ECON2125/8013*
Week 5 Tutorial Questions (20/3/2015)

Semester 1 2015

***Question* 1**

(1) Let $\mathbf{A} := \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ be such that

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 3 \\ 3 \\ 8 \end{bmatrix}$$

Is $\mathbf{A}$ a basis of $\mathbb{R}^3$? Why or why not?

(2) X is linearly independent if and only if $X_0 \subsetneq X \implies \text{span}(X_0) \subsetneq \text{span}(X)$. In the lecture, we have proved the sufficient part. Try to prove the necessary condition.

(3) Suppose

$$\mathbf{x}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \mathbf{x}_2 = \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}, \mathbf{x}_3 = \begin{bmatrix} 3 \\ 6 \\ 8 \end{bmatrix}$$

Use the conclusion in (2) to show that $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ are not linearly independent. (Hint: try to verify, for example, $\text{span}\{\mathbf{x}_1, \mathbf{x}_2\} = \text{span}\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$).

*Research School of Economics, Australian National University, Instructor: John Stachurski.

***Question* 2**

For given scalar α and general conformable matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, try to verify the following:

1. $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$
2. $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$
3. $(\mathbf{A} + \mathbf{B})\mathbf{C} = \mathbf{AC} + \mathbf{BC}$
4. $\mathbf{A}\alpha = \alpha\mathbf{AB}$
5. $\mathbf{AI} = \mathbf{IA} = \mathbf{A}$. (Both $\mathbf{A}$ and $\mathbf{I}$ are $N \times N$)
6. Find a counter-example that $\mathbf{AB} \neq \mathbf{BA}$.

Hint: To verify 1., for example, set $\mathbf{A} : m \times n$, $\mathbf{B} : n \times k$, $\mathbf{C} : k \times l$ to be general matrices

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{1k} \\ b_{21} & b_{22} & \dots & b_{2k} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \dots & b_{nk} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \dots & c_{1l} \\ c_{21} & c_{22} & \dots & c_{2l} \\ \vdots & \vdots & & \vdots \\ c_{k1} & c_{k2} & \dots & c_{kl} \end{bmatrix}$$

***Question* 3**

1. Prove that: A linear map $T : \mathbb{R}^K \to \mathbb{R}^N$, $\ker(T) = \{\mathbf{0}\}$ if and only if T is one-to-one.

2. Prove that: Suppose that $T : \mathbb{R}^K \to \mathbb{R}^N$ and $S : \mathbb{R}^N \to \mathbb{R}^M$ are linear maps, then $(S \circ T)$: $\mathbb{R}^K \to \mathbb{R}^M$ is also a linear map.