ECON2125/4021/8013*
Week 11 Tutorial Questions (8/5/2015)

Semester 1 2015

Question 1

Prove the following facts:

1. If $A \subset \mathbb{R}^K$ and $B \subset \mathbb{R}^K$ are bounded sets, then so is $C = A \cup B$.

2. For any two vectors $\mathbf{a}, \mathbf{b} \in \mathbb{R}^K$, if $\mathbf{a} \neq \mathbf{b}$, then there exists $\varepsilon > 0$, such that $B_\varepsilon(\mathbf{a}) \cap B_\varepsilon(\mathbf{b}) = \emptyset$.

Question 2

Prove the following facts:

1. Let $\{\mathbf{x}_n\}$ be a sequence in $\mathbb{R}^K$ and $\mathbf{x}, \mathbf{z} \in \mathbb{R}^K$. If $\mathbf{x}_n \to \mathbf{x}$, then $\mathbf{z}'\mathbf{x}_n \to \mathbf{z}'\mathbf{x}$.

2. Each sequence in $\mathbb{R}^K$ has at most one limit.

3. Every convergent sequence in $\mathbb{R}^K$ is bounded.

4. Every convergent sequence in $\mathbb{R}^K$ is Cauchy.

5. Every Cauchy sequence in $\mathbb{R}^K$ is convergent.

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***Question* 3**

Prove that: a sequence $\{\mathbf{x}_n\}$ in $\mathbb{R}^K$ converges to $\mathbf{a} \in \mathbb{R}^K$ if and only if each component sequence converges in $\mathbb{R}$.

That is,

$$\begin{pmatrix} x_n^1 \\ \vdots \\ x_n^K \end{pmatrix} \to \begin{pmatrix} a_1 \\ \vdots \\ a^K \end{pmatrix} \text{ in } \mathbb{R}^K \Leftrightarrow \begin{array}{c} x_n^1 \to a^1 \text{ in } \mathbb{R} \\ \vdots \\ x_n^K \to a^K \text{ in } \mathbb{R} \end{array}$$