3 notes on wealth distribution and social mobility

By Shu HU

0 intro

0.1 motivation

- wealth is unequally distrbuted
- high social mobility
    - intergenerational shorrocks mobility index: 0.88

0.2 econ question

- what factors
    - drive wealth dynamics
    - are consistent with the observed
        - cross-sectional wealth distribution
        - social mobility

0.3 methods

- develop a simple ifp model
- match the moments generated by model by empirical moments of
    - the observed wealth distribution
    - social mobility matrix
- use this matched model to quantitatively identify the effects of 3 critical factors on the tail of the distribution (generating thick tail) and social mobility
    - A. skew and persistent distribution of stochastic earnings ($:= Y$)
    - B. Differential saving rates across wealth levels ($:= s/a$)
        - higher saving and accumulation rates for the rich
        - forms in literature
            - takes the form of non-homogeneous bequests
            - bequests as a fraction of wealth that are increasing in wealth
    - C. stochastic idiosyncratic returns to wealth/capital income risk ($:= r = R - 1$)
        - allowing rates of return on wealth to be increasing in wealth might also add to the skewness of the distribution

0.4 Conclusions

- capital income risk
- differential savings
- stochastic earnings
    - limited role in filling the tail of wealth distribution (how to understand "filling")
    - fundamental in inducing enough mobility in the wealth process
- r increases in wealth itself
    - supported in the estimates
    - improve the fit of the model across the wealth distribution

0.5 drawbacks

- without directly observing return data, the mechanism above is poorly identified

1 model

1.1 setups

- each agent's lives T years
- every period t
    - consumers choose consumption c_t and
    - accumulate wealth a_t
    - subject to a no-borrowing constraint
- consumers leave wealth a_T as a bequest at the end of life T.
- each agent's preferences are composed of
    - a per-period utility from consumption, $u(c_t)$, at any period $t = 1, \dots, T$
        - $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$
    - a warm-glow utility from bequests at T, $e(a_T)$
        - $e(a_T) = A\frac{a_T^{1-\mu}}{1-\mu}$
- wealth accumulates from
    - savings
    - bequests
- idiosyncratic rates of returns r and life-time labor earnings y (w in his paper)
    - are drawn from a distribution at birth
        - possibly correlated with those of the parent
        - deterministic within each generatioin
    - stochastic over generations only
    - lifetime earnings processes are hump-shaped, with low earnings early in life
    - no-borrowing constraints limit how much agents can smoothing lifetime earnings

1.2 notations

- discount rate $\beta < 1$
- present discounted utility of an agent with wealth a_t at the beginning of period t: $V_t(a_t)$
- initial wealth: a_0
- earnings profiles: w
- rate of return: r

1.3 each agent's maximization problem

- $V_t(a) = \max_{c,a'} u(c) + \beta V_{t+1}(a')$
- s.t.
  - $a' = (1+r)a - c + w$
  - $c \in [0, a], t = 1, \ldots, T-1$
  - $V_T(a) = u(c) + e(a')$

1.4 optimal wealth processes

- $a_T = g(a_0; r, w)$
  - **CASE 1**: if $\mu = \sigma$, then $g(a_0; r, w) = \alpha(r, w)a_0 + \beta(r, w)$
- $\alpha(r, w)$ is the savings rate and independent of wealth
  - under general conditions (what conditions?), ${a^n}_n$ has a stationary distribution whose tail
  - is independent of the distribution of earnings
    - asymptotic to a Pareto law
- **CASE 2**: if $\mu < \sigma$, then $g''(a_0; r, w) > 0$.
  - keep σ constant, differential savings rate emerge
    - increasing with wealth
  - a stationary distribution might not exist
  - if stationary dist exist, then it displays a thick tail (proved?)
- stochastic process over generations: r^n, w^n
  - finite markov chain
  - r^n are w^n independent, but allowed to be serially correlated with transition matrix
  - life-cycle structure
    - $a^n = a_0^n = a_T^{n-1}$
- $a^n = g(a^{n-1}; r^n, w^n)$
  - stochastic process over generations: ${a^n}_n$ for initial wealth a

1.5 easy to extend to: Markov states of the stochastic process for r to depend on the initial wealth of the agent a

- optimal wealth processes have similar properties to **CASE 2** above.

2 quantitative analysis

2.1 methodology

- estimate parameters in the model using the method of simulated moments (MSM) estimators
  - fix (or externally calibrate several parameters of the model
  - select some relevant moments of the wealth process as target in the estimation
  - estimate the remaining parameters by matching the targeted moments generated by the stationary distribution induced by the model and those in the data
- assumption of quantitative exercise
  - wealth and social mobility observed in the data are generated by a stationary distribution

    Too strong?
- Simulated model can be solved by using **collocation method**, proposed by Miranda and Fackler (2004)
- quantitative exercise
  - 1. fix σ, T, β
    - stochastic individual earnings: w^n with transition matrix P (across generations) estimated from data
      - PSID
      - federal income tax records (Chetty et al, 2014)
  - 2. Target 12 moments
    - bottom 20% + 20-40% + 40-60% + 60-80% + 80-90% + 90-95% + 95-99% + top 1%|wealth shares
    - the diagonal of the (age-independent) social mobility markov chain transition matrix defined over quintiles
  - 3. Estimate 12 parameters
    - preference parameters: μ, A
    - parameters of r
      - defined
        - by 5 states r_i
        - 5 diagonal transition probabilities: $P(r_i|r_i)$
  - In 3-4,
    - modify r to allow returns to depend on the initial wealth a of the agent
    - experiment with an alternative social mobility matrix
      - defined over the same percentiles of the wealth distribution

2.2 data

- labor earnings

    - use 10 deterministic life-cycle household-level earnings profiles at different deciles (construct the earning profiles)
        - fix each of 6 age brackets, compute averages of the earnings deciles (Heathcote et al 2010) from PSID, 1967-2002 (Table 1)
        - assume agents stay in the same decile for their whole lifetime.
        - agentss randomly draw one of these earnings profiles at the beginning of life according to an intergenerational transition matrix (10 states) reduced from (from Chetty et al, 2014; 100 states; 1980-1982 us birth cohort and their parental income) (Figure 1)
- wealth distribution (Fig2+Table 2)
    - Survey of Consumer Finances 2007
    - wealth variable is net wealth (:=net financial wealth + housing - any debts)
    - skewed-to-right (heavy tail) distribution
        - take the shares from Diaz-Gimenez et al (2011)
- social mobility (wealth transition across generations)
    - mobility matrix calculated by Charles and Hurst (2003) from PSID (Table 3)
        - constructed by means of pairs of simultaneously alive parent and child of different ages
        - compute transitions from the residuals of the wealth of parents and children after conditioning on age and age squared
            - to eliminate age effects
        - shorrocks index: 0.88
- in 3-4, reproduce estimation
    - using an alternative social mobility matrix, using 2007-09 SCF panel data
    - with transitions computed for a synthetic agent over his/her age profile

3 estimation results

3.1 parameter estimates (baseline: (Table4))

- upper part
    - estimates of preference parameters
- lower part
    - estimated state space
    - diagonal of the transition matrix of the five-state Markov process for r we postulate
    - implied mean and standard deviation of the process
    - autocorrelation computed fitting an $AR(1)$ on simulated data from the estimated process
    - standard errors; obtained by bootstrapping

3.2 model fit (compare targeted simulated moments of the estimated model with counterpart in the data) (Table 5)

- compare moments in data with those in simulated
- compute age-independent social mobility moments after conditioning on age and age-squared

3.3 discussion and interpretation

- differential savings and bequests (Table 6
  - $\mu = 0.5993 < \sigma = 2$
    - rich save proportionally more than poor
  - μ depends on A
  - average saving rates
  - bequests
    - its dist implied by model is very skewed
    - should include inter vivos transfer (? a transfer made during one's lifetime)
- returns to wealth (Table 7)
  - significant standard deviation and wide dispersion in r
  - cautions
    - r is assumed constant over each agent's lifetime, disregarding the whole variation across the life cycle
    - difference in risk composition of investment portfolio might also contribute heterogeneous r in the data
- social mobility (Table 8)
  - fit well compared with Charles and Hurst (2003, Table 2; Table 3 in this paper)
  - slightly overestimate the mobility from the top to the bottom of the distribution vice versa
  - shorrocks index: 0.92

3.4 extensions and robustness

- re-estimate model
  - to allow for r dependent on wealth
  - to match an alternative social mobility matrix constructed using 2007-09 panel data
- rate of return positively dependent on wealth
  - positive correlation between r and a does not implies r increases with wealth a
    - think about average
  - this add to the skewness of the distribution
  - re-estimate the model using a r dependent on a
    - Assume $r = r_0 + b \times p(a)$
      - where
        - $p(a) = 1, 2, \ldots, 8$ moments
        - r_0 is a five-state Markov process
    - results

        - better parameters fits
        - better wealth distribution fits
            - top 20%
        - worse social mobility fits
            - dependence of r on wealth a is compensated by a reduced dependence of savings?
    - see Fagereng et al (2017)
- alternative social mobility matrix
    - baseline social mobility matrix
        - constructed by means of pairs of simultaneously alive parents and child
        - does not account for any transition induced by bequests
        - only available for wealth transitions between quintiles, but we care about the heavy tails
            - transitions in and out of the top 1 percent are one of the most relevant characteristics of the stochastic process of the wealth accumulation
    - re-estimate the model using a new social mobility matrix (Table11)
        - method
            - intergenerational transition matrix: each element of the matrix take the form $Pr(a_0^n \in p | a_0^{n-1} p')$, where p, p' are generic percentiles of the wealth distribution
            - intra-generational transition matrix: $Pr(a_T^{n-1} \in p | a_0^{n-1} \in p')$
            - divide agent's lifetime T into k-periods age groups
            - use the Markov assumption (?) to obtain $Pr(a_T^{n-1} \in p | a_0^{n-1} p')$ from the observation of $Pr(a_k^{n-1} \in p | a_0^{n-1} \in p')$, $Pr(a_{2k}^{n-1} \in p | a_k^{n-1} \in p')$ and so on for all age groups.
                - first construct age-dependent two-year transition matrices for age groups running from 30-31 to 66-67
                - multiply these age-dependent two-year transition matrices for all age groups
            - transitions computed for the same percentiles they use as wealth distribution moments
            - it produces spurious mobility due to measurement error in wealth
        - results
            - display substantial social mobility (shorrocks index: 0.98 vs 0.88 (baseline))
            - re-estimate model
                - gives estimates close to baselines (Table 12)
                - also give fits close to baselines (Table 13)

4 counterfactual estimates (by shutting down one factor at a time)

4.1 motives

- understand which mechanism mostly affects which dimension of the wealth distribution and mobility
- interpret the counterfactuals as informal tests of identification of these mechanism
    - lack of identification implies that shutting down one or more of the mechanism has limited effects on the fit for the targeted moments

4.2 re-estimation

- parameters estimates (Table 14)
- model fit (Table 15)

4.3 re-estimation results (see conclusion in intro part)

- no capital income risk (constant r)
- no stochastic earnings (constant w)
- homogeneous saving rates ($\mu = 2$)

5 transitional dynamics of the wealth distribution

5.1 assumption relaxation

- assumptions above: the observed distribution of wealth is a stationary distribution
    - our estimates are obtained by matching the data with the moments of the stationary distribution generated by the model.
- relax the stationary assumption on wealth distribution
    - try to match the transitional dynamics of the distribution of wealth

5.2 method

- use observed SCF 1962-1963 wealth distribution as initial condition
- estimate the parameters of the model by matching the implied distribution after 72 years (two iterations of the model) with (see Table 16)
    - observed SCF 2007 distribution
    - transition matrix adopted in previous analysis

5.3 results (see Table 17)

- such a dramatic increase in wealth inequality can be obtained by exploiting the power of capital income risk and differential savings
- A larger bequest motive

- a rate of return process with higher mean and volatility and much more auto-correlation
    - induce a simulated wealth distribution for 2007 is even more skewed at the top
    - the bottom 40% of the wealth distribution is very well matched
- not well fits on social mobility by overestimating mobility all across the distribution
    - mobility is the probability that children move away from their parents' wealth cell