ECON2125/8013*
Week 2 Tutorial Questions (27/2/2015)

Semester 1 2015

***Question* 1**

In this exercise, you are going to see how to use bits and pieces you've learned in class to sketch the graph of a function: Suppose we want to see the graph of the function:

$$f(x) = (x^3 - x^2 - x + 1)^{1/3} \qquad (-3 \le x \le 2) \tag{1}$$

Suppose also that we don't have our laptops at hand and thus can't seek for help from the powerful Python Language, what shall we do?

(1) Find the stationary point, and the points at which the first derivatives do not exist. Try to analyze the monotonicity properties of the function.

(2) Find the points where the second derivatives are zero or do not exist. Try to analyze the convexity and concavity properties of the function.

(3) If $\lim_{x\to\infty}[f(x) - (ax+b)] = 0$ then $y = ax + b$ is called the aymptotic line of $f(x)$. To find a and b, we see that $\lim_{x\to\infty} \frac{f(x)-(ax+b)}{x} = 0$, which tells us $a = \lim_{x\to\infty} \frac{f(x)}{x}$, and $b = \lim_{x\to\infty}[f(x) - ax]$. Find the asymptotic line $y = ax + b$ of the function f.

(4) Try to sketch the graph of the function f.

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***Question* 2**

Consider the following function

$$f(x) \quad = \quad \begin{cases} \mid x \mid, & -1 \leq x \leq 1 \\ 2- \mid x-2 \mid, & 1 \leq x \leq 3 \\ 2- \mid x-4 \mid, & 3 < x \leq 6 \end{cases}$$

Sketch the graph of the function, and try to find the maximizers and minimizers.

***Question* 3**

Theorem (The sufficient conditions for ***concavity/convexity*** in two dimensions)

Let $z = f(x, y)$ be a twice continuously differentiable function defined for all $(x, y) \in \mathbb{R}^2$. Then:

(1) f is convex $\Longleftrightarrow$ $f''_{11} \geq 0$, $f''_{22} \geq 0$, and $f''_{11}f''_{22} - (f''_{12})^2 \geq 0$.

(2) f is concave $\Longleftrightarrow$ $f''_{11} \leq 0$, $f''_{22} \leq 0$, and $f''_{11}f''_{22} - (f''_{12})^2 \geq 0$.

(3) $f''_{11} > 0$ and $f''_{11}f''_{22} - (f''_{12})^2 > 0 \Longrightarrow f$ is strictly convex.

(4) $f''_{11} < 0$ and $f''_{11}f''_{22} - (f''_{12})^2 > 0 \Longrightarrow f$ is strictly concave.

Find the largest domain S on which $f(x, y) = x^2 - y^2 - xy - x^3$ is concave. How about strictly concave?

***Question* 4**

Let $f(x, y) = 2x - y - x^2 + 2xy - y^2$ for all $(x, y) \in \mathbb{R}^2$. Is f concave/convex? (Hint: use the above theorem.)

***Question* 5**

(1) For what values of the constant a is the following function concave/convex?

$$f(x,y) = -6x^2 + (2a+4)xy - y^2 + 4ay \qquad \big((x,y) \in \mathbb{R}^2\big) \tag{2}$$

(2) Now, if we set $a = -3$, and the domain of f is $I = \{(x,y) \mid -5 \leq x \leq 3,\ -10 \leq y \leq 2\}$, then what is the stationary points of f on I, and what is the maximizer/minimizer of f on I?