$ECON2125/4021/8013^*$ Week 13 Tutorial Questions (29/5/2015)

Semester 1 2015

$Question\,1$

Let \mathbf{Q} , \mathbf{P} and \mathbf{A} be $N \times N$ matrices. \mathbf{A} and \mathbf{Q} are given, but the \mathbf{P} is unknown.

Give an example of how the discrete Lyapunov equation $\mathbf{P} = \mathbf{A}'\mathbf{P}\mathbf{A} + \mathbf{Q}$ can have no solution when the spectral radius condition $\rho(\mathbf{A}) < 1$ does not hold.

Question 2

Let **V** be an $N \times N$ matrix, and defined by $\mathbf{V} = \beta \mathbf{A}' (\mathbf{D} + \mathbf{V}) \mathbf{A}$ for some positive definite **D**.

Show that \mathbf{V} is positive definite whenever \mathbf{A} is nonsingular.

Question 3

Refer to the Lecture 24 slides P27/43.

Show that: $\operatorname{var}[\mathbf{x}_{t+1}] = \Sigma_{t+1} = \mathbf{A}\Sigma_t \mathbf{A}' + \mathbf{C}\mathbf{C}'$, where $\Sigma_t := \operatorname{var}[\mathbf{x}_t]$.

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