ECON2125/8013*
Week 3 Tutorial Questions (6/3/2015)

Semester 1 2015

***Question* 1**

Consider the following constrained optimization problem

$$\max_{x,y}(\min_{x,y}) f(x,y) = x^2 + y^2 \tag{1}$$

subject to

$$g(x,y) = x^2 + xy + y^2 = 3 \tag{2}$$

(a) Solve this optimization problem. (Hint: There are several solution candidates.)

(b) Try to find the maximizers and minimizers of this problem.

***Question* 2**

Consider the utility maximization problem

$$\max_{x,y} f(x,y) = xy + x + 2y \tag{3}$$

subject to

$$2x + y = m, \quad x \geq 0,\, y \geq 0 \tag{4}$$

where we have required that the amount of each good is nonnegative. Please solve this problem. (Note: "m" is a strictly positive parameter.)

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***Question* 3**

Let S be any set, $A \subset S$ and $K_\lambda \subset S$ for all $\lambda \in \Lambda$, try to prove the following properties:

$$A \backslash \left(\bigcup_{\lambda \in \Lambda} K_\lambda \right) = \bigcap_{\lambda \in \Lambda} (A \backslash K_\lambda) \text{ and } A \backslash \left(\bigcap_{\lambda \in \Lambda} K_\lambda \right) = \bigcup_{\lambda \in \Lambda} (A \backslash K_\lambda)$$

***Question* 4**

Find the composition $g \circ f$ of two functions f and g, if it exists:

(1) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \sin(x)$, and $g : \mathbb{R} \to \mathbb{R}$ defined by $g(x) = \frac{x}{1+x^2}$.

(2) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 1 - x^2$, and $g : [0, \infty) \to \mathbb{R}$ defined by $g(x) = \log(x)$. (Hint: Is there a composition in this case?)