ECON2125/8013*
Week 4 Tutorial Questions (13/3/2015)

Semester 1 2015

***Question* 1**

Consider the following two functions, f defined by

$$f(x) = x^{\frac{1}{2}} - 2 \tag{1}$$

is a function from $[0, \infty)$ to $\mathbb{R}$, and g defined by

$$g(x) = (x + 2)^{\frac{1}{2}} \tag{2}$$

is a function from $[-2, \infty)$ to $\mathbb{R}$.

1. Specify the range of f and g, respectively.
2. Find the composition $g \circ f$ of these two functions f and g, and the inverse of this composition $(g \circ f)^{-1}$.
3. Find f^{-1} and g^{-1}, then verify $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

***Question* 2**

Let $f : A \to B$ be a function, $C_1 \subset A$ and $C_2 \subset A$, define $f(C) := \{f(x)| \, x \in C \subset A\}$. Try to verify the following arguments:

1. $f(C_1 \cup C_2) = f(C_1) \cup f(C_2)$
2. If f is one-to-one, then $f(C_1 \cap C_2) = f(C_1) \cap f(C_2)$

Suppose f is bijection, which means f^{-1} exists (see the lecture slides). Define $f^{-1}(C) := \{x| \, f(x) \in C \subset B\}$. Show that

3. $f^{-1}(C_1 \cup C_2) = f^{-1}(C_1) \cup f^{-1}(C_2)$
4. $f^{-1}(C_1 \cap C_2) = f^{-1}(C_1) \cap f^{-1}(C_2)$

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***Question* 3**

For any $\alpha \in \mathbb{R}$ and any $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^N$. Try to verify the ***Parallelogram Equality*** holds:

$$\|\boldsymbol{x}+\boldsymbol{y}\|^2 + \|\boldsymbol{x}-\boldsymbol{y}\|^2 = 2(\|\boldsymbol{x}\|^2 + \|\boldsymbol{y}\|^2) \tag{3}$$

***Question* 4**

Let $S_1, S_2, S_3, ..., S_n$ be linear Subspaces of $\mathbb{R}^N$.

1. Try to verify $S_1 \cap S_2 \cap ... \cap S_n$ is always a linear subspace.

2. Is it true that $S_1 \cup S_2$ is always a linear subspace? If yes, prove it. Otherwise, give a counter example.

***Question* 5**

Let S be a linear subspace of $\mathbb{R}^N$, $M \subset \mathbb{R}^N$ be a subset. Show that:

1. $\mathbf{0} \in S$.
2. If $X \subset S$, then $\text{span}(X) \subset S$.
3. $\text{span}(S) = S$.
4. Let F be a linear subspace of $\mathbb{R}^N$, F is defined as the ***smallest*** linear subspace that contains M if for all linear subspace $Y \subset \mathbb{R}^N$ with $M \subset Y$, we have $M \subset F \subset Y$. Show that $\text{span}(M)$ is the ***smallest*** linear subspace that contains M.