ECON2125/8013*
Week 6 Tutorial Questions (27/3/2015)

Semester 1 2015

***Question* 1**

For conformable matrices $\mathbf{A}$ and $\mathbf{B}$, verify the following transpositions hold:

1. $(\mathbf{A}')' = \mathbf{A}$
2. $(\mathbf{AB})' = \mathbf{B}'\mathbf{A}'$
3. $(\mathbf{A} + \mathbf{B})' = \mathbf{A}' + \mathbf{B}'$
4. $(c\mathbf{A})' = c\mathbf{A}'$ for any constant c.

***Question* 2**

If $\mathbf{I}$ is the $N \times N$ identity, $\mathbf{A}$ and $\mathbf{B}$ are $N \times N$ matrices and scalar $\alpha \in \mathbb{R}$, then try to verify the following properties when the case is $N = 2$:

1. $\det(\mathbf{I}) = 1$
2. $\mathbf{A}$ is nonsingular if and only if $\det(\mathbf{A}) \neq 0$
3. $\det(\mathbf{AB}) = \det(\mathbf{A}) \det(\mathbf{B})$
4. $\det(\alpha\mathbf{A}) = \alpha^N \det(\mathbf{A})$
5. $\det(\mathbf{A}^{-1}) = (\det(\mathbf{A}))^{-1}$

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6. Is it true that $\det(\mathbf{A}+\mathbf{B}) = \det(\mathbf{A}) + \det(\mathbf{B})$? If yes, please verify. If not, then provide a counter example.

7. $\det(\mathbf{A}') = \det(\mathbf{A})$

8. If $\mathbf{A}$ is nonsingular, then so is $\mathbf{A}'$, and $(\mathbf{A}')^{-1} = (\mathbf{A}^{-1})'$

***Question* 3**

Prove that: If matrix $\mathbf{A}$ is $N \times M$ and matrix $\mathbf{B}$ is $M \times N$, then $\text{trace}(\mathbf{AB}) = \text{trace}(\mathbf{BA})$.

***Question* 4**

For conformable matrices $\mathbf{A}$ and $\mathbf{B}$, verify the following properties of the matrix norm:

1. $\|\mathbf{A}\| = \mathbf{0}$ if and only if all entries of $\mathbf{A}$ are zero.

2. $\|\alpha\mathbf{A}\| = |\alpha|\|\mathbf{A}\|$ for any scalar α.

3. $\|\mathbf{A}+\mathbf{B}\| \leq \|\mathbf{A}\| + \|\mathbf{B}\|$.

4. If $\mathbf{A}$ and $\mathbf{B}$ are square matrices, then $\|\mathbf{AB}\| \leq \|\mathbf{A}\|\|\mathbf{B}\|$.