ECON2125/4021/8013*
Week 13 Tutorial Questions (22/5/2015)

Semester 1 2015

***Question* 1**

Let $S \subset \mathbb{R}$ and let $x^* \in S$ be a steady state of (S, g).

Prove that: if g is continuously differentiable at x^* and $|g'(x^*)| < 1$, then $x^\star$ is locally stable for (S, g).

***Question* 2**

Let $S \subset \mathbb{R}$ and let $x^\star \in S$ be a steady state of (S, g).

Prove that: if $g^t(k) \to \hat{k}$ for some $k, \hat{k} \in S$, and g is continuous at $\hat{k}$, then $\hat{k}$ is a fixed point of g.

***Question* 3**

Prove that x^* is locally stable if and only if there exists an $\varepsilon > 0$ such that $B(\varepsilon, x^*) \subset O(x^*)$. Recall that $O(x^*)$ is the stable set of x^*.

***Question* 4**

Let (S, g) be a dynamical system.

Definition: g is **nonexpansive** on $S \subset \mathbb{R}$, if $\forall x, y \in S$, $|g(x) - g(y)| \leq |x - y|$.

Prove that: if g is nonexpansive and (S, g^N) is globally stable for some $N \in \mathbb{N}$, then (S, g) is globally stable.

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***Question* 5**

Let (S, g) be an arbitrary dynamical system. We order points in $\mathbb{R}^n$ by setting $x \leq y$ whenever $x_i \leq y_i$ for i in $\{1, ..., n\}$(i.e., each component of x is dominated by the corresponding component of y). In the lecture, we learned that if $g : S \to S$ is increasing on S and $S \subset \mathbb{R}$, then every trajectory is monotone. Now consider the case that $S \subset \mathbb{R}^n$ and $g : S \to S$ is an increasing function. Show that the same result no longer holds (i.e., g does not necessarily generate monotone trajectories.)