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A CLASS OF GENERALIZED MEASURES OF MOBILITY WITH APPLICATIONS

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A class of anonymous measures of mobility is defined that considers the relationship between the long-run and short-run distributions of income. Several measures of long-run income and inequality are explored. Michigan panel data are analyzed.

1. Introduction

A class of anonymous measures of mobility is defined that considers the relationship between the long-run and the short-run distributions of income or any other attribute. Several measures of long-run income are explored, and distributions are compared on the basis of several inequality measures. The data from the Michigan Panel Study of Income Dynamics is used to demonstrate.

The basic concept of mobility adopted here was introduced by Shorrocks (1978a). A comparison of inequality in the distribution of long-run incomes, measured over T periods, $t = 1, 2, \ldots, T$, with some representative value of inequality in the short-run (single period) distributions reveals the degree of mobility, stability, or equalization over time. Measurement of inequality in long-run income is of interest in itself since such measurements can be free of short-run transitory effects. For instance, the long-run measures are free of life-cycle income effects, particularly when long periods of accounting are used.

Defining y_{it} as the income of individual i; $i = 1, \ldots, N$, in period t; $t = 1, \ldots, T$, the simplest measure of long-run income is $y_i = \Sigma_{t=1}^{T} y_{it}$. To evaluate the inequality in the distribution $y = (y_1, \ldots, y_N)$ is equivalent to assessing equality preferring Social Welfare Functions (SWF's) which (i) are individualistic, and (ii) represent each individual by his (her) total income y_i [see, e.g., Atkinson (1970)]. A more general approach replaces y_i of (ii) with individual utility functions. Of course $y_i = \Sigma_t y_{it}$ can also be interpreted as a particularly simple, additive utility function over the T periods. In this vein, we propose more general utility (or aggregator) functions, $S_i = S_i(y_{i1}, y_{i2}, \ldots, y_{iT})$, which include the simple additive form as a special case. We do this in order to study the effects of different levels of income substitution over time. Of course, S_i can also be viewed as a measure of aggregate income over the accounting period.

However S_i is interpreted and measured, we define $I(S_1, \ldots, S_N) = I(S)$, say, as a measure of long-run inequality. Similarly, $I(y_{1t}, y_{2t}, \ldots, y_{Nt}) = I_t$, say, is the same measure of inequality for period t. Let $\bar{I} = \Sigma_t^T W_t I_t$ be a weighted average of the short-run inequalities. We then define the following measure of stability:

$$R_T = I(S) \Big/ \sum_t^T W_t I_t. \tag{1}$$

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Shorrocks (1978a) has shown that $0 \leqslant R_T \leqslant 1$ when $S_i = y_i$, the simple aggregate income, and $I(\ )$ is any convex measure of *relative* inequality. [1] For the more general functionals S_i, certain properties of multidimensional measures of inequality are used here to establish conditions under which $0 \leqslant R_T \leqslant 1$. Consequently we follow Maasoumi (1986, Propositions 1 and 2), and focus on a wide class of inequality indices known as the generalized entropy (GE) measures. For a discussion of the many desirable properties of GE and their decomposition, see, e.g., Shorrocks (1984). It may suffice to recall that GE contains both of Theil's (1967) information measures, as well as monotonic transformation of the rich family of measures proposed by Atkinson (1970). Also we consider the class of CES functions, $S_i = (\Sigma_t \alpha_t y_{it}^{-\beta})^{-1/\beta}$, $\Sigma_t \alpha_t = 1$, as our aggregate income. This includes the Cobb–Douglas and the linear forms. See Maasoumi (1986) for an interpretation of these aggregates as 'ideal' indices under certain information criteria including the Kullback–Leibler measure. For convenience, we also define the GE family of inequality measures,

$$I_\gamma(S) = \sum_{i=1}^{N} \frac{1}{N} \frac{\left[(NS_i^*)^{1+\gamma} - 1\right]}{\gamma(1+\gamma)}, \tag{2}$$

$$I_0(S) = \sum_{i=1}^{N} S_i^* \log(NS_i^*), \tag{3}$$

$$I_{-1}(S) = \sum_i \frac{1}{N} \log\left(\frac{1}{NS_i^*}\right), \tag{4}$$

where $S_i^* = S_i/\Sigma_{j=1}^N S_j$, and I_0 and I_{-1} are, respectively, Theil's (1967) first and second information measures of inequality whose particularly useful additive decomposability properties permit less ambiguous analysis of the incidence of inequality among population sub-groups.

2. An empirical demonstration

In this section panel data from the Michigan Study of Income Dynamics is analysed. The relative robustness of our findings is established by considering four different inequality measures from the GE family ($\gamma = 0, -\frac{1}{2}, -1$ and -2), representing a reasonable range of relative inequality aversion [see Atkinson (1970)], and four different CES functions for 'aggregate' income ($\beta = -1, -\frac{1}{2}, 0$ and 1, respectively) representing different degrees of substitution. Also, from (1), we compute 13 values for the stability measure as the period of accounting is extended from one to 13 years (1969–1981). Income is defined as the total real income from all sources, including wages and salaries, business income, interest income, transfer payments, etc. Current CPI was used for deflation. A further attempt at robustification of our findings was made by considering three different weighting schemes in measuring $S_i.\alpha_t$, the weight given to income at time t, is either equal for all t, equal to the ratio of total income at time t to total income over the entire accounting period [as in Shorrocks (1978b)], or equal to the (normalized) first principal component (PC) of the N-tuple income vectors for two periods, three periods,... and 13 periods. See Maasoumi (1985) and/or Ram (1982) for a discussion of the PC method as a means of constructing composite indices of several variables.

[1] Actually, Shorrocks considered only $W_t = \mu_t/\mu$, the ratio of mean income in period t and the mean income over the accounting period.

Table 1
Short-run inequality, 1969–1981 total family income.

Overall	Between	Within	18 to 29	30 to 39	40 to 49	50 to 59	60 to 99
Degree of inequality aversion = 2.0							
0.825	0.020	0.805	0.651	0.501	0.534	1.169	0.982
0.839	0.036	0.803	0.474	0.522	0.624	0.993	0.995
0.928	0.065	0.863	0.339	0.512	0.761	1.094	0.859
1.397	0.091	1.306	0.558	0.635	1.007	2.104	0.898
1.645	0.122	1.523	0.714	1.095	1.879	1.507	0.854
1.913	0.162	1.750	0.819	1.573	1.670	1.533	1.010
2.538	0.222	2.316	0.813	1.775	2.832	1.751	0.989
Degree of inequality aversion = 1.0							
0.337	0.018	0.319	0.186	0.246	0.298	0.402	0.513
0.376	0.029	0.347	0.200	0.269	0.322	0.454	0.537
0.403	0.049	0.354	0.179	0.266	0.347	0.491	0.513
0.482	0.067	0.415	0.226	0.306	0.414	0.570	0.589
0.549	0.088	0.462	0.242	0.349	0.472	0.653	0.596
0.615	0.115	0.500	0.253	0.360	0.504	0.715	0.689
0.711	0.155	0.556	0.290	0.439	0.579	0.799	0.649
Degree of inequality aversion = 0.50							
0.285	0.018	0.267	0.154	0.213	0.262	0.339	0.446
0.315	0.027	0.288	0.166	0.230	0.278	0.387	0.485
0.333	0.044	0.289	0.153	0.226	0.294	0.412	0.483
0.392	0.059	0.333	0.191	0.260	0.342	0.468	0.585
0.435	0.077	0.358	0.198	0.276	0.372	0.550	0.608
0.474	0.101	0.373	0.202	0.276	0.388	0.611	0.703
0.540	0.135	0.405	0.237	0.333	0.431	0.700	0.632
Degree of inequality aversion = 0.0							
0.266	0.017	0.249	0.139	0.199	0.250	0.317	0.429
0.293	0.025	0.268	0.151	0.214	0.263	0.367	0.494
0.307	0.039	0.268	0.141	0.208	0.278	0.387	0.514
0.362	0.053	0.309	0.178	0.246	0.321	0.438	0.669
0.396	0.069	0.327	0.183	0.250	0.341	0.527	0.728
0.422	0.090	0.331	0.182	0.245	0.348	0.596	0.844
0.482	0.121	0.361	0.219	0.298	0.383	0.713	0.704
SS 2305			359	525	641	471	309

Table 1 reports the basic inequality values for every other year, decomposed into between- and average-within-group components of inequality for five distinct age groups in the sample. [2] Among several interesting observations that are robust with respect to changes in the inequality measure we note:

[2] 2305 households who reported non-zero incomes over the entire period represent our sample. 527 of these households are known to be headed by females and the rest by males. In all the tables $\upsilon = -\gamma = 1 + \beta = 1/\sigma$, where σ is the constant elasticity of substitution in defining S_i.

(i) Inequality has generally increased over the accounting period, both within and between age groups.
(ii) Almost all of the total inequality is attributed to within-group inequality with $\gamma = -2$, with the between-group component explaining an increasing share of inequality as $\gamma \to 0$ [Theil's (1967) first measure].
(iii) There are numerous transitory fluctuations in short-run inequality values. Clearly any 'snap-shot' of income distribution (in any one period) gives misleading views of inequality.

Weighted averages of the single period inequality figures are computed but not reported here. We view these averages as reasonable representations of 'short-run' inequalities. Remarks (i)–(iii) are still

Table 2
Long-run inequality, 1969–1981 total family income.

Overall	Between	Within	18 to 29	30 to 39	40 to 49	50 to 59	60 to 99
Degree of inequality aversion = 2.0							
0.825	0.000	0.825	0.579	0.633	0.899	1.014	1.014
0.767	0.000	0.766	0.655	0.745	0.852	0.757	0.774
0.762	0.000	0.762	0.676	0.748	0.861	0.719	0.748
0.850	0.000	0.850	0.723	0.943	0.912	0.829	0.747
0.933	0.000	0.933	0.830	0.984	0.987	0.971	0.795
1.061	0.000	1.060	0.966	1.169	1.112	1.040	0.901
1.174	0.000	1.173	1.066	1.268	1.250	1.158	0.997
Degree of inequality aversion = 1.0							
0.337	0.000	0.337	0.320	0.299	0.379	0.336	0.334
0.337	0.000	0.337	0.331	0.320	0.376	0.316	0.322
0.343	0.000	0.343	0.344	0.331	0.382	0.317	0.322
0.363	0.000	0.362	0.370	0.355	0.400	0.333	0.333
0.389	0.000	0.389	0.407	0.379	0.427	0.358	0.353
0.420	0.000	0.419	0.443	0.412	0.459	0.383	0.378
0.455	0.000	0.455	0.475	0.451	0.494	0.418	0.413
Degree of inequality aversion = 0.50							
0.285	0.000	0.284	0.279	0.252	0.318	0.281	0.281
0.284	0.000	0.283	0.286	0.266	0.314	0.265	0.273
0.288	0.000	0.288	0.299	0.275	0.316	0.266	0.272
0.303	0.000	0.303	0.323	0.292	0.329	0.276	0.283
0.323	0.000	0.323	0.354	0.309	0.352	0.292	0.298
0.345	0.000	0.345	0.381	0.334	0.374	0.310	0.316
0.371	0.000	0.370	0.403	0.363	0.400	0.336	0.340
Degree of inequality aversion = 0.0							
0.266	0.000	0.265	0.263	0.234	0.297	0.259	0.262
0.263	0.000	0.262	0.270	0.245	0.290	0.242	0.255
0.266	0.000	0.265	0.286	0.252	0.288	0.243	0.251
0.278	0.000	0.278	0.312	0.265	0.299	0.250	0.263
0.297	0.000	0.296	0.341	0.277	0.322	0.262	0.277
0.315	0.000	0.314	0.366	0.298	0.339	0.276	0.291
0.338	0.001	0.337	0.382	0.325	0.364	0.299	0.310

Table 3
Income stability.

Overall	Between	Within	Overall	Between	Within
$v = 2.0$			$v = 0.5$		
1.000	0.000	1.000	1.000	0.001	0.999
0.945	0.000	0.944	0.947	0.001	0.946
0.915	0.000	0.915	0.928	0.001	0.928
0.877	0.000	0.877	0.916	0.001	0.916
0.843	0.000	0.843	0.910	0.001	0.909
0.805	0.000	0.805	0.906	0.001	0.905
0.762	0.000	0.761	0.896	0.001	0.895
$v = 1.0$			$v = 0.0$		
1.000	0.001	0.999	1.000	0.001	0.999
0.947	0.001	0.946	0.941	0.001	0.940
0.926	0.001	0.925	0.921	0.001	0.920
0.910	0.001	0.909	0.910	0.001	0.909
0.898	0.001	0.897	0.906	0.001	0.905
0.887	0.001	0.886	0.902	0.001	0.901
0.874	0.001	0.873	0.894	0.001	0.893

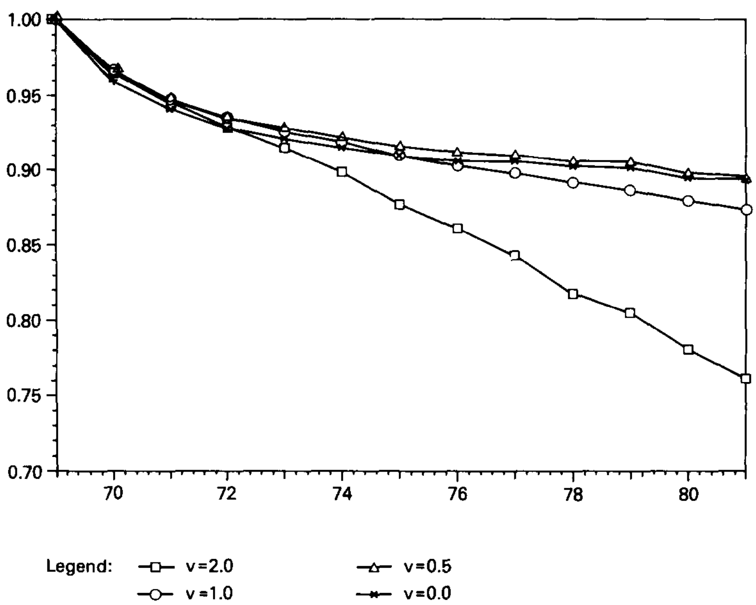

Fig. 1. Stability profiles – Real total family income 1969–1981 based on ratio of income weights.

applicable to these averages which, however, vary more slowly over time. The three weight schemes produced almost identical figures, even more so qualitatively. Using the ratio of incomes as weights we report 'long-run' inequalities in table 2. [3] We find that: (1) over longer periods of accounting

[3] In view of the space limitations we report the entries for only the first 1, 3, 5, ... and 13 years. Complete results may be obtained from the authors.

there remains practically no inequality *between* age groups, inequality *within* age groups being responsible for almost all the existing inequality, (2) clearly, long-run inequality increases over this period as the period of accounting is extended, (3) moving down in the table, the greater degree of income substitution (σ) over time is associated with more stable long-run inequality as the period of accounting is extended, and (4) long-run inequality is always smaller than the corresponding average short-run inequality. This is more clearly seen in table 3, where the stability profiles (R_1, $R_2, \ldots, R_{13}$) are reported. Mobility ($1 - R = M$, say) increases with the length of the accounting period. Put another way, degree of equalization increases in the long run.

The evidence reported above is a small part of an extensive study of the underlying data with decompositions according to gender, income level, educational attainment, family size, occupation and other criteria, and with several definitions of the income variable. Fig. 1 provides an example of the mobility profile. It is a graph of the entries in table 3.

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