Unravelling the contribution of financial and longevity risks to changes over time in life annuities

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stochastic change life annuities

interest or mortality?

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LIFE ANNUITY FACTORS

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How annuities respond to **changes in interest rates**? (Milevsky, 2013; Charupat, Kamstra, Milevsky, 2015)

What about changes in mortality?

Results from Demography

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- Comparison of **ageing patterns across species.** (Baudisch et al, 2011)

CHANGES OVER TIME IN $e_0(t)$

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Extended to causes-of-death and subpopulations (e.g. socio-economic groups).

Entropy of life annuity

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- Lin and Tsai (2020) sensitivity to changes in the *force of mortality-interest*: $\mu^* = \mu + \delta$.

Decomposition of changes over time in $\bar{a}_x(t)$

CHANGES OVER TIME IN $\bar{a}_x(t)$

Derivative of $\bar{a}_x(t)$ with respect to time t:

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Relative derivative of $\bar{a}_x(t)$:

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putting all the pieces together…

DECOMPOSITION OF CHANGES OVER TIME IN $\bar{a}_x(t)$

$$\acute{\bar{a}}_x(t) = \underbrace{\bar{\varphi}(t)D_x(t)}_{\text{financial component}} + \underbrace{\bar{\rho}(t)H_x(t)}_{\text{longevity component}}$$

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Historical contributions to changes in life annuities in the UK

Data

- **Long-term interest rates:** the yield on 2.5% Consols up to 1977, then by the yield on FTSE Actuaries Government Securities Irredeemable stocks up to 2014 and thereafter by the yield on FTSE Actuaries Government Securities 45 years stock (Bank of England, 2020),

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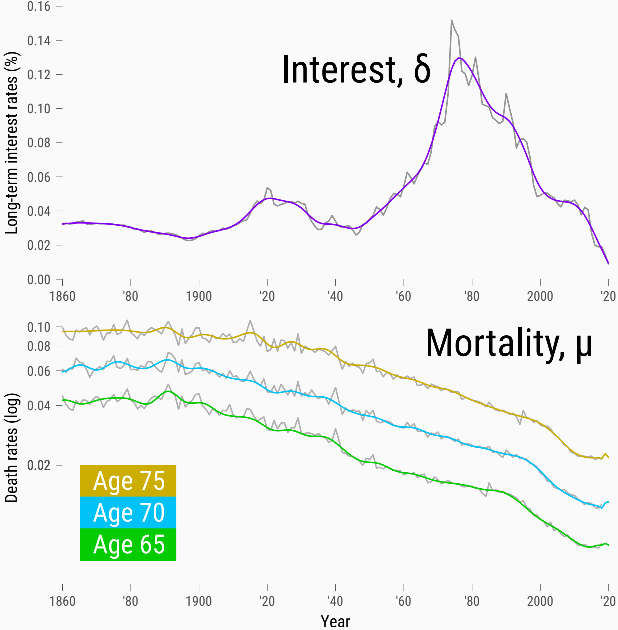
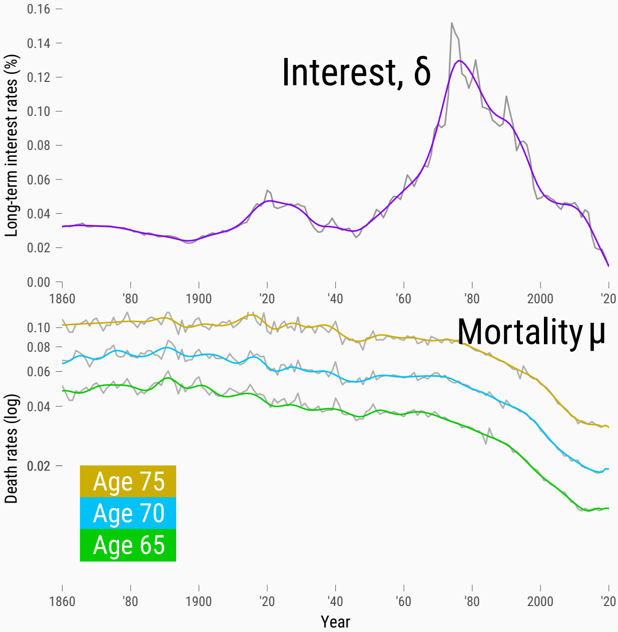
INTEREST AND MORTALITY RATES FOR FEMALES IN THE UK, 1841-2018

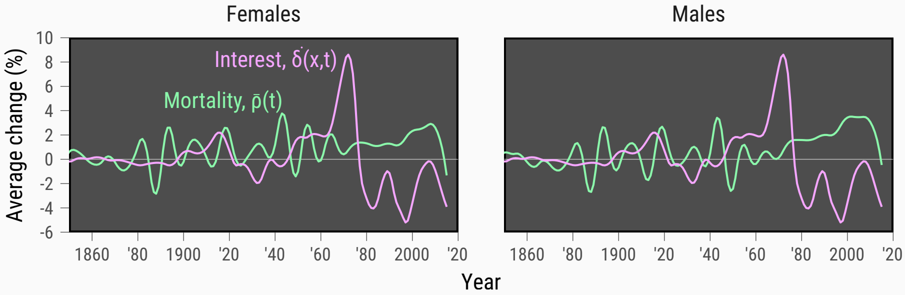


INTEREST AND MORTALITY RATES FOR MALES IN THE UK, 1841-2018

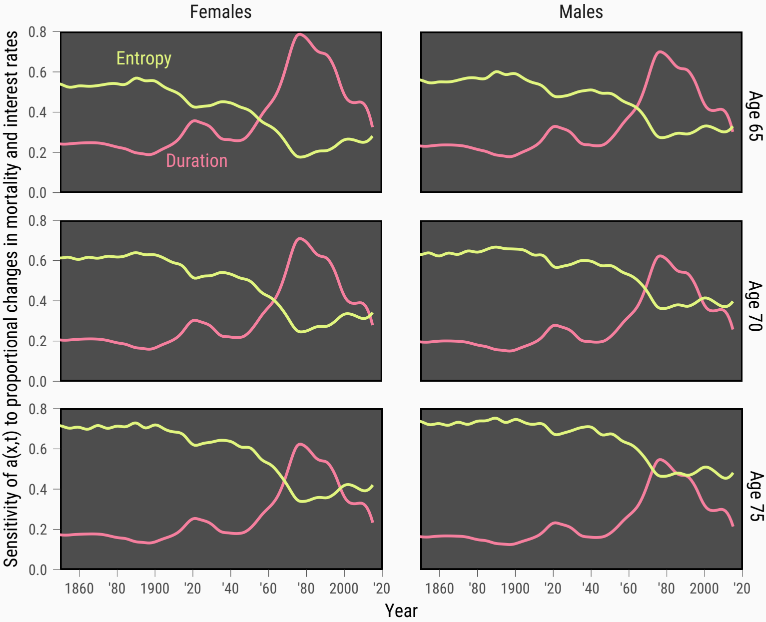


LIFE ANNUITY FACTORS AT AGES 65, 70 AND 75. UK, 1841-2018

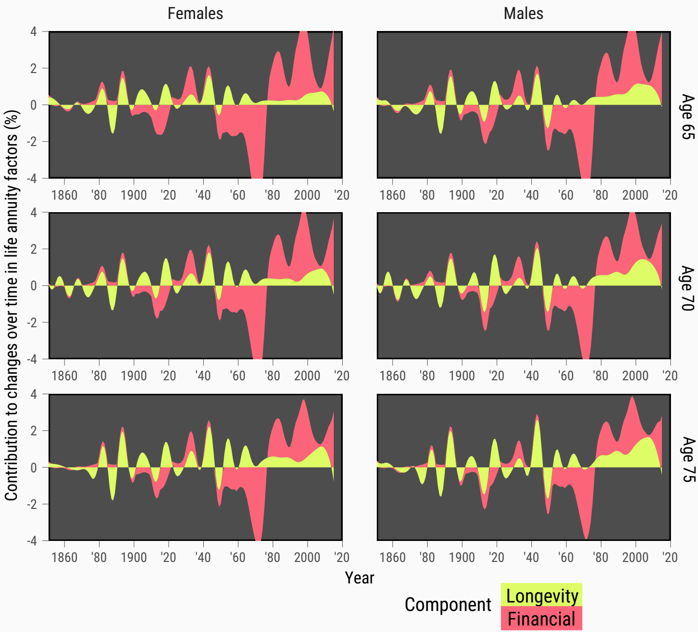
CHANGES OVER TIME IN INTEREST AND MORTALITY RATES. UK 1841-2018



DURATION AND ENTROPY AT AGES 65, 70 AND 75. UK, 1841-2018



DECOMPOSITION OF $\acute{\bar{a}}_x(t)$ AT AGES 65, 70 AND 75. UK, 1841-2018



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  - Policies aiming at **increasing retirement ages entail higher longevity risk** (e.g. Denmark, Alvarez et al (2020)).

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- **Causes of death** Vaupel and Canudas-Romo (2003).

Unravelling the contribution of financial and longevity risks to changes over time in life annuities

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