Age and year specific death counts are assumed to follow a Poisson distribution. P-splines and GLM are employed.

1. Fit model.
2. Predict log-hazard for very small intervals $x = 0, 0.01, 0.02...$ and $t = 0, 0.01, 0.02...$

Advantages

- In R, `MortalitySmooth` package.
- It can smooth in two dimensions (age and time).
- Can derive and integrate any function (with `trapz()` from `pracma`).
- It can produce confidence intervals.

Poisson counts with P-splines (Camarda, 2012)

Age and year specific death counts are assumed to follow a Poisson distribution. P-splines and GLM are employed.

1. Fit model.
2. Predict log-hazard for very small intervals $x = 0, 0.01, 0.02...$ and $t = 0, 0.01, 0.02...$

Advantages

- In R, `MortalitySmooth` package.
- It can smooth in two dimensions (age and time).
- Can derive and integrate any function (with `trapz()` from `pracma`).
- It can produce confidence intervals.

Disadvantages

Poisson counts with P-splines (Camarda, 2012)

Age and year specific death counts are assumed to follow a Poisson distribution. P-splines and GLM are employed.

1. Fit model.
2. Predict log-hazard for very small intervals $x = 0, 0.01, 0.02...$ and $t = 0, 0.01, 0.02...$

Advantages

- In R, `MortalitySmooth` package.
- It can smooth in two dimensions (age and time).
- Can derive and integrate any function (with `trapz()` from `pracma`).
- It can produce confidence intervals.

Disadvantages

- It is only designed for mortality data.

Poisson counts with P-splines (Camarda, 2012)

Age and year specific death counts are assumed to follow a Poisson distribution. P-splines and GLM are employed.

1. Fit model.
2. Predict log-hazard for very small intervals $x = 0, 0.01, 0.02...$ and $t = 0, 0.01, 0.02...$

Advantages

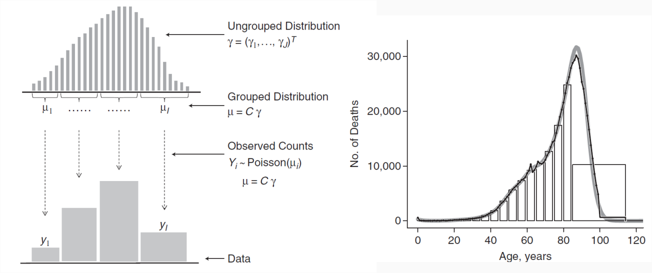
- In R, `MortalitySmooth` package.
- It can smooth in two dimensions (age and time).
- Can derive and integrate any function (with `trapz()` from `pracma`).
- It can produce confidence intervals.

Disadvantages

- It is only designed for mortality data.

Ungroup bins (Rizzi, 2015)

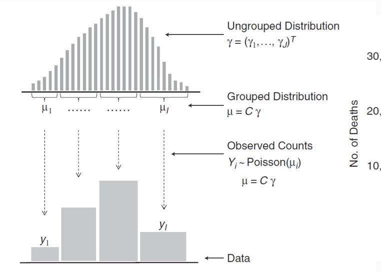
Ungroup age groups and smooth death counts:

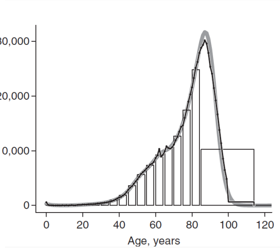


Advantages

- In R, `ungroup` package.

Ungroup age groups and smooth death counts:

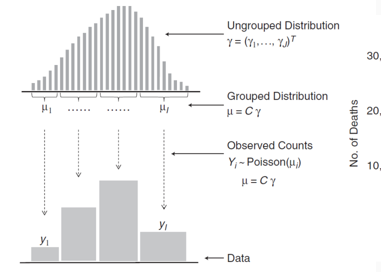


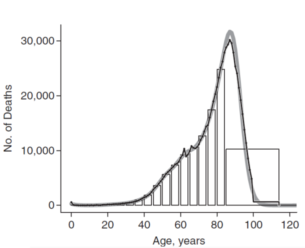


Advantages

- In R, `ungroup` package.
- It can smooth in two dimensions (age and time) .

Ungroup age groups and smooth death counts:





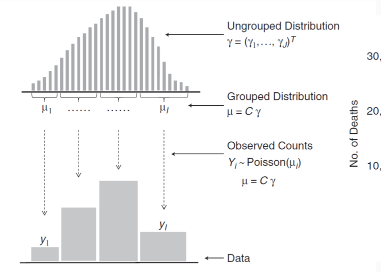
Advantages

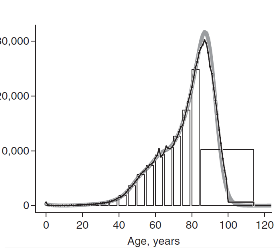
- In R, `ungroup` package.
- It can smooth in two dimensions (age and time) .

Disadvantages

UNGROUP BINS (RIZZI, 2015)

Ungroup age groups and smooth death counts:





Advantages

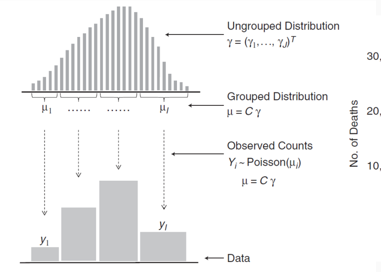
- In R, `ungroup` package.
- It can smooth in two dimensions (age and time) .

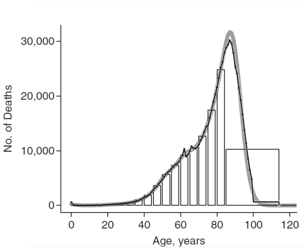
Disadvantages

- It is only designed for mortality data.

Ungroup bins (Rizzi, 2015)

Ungroup age groups and smooth death counts:





Advantages

- In R, `ungroup` package.
- It can smooth in two dimensions (age and time) .

Disadvantages

- It is only designed for mortality data.
- Not able to produce confidence intervals (so far).

Does Demography need differential equations?

by Thomas K. Burch

Does Demography need differential equations?

Why has demography made relatively little use of differential equations?

Does Demography need differential equations?

Why has demography made relatively little use of differential equations?

"Demography is or aspires to be an autonomous science, not just a branch of applied statistics".

Why has demography made relatively little use of differential equations?

"Demography is or aspires to be an autonomous science, not just a branch of applied statistics".

"The average demographer has little competence in the use of differential equations."

Does Demography need differential equations?

Why has demography made relatively little use of differential equations?

"Demography is or aspires to be an autonomous science, not just a branch of applied statistics".

"The average demographer has little competence in the use of differential equations."

"That level of mathematics has not been required for entrance into, or successful completion of, most graduate programs."

Why has demography made relatively little use of differential equations?

"Demography is or aspires to be an autonomous science, not just a branch of applied statistics".

"The average demographer has little competence in the use of differential equations."

"That level of mathematics has not been required for entrance into, or successful completion of, most graduate programs."

"Demographers generally were not schooled in differential equations, so we did not try to use them, and avoided topics that required their use even at the most elementary level."

Does Demography need differential equations?

"Demography is generally thought to be rich in data and technique, and poor in theory."

Does Demography need differential equations?

"Demography is generally thought to be rich in data and technique, and poor in theory."

"I think Demography might be an even stronger discipline if it had assimilated the regular use of differential equations in general, and systems dynamics software in particular."

Does Demography need differential equations?

"Demography is generally thought to be rich in data and technique, and poor in theory."

"I think Demography might be an even stronger discipline if it had assimilated the regular use of differential equations in general, and systems dynamics software in particular."

*"...their use would encourage us to **think more about dynamics and process, and not just cross-sectional relationships and equilibria.** They could help us think better about **complex social and demographic systems containing non-linear relationships and feedbacks**."*

Does Demography need differential equations?

"Demography is generally thought to be rich in data and technique, and poor in theory."

"I think Demography might be an even stronger discipline if it had assimilated the regular use of differential equations in general, and systems dynamics software in particular."

"...their use would encourage us to ***think more about dynamics and process, and not just cross-sectional relationships and equilibria.*** *They could help us think better about* ***complex social and demographic systems containing non-linear relationships and feedbacks****."*

"Our body of theory could be richer still if we were to take advantage of both classic (differential equations) and contemporary (software) tools for the statement and manipulation of theoretical ideas about demographic processes".

Does Demography need differential equations?

"Demography is generally thought to be rich in data and technique, and poor in theory."

"I think Demography might be an even stronger discipline if it had assimilated the regular use of differential equations in general, and systems dynamics software in particular."

"...their use would encourage us to ***think more about dynamics and process, and not just cross-sectional relationships and equilibria.*** *They could help us think better about* ***complex social and demographic systems containing non-linear relationships and feedbacks****."*

"Our body of theory could be richer still if we were to take advantage of both classic (differential equations) and contemporary (software) tools for the statement and manipulation of theoretical ideas about demographic processes".

Burch, Thomas K. Model-based demography: Essays on integrating data, technique and theory. Springer Nature, 2018.

Questions?