ECON2125/4021/8013*
Week 12 Tutorial Questions (15/5/2015)

Semester 1 2015

***Question* 1**

Let $A \subset \mathbb{R}^K$ be a convex and let f be a function from A to $\mathbb{R}^K$. Show that:

1. f is **concave** if and only if $-f$ is **convex**.

2. f is **strictly concave** if and only if $-f$ is **strictly convex**.

***Question* 2**

Show that:

1. $f : A \subset \mathbb{R}^K \to \mathbb{R}$ is convex if and only if its **epigraph**

$$E_f = \{(\mathbf{x}, y) \in A \times \mathbb{R} : f(\mathbf{x}) \leq y\}$$

is a convex subset of $\mathbb{R}^K \times \mathbb{R}$.

2. $f : A \subset \mathbb{R}^K \to \mathbb{R}$ is concave if and only if its **hypograph**

$$H_f = \{(\mathbf{x}, y) \in A \times \mathbb{R} : f(\mathbf{x}) \geq y\}$$

is a convex subset of $\mathbb{R}^K \times \mathbb{R}$.

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***Question* 3**

Use the definition of **concave** to show that the function f defined for all $(x_1, x_2) \in \mathbb{R}^2$ by

$$f(x_1, x_2) = 1 - x_1^2 \qquad \text{(so } x_2 \text{ does not appear in the formula for } f\text{)}$$

is concave. Is it strictly concave?

***Question* 4**

Provide the formal proof of the **Banach Contraction Mapping Theorem** based on the sketch of its proof in the lecture slides (See page 13-14 of Lecture 20). The theorem is repeated here for convenience:

If S is closed and T is a contraction mapping on S then

1. T has a unique fixed point $\bar{\mathbf{x}} \in S$

2. $T^n\mathbf{x} \to \bar{\mathbf{x}}$ as $n \to \infty$ for any $\mathbf{x} \in S$