ECE 358 Assignment

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1. The tight lower bound for the largest-sized routing table across all the n routers is 2. This is because a router in this IP network forwards packets between exactly 2 subnetworks. That means it has to be able to lookup at least two different subnet prefixes to know how to forward the datagram to each of the two subnetworks.

2.
```
def decision(P, k):
    P' = P
    for i <- 0 to P'.size() - 1:
        for j <- i + 1 to P'.size() - 1:
            if mergeable(P'[i], P'[j]):
                P'.add(merge(P'[i], P'[j]))
                P'.remove(P'[i])
                P'.remove(P'[j])
                i <- 0
                j <- 0
    return P'.size() <= k
```

3. Mike's go here

4. Yes, I concur with Alice.

   Let T be the initial minimum spanning tree of G. Let u be the node that leaves G, and let v be the node such that uv is the only edge incident to u in T. Let T' be T without u and uv. I want to show that T' is the minimum spanning tree of the new graph.

   Proof By Contradiction
   Assume for the sake of contradiction that T' is not the most minimum spanning tree of the new graph. That means there must exist some tree R that spans the new graph, and the total weight of its edges is less than that of T'.

   Now consider R with uv added to the tree. We can show that R with uv is a spanning

tree of G, because R is a tree that spans every node in G except u. However, we know v is in R so uv can be added to R to span all nodes in G while maintaining the tree structure.

Moreover, observe that R with uv has total weight less than that of T because:

$$weight(R) < weight(T')$$
$$weight(R) + weight(uv) < weight(T') + weight(uv)$$
$$weight(R) + weight(uv) < weight(T)$$

Now we have shown that R with uv is a spanning tree of G and its weight is less than that of the T, the minimum spanning tree. This is clearly a contradiction, so we can declare that there does not exist another spanning tree that has total weight less than T'. Therefore, we do not need to recompute the MST as when a node leaves the network and it only has one edge incident on it, the new MST is merely the original tree with the node and its incident edge removed.

5. Frank's go here